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Optimal Search for Efficient Estimator of Finite Population Mean Using Auxiliary Information

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Abstract In this manuscript, we present the optimal search for efficient estimator of finite population mean using auxiliary information. This seeks to develop efficient ratio and product type exponential estimators of population mean under predictive method of estimation utilizing auxiliary information. The large sample properties of the proposed estimators have been studied up to the first order of approximation. The expressions for the biases and mean square errors (MSE) have been obtained up to the first order of approximation. The minimum value of the MSEs of proposed estimators have also been obtained for the optimum values of the characterizing scalars known as kappas. A comparison has been made with previous estimators till optimal estimator having least MSE has been obtained leading to highest efficiency which has been demonstrated on numerical data under study.

Keywords Efficient Estimator, Predictive Estimation, Finite Population Mean, MSE, Efficiency

1. Introduction

The system of supplementary population information is modeled to rejig the entire spectrum of estimation process in the theory and practice of sample surveys. Presently, it is widely used by theorist and practitioners of estimation knowledge of population parameters of the given population. Availability of various approaches to construct improved efficient estimators for population parameters is not only the necessary condition but training of mind to them is also sufficient. Why do we use the theory of prediction is a quintessential aspect of enchantment for the researchers engaged in this field of investigation. The answer to this question of attraction of the method lies in the fact that it provides us general framework for statistical inference on the character of finite population, vide Cochran (1977).

In the theory of survey sampling, it is unanimously accepted that the suitable use of auxiliary information improves the efficiency of the estimators of the parameters of the population under consideration for the main characteristic (y) under study. The auxiliary variable (x) is the variable about which we have full information and which is highly positively or negatively correlated with the main variable under study. When the auxiliary variable is highly positively correlated with the main variable, then the ratio method of estimation is used in which the ratio type estimators are used for the estimation of the parameters. On the other hand product type estimators under product method of estimation are used for the estimation of the population parameters when the auxiliary variable is highly negatively correlated with the main variable under study; vide Murthy (1967) and Das (1988).

Many authors used auxiliary information for improved estimation of population mean through ratio and product type estimators for the main characteristic under discussion. The latest references can be made of Yan and Tian (2010), Yadav (2011), Pandey *et al*. (2011), Nayak and Sahoo (2012), Subramani and Kumarapandiyan (2012), Solanki *et al*. (2012), Onyeka (2012), Jeelani *et al.* (2013), Saini (2013), Yadav and Kadilar (2013), Singh et al. (2014) etc.. Optimal search for solution by using computational methods have been widely used in boiling down the numerical complexity including inventory populations in supply chain management etc for attaining efficient estimators of performance measures of inventory system vide for examples Mishra and Singh (2012, 2012, 2013), Mishra and Mishra (2013) and Yadav et al.(2012, 2014).

The fresh attempt has been made to present optimal search for efficient estimator of finite population mean using auxiliary information in which most efficient ratio and product type exponential estimators have been developed. Optimal search continues unless data computation provides us better efficiencies as compared to previously existing estimators. In the proposed estimator, kappa technique has been used which seeks to introduce a constant and further the optimal value is obtained by minimizing the mean square error of the estimator. Next, after putting this optimal value,

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minimum value of mean square error is obtained. Lastly, result discussion and conclusion are given in the last section.

2. Material and Methods

Let the finite population $U = (U_1, U_2, ..., U_N)$ consists of N distinct and identifiable units. Let the main variable under study is denoted by Y and the auxiliary variable by X. Thus (Y_i, X_i), $(i = 1, 2, ..., N)$ denote the ith observations for the main and auxiliary variables respectively. Thus we have

$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} y_i$, the population mean of the main variable to be estimated.

Further let S denote the set of all possible sample of fixed size from the population U. Let $\vartheta(s)$ denote the effective sample size that is the number of distinct elements in s and $\bar{s}$ denote the collection of all those units of U which are not in s. We denote,

$$\bar{Y}_s = \frac{1}{\vartheta(s)}\sum_{i \in s} y_i$$

$$\bar{Y}_{\bar{s}} = \frac{1}{[N - \vartheta(s)]}\sum_{i \in \bar{s}} y_i .$$

In predictive estimation a specific model is considered to predict the non sampled values of the population. The prediction theory is based on the model-based theory. It considers a general framework for statistical inferences to be drawn for the parameters of the population in question. The general prediction theory considers different ratio, product and regression type estimators of population parameters as the predictive estimators i.e, predictors of the unobserved units of the population under some specific model. Several authors used ratio, product and regression type estimators as predictors of population parameters in predictive estimation theory. In predictive estimation theory it is well known that the use of ratio and product type estimators as predictors of the parameters under consideration of the unobserved units of the population result in the corresponding estimators of the population parameters for the whole population.

