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Evaluation Some Reliability Characteristics of a System under Three Types of Failures with Repair-Replacement at Failure

Bashir Yusuf[1], Ibrahim Yusuf[2,*]

[1]Department of Mathematics, Federal University Dutse, Nigeria
[2]Department of Mathematical Sciences, Bayero University, Kano, Nigeria

Abstract In this paper, we studied a series system consisting of single unit. The system is subjected to three types of failures. Type I failure is minor in which the system is imperfectly repaired. Type II failure is major in which the entire system is replaced. Type III failure is called a partial failure in which the system works in reduced capacity and is perfectly repaired. Failure and repair time are assumed exponential. We developed the explicit expressions for mean time to system failure (MTSF), steady-state availability, busy period and profit function using Kolmogorov forward equations method. Special cases are studied to determine the impact of various system parameters on MTSF, busy period, steady-state availability and profit function.

Keywords Availability, Profit, Repair, Failure

1. Introduction

Reliability is a vital factor in ensuring system effectiveness, productivity, product quality and generated profit. Efforts have been made to optimize the system reliability which tremendous impact on profit generated. Studies on availability and profit generated from the system using are becoming more and richer day by day due to the fact that numbers of researchers in the field of system reliability are making huge contributions. Availability and profit are good evaluations of a system's performance. Expected profit is an important factor in economic evaluation of repairable systems. Expected profits have been obtained for different systems. Hajeeh[1] deals with availability of a system with different repair options. Long term performance of the system is investigated. Three special cases are derived to see the impact of imperfect repair, minimal repair or replacement at failure on system availability. Yusuf *et al*[2] obtained the availability and cost of a deterioration system. El-Damcese[3] analyzed the reliability availability of warm standby system with time varying failure and repair rates. Yusuf and Hussaini[4] obtained the reliability and availability characteristics of 2-out-of-3 system under a perfect repair condition. Expressions for steady-state availability and profit have been obtained. Bhardwaj and Chander[8], analyzed reliability models for 2-out-of-3 redundant system subject to conditional arrival time of the server. Chander and Bhardwaj [9] present reliability and economic analysis of 2-out-of-3 redundant system with priority to repair and Bhardwaj and Malik[10] studied MTSF and cost effectiveness of 2-out-of-3 cold standby system with probability of repair and inspection. Wang and kuo[11] studied the cost and probabilistic analysis of series system with mixed standby components while Wang *et al*[12] studied cost benefit analysis of series systems with warm standby components involving general repair time where the server is not subject to breakdowns. The failure time and repair time are assumed to have exponential distribution. Measures of system effectiveness such MTSF, steady-state availability, busy period and profit function are obtained. Yusuf and Bala[14] studied the evaluation of MTSF of 2-out-of-4 warm standby system attended by repair machines and repair men. Yusuf and Hussaini[15] studied the modeling of a redundant system with big and three small dissimilar units where various measures of system effectiveness have been obtained.

1.1. Objectives

In the present paper, we considered a series system and derived its corresponding mathematical models. Furthermore, we study reliability characteristics of system availability and profit generated model involving three types of failures using Kolmogorov forward equations method. The contribution of this paper is twofold. The first is to obtain explicit expression for availability, busy period and

* Corresponding author:
ibrahimyusif@yahoo.com (Ibrahim Yusuf)
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profit for the two configurations. The second is to capture the effect of both failure and repair rates on the measures of system effectiveness like availability and profit based on assumed numerical values given to the system parameters.

2. Methodology

2.1. Description of the System and Notations

2.1.1. Description of the System

In this paper, we consider a single series system exposes to three types of failures. Type I failure β_1 occurred to system when new, at this junction an imperfect repair α_1 is done to the system. From there the system continue to operate in a reduced capacity until type II a non repairable occurred. The entire system is replaced (α_2) at the occurrence of type II failure β_2. The system can also transit to partial type III failure state with rate β_3 and the system is perfectly repaired (α_3).

2.1.2. Notations

S_0 : The System's initial (new) state

S_1 : The System is under imperfect repair

S_2 : The System is working in a reduced capacity

S_3 : The System is under non repairable failure

S_4 : The System is under partial failure

α_1 : Imperfect repair rate

α_2 : Replacement rate

α_3 : Perfect repair rate

β_1 : Type I failure rate

β_2 : Type II failure rate

β_3 : Type III failure rate

2.2. Models Formulation

Let $P(t)$ be the probability row vector at time t, then the initial conditions for this problem are as follows:

$$P(0) = \left[P_0(0), P_1(0), P_2(0), P_3(0), P_4(0)\right] = \left[1, 0, 0, 0, 0\right]$$

we obtain the following system of differential equations:

