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Transforming and Solving Multi-objective Quadratic Fractional Programming Problems by Optimal Average of Maximin & Minimax Techniques

Nejmaddin A. Suleiman, Maher A. Nawkhass*

Department of Mathematics, College of Education, University of Salahaddin-Erbil-Iraq

Abstract This paper uses Optimal Average maximin and minimax value techniques to transform multi-objective quadratic fractional programming problems (MOQFPP) to single QFPP. An algorithm is proposed for solvingsuch problems. We also solved the problem by Chandra Sen. technique, mean & median techniques. The resultsfrom the latter techniquesare obtained and compared to the result of maximin and minimax technique. This workhas been tested through severalnumerical examples; only two of them are presented in this work. The numerical results in this paper indicate that our technique is promising.

Keywords MOQFPP, Chandra Sen, Mean, Median, Optimal Average of Maximin & Minimax Techniques

1. Introduction

A quadratic fractional form is a mathematical expression of the type $\frac{c^T x+\frac{1}{2}x^T G_1 x}{d^T x+\frac{1}{2}x^T G_2 x}$ where G_1 and G_2 are $(n \times n)$ matrix of coefficients and they are symmetric matrixes. All vectors are assumed to be column vectors unless transposed (T), x is an n-dimensional vector of decision variables, c, d are the n-dimensional vector of constants[10]. The term Programming refers to the processof determining a particular program or plan of action. So Quadratic Fractional Programming (QFP) is one of themost important optimization (maximization / minimization) techniques developed in the field ofOperations Research (OR).

A new modified simplex method[9] is used to solve special case of a quadratic fractional programming, a solution can be obtained by a set of simultaneous equations. However a unique solution for a set of simultaneous equations in n-variables $(x_1, x_2 \ldots x_n)$, at least one of them is non-zero, can be obtained if there are exactly n relations. When the number of relations is greater than or less than n, a unique solution cannot be exist but a number of trial solutions can be found.

In various practical situations, the problems are seen in which the number of relations is notequal to the number of variables and many of the relations are in the form ofinequalities $(\leq\ or\ \geq)$ to maximize or minimize a quadratic fractional function of the variables subject to suchconditions. Such problems are known as Quadratic Fractional Programming Problem (QFPP). Quadratic Fractional programming problemhave attracted considerable research and interest, since they are useful in production planning, financial and corporative planning, health care and hospital planning

This work develops a new model of (MOQFPP) and suggested an algorithm to solve it, byusinga special case of objective function for (QFPP)as a form $\frac{({c_1}^T x+\alpha)({c_2}^T x+\beta)}{(C^T x+\gamma)}$ where x, c_1, c_2, C are (n × 1) column vectors, andα, β, γ are scalars and prime (T) denoted the transpose of the vector.A new technique optimal average of maximin & minimax is suggestedto solve MOQFPP withseveral method (Chandra Sen, mean, median) to compare between the results.

In (1983), Chandra Sen[6] defined the multi-objective linear programming problem, and suggested an approach to construct multi-objective function under the limitation that the optimum values of individual problems are greater than zero.This technique use to solve MOLPP[7] and MOFPP[5] and apply the same technique to solve MOQFPP.

Mean, median methods is used to transforms multi - objective to one combined objective function and solve it by previous ways, In (1997) a reference direction approach to multiple objective quadratic-linear programming, that studied by GuangYuan Yu and Pekka Korhonen[2], proposes an interactive procedure for solving multiple criteria problems with one quadratic objective, several linear objectives, and a set of linear constraints. The procedure is based on the use of reference directions and weighted sums.[4] Sulaiman and Sadiq (2006) studied the multi - objective function by solving the multi - objective

* Corresponding author:
maher.nawkhass@gmail.com (Maher A. Nawkhass)
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programming problem, using mean and median value.(2006), Salwa and Emad Abass[8] studied and Construct mathematical models for solving multi-objective linear programming (MOLP) problem. Huey-Kuo Chen and Huey - Wen Chou[3] (1996) proposed a fuzzy approach, which induces some methodologies as special cases, for solving the multiple objective linear programming problem. Sulaiman & Abulrahim (2013) studied the transformation technique to solve multi-objective linear fractional programming problem [5]. (2008) Sulaiman and Hamadameen studied optimal transformation technique to solve multi-objective linear programming problem (MOLPP)[7].

