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A Partially Backlogging Inventory Model for Deteriorating Items with Ramp Type Demand Rate

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Abstract This paper presents an inventory model for deteriorating items with constant deterioration rate and ramp type demand rate. Shortages are allowed and completely backlogged for the next replenishment cycle. The backlogging rate of unsatisfied demand is assumed as a function of waiting time. The purpose of our study is to minimize the total variable inventory cost because if the total cost is minimum then profit will be maximum. A numerical example is given to demonstrate the developed model. Sensitivity analysis of the change of parameters is also given.

Keywords Deterioration, Partial-backlogging, Shortages and ramp type demand rate

1. Introduction

In most of the inventory models it has been observed that the demand rate is influenced by the amount of on hand inventory stock. Researchers developed their inventory models by considering demand rate is either constant or increasing, decreasing exponential functions of time, stock dependent and linear functions of time. The demand of newly launched products such as fashionable garments, electronic items and mobile phones etc. increases with time and later it becomes constant. In such type of products the concept of ramp type demand is introduced. The ramp type demand is a demand which increases up to a certain time and after that it becomes stable or constant.

Deterioration is a major problem in any business organization because the products such as food grains , vegetables, medicines and alcohols etc. that are stored for future use always loose part of their values with passage of time then this phenomenon is known as deterioration, so deterioration cannot be avoided in any business organization. Hill [1995] first time developed an inventory model with ramp type demand rate. Mandal and Pal [1998] extended the Hill [1995] model by allowing shortage. Wu and Ouyang [2000] developed an inventory model with ramp type demand rate. Wu [2001] devoloped an EOQ model for weibull deteriorating items with ramp type demand function of time by allowing shortage and the backlogging rate is a function of waiting time for next replenishment cycle. Giri et al. [2003] developed a single item single period EOQ model for weibull deteriorating items with ramp type demand rate by allowing shortages which are completely backlogged and an infinite planning horizon. Manna and Chaudhari [2006] developed an order level inventory model with finite production rate and time dependent deterioration rate and the demand rate is a ramp type function of time by allowing shortages which are completely backlogged. Deng et al. [2007] proposed an inventory model for deteriorating items with ramp type demand rate. Panda et al. [2008] considered an order level inventory model for seasonal products with ramp type demand rate by allowing shortage. Singh and Singh [2008] developed a partially backlogging inventory model for deteriorating items with quadratic demand rate. Skouri et al. [2009] developed a partially backlogging inventory model for weibull deteriorating items with ramp type demand rate. Singh and Singh [2010] developed an inflationary partially backlogging supply chain inventory model for deteriorating items with ramp type demand rate. Singh et al. [2010] determine a replenishment policy for non-instantaneous deteriorating items with stock dependent demand and partial backlogging. Jain and Kumar [2010] considered an inventory model for three parameters weibull deteriorating items with ramp type demand rate by allowing shortage. Chang [2011] developed a partially inventory model for two parameters weibull deteriorating items with ramp type demand rate. Yadav et al. [2012] determine an optimal replenishment policy for a partially backlogging inventory model with ramp type demand rate. Karmakar and Choudhuri [2013] developed a partially backlogging inventory model with time varying holding cost and ramp type demand rate. Sakrar and Chakrabarti [2013] developed an order level inventory model with fuzzy type demand under two level of shortage. Bansal and Garg [2014] considered a partially backlogging inventory model for non-instantaneous deteriorating items with ramp type demand rate. The table I show that the variation of total average cost with respect to the change in parameter θ, table II show that the variation of total average cost with

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respect to the change in parameter δ, table III show that the variation of total average cost with respect to the change in parameter b and the table IV show that the variation of total average cost with respect to the change in parameter s_C. Figures II, III and IV give the variation of parameter μ, T_1 and T with respect to the fixed value of $(\theta, T_1, T), (\theta, \mu, T)$ and (θ, μ, T_1), Figures V and VI give the variation of parameter δ and T with respect to the values of (μ, T_1, T) and (δ, μ, T_1), Figures VII, VIII and IX give the variation of parameter μ, T_1 and T with respect to the fixed value of $(b, T_1, T), (b, \mu, T)$ and (b, μ, T_1). Figures X and XI give the variation of parameter s_C and T with respect to the fixed values of (μ, T_1, T) and (s_C, μ, T_1).