$$\frac{dP_0(t)}{dt} = -(\beta_1 + \beta_2 + \beta_3)P_0(t) + \alpha_2 P_3(t) + \alpha_3 P_4(t)$$

$$\frac{dP_1(t)}{dt} = -\alpha_1 P_1(t) + \beta_1 P_2(t) + \beta_1 P_0(t)$$

$$\frac{dP_2(t)}{dt} = -(\beta_1 + \beta_2 + \beta_3)P_2(t) + \alpha_3 P_4(t) + \alpha_1 P_1(t)$$

$$\frac{dP_3(t)}{dt} = -\alpha_2 P_3(t) + \beta_2 P_0(t) + \beta_2 P_2(t) + \beta_2 P_4(t)$$

$$\frac{dP_4(t)}{dt} = -(2\alpha_3 + \beta_2)P_4(t) + \beta_3 P_0(t) + \beta_3 P_2(t) \quad (1)$$

The differential equations in (1) above is transformed into matrix as

$$P' = TP \quad (2)$$

where

$$T = \begin{bmatrix} -(\beta_1+\beta_2+\beta_3) & 0 & 0 & \alpha_2 & \alpha_3 \\ \beta_1 & -\alpha_1 & \beta_1 & 0 & 0 \\ 0 & \alpha_1 & -(\beta_1+\beta_2+\beta_3) & 0 & \alpha_3 \\ \beta_2 & 0 & \beta_2 & -\alpha_2 & \beta_2 \\ \beta_3 & 0 & \beta_3 & 0 & -(2\alpha_3+\beta_2) \end{bmatrix}$$

2.2.1. Availability and Busy Period Modeling

For the availability case of Fig. 1 using the initial condition in section 3 for this system, $P(0) = [P_0(0), P_1(0), P_2(0), P_3(0), P_4(0)] = [1, 0, 0, 0, 0]$

The system of differential equations in (1) for the system above can be expressed in matrix form as:

$$\begin{bmatrix} P_0'(t) \\ P_1'(t) \\ P_2'(t) \\ P_3'(t) \\ P_4'(t) \end{bmatrix} = \begin{bmatrix} -(\beta_1+\beta_2+\beta_3) & 0 & 0 & \alpha_2 & \alpha_3 \\ \beta_1 & -\alpha_1 & \beta_1 & 0 & 0 \\ 0 & \alpha_1 & -(\beta_1+\beta_2+\beta_3) & 0 & \alpha_3 \\ \beta_2 & 0 & \beta_2 & -\alpha_2 & \beta_2 \\ \beta_3 & 0 & \beta_3 & 0 & -(2\alpha_3+\beta_2) \end{bmatrix} \begin{bmatrix} P_0(t) \\ P_1(t) \\ P_2(t) \\ P_3(t) \\ P_4(t) \end{bmatrix}$$

Let V be the time to failure of the system. The steady-state availability is given by

$$A_V = P_0(\infty) + P_2(\infty) + +P_4(\infty) \tag{3}$$

In steady state, the derivatives of state probabilities become zero, thus (2) becomes

$$TP(\infty) = 0 \tag{4}$$

which in matrix form is

$$\begin{bmatrix} -(\beta_1+\beta_2+\beta_3) & 0 & 0 & \alpha_2 & \alpha_3 \\ \beta_1 & -\alpha_1 & \beta_1 & 0 & 0 \\ 0 & \alpha_1 & -(\beta_1+\beta_2+\beta_3) & 0 & \alpha_3 \\ \beta_2 & 0 & \beta_2 & -\alpha_2 & \beta_2 \\ \beta_3 & 0 & \beta_3 & 0 & -(2\alpha_3+\beta_2) \end{bmatrix} \begin{bmatrix} P_0(t) \\ P_1(t) \\ P_2(t) \\ P_3(t) \\ P_4(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

using the normalizing condition

$$P_0(\infty) + P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) = 1 \tag{5}$$

we substitute (5) in the last row of (4) following[5,6,7]. The resulting matrix is

$$\begin{bmatrix} -(\beta_1+\beta_2+\beta_3) & 0 & 0 & \alpha_2 & \alpha_3 \\ \beta_1 & -\alpha_1 & \beta_1 & 0 & 0 \\ 0 & \alpha_1 & -(\beta_1+\beta_2+\beta_3) & 0 & \alpha_3 \\ \beta_2 & 0 & \beta_2 & -\alpha_2 & \beta_2 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} P_0(t) \\ P_1(t) \\ P_2(t) \\ P_3(t) \\ P_4(t) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$A_V(\infty) = \frac{N_1}{D_1}$$