In this article, we aim to solve a multi objective quadratic fractional programming problem by optimal average of maximin & minimax (OAxn) which is reported in section (5.3) to minimizes cost and maximizes profit, Irrespective of the number objectives with less computational burden, Computer applications for the algorithm will been discussed by solving numerical examples.

2. Quadratic Programming

If the optimization problem assumes the form

$$max.z \text{ (or min. z)} = \alpha + C^T x + x^T G x$$

subject to:

$$\mathrm{Ax}\begin{pmatrix} \leq \\ \geq \\ = \end{pmatrix} \mathrm{b}$$

$x \geq 0$

Where $A = (a_{ij})_{m\times n}$, matrix of coefficients, $\forall i = 1,2,\dots,m$ and $= 1,2,\dots,n$,

$b = (b_1,b_2,\dots,b_m)^T$, $x = (x_1,x_2,\dots,x_n)^T$, $C^T = (c_1,c_2,\dots,c_n)^T$,

and $G = (g_{ij})_{n\times n}$ is a positive definite or positive semi-definite symmetric square matrix, moreover the constraints are linear and the objective function is quadratic. Such optimization problem is said to be a quadratic programming (QP) problem.[1]

3. Mathematical Form of QFPP

The mathematical form of this type of problems is given as follows:

$$max.z = \frac{(c^T x + \delta + \frac{1}{2} x^T G x)}{(C^T x + \gamma)}$$

Subject to:

$$Ax\begin{bmatrix} \geq \\ \leq \\ = \end{bmatrix} b$$

$$x \geq o$$

Where G is $(n\times n)$ matrix of coefficients with G is symmetric matrixes. All vectors are assumed to be column vectors unless transposed(T). Where$x$ is an n-dimensional vector of decision variables, c, C is the n-dimensional vector of constants, b is n -dimensional vector of constants.γ, δ are scalars.

In this work the problem that has objective function is tried to be solved has thefollowing form:

$$max.z = \frac{({c_1}^T x + \alpha)({c_2}^T x + \beta)}{(C^T x + \gamma)}$$

Subject to:

$$Ax\begin{bmatrix} \geq \\ \leq \\ = \end{bmatrix} b$$

$$x \geq o$$

A is m × n matrix ,all vectors are assumed to be column vectors unless transposed (T). where x is an n-dimensional vector of decision variables, c_1, c_2, C are the n-dimensional vector of constants, b is n-dimensional vector of constants, α, β, γ are scalars.[9]

3.1. Solving QFPP by the Following Method

A new Modified Simplex Method is used to solve the numerical exampleto apply simplex process, first we find $\Delta \mathrm{k}_1^i$, $\Delta \mathrm{k}_2$ from the coefficient of numerator and denominator of objective function respectively, by using the following formula:

$$\Delta \mathrm{k}_1^{ij} = c_{Bi} x_j - c_{ij}, \qquad \mathrm{i} = 1,2, \qquad \mathrm{j} = 1,2,\dots,\mathrm{m}+\mathrm{n}$$

$$\Delta \mathrm{k}_2^{1j} = C_B x_j - C_{1j}\ , \qquad j = 1,2,\dots,m+n$$

$$z_1^1 = c_{B1} x_B + \alpha, z_1^2 = c_{B2} x_B + \beta, z_2 = C_B x_B + \gamma$$

$$Z_1 = z_1^1 z_1^2,\ Z_2 = z_2,$$

$$\Delta\xi_{1j} = z_1^2 \Delta \mathrm{k}_1^{1j} + z_1^1 \Delta \mathrm{k}_1^{2j}$$

$$\Delta\xi_{2j} = \Delta \mathrm{k}_2^{1j}$$

$$Z = Z_1 / Z_2$$

In this approach we define the formula to find Δ_j from $Z_1, Z_2, \Delta\xi_{1j}$ and $\Delta\xi_{2j}$ as follows:

$\Delta_j = Z_2 \Delta\xi_{1j} - Z_1 \Delta\xi_{2j}$. For more detail see[9] p. 3756.

