American Journal of Operational Research 2015, 5(4): 96-101
DOI: 10.5923/j.ajor.20150504.03

Estimation Approach to Ratio of Two Inventory Population Means in Stratified Random Sampling

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Abstract The present paper proposes a new approach of estimation to the inventory system which undergoes the process of stratification by using the technique of stratified random sampling. This seeks to develop the estimation of ratio of two population means using auxiliary variable under stratified random sampling through generalized ratio type estimator. The statistical analyses of existing estimators and proposed estimator have been discussed and condition of efficient estimator has been attained. With the help of computing algorithm, a numerical illustration of the problem has been also presented to meet the scientific footings of theory of estimation.

Keywords Stratified Inventory, Ratio Estimator, Auxiliary Variable and Computing Algorithm

1. Introduction

Usable and idle resources referring to man, material, machine and money are called inventory items; vide Ackoff and Sasieni (1993). Heterogeneous inventory items are stratified for optimal control of inventory system. Stratification of inventory system attempts to provide the mechanism to optimize the inventory control for each stratum which is believed to be very efficient and easily minimizes the costs. This also makes handling of inventory easy for each stratum and accordingly the stratum domain experts are supposed to supervise the field works of the inventory handling and management, vide for example Alex and Benny (2009). In this situation, stratified random sampling is used to develop the efficient estimator to estimate the population parameters of stratified inventory items for characteristics under study is not homogeneous, vide for example Cochran (1940, 1977). A few researches are available in this field and therefore demands deep exploration in this area of interdisciplinary research. Under this sampling scheme, the whole population of inventory items is divided into relatively homogeneous groups. Then the required sample is drawn by taking appropriate subsamples from these stratums using simple random sampling technique.

The fresh objective of this paper is to develop estimation approach to the ratio of two population means in stratified inventory system by evolving the efficient estimator using stratified random sampling technique for inventory populations under consideration. This proposes estimation of ratio of two population means using auxiliary variable under stratified random sampling through generalized ratio type estimator. The expressions for the bias and mean square error (MSE) have been obtained up to the first order of approximation. The minimum MSE of proposed estimator is also obtained. An empirical study is also carried out to meet the theoretical findings.

2. Statistical Analysis

The estimation of the ratio of two population means has been studied by many authors in the literature including Murthy and Singh (1965), Rao and Pareira (1968), Shah and Shah (1978), Ray and Singh (1985), Upadhyaya and Singh (1985), Upadhyaya et.al (1985), Singh and Rani (2005, 2006) and Sindhu et. al (2009) etc.

Let us consider the population of inventory items under study consisting of N units and this population of size N is divided into L stratums each of size N_h $(h=1,2,...,L)$ and the required sample of size n is drawn by taking n_h ($h=1,2,...,L$) units from corresponding strata. Thus $N=\sum_{h=1}^{L} N_h$ and $n=\sum_{h=1}^{L} n_h$. Let y_0 and y_1 be the two main variables under study and x be the auxiliary variable. y_{0hi} , y_{1hi} and x_{hi} ($h=1,2,...,L$, $i=1,2,...,N_h$) are the observations on the i[th] unit of the h[th] stratum for the variables y_0 , y_1 and x respectively.

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Published online at http://journal.sapub.org/ajor

Now we have the following notations which have been used throughout this paper.

$\overline{Y}_{0h} = \frac{1}{N_h}\sum_{i=1}^{N_h} y_{0hi}$, the h[th] strata mean for the main variable under study y_0.

$\overline{Y}_{1h} = \frac{1}{N_h}\sum_{i=1}^{N_h} y_{1hi}$, the h[th] strata mean for the main variable under study y_1.

$\overline{X}_{h} = \frac{1}{N_h}\sum_{i=1}^{N_h} x_{hi}$, the h[th] strata mean for the auxiliary variable under study x.

$\overline{Y}_0 = \frac{1}{N}\sum_{h=1}^{L} N_h \overline{Y}_{0h} = \sum_{h=1}^{L} W_h \overline{Y}_{0h}$, the inventory population mean for the main variable under study y_0.

$\overline{Y}_1 = \frac{1}{N}\sum_{h=1}^{L} N_h \overline{Y}_{1h} = \sum_{h=1}^{L} W_h \overline{Y}_{1h}$, the inventory population mean for the main variable under study y_1.