$$\begin{aligned} N_1 = {} & (\alpha_1\alpha_2(2\beta_1\beta_2 + 2\alpha_3\beta_1 + \alpha_3\beta_3 + \beta_2\beta_3 + \beta_2^2 + 2\alpha_3\beta_2) + \alpha_1\alpha_2(\alpha_3\beta_3 - \beta_1\beta_2))(2\alpha_1\beta_1\beta_2 - \alpha_2\beta_1\beta_2 \\ & +2\alpha_1\alpha_2\beta_1 - 2\alpha_2\alpha_3\beta_1 + \alpha_1\beta_2^2 + \alpha_1\alpha_2\beta_2 + \alpha_1\beta_2\beta_3 + 2\alpha_1\alpha_3\beta_2 + \alpha_1\alpha_2\beta_3 + 2\alpha_1\alpha_2\alpha_3) + 2\alpha_1\alpha_2(2\beta_1 + \beta_3) \\ & (2\alpha_1\beta_1^2\beta_2 - \alpha_2\beta_1^2\beta_2 + 2\alpha_1\alpha_2\beta_1^2 - 2\alpha_2\alpha_3\beta_1^2 + 3\alpha_1\beta_1\beta_2^2 - \alpha_2\beta_1\beta_2^2 - 2\alpha_2\alpha_3\beta_1\beta_2 + 3\alpha_1\alpha_2\beta_1\beta_2 + 3\alpha_1\beta_1\beta_2\beta_3 \\ & +2\alpha_1\alpha_3\beta_1\beta_2 - \alpha_2\beta_1\beta_2\beta_3 + 3\alpha_1\alpha_2\beta_1\beta_3 + 2\alpha_1\alpha_2\alpha_3\beta_1 - 2\alpha_2\alpha_3\beta_1\beta_3 + \alpha_1\beta_2^3 + 2\alpha_1\beta_2^2\beta_3 + 2\alpha_1\alpha_3\beta_2^2 + \alpha_1\alpha_2\beta_2^2 \\ & +2\alpha_1\alpha_2\beta_2\beta_3 + 2\alpha_1\alpha_3\beta_2\beta_3 + \alpha_1\beta_2\beta_3^2 + 2\alpha_1\alpha_2\alpha_3\beta_2 + 2\alpha_1\alpha_2\alpha_3\beta_3 + \alpha_1\alpha_2\beta_3^2) \end{aligned}$$

$$\begin{aligned} D_1 = {} & (2\alpha_1\beta_1\beta_2 - \alpha_2\beta_1\beta_2 + 2\alpha_1\alpha_2\beta_1 - 2\alpha_2\alpha_3\beta_1 + \alpha_1\beta_2^2 + \alpha_1\alpha_2\beta_2 + \alpha_1\beta_2\beta_3 + 2\alpha_1\alpha_3\beta_2 + \alpha_1\alpha_2\beta_3 + 2\alpha_1\alpha_2\alpha_3) \\ & (2\alpha_1\beta_1^2\beta_2 - \alpha_2\beta_1^2\beta_2 + 2\alpha_1\alpha_2\beta_1^2 - 2\alpha_2\alpha_3\beta_1^2 + 3\alpha_1\beta_1\beta_2^2 - \alpha_2\beta_1\beta_2^2 - 2\alpha_2\alpha_3\beta_1\beta_2 + 3\alpha_1\alpha_2\beta_1\beta_2 + 3\alpha_1\beta_1\beta_2\beta_3 \\ & +2\alpha_1\alpha_3\beta_1\beta_2 - \alpha_2\beta_1\beta_2\beta_3 + 3\alpha_1\alpha_2\beta_1\beta_3 + 2\alpha_1\alpha_2\alpha_3\beta_1 - 2\alpha_2\alpha_3\beta_1\beta_3 + \alpha_1\beta_2^3 + 2\alpha_1\beta_2^2\beta_3 + 2\alpha_1\alpha_3\beta_2^2 + \alpha_1\alpha_2\beta_2^2 \\ & +2\alpha_1\alpha_2\beta_2\beta_3 + 2\alpha_1\alpha_3\beta_2\beta_3 + \alpha_1\beta_2\beta_3^2 + 2\alpha_1\alpha_2\alpha_3\beta_2 + 2\alpha_1\alpha_2\alpha_3\beta_3 + \alpha_1\alpha_2\beta_3^2) \end{aligned}$$