3.2. Algorithm for Solving QFPP by New Modified Simplex Method

An algorithm to solve QFPP by Modified Simplex Method is presented in reference[9] p.3756 and p. 3757.

4. Multi-Objective Quadratic Fractional Programming Problem

Multi-Objective functions are the ratio of two objective functions thathave quadratic objective function in numerator and linear objective function in denominator, this is said to be MOQFPP then can be defined:

$$\left.\begin{array}{l} Max. Z_1 = \frac{(c_{11}{}^T x+\alpha_1)(c_{21}{}^T x+\beta_1)}{(C_1{}^T x+\gamma_1)} \\ Max. Z_2 = \frac{(c_{12}{}^T x+\alpha_2)(c_{22}{}^T x+\beta_2)}{(C_2{}^T x+\gamma_2)} \\ \vdots \\ Max. Z_r = \frac{(c_{1r}{}^T x+\alpha_r)(c_{2r}{}^T x+\beta_r)}{(C_r{}^T x+\gamma_r)} \\ Min. Z_{r+1} = \frac{(c_{1r+1}{}^T x+\alpha_{r+1})(c_{2r+1}{}^T x+\beta_{r+1})}{(C_{r+1}{}^T x+\gamma_{r+1})} \\ \vdots \\ Min. Z_s = \frac{(c_{1s}{}^T x+\alpha_s)(c_{2s}{}^T x+\beta_s)}{(C_s{}^T x+\gamma_s)} \end{array}\right] \quad (4.1)$$

Subject to:

$$Ax = b \quad (4.2)$$

$$x \geq 0 \quad (4.3)$$

Where b is $m-$dimensional vector of constants, x is $n-$dimensional vector of decision variables, A is a $m \times n$ matrix of constants, r is number of objective functions to be maximized, s is the number of objective functions to be maximized and minimized and $(s-r)$ is the number of objective functions that is minimized. A is m × n matrix, all vectors are assumed to be column vectors unless transposed (T). c_{1i}, c_{2i}, C_i where i=1,…,s are the n-dimensional vector of constants, $\alpha_i, \beta_i, \gamma_i$ where i =1,…,s are scalars.

5. Solving MOQFPP by Using the Following Technique

5.1. Chandra Sen. Technique

The same approach which was taken by Sen. (1983)[6] is followed here to formulate the constraint objective function for the MOQFPP. Suppose we obtain a single value corresponding to each of the objective functions of the MOQFPP of equation (4.1).They are being optimized individually subject to the constraints (4.2) and (4.3) as follows:

$$\left.\begin{array}{l} Max. Z_1 = \varphi_1 \\ Max. Z_2 = \varphi_2 \\ \vdots \\ Max. Z_r = \varphi_r \\ Min. Z_{r+1} = \varphi_{r+1} \\ \vdots \\ Min. Z_s = \varphi_s \end{array}\right] \quad (5.1.1)$$

Where $\varphi_1, \varphi_2, \ldots, \varphi_s$ are values of the objective functions, which each objective function in 4.1 solved by section 3.3, the level of the decision variable may not necessarily be the same for all optimal solutions in presence of conflicts among objectives. But we require the common set of decision variables to be the best compromising optimal solution that we can determine for the common set of the decision variables from the following combined objective function, which formulate the MOQFPP given in following equation.

$$Max. Z = \sum_{i=1}^{r} \frac{Z_i}{|\varphi_i|} - \sum_{i=r+1}^{s} \frac{Z_i}{|\varphi_i|} \quad (5.1.2)$$

Where $\varphi_i \neq 0, i = 1,2,\ldots,s$. Subject to constraints (4.2) and (4.3), the optimum value of the objective functions $\varphi_i, i = 1,2,\ldots,s$ may be positive or negative. And Z_i when i=1,…,r represents the max.z objective functions and when i= r+1 ,…,s min.z objective function in (4.1). Finally solve equation (5.1.2) with same constraint (4.2) and (4.3) by section 3.2.