In this paper we developed a partially backlogging inventory model for deteriorating items with constant deterioration rate and ramp type demand rate. Shortages are allowed and completely backlogged for next replenishment cycle, the backlogging rate of unsatisfied demand is assumed as a function of waiting time. The ramp type demand is a demand which increases up to a certain time and after that it becomes stable or constant in the case of real estate, electronics items, fashionable garments and food grains etc. The purpose of our study is to minimize the total variable inventory cost for maximize the profit.

2. Assumptions and Notations

We consider the following assumptions and notations corresponding to the developed model

1. The ramp type demand rate is $R(t) = ae^{b[t-(t-\mu)H(t-\mu)]}$, where a is the initial demand rate and b a constant governing exponential demand rate and $H(t-\mu)$ is the Heaviside unit step function of time defined by $H(t-\mu) = \begin{cases} 1, t \geq \mu \\ 0, t \prec \mu \end{cases}$
2. The demand dependent production rate is $P(t) = kD(t)$, where k is a constant.
3. θ is the constant deterioration rate .
4. δ is the backlogging parameter.
5. h_C is the holding cost.
6. d_C is the deterioration cost.
7. s_C is the shortage cost.
8. μ is the time of maximum inventory level.
9. T_1 is the time of zero inventory level.
10. T is the length of inventory cycle.
11. The replenishment rate is infinite.
12. Shortages are allowed and partially backlogged.
13. The lead time is zero.
14. $I(t)$ is the inventory at any time in [0,T].

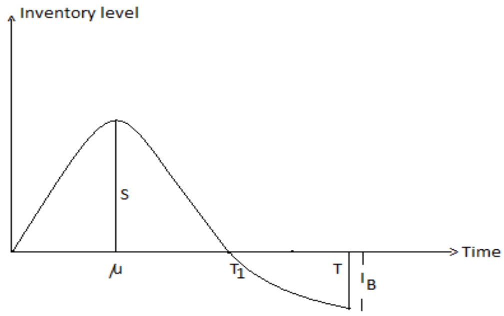

Figure 1

3. Mathematical Formulation

Let us consider an inventory system in which the production starts at t=0 and stop at $t = \mu$. During the period $[0, \mu]$ the inventory level grows due to both production and deterioration and during the period $[\mu, T_1]$ the maximum inventory level decreases due to both demand and deterioration and becomes zero at $t = T_1$. The shortage starts at $t = T_1$ and shortage interval is the end of current order cycle. The whole process is repeated and during the shortage interval $[T_1, T]$ the unsatisfied demand is backlogged at a rate of $\delta(T-t)$, where t is the waiting time and the positive constant δ is the backlogging parameter.

The instantaneous inventory level at any time t in $[0, T]$ is governed by the following differential equations

$$\frac{dI}{dt} + \theta I = kP(t) - R(t)$$

$$\frac{dI}{dt} + \theta I = a(k-1)e^{bt}, \qquad 0 \leq t \leq \mu \qquad (1)$$

With boundary condition

$$I(0) = 0$$

$$\frac{dI}{dt} + \theta I = -ae^{b\mu}, \qquad \mu \leq t \leq T_1 \qquad (2)$$

With boundary condition

$$I(T_1) = 0$$

$$\frac{dI}{dt} = -a\, e^{b\mu}\delta(T-i), \qquad T_1 \leq t \leq T \qquad (3)$$

With boundary condition

$$I(T_1) = 0$$

The solution of equation (1) is

$$I = a(k-1)[t + \frac{(b-\theta)t^2}{2}], \qquad 0 \le t \le \mu \tag{4}$$

The solution of equation (2) is

$$I = -a[(1+b\mu)t - \frac{\theta}{2}t^2] + a[(1+b\mu)T_1 - \frac{\theta}{2}T_1^2], \quad \mu \le t \le T_1 \tag{5}$$

The solution of equation (3) is

$$I = a\delta(1+b\mu)[TT_1 - \frac{1}{2}T_1^2 - tT + \frac{1}{2}t^2], \quad T_1 \le t \le T \tag{6}$$