The steady-state busy period is given by

$$B_V = P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) \tag{6}$$

$$B(\infty) = P_1(\infty) + P_2(\infty) + P_3(\infty) + P_4(\infty) = \frac{N_2}{D_1}$$

$$\begin{aligned} N_2 = {} & \alpha_1\alpha_2(-\beta_1\beta_2 + \alpha_3\beta_3)(2\alpha_1\beta_1\beta_2 - \alpha_2\beta_1\beta_2 + 2\alpha_1\alpha_2\beta_1 - 2\alpha_2\alpha_3\beta_1 + \alpha_1\beta_2^2 + \alpha_1\alpha_2\beta_2 + \alpha_1\beta_2\beta_3 \\ & +2\alpha_1\alpha_3\beta_2 + \alpha_1\alpha_2\beta_3 + 2\alpha_1\alpha_2\alpha_3) + ((-\alpha_2\beta_1(\beta_2 + 2\alpha_3) + \alpha_1\beta_2(2\beta_1 + \beta_2 + 2\alpha_3 + \beta_3) + \alpha_1\alpha_2(2\beta_1 + \beta_3)) \\ & (2\alpha_1\beta_1^2\beta_2 - \alpha_2\beta_1^2\beta_2 + 2\alpha_1\alpha_2\beta_1^2 - 2\alpha_2\alpha_3\beta_1^2 + 3\alpha_1\beta_1\beta_2^2 - \alpha_2\beta_1\beta_2^2 - 2\alpha_2\alpha_3\beta_1\beta_2 + 3\alpha_1\alpha_2\beta_1\beta_2 + 3\alpha_1\beta_1\beta_2\beta_3 \end{aligned}$$

$$+2\alpha_1\alpha_3\beta_1\beta_2 - \alpha_2\beta_1\beta_2\beta_3 + 3\alpha_1\alpha_2\beta_1\beta_3 + 2\alpha_1\alpha_2\alpha_3\beta_1 - 2\alpha_2\alpha_3\beta_1\beta_3 + \alpha_1\beta_2^3 + 2\alpha_1\beta_2^2\beta_3 + 2\alpha_1\alpha_3\beta_2^2$$
$$+\alpha_1\alpha_2\beta_2^2 + 2\alpha_1\alpha_2\beta_2\beta_3 + 2\alpha_1\alpha_3\beta_2\beta_3 + \alpha_1\beta_2\beta_3^2 + 2\alpha_1\alpha_2\alpha_3\beta_2 + 2\alpha_1\alpha_2\alpha_3\beta_3 + \alpha_1\alpha_2\beta_3^2)$$

2.2.2. Profit Modeling

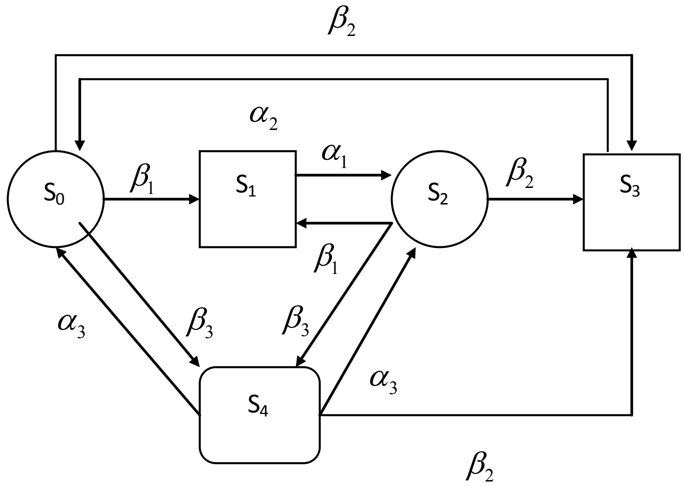

Figure 1. Transition diagram of the system

The system is under corrective maintenance (repair) at failure as can be observed in states 1,3 and 4. From Fig. 1 the repairman is busy performing corrective maintenance action to the system at failure in states 1, 3 and 4. The expected profit per unit time incurred to the system in the steady-state is given by:

$$PF(\infty) = C_0 A_T(\infty) - C_1 B(\infty) \tag{7}$$

2.2.3. Mean time to System Failure Modeling

It is difficult to evaluate the transient solutions, hence we follow El-said[5], Haggag[6, 13], and Wang[9], the procedure to develop the explicit expression for *MTSF* is to delete the rows and columns of an absorbing state in matrix T and take the transpose to produce a new matrix, say Q. The expected time to reach an absorbing state is obtained from

$$E\left[T_{P(0)\to P(absorbing)}\right] = MTSF = P(0)(-Q^{-1})\begin{bmatrix}1\\1\\1\end{bmatrix} = \frac{N_3}{D_2} \tag{8}$$

