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Improved Dual to Ratio Cum Dual to Product Estimator in the Stratified Random Sampling

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Abstract In this article, we propose an improved dual to ratio cum dual to product estimator of the population mean under the stratified random sampling scheme. The expressions for the bias and mean squared error (MSE) of the proposed estimator are found by the first degree of approximation. The optimum value of the constant, which minimizes the MSE of the proposed estimator, is also obtained. Efficiency comparisons are performed between the proposed estimator and many estimators in Literature under the stratified random sampling and the efficiency conditions of the proposed estimator are determined. Finally, an empirical study is carried out which shows the performance of the proposed estimator along with the existing estimators under the stratified random sampling.

Keywords Ratio and Product estimators, Stratified Random Sampling, Bias, MSE, Efficiency

1. Introduction

In practice, it is very common that the use of the auxiliary variable (x or z) improves the efficiency of the estimators for the population parameters of the study variable (y). The auxiliary information supplied by the auxiliary variables is used both at the design and estimation stages of the survey. We have used it in the estimation stage in this article. The auxiliary variable is highly correlated (positively or negatively) with the study variable. The ratio method of estimation providing ratio type estimators is used for the estimation of population parameters when the study variable and the auxiliary variables are highly positively correlated to each other; whereas, the product method of estimation giving product type estimators is used when y and x are highly negatively correlated to each other. As in practice, we find that the study variable has both positive and negative correlations with two different variables at a time. This encourages us to use both positively and negatively correlated variables in our study and we propose the dual to ratio and dual to product estimator of the population mean in the stratified random sampling. The improvement is a continuous process of research and the form of the estimator using the scalar α has always improved the estimators of population parameters in the simple random sampling. Being inspired from this truth, we have proposed the dual to ratio and dual to product estimator of the population mean in the stratified random sampling.

As we deal with the methods in the stratified random sampling, assume that the finite population, $\mathbf{u}=(u_1,u_2,...,u_N)$, consists of N distinct and identifiable units which are heterogeneous from each other. Let the whole population be divided into L strata of sizes, N_h ($h=1,2,...,L$), in which units are relatively homogeneous to each other. In addition, the study variable, y, and two auxiliary variables, x and z, take the values y_{hi}, x_{hi}, and z_{hi} ($h=1,2,...,L$; $i=1,2,...,N_h$), respectively, for the i^{th} unit of the h^{th} stratum. It is clear that the sub-samples of sizes, n_h ($h=1,2,...,L$), are drawn from each stratum using the proportional allocation method, constituting of the required sample of size as $n=\sum_{h=1}^{L} n_h$.

Following common notations of the stratified random sampling, it can be given by

$\bar{Y}_h=\frac{1}{N_h}\sum_{i=1}^{N_h} y_{hi}$: The h^{th} stratum population mean for the study variable, y,

$\bar{X}_h=\frac{1}{N_h}\sum_{i=1}^{N_h} x_{hi}$: The h^{th} stratum population mean for the auxiliary variable, x,

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$\bar{Z}_h = \frac{1}{N_h}\sum_{i=1}^{N_h} z_{hi}$: The h^{th} stratum population mean for the auxiliary variable, z,

$\bar{Y} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} y_{hi} = \frac{1}{N}\sum_{h=1}^{L} N_h\bar{Y}_h = \sum_{h=1}^{L} W_h\bar{Y}_h$: The population mean of the study variable, y,

$\bar{X} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} x_{hi} = \frac{1}{N}\sum_{h=1}^{L} N_h\bar{X}_h = \sum_{h=1}^{L} W_h\bar{X}_h$: The population mean of the auxiliary variable, x,

$\bar{Z} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} z_{hi} = \frac{1}{N}\sum_{h=1}^{L} N_h\bar{Z}_h = \sum_{h=1}^{L} W_h\bar{Z}_h$: The population mean of the auxiliary variable, z,

$\bar{y}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} y_{hi}$: The h^{th} stratum sample mean of the study variable, y,

$\bar{x}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} x_{hi}$: The h^{th} stratum sample mean of the auxiliary variable, x,

$\bar{z}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} z_{hi}$: The h^{th} stratum sample mean of the auxiliary variable, z,

$W_h = \frac{N_h}{N}$: Weight of the h^{th} stratum.