$\overline{X} = \frac{1}{N}\sum_{h=1}^{L} N_h \overline{X}_{h} = \sum_{h=1}^{L} W_h \overline{X}_{h}$, the inventory population mean for the auxiliary variable under study x.

$\overline{y}_{0h} = \frac{1}{n_h}\sum_{i=1}^{n_h} y_{0hi}$, the sample mean of y_0 for h[th] strata.

$\overline{y}_{1h} = \frac{1}{n_h}\sum_{i=1}^{n_h} y_{1hi}$, the sample mean of y_1 for h[th] strata.

$\overline{x}_{h} = \frac{1}{n_h}\sum_{i=1}^{n_h} x_{hi}$, the sample mean of x for h[th] strata.

$W_h = \frac{N_h}{N}$, the weight of h[th] strata.

$R = \frac{\overline{Y}_0}{\overline{Y}_1}$, the ratio of two population means of study variables.

It is well known that the appropriate estimators for the estimation of inventory population parameters are the corresponding statistics. Thus appropriate estimators for population means $\overline{Y}_0$, $\overline{Y}_1$ and $\overline{X}$ are the usual unbiased estimators, the sample means in stratified random sampling $\overline{y}_{0st} = \sum_{h=1}^{L} W_h \overline{y}_{0h}$, $\overline{y}_{1st} = \sum_{h=1}^{L} W_h \overline{y}_{1h}$ and $\overline{x}_{st} = \sum_{h=1}^{L} W_h \overline{x}_{h}$ respectively.

The conventional estimator for the ratio of two population means R in stratified random sampling is defined as,

$$\hat{R}_{st} = \frac{\overline{y}_{0st}}{\overline{y}_{1st}} \tag{2.1}$$

As it is well known that the use of auxiliary information supplied by the auxiliary variable enhances the efficiency of the estimator of any parameter, so Singh (1965) proposed the traditional ratio type estimator of ratio of two population means using auxiliary information in stratified random sampling as,

$$G_{1st} = \left(\frac{\overline{y}_{0st}}{\overline{y}_{1st}}\right)\left(\frac{\overline{X}}{\overline{x}_{st}}\right) \tag{2.2}$$

The bias and mean square error of G_{1st}, up to the first order of approximations respectively are,

$$B(G_{1st}) = R\sum_{h=1}^{L} W_h^2 \gamma_h \left[\begin{array}{l} \frac{S_{1h}^2}{\overline{Y}_1^2} + \frac{S_{xh}^2}{\overline{X}^2} - \frac{S_{01h}}{\overline{Y}_0\overline{Y}_1} \\ + \frac{S_{0xh}}{\overline{Y}_0\overline{X}} + \frac{S_{1xh}}{\overline{Y}_1\overline{X}} \end{array}\right] \tag{2.3}$$

$$MSE(G_{1st}) = R^2\sum_{h=1}^{L} W_h^2 \gamma_h \left[\begin{array}{l} \frac{S_{0h}^2}{\overline{Y}_0^2} + \frac{S_{1h}^2}{\overline{Y}_1^2} + \frac{S_{xh}^2}{\overline{X}^2} \\ -2\left(\frac{S_{01h}}{\overline{Y}_0\overline{Y}_1} + \frac{S_{0xh}}{\overline{Y}_0\overline{X}} - \frac{S_{1xh}}{\overline{Y}_1\overline{X}}\right) \end{array}\right] \tag{2.4}$$

Jan et.al (2014) proposed the following modified estimator using median (M_d) of the auxiliary information as,

$$G_{2st} = \left(\frac{\overline{y}_{0st}}{\overline{y}_{1st}}\right)\left(\frac{\overline{X}+M_d}{\overline{x}_{st}}\right) \tag{2.5}$$

The bias and mean square error of G_{2st}, up to the first order of approximation is,

$$B(G_{2st}) = R\sum_{h=1}^{L} W_h^2 \gamma_h \left[\begin{array}{l} \frac{S_{1h}^2}{\overline{Y}_1^2} + \frac{S_{xh}^2}{(\overline{X}+M_d)^2} \\ -\frac{S_{01h}}{\overline{Y}_0\overline{Y}_1} + \frac{S_{0xh}}{\overline{Y}_0(\overline{X}+M_d)} \\ +\frac{S_{1xh}}{\overline{Y}_1(\overline{X}+M_d)} \end{array}\right] \tag{2.6}$$