Where

$$Q = \begin{bmatrix} -(\beta_1+\beta_2+\beta_3) & 0 & \beta_3 \\ 0 & -(\beta_1+\beta_2+\beta_3) & \beta_3 \\ \alpha_3 & \alpha_3 & -(2\alpha_3+\beta_2) \end{bmatrix}$$

$$N_3 = (\beta_1\beta_2 + 2\alpha_3\beta_1 + \beta_2^2 + 2\alpha_3\beta_2 + \beta_2\beta_3 + \alpha_3\beta_3)(\beta_1\beta_2 + 2\alpha_3\beta_1 + \beta_2\beta_3 + 2\alpha_3\beta_2 + \beta_2^2) + \alpha_3\beta_3(\beta_1\beta_2 + 2\alpha_3\beta_1$$
$$+\beta_2\beta_3 + 2\alpha_3\beta_2 + \beta_2^2) + \beta_3(\beta_1^2\beta_3 + 2\alpha_3\beta_1^2 + 2\beta_1\beta_2^2 + 4\alpha_3\beta_1\beta_2 + 2\beta_1\beta_2\beta_3 + 2\alpha_3\beta_1\beta_3 + \beta_2^3 + 2\alpha_3\beta_2^2$$
$$+2\beta_2^2\beta_3 + 2\alpha_3\beta_2\beta_3 + \beta_2\beta_3^2$$

$$D_2 = (\beta_1\beta_2 + 2\alpha_3\beta_1 + \beta_2\beta_3 + 2\alpha_3\beta_2 + \beta_2^2)(\beta_1^2\beta_3 + 2\alpha_3\beta_1^2 + 2\beta_1\beta_2^2 + 4\alpha_3\beta_1\beta_2 + 2\beta_1\beta_2\beta_3$$
$$+ 2\alpha_3\beta_1\beta_3 + \beta_2^3 + 2\alpha_3\beta_2^2 + 2\beta_2^2\beta_3 + 2\alpha_3\beta_2\beta_3 + \beta_2\beta_3^2)$$

3. Results

Figure Number	Parameter/Range	Relationship
2	$0 \le \alpha_3 \le 1$	Relationship between α_3 and availability
3	$0 \le \alpha_2 \le 1$	Relationship between α_2 and availability
4	$0 \le \beta_2 \le 1$	Relationship between β_2 and availability
5	$0 \le \beta_3 \le 1$	Relationship between β_3 and availability
6	$0 \le \beta_3 \le 1$	Relationship between β_3 and profit
7	$0 \le \beta_2 \le 1$	Relationship between β_2 and profit
8	$0 \le \alpha_2 \le 1$	Relationship between α_2 and profit
9	$0 \le \alpha_3 \le 1$	Relationship between β_1 and MTSF
10	$0 \le \beta_1 \le 1$	Relationship between α_3 and availability
11	$0 \le \alpha_3 \le 1$	Relationship between α_3 and busy period
12	$0 \le \alpha_2 \le 1$	Relationship between α_2 and busy period
13	$0 \le \beta_2 \le 1$	Relationship between β_2 and busy period
14	$0 \le \beta_3 \le 1$	Relationship between β_3 and busy period

4. Discussion and Conclusions

4.1. Discussion

In this section, we numerically obtained the results for system availability and profit function for all the developed models. For the models analysis, the following set of parameters values are fixed throughout the simulations for consistency:

1. $\beta_1 = 0.2$, $\beta_2 = 0.4$, $\beta_3 = 0.5$, $\alpha_1 = 0.3$, $\alpha_2 = 0.8$ for Figures 2 and 11
2. $\beta_1 = 0.2$, $\beta_2 = 0.4$, $\beta_3 = 0.5$, $\alpha_1 = 0.3$, $\alpha_3 = 0.4$ for Figures 3 and 12
3. $\beta_1 = 0.2$, $\beta_3 = 0.5$, $\alpha_1 = 0.2$, $\alpha_2 = 0.05$, $\alpha_3 = 0.004$ for Figures 4 and 13
4. $\beta_1 = 0.2$, $\beta_2 = 0.5$, $\alpha_1 = 0.2$, $\alpha_2 = 0.05$, $\alpha_3 = 0.004$ for Figures 5 and 14
5. $\beta_1 = 0.2$, $\beta_2 = 0.5$, $\beta_3 = 0.5$, $\alpha_1 = 0.2$, $\alpha_2 = 0.05$, $\alpha_3 = 0.004$, $C_0 = 100{,}000$, $C_1 = 10{,}000$ for Figures 6,7,8 and 9
6. $\beta_1 = 0.2$, $\beta_2 = 0.2$, $\beta_3 = 0.0003$, $\alpha_3 = 0.4$ for Fig. 10

The impact of α_3 on system availability and busy period of repairman can be observed in Figures 2,9 and 11. From this Figures 2 and 9, it is evident that the availability and profit increases as α_3 increases while from Fig. 11 busy period decreases with increase in α_3. Similar results can be observed in Figures 3, 8 and 12 of availability, profit and busy period with respect to α_2. From these Figures 3 and 8, the availability and profit increases as α_2 increase while from Fig. 12 busy period decreases with increase in α_2. Results of availability, profit and busy period with respect to β_3 are given in Figures 5, 6 and 13. It is evident from these Figures that as β_3 increases, the availability and profit decreases while busy period increases. Figures 4,7 and 14 reveal the effect of β_2 on availability, profit and busy period. The results obtained in these figures reveal similar to those of Figures 5, 6 and 13 that availability and profit decreases with increase in β_2 while busy period increases with increase in β_2 .Figure 10 shows that the mean time to system failure decreases with increase in failure rate β_1.