For a specific sample $s \in S$, the population mean $\bar{Y}$ can be written as,

$$\bar{Y} = \left[\frac{\vartheta(s)}{N}\bar{Y}_s + \frac{[N - \vartheta(s)]}{N}\bar{Y}_{\bar{s}}\right] \tag{2.1}$$

The sample mean of the sample of size n (i.e. $\vartheta(s) = n$) in simple random sampling is,

$\bar{y} = \frac{1}{n}\sum_{i \in s} y_i$ that is $\bar{Y}_s = \bar{y}$.

Thus, the population mean $\bar{Y}$ in (1) may be rewritten as,

$$\bar{Y} = \left[\frac{n}{N}\bar{Y}_s + \frac{(N-n)}{N}\bar{Y}_{\bar{s}}\right] \tag{2.2}$$

The suitable estimator for $\bar{Y}$ using (2.2) can be considered as,

$$t = \left[\frac{n}{N}\bar{y} + \frac{(N-n)}{N}T\right] \tag{2.3}$$

where T is the predictor of the population mean $\bar{Y}_{\bar{s}}$ of unobserved units of the population.

Srivastava (1983) considered the following estimators as the predictors of $\bar{Y}_{\bar{s}}$ as,

$$T = \bar{y} = \frac{1}{n}\sum_{i \in \bar{s}} y_i ,$$

$$T = \bar{y}_R = \bar{X}_{\bar{s}}\left(\frac{\bar{y}}{\bar{x}}\right),$$

where,

$$\bar{x} = \frac{1}{n}\sum_{i \in s} x_i ,\ \bar{X} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

and $\bar{X}_{\bar{s}} = \frac{1}{(N-n)}\sum_{i=1}^{N} x_i = \frac{(N\bar{X} - n\bar{x})}{(N-n)}$.

He has shown that whenever these estimators are used as predictive estimators of $\bar{Y}_{\bar{s}}$, then the estimator t given in (2.3) results in corresponding classical estimators,

$\bar{y} = \frac{1}{n}\sum_{i \in s} y_i$ and $\bar{y}_R = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)$ respectively.

However, he has shown that for the product predictive estimator, $T = \bar{y}_P = \bar{y}\left(\frac{\bar{x}}{\bar{X}_s}\right)$ of $\bar{Y}_{\bar{s}}$, the estimator does not result in the classical product estimator $\bar{y}_P = \bar{y}\left(\frac{\bar{x}}{\bar{X}}\right)$ of population mean $\bar{Y}$.

Thus, for $T = \bar{y}_P = \bar{y}\left(\frac{\bar{x}}{\bar{X}_s}\right)$, we have

$$t = \frac{n\bar{X} + (N-2n)\bar{x}}{(N\bar{X} - n\bar{x})} = t_p \tag{2.4}$$

The biases and the mean square errors of the estimators $\bar{y}_R$, $\bar{y}_P$ and t_p, up to the first order of approximations respectively are,

$$Bias(\bar{y}_R) = \theta\bar{Y}C_x^2(1-C) ,$$

$$Bias(\bar{y}_P) = \theta \bar{Y} C C_x^2,$$

$$Bias(t_p) = \theta \bar{Y} C_x^2 [C + f(1-f)^{-1}],$$

$$MSE(\bar{y}_R) = \theta \bar{Y}^2 [C_y^2 + C_x^2 (1-2C)], \quad (2.5)$$

$$MSE(\bar{y}_P) = MSE(t_p) = \theta \bar{Y}^2 [C_y^2 + C_x^2 (1+2C)] \quad (2.6)$$

where,

$$\theta = (1-f)^{-1},\ f = (n/N),\ C_y^2 = (S_y^2 / \bar{Y}^2),$$

$$C_x^2 = (S_x^2 / \bar{X}^2), C = \rho (C_y / C_x),\ \rho = (S_{yx} / S_y S_x),$$

$$S_y^2 = (N-1)^{-1} \sum_{i=1}^{N} (y_i - \bar{Y})^2,\ S_x^2 = (N-1)^{-1} \sum_{i=1}^{N} (x_i - \bar{X})^2$$

and $S_{yx} = (N-1)^{-1} \sum_{i=1}^{N} (y_i - \bar{Y})(x_i - \bar{X})$.