$$MSE(G_{2st}) = R^2 \sum_{h=1}^{L} W_h^2 \gamma_h \left[\begin{array}{l} \frac{S_{0h}^2}{\bar{Y}_0^2} + \frac{S_{1h}^2}{\bar{Y}_1^2} + \frac{S_{xh}^2}{\theta^2} \\ -2\left(\frac{S_{01h}}{\bar{Y}_0 \bar{Y}_1} + \frac{S_{0xh}}{\bar{Y}_0 \theta} - \frac{S_{1xh}}{\bar{Y}_1 \theta} \right) \end{array} \right] \quad (2.7)$$

where,

$$\theta = (\bar{X} + M_d),$$

$$S_{0h}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (y_{0hi} - \bar{y}_{0h})^2,$$

$$S_{1h}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (y_{1hi} - \bar{y}_{1h})^2,$$

$$S_{xh}^2 = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (x_{hi} - \bar{x}_h)^2,$$

$$S_{01h} = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (y_{0hi} - \bar{y}_{0h})(y_{1hi} - \bar{y}_{1h}),$$

$$S_{0xh} = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (y_{0hi} - \bar{y}_{0h})(x_{hi} - \bar{x}_h),$$

$$S_{1xh} = \frac{1}{N_h - 1} \sum_{i=1}^{N_h} (y_{1hi} - \bar{y}_{1h})(x_{hi} - \bar{x}_h)$$

3. Statistical Analysis of Proposed Estimator

Motivated by Jan et.al (2014), we propose the following generalized estimator for the ratio of two inventory population means as,

$$\tau_{st} = \left(\frac{\bar{y}_{0st}}{\bar{y}_{1st}} \right) \left(\frac{\bar{X} + M_d}{\bar{x}_{st} + M_d} \right)^{\alpha} \quad (3.1)$$

Where α is a suitable constant be determined such that the mean square error of τ_{st} is minimum.

In order to study the large sample properties of the proposed estimator we have assumed that,

$\bar{y}_{0h} = \bar{Y}_{0h}(1 + e_{0h})$, $\bar{y}_{1h} = \bar{Y}_{1h}(1 + e_{1h})$ and $\bar{x}_h = \bar{X}(1 + e_{2h})$ such that $E(e_{0h}) = E(e_{1h}) = E(e_{2h})$

And

$$E(e_{0h}^2) = \sum_{h=1}^{L} W_h^2 \gamma_h \frac{S_{0h}^2}{\bar{Y}_0^2},$$

$$E(e_{1h}^2) = \sum_{h=1}^{L} W_h^2 \gamma_h \frac{S_{1h}^2}{\bar{Y}_1^2},$$

$$E(e_{2h}^2) = \sum_{h=1}^{L} W_h^2 \gamma_h \frac{S_{xh}^2}{\bar{X}^2},$$

$$E(e_{0h} e_{1h}) = \sum_{h=1}^{L} W_h^2 \gamma_h \frac{S_{01h}}{\bar{Y}_0 \bar{Y}_1},$$

$$E(e_{0h} e_{2h}) = \sum_{h=1}^{L} W_h^2 \gamma_h \frac{S_{0xh}}{\bar{Y}_0 \bar{X}},$$

$$E(e_{1h} e_{2h}) = \sum_{h=1}^{L} W_h^2 \gamma_h \frac{S_{1xh}}{\bar{Y}_1 \bar{X}}.$$

Using above expressions, the bias and the mean square error of the proposed estimator, up to the first order of approximation respectively are,

$$B(\tau_{st}) = R \sum_{h=1}^{L} W_h^2 \gamma_h \left[\begin{array}{l} \frac{S_{1h}^2}{2\bar{Y}_1^2} + \frac{\alpha(1+\alpha)\theta_1^2 S_{xh}^2}{2\bar{X}^2} \\ -\left(\frac{S_{01h}}{\bar{Y}_0 \bar{Y}_1} + \frac{\alpha \theta_1 S_{0xh}}{\bar{Y}_0 \bar{X}} - \frac{\alpha \theta_1 S_{1xh}}{\bar{Y}_1 \bar{X}} \right) \end{array} \right] \quad (3.2)$$

$$MSE(\tau_{st}) = R^2 \sum_{h=1}^{L} W_h^2 \gamma_h \left[\begin{array}{l} \frac{S_{0h}^2}{\bar{Y}_0^2} + \frac{S_{1h}^2}{\bar{Y}_1^2} + \frac{\alpha^2 \theta_1^2 S_{xh}^2}{\bar{X}^2} \\ -2\left(\begin{array}{l} \frac{S_{01h}}{\bar{Y}_0 \bar{Y}_1} + \alpha \theta_1 \frac{S_{0xh}}{\bar{Y}_0 \bar{X}} \\ -\alpha \theta_1 \frac{S_{1xh}}{\bar{Y}_1 \bar{X}} \end{array} \right) \end{array} \right] \quad (3.3)$$

where, $\theta_1 = \frac{\bar{X}}{\bar{X} + M_d}$.