The maximum inventory level S is obtained by putting $t = \mu$ in equation (4)

$$S = a(k-1)[\mu + \frac{(b-\theta)}{2}\mu^2] \tag{7}$$

The total number of units deteriorated during the interval $[0, T_1]$ is

$$n_d = [S - \int_0^{\mu} I(t)dt - \int_{\mu}^{T_1} I(t)dt]$$

$$n_d = a(k-1)[\mu + \frac{(b-\theta-1)}{2}\mu^2 - \frac{(b-\theta)}{6}\mu^3] + a[\mu T_1 + bT_1\mu^2 - \frac{1}{2}T_1^2 - \frac{b}{2}\mu T_1^2 - \frac{1}{2}\mu^2 - \frac{\theta}{2}\mu T_1^2 + \frac{\theta}{6}T_1^3 - \frac{b}{2}\mu^3 + \frac{\theta}{6}\mu^3], \tag{8}$$

The total number of units shortage during the interval $[T_1, T]$ is

$$n_S = -\int_{T_1}^{T} I(t)dt$$

$$n_S = -a\delta(1+b\mu)[T_1T^2 - TT_1^2 + \frac{1}{3}T_1^3 - \frac{1}{2}T^3], \tag{9}$$

The inventory holding cost per cycle in $[0, T_1]$ is

$$H_C = h_C[\int_0^{\mu} I(t)dt + \int_{\mu}^{T_1} I(t)dt]$$

$$H_C = ah_C[(k-1)\{\frac{1}{2}\mu^2 + \frac{(b-\theta)}{6}\mu^3\} - \{\mu T_1 + b\mu^2 T_1 - \frac{1}{2}T_1^2 - \frac{b}{2}\mu T_1^2 - \frac{\theta}{2}\mu T_1^2 - \frac{1}{2}\mu^2 - \frac{b}{2}\mu^3 - \frac{\theta}{6}T_1^3 + \frac{\theta}{6}\mu^3\}], \tag{10}$$

The deterioration cost per cycle is

$$D_C = d_C[S - \int_0^{\mu} I(t)dt + \int_{\mu}^{T_1} I(t)dt]$$

$$D_C = ad_C[(k-1)\{\mu + \frac{(b-\theta-1)}{2}\mu^2 - \frac{(b-\theta)}{6}\mu^3\} + \{\mu T_1 + bT_1\mu^2 - \frac{1}{2}T_1^2 - \frac{b}{2}\mu T_1^2 - \frac{\theta}{2}\mu T_1^2 - \frac{1}{2}\mu^2 + \frac{\theta}{6}T_1^3 - \frac{b}{2}\mu^3 + \frac{\theta}{6}\mu^3\}], \tag{11}$$

The shortage cost per cycle is

$$S_C = -a\delta(1+b\mu)s_C[T_1T^2 - TT_1^2 + \frac{1}{3}T_1^3 - \frac{1}{2}T^3], \tag{12}$$

The total variable inventory cost per unit cycle is

$$TC(\mu,T_1,T)=\frac{1}{T}[H_C+D_C+S_C]$$

$$TC(\mu,T_1,T)=\frac{ah_C}{T}[(k-1)\{\frac{1}{2}\mu^2+\frac{(b-\theta)}{6}\mu^3\}-\{\mu T_1+b\mu^2T_1-\frac{1}{2}T_1^2-\frac{b}{2}\mu T_1^2-\frac{\theta}{2}\mu T_1^2-\frac{1}{2}\mu^2$$
$$-\frac{b}{2}\mu^3-\frac{\theta}{6}T_1^3+\frac{\theta}{6}\mu^3\}]+\frac{ad_C}{T}[(k-1)\{\mu+\frac{(b-\theta-1)}{2}\mu^2-\frac{(b-\theta)}{6}\mu^3\}$$
$$+\{\mu T_1+bT_1\mu^2-\frac{1}{2}T_1^2-\frac{b}{2}\mu T_1^2-\frac{\theta}{2}\mu T_1^2-\frac{1}{2}\mu^2+\frac{\theta}{6}T_1^3-\frac{b}{2}\mu^3+\frac{\theta}{6}\mu^3\}]$$
$$-\frac{a\delta(1+b\mu)s_C}{T}[T_1T^2-TT_1^2+\frac{1}{3}T_1^3-\frac{1}{2}T^3], \tag{13}$$