Singh et al. (2014) suggested two ratio and product type exponential estimators of population mean $\bar{Y}$ using Bahl and Tuteja (1991) ratio and product types exponential estimators of population mean as the predictive estimators of $\bar{Y}_{\bar{s}}$ respectively as,

$$t = t_{\text{Re}} = \left[\frac{n}{N} \bar{y} + \left(\frac{N-n}{N} \right) \bar{y} \exp\left(\frac{\bar{X}_{\bar{s}} - \bar{x}}{\bar{X}_{\bar{s}} + \bar{x}} \right) \right]$$

$$= \left[\frac{n}{N} \bar{y} + \left(\frac{N-n}{N} \right) \bar{y} \exp\left(\frac{N(\bar{X} - \bar{x})}{N(\bar{X} - \bar{x}) - 2n\bar{x}} \right) \right] \quad (2.7)$$

$$t = t_{Pe} = \left[\frac{n}{N} \bar{y} + \left(\frac{N-n}{N} \right) \bar{y} \exp\left(\frac{\bar{x} - \bar{X}_{\bar{s}}}{\bar{x} + \bar{X}_{\bar{s}}} \right) \right]$$

$$= \left[\frac{n}{N} \bar{y} + \left(\frac{N-n}{N} \right) \bar{y} \exp\left(\frac{N(\bar{x} - \bar{X})}{N\bar{X} + (N-2n)\bar{x}} \right) \right] \quad (2.8)$$

The biases and the mean square errors of above estimators up to the first order of approximations respectively are,

$$Bias(t_{\text{Re}}) = \frac{\theta}{8} \bar{Y} C_x^2 [3 - 4(C+f)],$$

$$Bias(t_{Pe}) = \frac{\theta}{8} \bar{Y} C_x^2 \left[4C - \frac{1}{(1-f)} \right],$$

$$MSE(t_{\text{Re}}) = \theta \bar{Y}^2 [C_y^2 + \frac{C_x^2}{4} (1-4C)], \quad (2.9)$$

$$MSE(t_{Pe}) = \theta \bar{Y}^2 [C_y^2 + \frac{C_x^2}{4} (1+4C)], \quad (2.10)$$

3. Proposed Estimators

Motivated by Singh et al. (2014) and Prasad (1989), we attempt to propose the following efficient ratio and product type exponential estimator in predictive estimation as,

$$\tau_{\text{Re}} = \kappa_1 \left[\frac{n}{N} \bar{y} + \left(\frac{N-n}{N} \right) \bar{y} \exp\left(\frac{\bar{X}_{\bar{s}} - \bar{x}}{\bar{X}_{\bar{s}} + \bar{x}} \right) \right]$$

$$= \kappa_1 \left[\frac{n}{N} \bar{y} + \left(\frac{N-n}{N} \right) \bar{y} \exp\left(\frac{N(\bar{X} - \bar{x})}{N(\bar{X} - \bar{x}) - 2n\bar{x}} \right) \right] \quad (3.1)$$

$$\tau_{Pe} = \kappa_2 \left[\frac{n}{N} \bar{y} + \left(\frac{N-n}{N} \right) \bar{y} \exp\left(\frac{\bar{x} - \bar{X}_{\bar{s}}}{\bar{x} + \bar{X}_{\bar{s}}} \right) \right]$$

$$= \kappa_2 \left[\frac{n}{N} \bar{y} + \left(\frac{N-n}{N} \right) \bar{y} \exp\left(\frac{N(\bar{x} - \bar{X})}{N\bar{X} + (N-2n)\bar{x}} \right) \right] \quad (3.2)$$

where κ_1 and κ_2 are the constants known as kappas to be determined such that the mean square errors of the estimators τ_1 and τ_2 are minimum respectively.