2. Estimators in Literature

Hansen *et al.* [1] proposed the classical combined ratio estimator for the population mean under the stratified random sampling as

$$\bar{y}_{RC} = \bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right), \tag{2.1}$$

where $\bar{y}_{st} = \sum_{h=1}^{L} W_h\bar{y}_h$ and $\bar{x}_{st} = \sum_{h=1}^{L} W_h\bar{x}_h$.

The MSE of the combined ratio estimator in (2.1), up to the first order of approximation, is

$$MSE(\bar{y}_{RC}) = \sum_{h=1}^{L} W_h^2\lambda_h\left(S_{yh}^2 - 2R_x S_{yxh} + R_x^2 S_{xh}^2\right), \tag{2.2}$$

where $\lambda_h = \left(\frac{1}{n_h} - \frac{1}{N_h}\right)$, $R_x = \frac{\bar{Y}}{\bar{X}} = \frac{\bar{Y}_{st}}{\bar{X}_{st}}$ is the population ratio, S_{yh}^2 is the population variance of the study variable, S_{xh}^2 is the population variance of the auxiliary variable, and S_{yxh} is the population covariance between the study and auxiliary variables in the h^{th} stratum.

The combined product estimator of the population mean in the stratified random sampling is defined as

$$\bar{y}_{PC} = \bar{y}_{st}\left(\frac{\bar{z}_{st}}{\bar{Z}}\right), \tag{2.3}$$

where $\bar{z}_{st} = \sum_{h=1}^{L} W_h\bar{z}_h$.

The MSE of the combined product estimator in (2.3), up to the first order of approximation, is

$$MSE(\bar{y}_{PC}) = \sum_{h=1}^{L} W_h^2\lambda_h\left(S_{yh}^2 + 2R_z S_{yzh} + R_z^2 S_{zh}^2\right), \tag{2.4}$$

where $R_z = \frac{\bar{Y}}{\bar{Z}} = \frac{\bar{Y}_{st}}{\bar{Z}_{st}}$.

Many authors, such as Kadilar and Cingi ([2], [3]), Shabbir and Gupta [4], Singh and Vishwakarma [5], Koyuncu and Kadilar ([6], [7], [8], [9]), Sanaullah *et al.* [10] have improved the ratio and product estimators, given in (2.1) and (2.3), for the population mean of the study variable in the stratified random sampling. However, in this article, we examine only dual estimators for the population mean in Literature that can be summarized as follows:

Using the combined ratio and product estimators, Kushwaha *et al.* [11] proposed the following dual to ratio and dual to product estimators by the Srivenkataramana [12] transformation as

$$\bar{y}_{RC}^{*} = \bar{y}_{st}\left(\frac{\bar{x}_{st}^{*}}{\bar{X}}\right), \tag{2.5}$$

$$\bar{y}_{PC}^{*} = \bar{y}_{st}\left(\frac{\bar{Z}}{\bar{z}_{st}^{*}}\right), \tag{2.6}$$

respectively, where $\bar{x}_{st}^{*} = \sum_{h=1}^{L} W_h\bar{x}_h^{*}$ and $\bar{z}_{st}^{*} = \sum_{h=1}^{L} W_h\bar{z}_h^{*}$. Here, the Srivenkataramana [12] transformations are $\bar{x}_h^{*} = \frac{\bar{X}_h N_h - \bar{x}_h n_h}{N_h - n_h}$ and $\bar{z}_h^{*} = \frac{\bar{Z}_h N_h - \bar{z}_h n_h}{N_h - n_h}$.

The MSE of dual to ratio and dual to product estimators in (2.5) and (2.6), respectively, up to the first order of approximation, are, respectively, given by

$$MSE(\bar{y}_{RC}^{*}) = \sum_{h=1}^{L} W_h^2\lambda_h\left(S_{yh}^2 + R_x^2 g_h^2 S_{xh}^2 - 2R_x g_h S_{yxh}\right), \tag{2.7}$$

$$MSE(\bar{y}^*_{PC}) = \sum_{h=1}^{L} W_h^2 \lambda_h \left(S_{yh}^2 + R_z^2 g_h^2 S_{zh}^2 + 2R_z g_h S_{yzh} \right), \quad (2.8)$$

where $g_h = \dfrac{n_h}{N_h - n_h}$.