5.2. Mean and Median Modified Approach

We formulate the combined objective function as follows to determine the common set of decision variables,to solving the MOQFPP by modified approach (using mean and median value),[7],[5].

$$Max. Z = \sum_{i=1}^{r} \frac{Z_i}{mean\ (MV_i)} - \sum_{i=r+1}^{s} \frac{Z_i}{mean\ (MU_i)} \quad (5.2.1)$$

Subject to the same constraints (4.2), (4.3);
Where $MV_i = |\varphi_i|$, for all $i = 1, 2, \ldots, r$;
$MU_i = |\varphi_i|$, for all $i = r+1, r+2, \ldots, s$

$$Max. Z = \sum_{i=1}^{r} \frac{Z_i}{mean\ (MV_i)} - \sum_{i=r+1}^{s} \frac{Z_i}{mean\ (MU_i)} \quad (5.2.2)$$

Subject to the same constraints (4.2), (4.3);
Where $MV_i = |\varphi_i|$, for all $i = 1, 2, \ldots, r$.
$MU_i = |\varphi_i|$, for all $i = r+1, r+2, \ldots, s$.

5.3. Optimal Average of Maximin & Minimax (OAxn) Techniques

We will define some definitions related with the Optimal Average (OAxn) techniques and introduce an algorithm for it.

Definition1: let y_1=min { MV_i }, where $MV_i = |\varphi_i|$, and φ_i is the maximum value of Z_i, for all $i = 1,2, \ldots r$.

Definition 2: let y_2=max { MU_i }, where $MU_i = |\varphi_i|$, and φ_i is the minimum value of Z_i, for all $i = r+1, r+2, \ldots, s$.

Definition 3: We denote the optimal average of maximin &minimax (OAxn), as follows:

OAxn = $(|y_1| + |y_2|)/2$, where y_j defined by definition(j), for all j=1,2 respectively

5.3.1. The Algorithm

The following algorithm is to obtain the optimal average of maximin & minimax for the multiobjective quadratic fractional programming problem and can be summarized as follows:-

Step1: Write the standard form of the problem, by introducing slack and artificial variables to constraints, and write starting Simplex table.

Step2: Calculate the Δ_j by the following formula $\Delta_j = Z2\Delta\xi1j - Z1\Delta\xi2j$, then write it in the starting Simplex table.

Step3: Find the solution of first objective problem by using Simplex process.

Step4: Check the solution for feasibility in step3, if it is feasible then go to step5, otherwise use dual Simplex method to remove infeasibility.

Step5: Check the solution for optimality if all $\Delta_j \geq 0$, then go to step6, otherwise back to step3.

Step6: Assign a name to the optimum value of the maximum objective function Z_i say φV_i where ∀i= i=1,2,…,r. and assign a name to the optimum value of the minimum objective function Z_i say φU_i where ∀i= $r+1, r+2, \ldots, s$.

Step7: Repeat process from the step 3; for i =2,…, s to be include all the objective problem.

Step8: Select $y_1 = min\ \{\ \varphi V_i\ \},\ \forall\ i = 1,2,\ldots,r, y_2 = max\ \{\ \varphi U_i\ \},\ \forall\ i = r+1, r+2, \ldots, s$ then calculate, OAxn= $(|y_1| + |y_2|)/2$,

Step9: Optimize the combined objective function order the same constrains

(4.2),(4.3) as: $Max.Z = (\sum_{i=1}^{r} Z_i - \sum_{i=r+1}^{s} Z_i)/$ OAxn, (5.3.1.1)

5.3.2. Program Notation

The following notations, which are used in computer program, aredefined as follows:

φV_i = The value of objective function which is to be maximized.

φU_i = The value of objective function which is to be minimized.

$MV_i = |\varphi V_i|$; ∀i= i=1,2,…,r.

$MU_i = |\varphi U_i|$; ∀i=$r+1, r+2, \ldots, s$.

SM=$\sum_{i=1}^{r} Z_i$.

SN=$\sum_{i=r+1}^{s} Z_i$.

$Max.Z = (\text{SM} - SN)/\text{OAxn}$.