To study the large sample properties of the proposed estimators τ_{Re} and τ_{Pe}, we define,

$\bar{y} = \bar{Y}(1+e_0)$ and $\bar{x} = \bar{X}(1+e_1)$ such that $E(e_0) = E(e_1) = 0$ and up to the first order of approximation,

$$E(e_0^2) = \theta C_y^2,\ E(e_1^2) = \theta C_x^2 \text{ and } E(e_0 e_1) = \theta C C_x^2.$$

Expressing (3.1) in terms of e_i's and simplifying, we have

$$\tau_{\text{Re}} = \kappa_1 \bar{Y} \left[1 + e_0 - \frac{e_1}{2} - \frac{e_0 e_1}{2} + \frac{e_1^2}{8} (3-4f) \right] \quad (3.3)$$

Subtracting $\bar{Y}$ on both sides of (3.3), simplifying and taking expectation both sides, we get the bias of τ_{Re} as,

$$B(\tau_{\text{Re}}) = \bar{Y} \left[(\kappa_1 - 1) + \kappa_1 \left\{ E(e_0) - \frac{E(e_1)}{2} - \frac{E(e_0 e_1)}{2} + (3-4f) \frac{E(e_1^2)}{8} \right\} \right]$$

Putting the values of different expectations, we get

$$B(\tau_{\text{Re}}) = \bar{Y} \left[(\kappa_1 - 1) + \kappa_1 \theta \left\{ (3-4f) \frac{C_x^2}{8} - \frac{C C_x^2}{2} \right\} \right] \quad (3.4)$$

Squaring on both sides of (3.3) after subtracting $\overline{Y}$, expanding, simplifying up to the first order of approximation and taking expectation on both sides, we have the mean square error of τ_{Re} as,

$$MSE(\tau_{\text{Re}}) = \overline{Y}^2\left[(\kappa_1 - 1)^2 + \kappa_1^2\left\{E(e_0^2) - 2E(e_0e_1) + (5-4f)\frac{E(e_1^2)}{8}\right\} - 2\kappa_1\left\{(3-4f)\frac{E(e_1^2)}{8} - \frac{E(e_0e_1)}{2}\right\}\right]$$

which is minimum for

$$\kappa_1 = \frac{\left[1 + (3-4f)\dfrac{E(e_1^2)}{8} - \dfrac{E(e_0e_1)}{2}\right]}{\left[1 + E(e_0^2) - 2E(e_0e_1) + (5-4f)\dfrac{E(e_1^2)}{8}\right]} = \frac{A_1}{B_1}$$

where

$$A_1 = \left[1 + (3-4f)\frac{E(e_1^2)}{8} - \frac{E(e_0e_1)}{2}\right]$$

$$B_1 = \left[1 + E(e_0^2) - 2E(e_0e_1) + (5-4f)\frac{E(e_1^2)}{8}\right]$$

And the minimum mean square error of τ_{Re} is,

$$MSE_{\min}(\tau_{\text{Re}}) = \overline{Y}^2\left[1 - \frac{A_1^2}{B_1}\right] \tag{3.5}$$

Similarly, the bias and mean square error of τ_{Pe}, up to the first order of approximation respectively are,

$$B(\tau_{Pe}) = \overline{Y}\left[(\kappa_2 - 1) + \kappa_2\theta\left\{\frac{CC_x^2}{2} - \frac{C_x^2}{8(1-f)}\right\}\right] \tag{3.6}$$

$$MSE(\tau_{Pe}) = \overline{Y}^2\left[(\kappa_2 - 1)^2 + \kappa_2^2\left\{E(e_0^2) + 2E(e_0e_1) - \frac{f}{4(1-f)}E(e_1^2)\right\} - 2\kappa_2\left\{\frac{E(e_0e_1)}{2} - \frac{E(e_1^2)}{8(1-f)}\right\}\right]$$

which is minimum for

$$\kappa_2 = \frac{\left[1 + \dfrac{E(e_0e_1)}{2} - \dfrac{E(e_1^2)}{8(1-f)}\right]}{\left[1 + E(e_0^2) + 2E(e_0e_1) - \dfrac{f}{4(1-f)}E(e_1^2)\right]} = \frac{A_2}{B_2}$$

where

$$A_2 = \left[1 + \frac{E(e_0e_1)}{2} - \frac{E(e_1^2)}{8(1-f)}\right]$$

$$B_2 = \left[1 + E(e_0^2) + 2E(e_0e_1) - \frac{f}{4(1-f)}E(e_1^2)\right]$$

And the minimum mean square error of τ_{Pe} is,

$$MSE_{\min}(\tau_{Pe}) = \bar{Y}^2\left[1-\frac{A_2^2}{B_2}\right] \tag{3.7}$$

4. Conditions of Optimal Search and Efficiency Comparison

Using (2.5) and (3.5), we fairly get

$$MSE(\bar{y}_R) - MSE(\tau_{\text{Re}}) = \bar{Y}^2\left[\theta\left\{C_y^2 + C_x^2(1-2C)\right\} - \left\{1-\frac{A_1^2}{B_1}\right\}\right] > 0 \tag{4.1}$$