Singh *et al*. [13] suggested following exponential ratio and product type estimators in the stratified random sampling based on Bahl and Tuteja [14] estimators of the population mean under the simple random sampling, respectively, as follows:

$$\bar{y}^{st}_{\mathrm{Re}} = \bar{y}_{st} \exp\left(\frac{\bar{X} - \bar{x}_{st}}{\bar{X} + \bar{x}_{st}} \right) = \bar{y}_{st} \exp\left(\frac{\sum_{h=1}^{L} W_h (\bar{X}_h - \bar{x}_h)}{\sum_{h=1}^{L} W_h (\bar{X}_h + \bar{x}_h)} \right), \quad (2.9)$$

$$\bar{y}^{st}_{\mathrm{Pe}} = \bar{y}_{st} \exp\left(\frac{\bar{z}_{st} - \bar{Z}}{\bar{z}_{st} + \bar{Z}} \right) = \bar{y}_{st} \exp\left(\frac{\sum_{h=1}^{L} W_h (\bar{z}_h - \bar{Z}_h)}{\sum_{h=1}^{L} W_h (\bar{z}_h + \bar{Z}_h)} \right). \quad (2.10)$$

The MSE of the estimators in (2.9) and (2.10), up to the first order of approximation, are, respectively,

$$MSE(\bar{y}^{st}_{\mathrm{Re}}) = \sum_{h=1}^{L} W_h^2 \lambda_h \left[S_{yh}^2 - R_x S_{yxh} + \frac{R_x^2}{4} S_{xh}^2 \right], \quad (2.11)$$

$$MSE(\bar{y}^{st}_{\mathrm{Pe}}) = \sum_{h=1}^{L} W_h^2 \lambda_h \left[S_{yh}^2 + \frac{R_z^2}{4} S_{zh}^2 + R_z S_{yzh} \right]. \quad (2.12)$$

Tailor *et al*. [15] proposed the following dual to ratio and product type estimators using the Singh *et al*. [13] estimators as follows:

$$\bar{y}^{*st}_{\mathrm{Re}} = \bar{y}_{st} \exp\left(\frac{\bar{x}^*_{st} - \bar{X}}{\bar{x}^*_{st} + \bar{X}} \right) = \bar{y}_{st} \exp\left(\frac{\sum_{h=1}^{L} W_h (\bar{x}^*_h - \bar{X}_h)}{\sum_{h=1}^{L} W_h (\bar{x}^*_h + \bar{X}_h)} \right), \quad (2.13)$$

$$\bar{y}^{*st}_{Pe} = \bar{y}_{st} \exp\left(\frac{\bar{Z} - \bar{z}^*_{st}}{\bar{Z} + \bar{z}^*_{st}} \right) = \bar{y}_{st} \exp\left(\frac{\sum_{h=1}^{L} W_h (\bar{Z}_h - \bar{z}^*_h)}{\sum_{h=1}^{L} W_h (\bar{Z}_h + \bar{z}^*_h)} \right). \quad (2.14)$$

The MSE of the estimators in (2.13) and (2.14), up to the first order of approximation are, respectively,

$$MSE(\bar{y}^{*st}_{\mathrm{Re}}) = \sum_{h=1}^{L} W_h^2 \lambda_h \left[S_{yh}^2 - R_x g_h S_{yxh} + \frac{R_x^2}{4} g_h^2 S_{xh}^2 \right], \quad (2.15)$$

$$MSE(\bar{y}^{*st}_{\mathrm{Pe}}) = \sum_{h=1}^{L} W_h^2 \lambda_h \left[S_{yh}^2 + R_z g_h S_{yzh} + \frac{R_z^2}{4} g_h^2 S_{zh}^2 \right]. \quad (2.16)$$

3. Proposed Estimator

Motivated by the estimators, mentioned in Section 2 and the fact that the use of a scalar in the current forms (2.13) and (2.14) always improves the estimator, we propose the following dual to ratio cum dual to product estimator of the population mean in the stratified random sampling by combining the dual to ratio and dual to product estimators as

$$t = \alpha\, \bar{y}_{st} \exp\left(\frac{\bar{x}^*_{st} - \bar{X}}{\bar{x}^*_{st} + \bar{X}} \right) + (1-\alpha)\bar{y}_{st} \exp\left(\frac{\bar{Z} - \bar{z}^*_{st}}{\bar{Z} + \bar{z}^*_{st}} \right), \quad (3.1)$$

where α is a suitable constant to be determined such that the MSE of the proposed estimator, t, is minimum.

It is worth notable that it becomes the dual to product estimator, given in (2.14), for $\alpha = 0$ and that it reduces to dual to ratio estimator, given in (2.13), for $\alpha = 1$.