6. Numerical Examples

We solved several numerical examples, but only two of them are presented in this work.

Example 1:

$$Max.Z_1 = \frac{(2x_1+x_2+1)(2x_1+x_2+2)}{(3x_1+3x_2+3)}$$

$$Max.Z_2 = \frac{(6x_1+3x_2+3)(4x_1+2x_2+4)}{(2x_1+2x_2+2)}$$

$$Max.Z_3 = \frac{(8x_1+4x_2+4)(6x_1+3x_2+6)}{(5x_1+5x_2+5)}$$

$$Min.Z_4 = \frac{(10x_1+5x_2+5)(-8x_1-4x_2-8)}{(7x_1+7x_2+7)}$$

$$Min.Z_5 = \frac{(-4x_1-2x_2-2)(6x_1+3x_2+6)}{(6x_1+6x_2+6)}$$

$$Min.Z_6 = \frac{(-2x_1-x_2-1)(4x_1+2x_2+4)}{(9x_1+9x_2+9)}$$

Subject to:

$$x_1 + 2x_2 \leq 4$$
$$3x_1 + x_2 \leq 6$$
$$x_1, x_2 \geq 0$$

Solution: After finding the value of each of individual objective functions by an algorithm of section (3.2) the results are as below in table 1:

Table 1. Results of example (1)

i	Z_i	x_i	φ_i	$MV_i = \lvert\varphi V_i\rvert \forall i = 1,2,\ldots,r.$	$MU_i = \lvert\varphi U_i\rvert\ \forall i = r+1, r+1, \ldots, s.$
1	3.33	(2,0)	3.33	3.33	
2	30	(2,0)	30	30	
3	24	(2,0)	24	24	
4	-28.571	(2,0)	-28.571		28.571
5	-10	(2,0)	-10		10
6	-2.22	(2,0)	-2.22		2.22

(5.3.1.1) in section (5.3.1) is utilized insolving example (1) to find MOQFPP by usingoptimalaverage of maximin &minimax techniques:

$$\text{OAxn} = (|y_1| + |y_2|)/2 = \frac{|3.33| + |-2.22|}{2} = 2.775$$

And SM $= \sum_{i=1}^{r} Z_i = \frac{(16x_1+8x_2+8)(43x_1+21.5x_2+43)}{(30x_1+30x_2+30)}$ and

$$SN = \sum_{i=r+1}^{s} Z_i = \frac{(-80x_1-40x_2-40)(77.1x_1+38.55x_2+77.1)}{(378x_1+378x_2+378)}$$

Then $Max.Z = (\text{SM} - SN)/\text{OAxn}$,

$$Max.Z = \left(\frac{(16x_1+8x_2+8)(43x_1+21.5x_2+43)}{(30x_1+30x_2+30)} - \frac{(-80x_1-40x_2-40)(77.1x_1+38.55x_2+77.1)}{(378x_1+378x_2+378)}\right)/2.775$$

Then write the objective functions as following.

$$Max.Z = \frac{(24x_1+12x_2+12)(73.666x_1+36.833x_2+73.666)}{(125x_1+125x_1+125)} \quad (A1)$$

Subject to given constraint

$$x_1 + 2x_2 \le 4$$
$$3x_1 + x_2 \le 6$$
$$x_1, x_2 \ge 0$$

Now (A1) issolved by algorithm (3.2),then the optimal solution is found as follow

$$Max.Z = 35.359, x_1 = 2,\ x_2 = 0.$$

When (5.1.2) in section (5.1) is used tosolve example (1), to find MOQFPP by Chandra Sen. technique:
We have $Max.Z = \sum_{i=1}^{r} \frac{Z_i}{|\varphi_i|} - \sum_{i=r+1}^{s} \frac{Z_i}{|\varphi_i|}$,the combined objective quadratic fractional function is

$$Max.Z = \frac{(4x_1+2x_2+2)(6x_1+3x_2+6)}{(5x_1+5x_2+5)} - \frac{(-6x_1-3x_2-3)(2x_1+x_2+2)}{(10x_1+10x_2+10)}$$