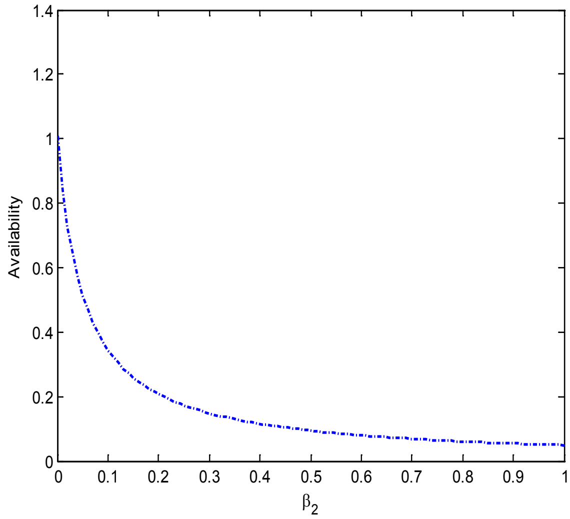

Figure 4. Effect of β_2 on Availability

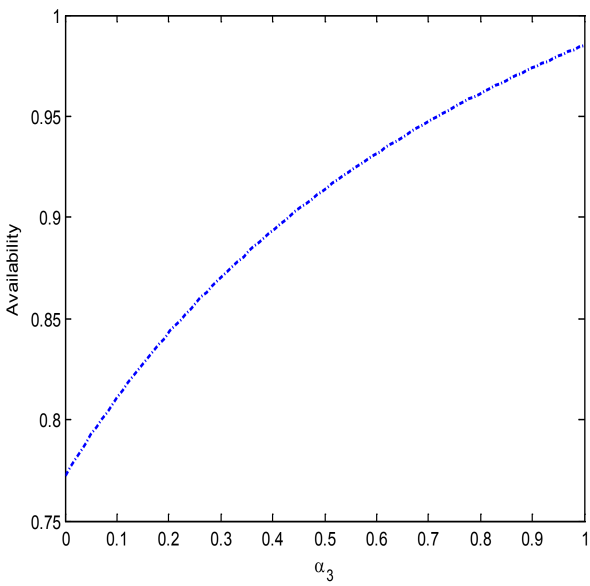

Figure 2. Effect of α_3 on Availability

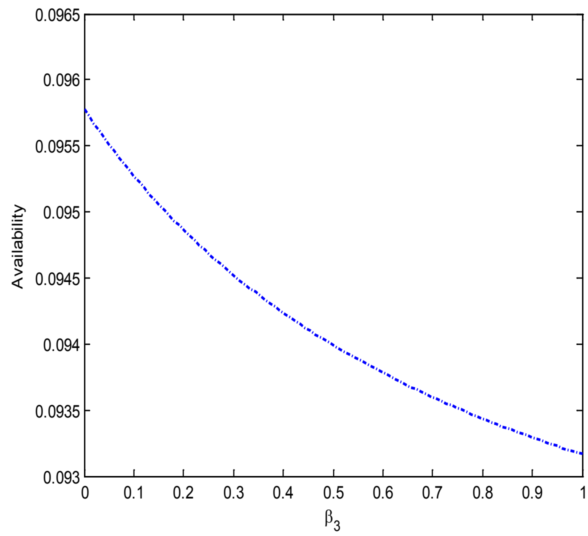

Figure 5. Effect of β_3 on Availability

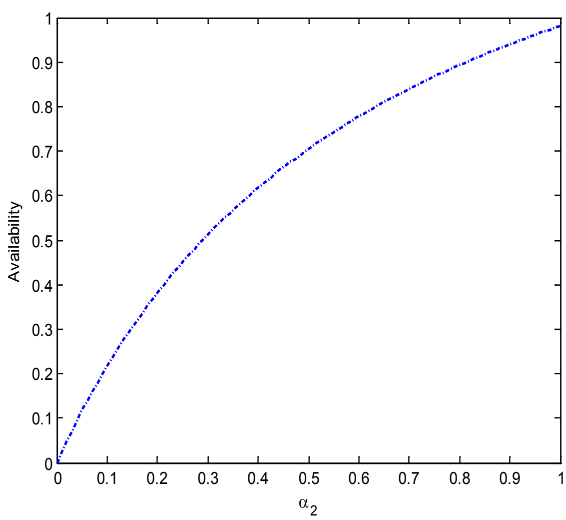

Figure 3. Effect of α_2 on Availability