To study the large sample properties of the proposed estimator, t, let us define the following notations:

$\bar{y}_h = \bar{Y}_h \left(1 + \mathrm{e}_{0h}\right)$, $\bar{x}_h = \bar{X}_h \left(1 + \mathrm{e}_{1h}\right)$, and $\bar{z}_h = \bar{Z}_h \left(1 + \mathrm{e}_{2h}\right)$ such that $\mathrm{E}(\mathrm{e}_{0h}) = \mathrm{E}(\mathrm{e}_{1h}) = \mathrm{E}(\mathrm{e}_{2h}) = 0$ and $\mathrm{E}\left(\mathrm{e}_{0h}^2\right) = \lambda_h C_{yh}^2$, $\mathrm{E}\left(\mathrm{e}_{1h}^2\right) = \lambda_h C_{xh}^2$, $\mathrm{E}\left(\mathrm{e}_{2h}^2\right) = \lambda_h C_{zh}^2$,

$\mathrm{E}\left(\mathrm{e}_{0h}\mathrm{e}_{1h}\right) = \lambda_h \rho_{yxh} C_{yh} C_{xh}$,

$\mathrm{E}\left(\mathrm{e}_{0h}\mathrm{e}_{2h}\right) = \lambda_h \rho_{yzh} C_{yh} C_{zh}$,

$\mathrm{E}\left(\mathrm{e}_{1h}\mathrm{e}_{2h}\right) = \lambda_h \rho_{xzh} C_{xh} C_{zh}$.

Expressing the proposed estimator, in (3.1), in terms of $\mathrm{e}_{ih}\ (i = 0,1,2)$, we have

$$t = \alpha \sum_{h=1}^{L} W_h \bar{y}_h \exp\left(\frac{\sum_{h=1}^{L} W_h (\bar{x}^*_h - \bar{X}_h)}{\sum_{h=1}^{L} W_h (\bar{x}^*_h + \bar{X}_h)} \right) + (1-\alpha) \sum_{h=1}^{L} W_h \bar{y}_h \exp\left(\frac{\sum_{h=1}^{L} W_h (\bar{Z}_h - \bar{z}^*_h)}{\sum_{h=1}^{L} W_h (\bar{Z}_h + \bar{z}^*_h)} \right)$$

$$= \alpha \sum_{h=1}^{L} W_h \bar{Y}_h (1+e_{0h}) \exp\left(\frac{-\sum_{h=1}^{L} W_h g_h \bar{X}_h e_{1h}}{2\sum_{h=1}^{L} W_h \bar{X}_h - \sum_{h=1}^{L} W_h g_h \bar{X}_h e_{1h}} \right)$$

$$+(1-\alpha) \sum_{h=1}^{L} W_h \bar{Y}_h (1+e_{0h}) \exp\left(\frac{\sum_{h=1}^{L} W_h g_h \bar{Z}_h e_{2h}}{2\sum_{h=1}^{L} W_h \bar{Z}_h - \sum_{h=1}^{L} W_h g_h \bar{Z}_h e_{2h}} \right)$$

$$= \alpha \bar{Y}(1+e_0) \exp\left[\left(-\frac{e_1}{2}\right)\left(1-\frac{e_1}{2}\right)^{-1} \right] + (1-\alpha)\bar{Y}(1+e_0)\exp\left[\left(\frac{e_2}{2}\right)\left(1-\frac{e_2}{2}\right)^{-1} \right], \tag{3.2}$$

where $e_0 = \frac{\sum_{h=1}^{L} W_h \bar{Y}_h e_{0h}}{\bar{Y}}$, $e_1 = \frac{\sum_{h=1}^{L} W_h g_h \bar{X}_h e_{1h}}{\bar{X}}$, and $e_2 = \frac{\sum_{h=1}^{L} W_h g_h \bar{Z}_h e_{1h}}{\bar{Z}}$ such that $E(e_0) = E(e_1) = E(e_2) = 0$

and

$$\mathrm{E}\left(e_0^2\right) = \frac{1}{\bar{Y}^2} \sum_{h=1}^{L} W_h^2 \lambda_h S_{yh}^2, \quad \mathrm{E}\left(e_1^2\right) = \frac{1}{\bar{X}^2} \sum_{h=1}^{L} W_h^2 \lambda_h g_h^2 S_{xh}^2, \quad \mathrm{E}\left(e_2^2\right) = \frac{1}{\bar{Z}^2} \sum_{h=1}^{L} W_h^2 \lambda_h g_h^2 S_{zh}^2,$$