This mean square error is minimum for,

$$\alpha = \frac{\sum_{h=1}^{L} W_h^2 \gamma_h \left[\frac{S_{0xh}}{\bar{Y}_0 \bar{X}} - \frac{S_{1xh}}{\bar{Y}_1 \bar{X}} \right]}{\theta_1 \sum_{h=1}^{L} W_h^2 \gamma_h \frac{S_{xh}^2}{\bar{X}^2}} \quad (3.4)$$

And the minimum mean square error is,

$$MSE_{\min}(\tau_{st}) = R^2 \left[\begin{array}{l} \sum_{h=1}^{L} W_h^2 \gamma_h \left(\frac{S_{0h}^2}{\bar{Y}_0^2} + \frac{S_{1h}^2}{\bar{Y}_1^2} - 2\frac{S_{01h}}{\bar{Y}_0 \bar{Y}_1} \right) \\ -\frac{\left[\sum_{h=1}^{L} W_h^2 \gamma_h \left(\frac{S_{0xh}}{\bar{Y}_0 \bar{X}} - \frac{S_{1xh}}{\bar{Y}_1 \bar{X}} \right) \right]^2}{\sum_{h=1}^{L} W_h^2 \gamma_h \frac{S_{xh}^2}{\bar{X}^2}} \end{array} \right] \quad (3.5)$$

4. Efficiency Comparison

From (2.4) and (3.5) we have that the proposed estimator τ_{st} is better than the estimator G_{1st}, if

$MSE(G_{1st}) - MSE_{\min}(\tau_{st}) > 0$, if

$$R^2 \left[\frac{\left[\sum_{h=1}^{L} W_h^2 \gamma_h \left(\frac{S_{0xh}}{\overline{Y}_0 \overline{X}} - \frac{S_{1xh}}{\overline{Y}_1 \overline{X}} \right) \right]^2}{\sum_{h=1}^{L} W_h^2 \gamma_h \frac{S_{xh}^2}{\overline{X}^2}} + \sum_{h=1}^{L} W_h^2 \gamma_h \left[\frac{S_{xh}^2}{\overline{X}^2} - 2\left(\frac{S_{0xh}}{\overline{Y}_0 \overline{X}} - \frac{S_{1xh}}{\overline{Y}_1 \overline{X}} \right) \right] \right] > 0 \quad (4.1)$$

Under the above laid down condition, the proposed estimator is believed to perform more efficiently than the estimator G_{1st} considered in (2.2) of ratio of two population means for it will have lesser mean squared error as compared to G_{1st}.

From (2.7) and (3.5) we have that the proposed estimator τ_{st} is better than the estimator G_{2st}, if

$MSE(G_{2st}) - MSE_{\min}(\tau_{st}) > 0$, if

$$R^2 \left[\frac{\left[\sum_{h=1}^{L} W_h^2 \gamma_h \left(\frac{S_{0xh}}{\overline{Y}_0 \overline{X}} - \frac{S_{1xh}}{\overline{Y}_1 \overline{X}} \right) \right]^2}{\sum_{h=1}^{L} W_h^2 \gamma_h \frac{S_{xh}^2}{\overline{X}^2}} + \sum_{h=1}^{L} W_h^2 \gamma_h \left[\frac{S_{xh}^2}{\theta^2} - 2\left(\frac{S_{0xh}}{\overline{Y}_0 \theta} - \frac{S_{1xh}}{\overline{Y}_1 \theta} \right) \right] \right] > 0 \quad (4.2)$$

The proposed estimator τ_{st} given in (3.1) is supposed to provide to lesser mean squared error as compared to G_{2st} under the laid down condition in (4.2).

In brief, the physical significance of expressions given in (4.1) and (4.2) attempts to seek exceedingly better estimator having least mean square error as compared to previously existing ones.