The necessary condition for $TC(\mu,T_1,T)$ to be minimum is that

$$\frac{\partial TC}{\partial\mu}=0,\quad \frac{\partial TC}{\partial T_1}=0,\quad \frac{\partial TC}{\partial T}=0$$

$$H=\begin{vmatrix}\frac{\partial^2TC}{\partial\mu^2} & \frac{\partial^2TC}{\partial\mu T_1} & \frac{\partial^2TC}{\partial\mu\partial T}\\ \frac{\partial^2TC}{\partial T_1\partial\mu} & \frac{\partial^2TC}{\partial T_1^2} & \frac{\partial^2TC}{\partial T_1\partial T}\\ \frac{\partial^2TC}{\partial T\,\partial\mu} & \frac{\partial^2TC}{\partial T\,\partial T_1} & \frac{\partial^2TC}{\partial T^2}\end{vmatrix}$$

$$H=\begin{vmatrix}1596.79273 & 2088.28500 & 23196.3335\\ -3150.2350 & 5431.6240 & -4829.9380\\ -8958.4333 & -4907.58430 & 5349.2910\end{vmatrix}$$

and the sufficient condition is the determinant of the principal minors H_1, H_2, H_3...... of Hessian matrix (H-matrix), of $TC(\mu,T_1,T)$ are positive definite.

$$\frac{\partial TC}{\partial\mu}=\frac{ah_C}{T}[(k-1)\{\mu+\frac{(b-\theta)}{2}\mu^2\}+\{\mu-T_1-2b\mu T_1+\frac{b}{2}T_1^2+\frac{\theta}{2}T_1^2+\frac{3b}{2}\mu^2-\frac{\theta}{2}\mu^2\}]$$
$$+\frac{ad_C}{T}[(k-1)\{1+(b-\theta-1)\mu-\frac{(b-\theta)}{2}\mu^2\}+\{T_1+2b\mu T_1-\mu-\frac{b}{2}T_1^2$$
$$-\frac{\theta}{2}T_1^2-\frac{3b}{2}\mu^2+\frac{\theta}{2}\mu^2\}]-\frac{ab\delta s_C}{T}[T_1T^2-TT_1^2+\frac{1}{3}T_1^3-\frac{1}{2}T^3], \tag{14}$$

$$\frac{\partial^2TC}{\partial\mu^2}=\frac{ah_C}{T}[(k-1)\{1+(b-\theta)\mu\}+1+3b\mu-\theta\mu-2bT_1]+\frac{ad_C}{T}[(k-1)\{b-\theta-1$$
$$-(b-\theta)\mu\}+\theta\mu+2bT_1-1-3b\mu], \tag{15}$$

$$\frac{\partial TC}{\partial T_1}=\frac{ah_C}{T}[T_1-\mu+b\mu T_1+\theta\mu T_1-b\mu^2+\frac{\theta}{2}T_1^2]+\frac{ad_C}{T}[\mu-T_1-b\mu T_1$$
$$-\mu\theta T_1+b\mu^2+\frac{3\theta}{2}T_1^2]+\frac{2a\delta(1+b\mu)s_C}{T}[T_1-t], \tag{16}$$

$$\frac{\partial^2 TC}{\partial T_1^2}=\frac{ah_C}{T}[1+(b+\theta)\mu+\theta T_1]-\frac{ad_C}{T}[1+(b+\theta)\mu-3\theta T_1]-\frac{2a\delta(1+b\mu)s_C}{T}[T_1-T], \tag{17}$$

$$\frac{\partial TC}{\partial T}=-\frac{ah_C}{T^2}[(k-1)\{\frac{1}{2}\mu^2+\frac{(b-\theta)}{6}\mu^3\}-\mu T_1-bT_1\mu^2+\frac{1}{2}T_1^2+\frac{b}{2}\mu T_1^2+\frac{\theta}{2}\mu T_1^2$$
$$+\frac{1}{2}\mu^2+\frac{b}{2}\mu^3+\frac{\theta}{6}T_1^3-\frac{\theta}{6}\mu^3]-\frac{ad_C}{T^2}[(k-1)\{\mu+\frac{(b-\theta-1)}{2}\mu^2-\frac{(b-\theta)}{6}\mu^3\}$$
$$+\mu T_1+bT_1\mu^2-\frac{1}{2}T_1^2-\frac{1}{2}\mu^2-\frac{b}{2}\mu T_1^2-\frac{\theta}{2}\mu T_1^2+\frac{\theta}{2}T_1^3-\frac{b}{2}\mu^3+\frac{\theta}{6}\mu^3]$$
$$-\frac{a\delta(1+b\mu)s_C}{T}[2TT_1-T_1^2-\frac{3}{2}T^2]+\frac{a\delta(1+b\mu)s_C}{T^2}[T_1T^2-TT_1^2-\frac{1}{3}T_1^3-\frac{1}{2}T^3], \tag{18}$$