Then

$$Max.Z = \frac{(10x_1+5x_2+5)(6x_1+3x_2+6)}{(10x_1+10x_2+10)} \quad (B1)$$

Therefore. After solving (B1)by given a subjectwiththe same constraints as before, we find

$$Max.Z = 15, x_1 = 2,\ x_2 = 0.$$

When (5.2.1) and (5.2.2) in section (5.2) are used to solve example (1) to find MOQFPP by using mean and median respectively:

First when we solve the example by using mean modified approach (5.2.1)

Then the combined objective quadratic fractional function is

$$Max.Z = \frac{(6x_1+3x_2+3)(2x_1+x_2+2)}{(x_1+x_2+1)} - \frac{(-6x_1-3x_2-3)(2x_1+x_2+2)}{(10x_1+10x_2+10)}$$

Then $Max.Z = \frac{(8x_1+4x_2+4)(16x_1+8x_2+16)}{(10x_1+10x_2+10)}$

Therefore. After solving $Max.Z$by given a subjectwith the same constraints as before, we find

$$Max.Z = 32, x_1 = 2,\ x_2 = 0.$$

Second when we solve the example by using medianmodified approach (5.2.2)

Then the combined objective quadratic fractional function is

$$Max.Z = \frac{(0.46x_1+0.23x_2+0.23)(2x_1+x_2+2)}{(x_1+x_2+1)} - \frac{(-50x_1-25x_2-25)(32.56x_1+16.28x_2+32.56)}{(1000x_1+1000x_2+1000)}$$

Then $Max.Z = \frac{(60x_1+30x_2+30)(42.46x_1+21.23x_2+42.46)}{(1000x_1+1000x_2+1000)}$

Therefore. After solving $Max.Z$by given a subjectwith the same constraints as before, we find
$Max.Z$= 6.369,$x_1 = 2,\ x_2 = 0$ when using median modified approach

Example 2:

$$Max.Z_1 = \frac{(3x_1+3x_2+2)(2x_1+3x_2+3)}{(5x_1+5x_2+5)}$$
$$Max.Z_2 = \frac{(6x_1+6x_2+4)(6x_1+9x_2+9)}{(6x_1+6x_2+6)}$$
$$Max.Z_3 = \frac{(12x_1+12x_2+8)(4x_1+6x_2+6)}{(8x_1+8x_2+8)}$$
$$Max.Z_4 = \frac{(9x_1+9x_2+6)(8x_1+12x_2+12)}{(7x_1+7x_2+7)}$$
$$Min.Z_5 = \frac{(15x_1+15x_2+10)(-12x_1-18x_2-18)}{(3x_1+3x_2+3)}$$
$$Min.Z_6 = \frac{(-21x_1-21x_2-28)(10x_1+15x_2+15)}{(4x_1+4x_2+4)}$$
$$Min.Z_7 = \frac{(-18x_1-18x_2-12)(14x_1+21x_2+21)}{(6x_1+6x_2+6)}$$

Subject to:

$$x_1 + 4x_2 \le 4$$
$$2x_1 + x_2 \le 2$$
$$x_1, x_2 \ge 0$$

Solution: After finding the value of each of individual objective functions by an algorithm of section (3.2) the results as below in table 2:

Table 2. Results of example (2)

i	Z_i	x_i	φ_i	$MV_i = \lvert\varphi V_i\rvert \forall i = 1,2,\ldots,r.$	$MU_i = \lvert\varphi U_i\rvert$ $\forall i = r+1, r+2, \ldots, s.$
1	3.466	$(4/7, 6/7)$	3.466	3.466	
2	17.377	$(4/7, 6/7)$	17.377	17.377	
3	17.378	$(4/7, 6/7)$	17.378	17.378	
4	29.79	$(4/7, 6/7)$	29.79	29.79	
5	-173.792	$(4/7, 6/7)$	-173.792		173.792
6	-200.444	$(4/7, 6/7)$	-200.444		200.444
7	-121	$(4/7, 6/7)$	-121		121