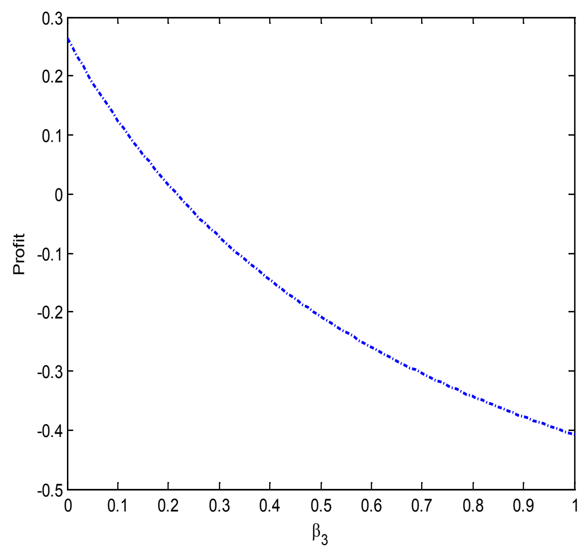

Figure 6. Effect of β_3 on Profit

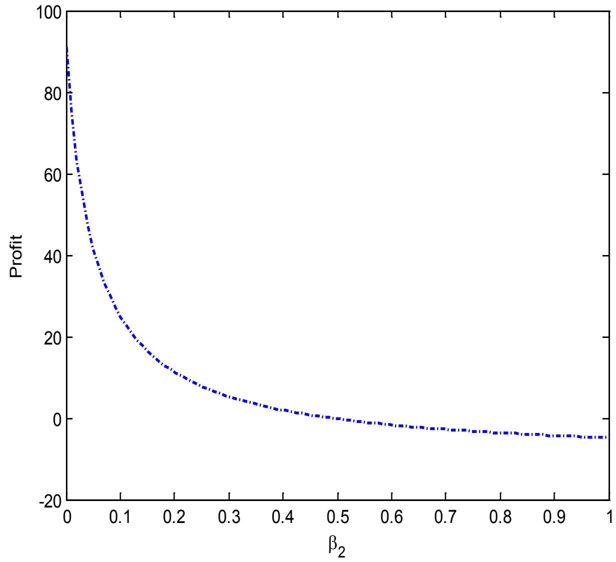

Figure 7. Effect of β_2 on Profit

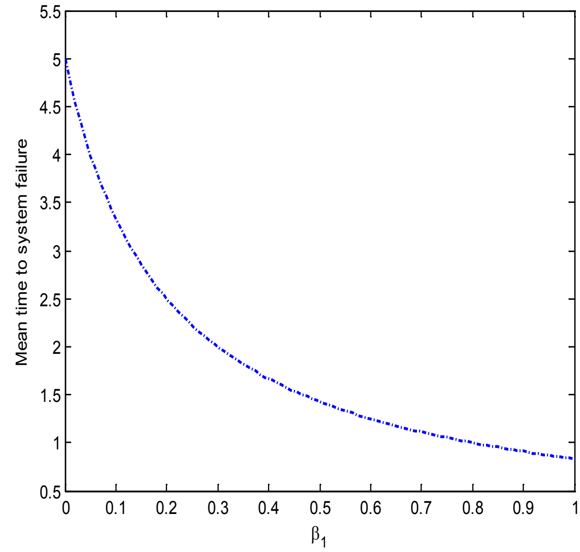

Figure 10. Effect of β_1 on MTSF

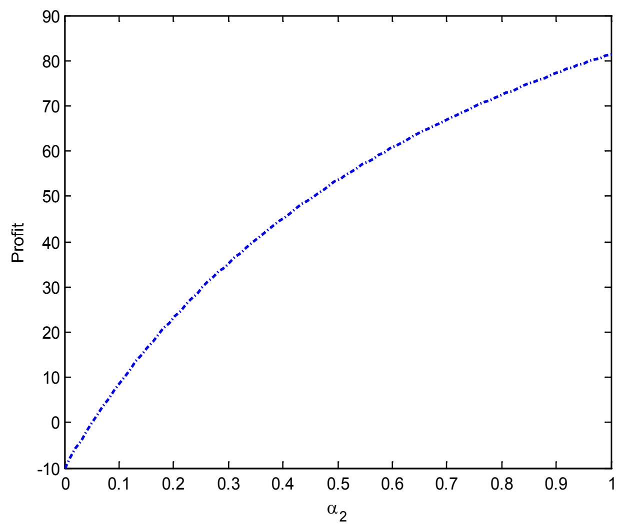

Figure 8. Effect of α_2 on Profit