$$\frac{\partial^2 TC}{\partial T^2}=\frac{2ah_C}{T^3}[(k-1)\{\frac{1}{2}\mu^2+\frac{(b-\theta)}{6}\mu^3\}-\mu T_1+\frac{1}{2}T_1^2+\frac{1}{2}\mu^2-bT_1\mu^2+\frac{b}{2}\mu T_1^2$$
$$+\frac{\theta}{2}\mu T_1^2+\frac{\theta}{6}T_1^3+\frac{b}{2}\mu^3-\frac{\theta}{6}\mu^3]+\frac{2ad_C}{T^3}[(k-1)\{\mu+\frac{(b-\theta-1)}{2}\mu^2-\frac{(b-\theta)}{6}\mu^3\}$$
$$+\mu T_1-\frac{1}{2}T_1^2-\frac{1}{2}\mu^2+bT_1\mu^2-\frac{b}{2}\mu T_1^2-\frac{\theta}{2}\mu T_1^2+\frac{\theta}{2}T_1^3-\frac{b}{2}\mu^3+\frac{\theta}{6}\mu^3]$$
$$+\frac{a\delta(1+b\mu)s_C}{T}[3T-2T_1]+\frac{2a\delta(1+b\mu)s_C}{T^2}[2TT_1-T_1^2-\frac{3}{2}T^2]$$
$$-\frac{2a\delta(1+b\mu)s_C}{T^3}[T_1T^2-TT_1^2-\frac{1}{2}T^3+\frac{1}{3}T_1^3], \tag{19}$$

$$\frac{\partial^2 TC}{\partial\mu\partial T_1}=\frac{ah_C}{T}[bT_1-1-2b\mu+\theta T_1]+\frac{ad_C}{T}[1+2b\mu-bT_1-\theta T_1]-\frac{ab\delta s_C}{T}(T-T_1)^2 \tag{20}$$

$$\frac{\partial^2 TC}{\partial\mu\partial T}=-\frac{ah_C}{T^2}[(k-1)\{\mu+\frac{(b-\theta)\mu^2}{2}\}+\mu-T_1+\frac{bT_1^2}{2}-2b\mu T_1+\frac{\theta T_1^2}{2}+\frac{3b\mu^2}{2}-\frac{\theta\mu^2}{2}]$$
$$-\frac{ad_C}{T^2}[(k-1)\{1+(b-\theta-1)\mu-\frac{(b-\theta)\mu^2}{2}\}+T_1-\mu+2b\mu T_1-\frac{bT_1^2}{2}-\frac{\theta T_1^2}{2}$$
$$-\frac{3b\mu^2}{2}+\frac{\theta\mu^2}{2}]-\frac{ab\delta s_C}{2T}\{4TT_1-2T_1^2-3T^2\}+\frac{ab\delta s_C}{6T^2}\{6T_1T^2-6TT_1^2+\frac{T_1^3}{3}-\frac{T^3}{2}\} \tag{21}$$

$$\frac{\partial^2 TC}{\partial T_1\partial\mu}=\frac{ah_C}{T}[bT_1+\theta T_1-1-2b\mu]+\frac{ad_C}{T}[1+2b\mu-bT_1-\theta T_1]-\frac{ab\delta s_C}{T}[(T-T_1)^2] \tag{22}$$

$$\frac{\partial^2 TC}{\partial T_1\partial T}=-\frac{ah_C}{T^2}[T_1-\mu+b\mu T_1-b\mu^2+\theta\mu T_1+\frac{\theta T_1^2}{2}]-\frac{ad_C}{T^2}[\mu-T_1+b\mu^2-b\mu T_1-\theta\mu T_1+\frac{3\theta T_1^2}{2}]$$
$$-\frac{2a\delta s_C(1+b\mu)}{T}[T-T_1]+\frac{a\delta s_C(1+b\mu)}{T^2}[(T-T_1)^2] \tag{23}$$