5. Computing Algorithm and Numerical Illustration

The following algorithm has been developed to compute the estimator and its efficiency.

i. Begin
ii. Data input
iii. Compute sample mean of first inventory population
iv. Compute sample mean of second inventory population
v. Compute sample and population means of auxiliary inventory population
vi. Compute estimator for ratio of two inventory populations
vii. Compute biases for all estimators
viii. Compute MSE
ix. Compute efficiency (Percentage Relative Efficiency-PRE)
x. If PRE is greater than previous ones
xi. Find efficient estimator
xii. Data output
xiii. End

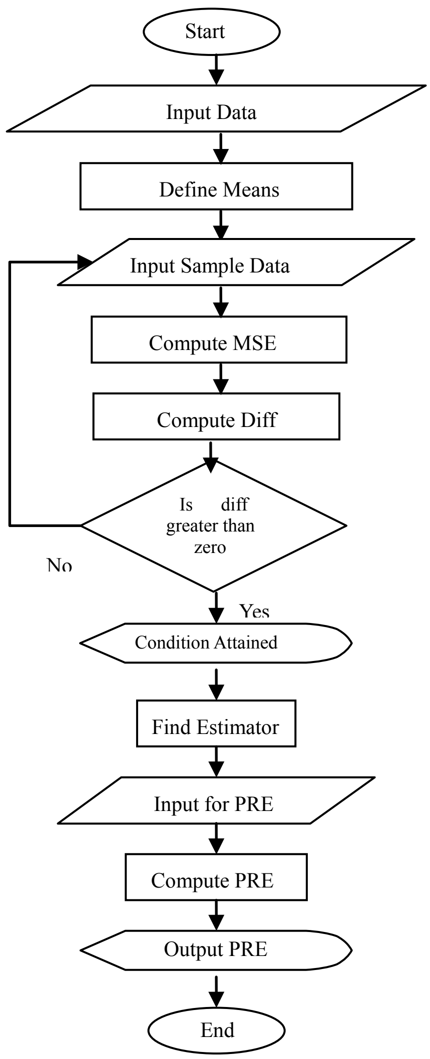

Figure 5.1. Tabular form of the algorithm (Computing flow chart)

Table 5.1. Computed Data Statistics

$\overline{Y}_{01} = 91.40$	$\overline{Y}_{02} = 109.20$	$\overline{Y}_{11} = 577.80$	$\overline{Y}_{12} = 609.60$
$\overline{X}_1 = 3757$	$\overline{X}_2 = 4049.60$	$S_{01}(y) = 4.0373$	$S_{02}(y) = 6.1400$
$S_{11}(y) = 38.78402$	$S_{12}(y) = 43.0848$	$S_{x1} = 40.5401$	$S_{x2} = 126.7292$
$S_{011} = 84.60$	$S_{012} = 245.10$	$S_{0x1} = 139.50$	$S_{0x2} = 761.35$
$S_{1x1} = 860.50$	$S_{1x2} = 4597.55$	$M_d(x) = 3853.50$	

Table 5.2. Computed Bias, MSE and PRE of different estimators w.r.t. G_{1st}

Estimators	BIAS	M S E	PRE
G_{1st}	1.24E-04	1.09141E-05	100
G_{2st}	8.41E-05	8.94115E-06	122.06
τ_{st}	**-9.94E-06**	**8.18904E-06**	**133.27**

To meet out the theoretical findings, we have considered the data in Murthy (1967) where the main variables y_0 and y_1 are the number of workers and the fixed capital respectively along with the output as the auxiliary variable. The size of the population is 10 and is divided into two stratums each of size 5. The sample size is 4 by taking 2 from each stratum. Following are the parameters computed.

6. Observations and Conclusions

When inventory items are heterogeneous and huge, the estimation approach is only panacea for estimating the characteristics of inventory populations under consideration which otherwise seems difficult to control and manage for any organization. So for the estimation of ratio of two inventory population means have been discussed in paper, one is man-inventory population and another is money-inventory population as defined by Ackoff and Sasieni (1993).

Under the section numerical illustration, two heterogeneous inventory populations are given whose estimators are given thereafter final estimator as ratio has been developed whose bias and mean square errors are lesser than all previous estimators and percentage relative efficiency is given which is higher than previous ones, vide table 5.2, row 3.

Thus proposed estimator is most efficient estimator of ratio of two inventory population means using auxiliary information among such estimators as it has lesser mean square error. Thus the proposed estimator should be preferred for the estimation of ratio of two inventory population means.

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