$$\mathrm{E}(e_0 e_1) = \frac{1}{\bar{Y}\,\bar{X}} \sum_{h=1}^{L} W_h^2 \lambda_h g_h S_{yxh}, \quad \mathrm{E}(e_0 e_2) = \frac{1}{\bar{Y}\,\bar{Z}} \sum_{h=1}^{L} W_h^2 \lambda_h g_h S_{yzh}, \quad E(e_1 e_2) = \frac{1}{\bar{X}\,\bar{Z}} \sum_{h=1}^{L} W_h^2 \lambda_h g_h^2 S_{xzh}.$$

On simplifying the expressions after the expansion on the right hand side of (3.2), up to the first order of approximation, we have

$$t = \left[\alpha \bar{Y} \left(1 + e_0 - \frac{e_1}{2} - \frac{e_0 e_1}{2} - \frac{e_1^2}{8} \right) + (1-\alpha)\bar{Y}\left(1 + e_0 + \frac{e_2}{2} + \frac{e_0 e_2}{2} + \frac{3e_2^2}{8} \right) \right]$$

$$= \bar{Y}\left[1 + e_0 - \alpha\frac{e_1}{2} - \alpha\frac{e_0 e_1}{2} - \alpha\frac{e_1^2}{8} + (1-\alpha)\frac{e_2}{2} + (1-\alpha)\frac{e_0 e_2}{2} + (1-\alpha)\frac{3e_2^2}{8} \right]. \tag{3.3}$$

We can write (3.3) as

$$t - \bar{Y} = \bar{Y}\left[e_0 - \alpha\frac{e_1}{2} - \alpha\frac{e_0 e_1}{2} - \alpha\frac{e_1^2}{8} + (1-\alpha)\frac{e_2}{2} + (1-\alpha)\frac{e_0 e_2}{2} + (1-\alpha)\frac{3e_2^2}{8} \right]$$

$$= \bar{Y}\left[e_0 - \alpha\frac{e_1}{2} - \alpha\frac{e_0 e_1}{2} - \alpha\frac{e_1^2}{8} + \alpha_1\frac{e_2}{2} + \alpha_1\frac{e_0 e_2}{2} + \alpha_1\frac{3e_2^2}{8} \right]. \tag{3.4}$$

Taking the expectation on both sides of (3.4), we have the bias of the proposed estimator, up to the first order of approximation, as

$$B(t) = \sum_{h=1}^{L} W_h^2 \lambda_h \left[\frac{\alpha_1}{2}\frac{g_h S_{yzh}}{\bar{Y}\,\bar{Z}} - \frac{\alpha}{2}\frac{g_h S_{yxh}}{\bar{Y}\,\bar{X}} + \frac{3\alpha_1}{8}\frac{g_h^2 S_{zh}^2}{\bar{Z}^2} - \frac{\alpha}{8}\frac{g_h^2 S_{xh}^2}{\bar{X}^2} \right]. \tag{3.5}$$

From (3.4), up to the first order of approximation, the MSE of the proposed estimator is

$$MSE(t) \cong E\left[\bar{Y}\left(e_0 - \alpha\frac{e_1}{2} + \alpha_1\frac{e_2}{2} \right) \right]^2$$

$$= \bar{Y}^2 E\left[e_0 - \alpha\frac{e_1}{2} + \alpha_1\frac{e_2}{2} \right]^2$$

$$=\bar{Y}^2 E\left[e_0^2+\frac{\alpha^2}{4}e_1^2+\frac{\alpha_1^2}{4}e_2^2-\alpha e_0 e_1+\alpha_1 e_0 e_2-\frac{1}{2}\alpha\alpha_1 e_1 e_2\right] \tag{3.6}$$

which is minimum for

$$\alpha=\frac{E(e_2^2)+2E(e_0e_1)+2E(e_0e_2)+E(e_1e_2)}{E(e_1^2)+E(e_2^2)+2E(e_1e_2)}=\frac{A}{B},$$

where

$$A=E(e_2^2)+2E(e_0e_1)+2E(e_0e_2)+E(e_1e_2)=\sum_{h=1}^{L}W_h^2\lambda_h\left[\frac{g_h^2S_{zh}^2}{\bar{Z}^2}+2\frac{g_hS_{yxh}}{\bar{Y}\,\bar{X}}+2\frac{g_hS_{yzh}}{\bar{Y}\,\bar{Z}}+\frac{g_h^2S_{xzh}}{\bar{X}\,\bar{Z}}\right],$$