$$\frac{\partial^2 TC}{\partial T \partial \mu} = -\frac{ah_C}{T^2}[(k-1)\{\mu + \frac{(b-\theta)\mu^2}{2}\} + \{\mu - T_1 - 2b\mu T_1 + \frac{bT_1^2}{2} + \frac{\theta T_1^2}{2} - \frac{\theta\mu^2}{2} + \frac{3b\mu^2}{2}\}]$$

$$-\frac{ad_C}{T^2}[(k-1)\{1 + (b-\theta-1)\mu - \frac{(b-\theta)\mu^2}{2}\} + \{T_1 - \mu + 2b\mu T_1 - \frac{bT_1^2}{2} - \frac{\theta T_1^2}{2}$$

$$-\frac{3b\mu^2}{2} + \frac{\theta\mu^2}{2}\}] + \frac{ab\delta s_C}{T^2}[T_1 T^2 - TT_1^2 + \frac{T_1^3}{3} - \frac{T^3}{3}] + \frac{ab\delta s_C}{T}[(T-T_1)^2] \quad (24)$$

$$\frac{\partial^2 TC}{\partial T \partial T_1} = -\frac{ah_C}{T^2}[T_1 - \mu + b\mu T_1 + \theta\mu T_1 - b\mu^2 + \frac{\theta T_1^2}{2}] - \frac{ad_C}{T^2}[\mu - T_1 - b\mu T_1 - \theta\mu T_1$$

$$+ b\mu^2 + \frac{3\theta T_1^2}{2}] + \frac{a\delta s_C(1+b\mu)}{T^2}[(T-T_1)^2] - \frac{2a\delta s_C(1+b\mu)}{T}[T - T_1] \quad (25)$$

4. Numerical Example

Let us consider an inventory system with the given data in appropriate units as follows:

$a = 100$, $b = 4$, $\theta = 0.05$, $\delta = 3$, $k = 2$, $h_C = 5$, $d_C = 8$, $S_C = 10$, and the total variable inventory cost is TC = 76969.433039.

Table 1 show the variation of total variable inventory cost with respect to the change in parameter θ.

Table 1

θ	μ	T_1	T	TC
0.05	0.935696	4.58769	5.70344	76969.433039
0.10	0.899044	4.41938	5.26421	63084.906820
0.15	0.845138	4.18073	4.67777	47122.554102

As we increase the parameter θ then the values of the parameters μ, T_1 and T decreases and the value of TC also decreases.

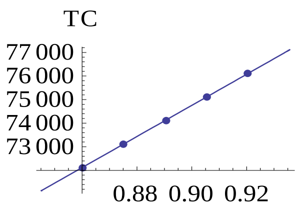

Figure 2. (θ, T_1 and T are const.)

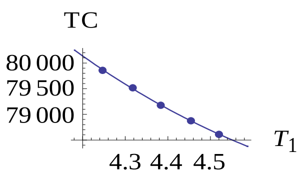

Figure 3. (θ, μ and T are const.)

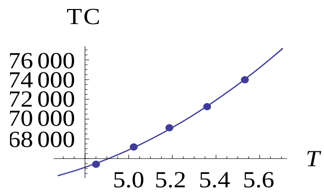

Figure 4. (θ, μ and T_1 are const.)

Table 2 show the variation of total variable inventory cost with respect to the change in parameter δ.

Table 2

δ	μ	T_1	T	TC
3	0.935696	4.58769	5.70344	76969.433039
6	0.935696	4.58769	5.37665	136567.28510
9	0.935696	4.58769	5.23187	194014.38186
12	0.935696	4.58769	5.14557	250319.14159
20	0.935696	4.58769	5.01982	397374.36731

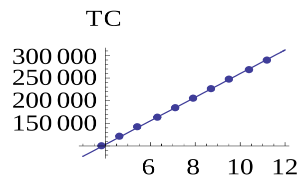

Figure 5. (μ, T_1 and T are const.)