(5.3.1.1) in section (5.3.1) is used tosolve the example (2) to find MOQFPP by usingoptimalaverage of maximin &minimax techniques:

$$Max.Z = \left(\frac{(24x_1+24x_2+16)(34.25x_1+51.375x_2+51.375)}{(35x_1+35x_2+35)} - \frac{(-15x_1-15x_2-10)(10.3x_1+15.45x_2+15.45)}{(x_1+x_2+1)}\right)/62.557$$

Then write the objective functions as following.

$$Max.Z = \frac{(36x_1+36x_2+24)(39.5x_1+39.5x_2+59.25)}{(500x_1+500x_2+500)} \quad \text{(B1)}$$

Subject to

$$x_1 + 4x_2 \le 4$$
$$2x_1 + x_2 \le 2$$
$$x_1, x_2 \ge 0$$

Now solving (B1) by algorithm (3.2) we get the optimal solution as follows.

$$Max.Z = 8.237, x_1 = \frac{4}{7},\ x_2 = \frac{6}{7}.$$

When (5.1.2) in section (5.1)is used to solve example (2),to find MOQFPP by usingChandra Sen. Technique:

$$Max.Z = \frac{(12x_1+12x_2+8)(11.5x_1+17.25x_2+17.25)}{(100x_1+100x_2+100)} - \frac{(-27x_1-27x_2-18)(17.554x_1+26.331x_2+26.331)}{(500x_1+500x_2+500)}$$

Then we have. $Max.Z = \frac{(45x_1+45x_2+30)(25.866x_1+38.799x_2+38.977)}{(500x_1+500x_2+500)}$

Therefore. After solving $Max.Z$by given a subjectwith the same constraints as before, we find

$$Max.Z = 6.742, x_1 = \frac{4}{7},\ x_2 = \frac{6}{7}.$$

When (5.2.1) and (5.2.2) in section (5.2) are used to solve example (2) to find MOQFPP by using mean and median respectively :

First when we solve the example by using mean modified approach (5.2.1)

Then the combined objective quadratic fractional function is

$$Max.Z = \frac{(12x_1+12x_2+8)(11.5x_1+17.25x_2+17.25)}{(100x_1+100x_2+100)} - \frac{(-12x_1-12x_2-8)(12.874x_1+19.311x_2+19.311)}{(165.295x_1+165.295x_2+165.295)}$$

Then $Max.Z = \frac{(24x_1+24x_2+16)(19.25x_1+28.875x_2+28.875)}{(200x_1+200x_2+200)}$

Therefore. After solving $Max.Z$by given a subjectwith the same constraints as before, we find

$$Max.Z = 6.6905, x_1 = \frac{4}{7},\ x_2 = \frac{6}{7}.$$

Second when we solve the example by using median modified approach (5.2.2)

Then the combined objective quadratic fractional function is

$$Max.Z = \frac{(9x_1+9x_2+6)(6x_1+9x_2+9)}{(40x_1+40x_2+40)} - \frac{(-12x_1-12x_2-8)(12.874x_1+19.311x_2+19.311)}{(173.792x_1+173.792x_2+173.792)}$$

Then $Max.Z = \frac{(24x_1+24x_2+16)(11.75x_1+17.625x_2+17.625)}{(100x_1+100x_2+100)}$

Therefore. After solving $Max.Z$by given a subjectwith the same constraints as before, we find

$$Max.Z = 8.1677, x_1 = \frac{4}{7},\ x_2 = \frac{6}{7}$$

7. Comparison of the Numerical Results

Now, we are going to comparethe numerical results which are obtained of the examples above. The comparison is presented in in table 3 below

The table 3 shows the results whichsolved by optimal Average approach was better than theResults which solved by other approaches.

Table 3. Comparison between results of the numerical techniques

Examples	Example 1	Example 2
Chandra Sen. Technique	15	6.742
Mean Technique	32	6.6905
Median Technique	6.369	8.1677
Optimal Average of Maximin & Minimax Techniques	35.359	8.237

8. Conclusions

In This work, we have defined and discussed a number of techniques,the comparisons of these methods are based on the value of the objective function. After solving the numerical examples, we found that $max.z$ which obtained by our technique is better than other techniques (Chandra Sen.,Mean&Median).

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