$$B=E(e_1^2)+E(e_2^2)+2E(e_1e_2)=\sum_{h=1}^{L}W_h^2\lambda_h\left[\frac{g_h^2S_{xh}^2}{\bar{X}^2}+\frac{g_h^2S_{zh}^2}{\bar{Z}^2}+2\frac{g_h^2S_{xzh}}{\bar{X}\,\bar{Z}}\right].$$

Finally, the minimum MSE of the proposed estimator is

$$MSE_{\min}(t)=\bar{Y}^2\left[\sum_{h=1}^{L}W_h^2\lambda_h\left(\frac{S_{yh}^2}{\bar{Y}^2}+\frac{g_h^2S_{zh}^2}{4\bar{Z}^2}+\frac{g_hS_{yzh}}{\bar{Y}\,\bar{Z}}\right)-\frac{A^2}{4B}\right]$$

$$=\sum_{h=1}^{L}W_h^2\lambda_h\left(S_{yh}^2+\frac{1}{4}R_z^2g_h^2S_{zh}^2+R_zg_hS_{yzh}\right)-\bar{Y}^2\frac{A^2}{4B}. \tag{3.7}$$

4. Efficiency Comparisons

The variance of the sample mean in the stratified random sampling, $\bar{y}_{st}$, is given by

$$V(\bar{y}_{st})=\sum_{h=1}^{L}W_h^2\lambda_hS_{yh}^2. \tag{4.1}$$

We obtain the following conditions under which the proposed dual to ratio cum dual to product type exponential estimator is better than the estimators, $\bar{y}_{st}$, $\hat{\bar{Y}}_{RC}$, $\hat{\bar{Y}}_{PC}$, $\hat{\bar{Y}}_{RC}^{*}$, $\hat{\bar{Y}}_{PC}^{*}$ $\hat{\bar{Y}}_{\mathrm{Re}}^{st}$, $\hat{\bar{Y}}_{Pe}^{st}$, $\hat{\bar{Y}}_{\mathrm{Re}}^{*st}$, and $\hat{\bar{Y}}_{PC}^{*st}$, respectively.

The proposed estimator, t, is more efficient than $\bar{y}_{st}$ if

$$\bar{Y}^2\frac{A^2}{4B}-\sum_{h=1}^{L}W_h^2\lambda_h\left[\frac{R_z^2}{4}g_h^2S_{zh}^2+R_zg_hS_{yzh}\right]>0. \tag{4.2}$$

It is more efficient than the estimator, $\bar{y}_{RC}$, if

$$\sum_{h=1}^{L}W_h^2\lambda_h\left[R_x^2S_{xh}^2-2R_xS_{yxh}-\frac{R_z^2}{4}g_h^2S_{zh}^2-R_zg_hS_{yzh}\right]+\bar{Y}^2\frac{A^2}{4B}>0. \tag{4.3}$$

It is more efficient than the estimator, $\bar{y}_{PC}$, if

$$\sum_{h=1}^{L}W_h^2\lambda_h\left[(1-\frac{g_h^2}{4})R_z^2S_{zh}^2+(2-g_h)R_zS_{yzh}\right]+\bar{Y}^2\frac{A^2}{4B}>0. \tag{4.4}$$

It is more efficient than the estimator, $\bar{y}_{RC}^{*}$, if

$$\sum_{h=1}^{L}W_h^2\lambda_h\left[R_x^2g_h^2S_{xh}^2-2R_xg_hS_{yxh}-\frac{R_z^2}{4}g_h^2S_{zh}^2-R_zg_hS_{yzh}\right]+\bar{Y}^2\frac{A^2}{4B}>0. \tag{4.5}$$