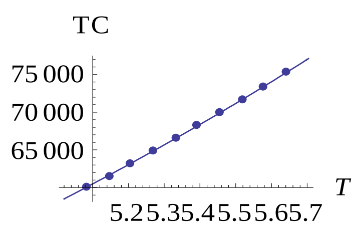

Figure 6. (δ, μ and T_1 are const.)

As we increase the parameter δ then the values of the parameters μ and T_1 remains constant, the value of the parameter T decreases and the value of TC increases.

Table 3 show the variation of total variable inventory cost with respect to the change in demand parameter b.

Table 3

b	μ	T_1	T	TC
4	0.935696	4.58769	5.70344	76969.433039
8	0.336864	2.35896	3.14422	25641.26890
12	0.196937	1.69609	2.35666	9431.469874
16	0.137035	1.36618	1.95630	6202.615740

As we increase the parameter b then the values of the parameters μ, T_1 and T decreases and the value of TC also decreases.

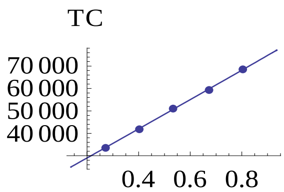

Figure 7. (b, T_1 and T are const.)

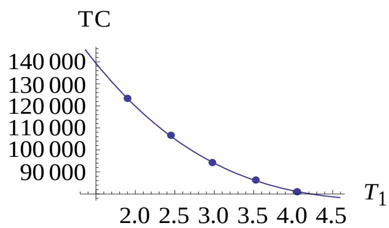

Figure 8. (b, μ and T are const.)

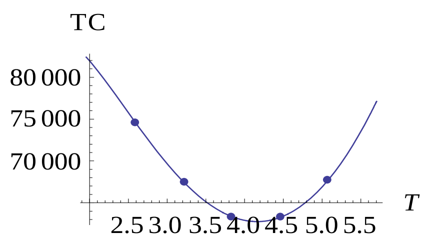

Figure 9. (b, μ and T_1 are const.)

Table 4 show the variation of total variable inventory cost with respect to the change in shortage cost parameter s_C.

As we increase the parameter s_C then the values of the parameters μ and T_1 remains constant, the value of the parameter T decreases and the value of TC increases.

Table 4

s_C	μ	T_1	T	TC
10	0.935696	4.58769	5.70344	76969.433039
15	0.935696	4.58769	5.49870	107301.59939
20	0.935696	4.58769	5.37665	130478.72470
25	0.935696	4.58769	5.29335	165624.98449

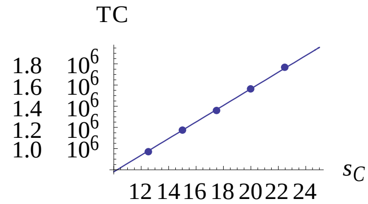

Figure 10. (μ, T_1 and T are const.)

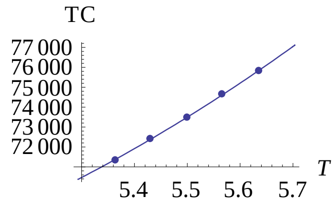

Figure 11. (s_C, μ and T_1 are const.)

Sensitivity Analysis- From the table I we see that as we increase the deterioration parameter θ then the values of the parameters μ, T_1,T and TC decreases. From the table II we see that as we increase the backlogging parameter δ then the values of the parameters μ and T_1 remains constant and T decreases and TC increases.

From the table III we see that as we increase the demand parameter b then the values of the parameters μ,T_1,T and TC decreases. From the table IV we see that as we increase the shortage cost s_C then the values of the parameters μ and T_1 remains constant and T decreases and TC increases.

Thus the parameters δ and s_C are more sensitive than the parameters θ , b, h_C and d_C (because for the change of the values h_C and d_C the values of μ,T_1 and T comes out to imaginary).

5. Conclusions

In this paper we consider a partially backlogging inventory model for deteriorating items with constant deterioration rate and ramp type demand rate. Shortages are allowed and completely backlogged for next replenishment cycle, the backlogging rate of unsatisfied demand is assumed as a function of waiting time. The change of the values of backlogging parameter δ and shortage cost s_C gives the maximum value of total cost. Further this model can be generalized by considering time dependent holding and deterioration cost.

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