In view of (2.6) and (3.5), we have

$$MSE(\bar{y}_P\ or\ t_p) - MSE(\tau_{\text{Re}}) = \bar{Y}^2\left[\theta\left\{C_y^2 + C_x^2(1+2C)\right\} - \left\{1-\frac{A_1^2}{B_1}\right\}\right] > 0 \tag{4.2}$$

Further using (2.9) and (3.5), we find

$$MSE(t_{\text{Re}}) - MSE(\tau_{\text{Re}}) = \bar{Y}^2\left[\theta\left\{C_y^2 + \frac{C_x^2}{4}(1-4C)\right\} - \left\{1-\frac{A_1^2}{B_1}\right\}\right] > 0 \tag{4.3}$$

Finally, in the light of (2.10) and (3.5), we happen to obtain

$$MSE(t_{Pe}) - MSE(\tau_{\text{Re}}) = \bar{Y}^2\left[\theta\left\{C_y^2 + \frac{C_x^2}{4}(1+4C)\right\} - \left\{1-\frac{A_1^2}{B_1}\right\}\right] > 0 - \left\{1-\frac{A_1^2}{B_1}\right\}\Bigg] > 0 \tag{4.4}$$

Table 1. The data description

Population	N	n	C_y	C_x	ρ	C
I. Steel and Torrie (1960) y: Log of leaf burn in sec. x: Chlorine percentages	30	6	0.7493	0.7000	0.4996	0.4667
II. Das (1988) y: The number of agricultural laborers for 1961 x: The number of agricultural laborers for 1971	278	60	1.6198	1.4451	0.7213	0.6435
III. Cochran (1977) y: The number of persons per block x: The number of rooms per block	20	8	0.1281	0.1445	0.6500	0.7332

Table 2. The PREs of different estimators with respect to $\bar{y}$

Population	PRE($\bar{y}_R$, $\bar{y}$)	PRE($\bar{y}_P$ or t_p, $\bar{y}$)	PRE(t_{Re}, $\bar{y}$)	PRE(t_{Pe}, $\bar{y}$)	PRE(τ_{Pe}, $\bar{y}$)	PRE(τ_{Re}, $\bar{y}$)
I	92.9156	31.1004	133.0374	54.9076	122.422	304.651
II	156.3967	25.8171	197.7846	47.1121	384.722	688.195
III	157.8695	34.0327	161.2267	56.4111	60.396	209.135

PRE used in the above table is computed by the following formula.

PRE (., $\bar{y}$) = MSE($\bar{y}$)x100/MSE(.)

Above conditions are conditions of under which the proposed estimator performs better than the above mentioned estimators of population mean.

Remark: Similar conditions are also drawn for the estimator τ_{Pe} to be more efficient than the above mentioned estimators.

5. Empirical Study

An empirical study has been carried out numerically to show the usefulness of suggested methodology in this paper. To verify the theoretical findings of the proposed estimator τ over the estimators $\overline{y}$, $\overline{y}_R$, $\overline{y}_P$, t_p, t_{Re} and t_{Pe} of population mean in predictive estimation, we have taken the following three populations in table 1.

The following table2 represents the percentage relative efficiency (PRE) of different mentioned estimators with respect to mean per unit estimator $\overline{y}$ of population mean $\overline{Y}$ in predictive estimation approach.

6. Results and Conclusions

In the paper, we have succeeded in developing the improved efficient estimators for positively and negatively correlated data for estimating population mean in predictive estimation approach. Upon drawing the observations from the theoretical discussion of section-4 and the results in table-2, it is evident that the proposed estimators τ_{Re} and τ_{Pe} are better than the Srivastava (1983) estimators $\overline{y}_R$, $\overline{y}_P$, t_p and the Singh et.al (2014) estimators t_{Re}, t_{Pe} as they have lesser mean square error than all these estimators. Therefore the proposed estimators τ_{Re} and τ_{Pe} should be preferred for the estimation of population mean in predictive estimation approach for positively and negatively correlated data respectively.

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