It is more efficient than the estimator, $\overline{y}_{PC}^{*}$, if

$$\sum_{h=1}^{L} W_h^2 \lambda_h \left[\frac{3g_h^2}{4} R_z^2 S_{zh}^2 - 3R_z S_{yzh} \right] + \overline{Y}^2 \frac{A^2}{4B} > 0 . \tag{4.6}$$

t is more efficient than $\overline{y}_{\mathrm{Re}}^{st}$ if

$$\sum_{h=1}^{L} W_h^2 \lambda_h \left[\frac{R_x^2}{4} S_{xh}^2 - R_x S_{yxh} - \frac{R_z^2}{4} g_h^2 S_{zh}^2 - R_z g_h S_{yzh} \right] + \overline{Y}^2 \frac{A^2}{4B} > 0 . \tag{4.7}$$

t is more efficient than $\overline{y}_{Pe}^{st}$ if

$$\sum_{h=1}^{L} W_h^2 \lambda_h \left[(1-g_h^2) \frac{R_z^2}{4} S_{zh}^2 + (1-g_h) R_z S_{yzh} \right] + \overline{Y}^2 \frac{A^2}{4B} > 0 . \tag{4.8}$$

The proposed estimator, t, is more efficient than $\overline{y}_{\mathrm{Re}}^{*st}$ if

$$\sum_{h=1}^{L} W_h^2 \lambda_h \left[\frac{R_x^2}{4} g_h^2 S_{xh}^2 - R_x g_h S_{yxh} - \frac{R_z^2}{4} g_h^2 S_{zh}^2 - R_z g_h S_{yzh} \right] + \overline{Y}^2 \frac{A^2}{4B} > 0 . \tag{4.9}$$

The proposed estimator, t, is more efficient than $\overline{y}_{PC}^{*st}$ if

$$\overline{Y}^2 \frac{A^2}{4B} > 0 . \tag{4.10}$$

5. Empirical Study

Table 1. Data Statistics

	Stratum I	**Stratum II**
n_h	3	2
N_h	5	5
$\overline{Y}_h$	1925.80	3115.60
$\overline{X}_h$	214.40	333.80
$\overline{Z}_h$	51.80	60.60
S_{yh}	615.92	340.38
S_{xh}	74.87	66.35
S_{zh}	0.75	4.84
S_{yxh}	39360.68	223563.50
S_{xzh}	38.08	287.92
S_{yzh}	411.16	1536.24

To examine the efficiency of the proposed estimator over other estimators, we consider the data set in Murthy [16] whose statistics are given in Table 1. The MSE and percent relative efficiency (PRE) values are given in Table 2.

Table 2. MSE and PRE Values of Estimators

Estimator	**MSE**	**PRE**
$\overline{y}_{st}$	21334.64	100.00
$\overline{y}_{RC}$	10089.28	211.46
$\overline{y}_{PC}$	36471.83	58.49
$\overline{y}_{RC}^{*}$	12543.21	170.08
$\overline{y}_{PC}^{*}$	31724.91	67.25
$\overline{y}_{Re}^{st}$	4780.56	446.28
$\overline{y}_{Pe}^{st}$	28010.19	76.17
$\overline{y}_{Re}^{*st}$	4947.57	431.21
$\overline{y}_{Pe}^{*st}$	26115.84	81.69
t	4306.88	495.36

We wish to elaborate the tabulated values in various columns of the Table 1. In the first column of the table, the various parameters of main and auxiliary variables, such as population size, sample size, population means, variances and covariances of both strata, respectively, have been given. Second and third columns present respective parameter values for first and second strata. Table 2 presents the mean square error and the Percentage Relative Efficiency (PRE) for the existing and the proposed estimators. The PRE of an estimator with respect to the sample mean is defined by

$$PRE(.,\overline{y}) = \frac{MSE(\overline{y})}{MSE(.)} \times 100 .$$

6. Conclusions

Sample surveys are legitimately considered as cost effective apparatus for estimation of the population parameter. The Statistician wishes to minimize the mean square error of the estimator to ideally infer about the parameter of the given population. In the present problem, we have proposed a dual to ratio cum dual to product estimator of population mean in stratified random sampling as in many situations we need both positively and negatively correlated information with the main variable under study. Further large sample properties of the proposed estimator have been studied up to the first order of approximation. We have also made the comparisons of desired results with previous researchers. Finally we have judged the performances of different estimators along with the proposed estimator through an empirical study under stratified random sampling. From the theoretical discussion and the numerical results from Table 2, we conclude that the proposed estimator is better than the mentioned estimators, in Section 2, under the stratified random sampling scheme as the proposed estimator has smaller mean squared error. By this numerical example, we also show that the efficiency conditions in theory for the proposed estimator, obtained in (4.2)-(4.10), are satisfied in practice, as well. Thus, the proposed estimator should be preferred for the estimation of the population mean under the stratified random sampling.

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