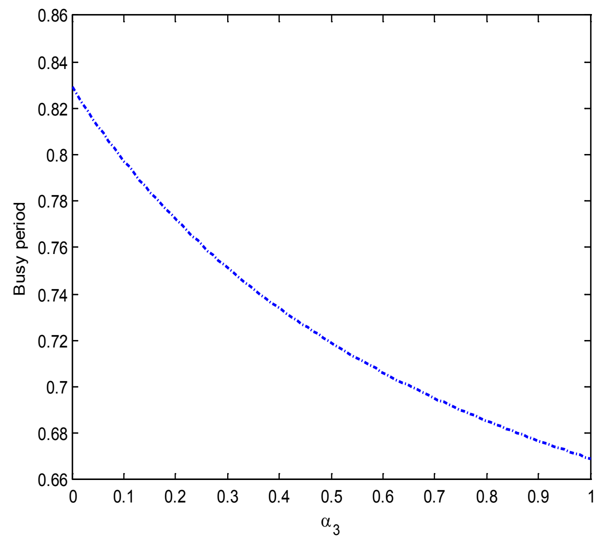

Figure 11. Effect of α_3 on Busy period

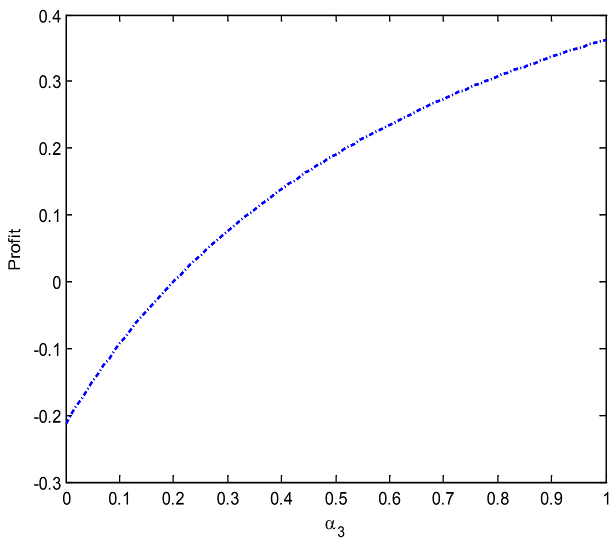

Figure 9. Effect of α_3 on Profit

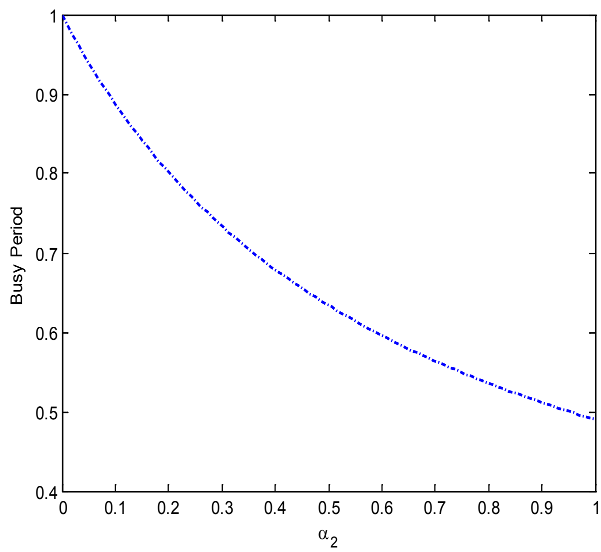

Figure 12. Effect of α_2 on Busy period

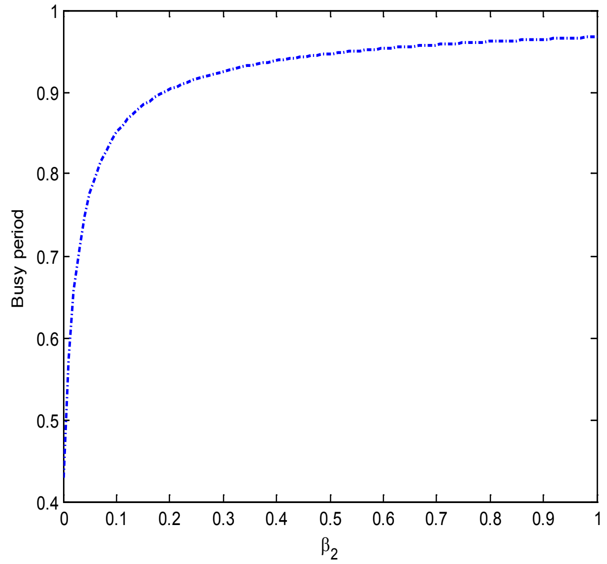

Figure 13. Effect of β_2 on Busy period

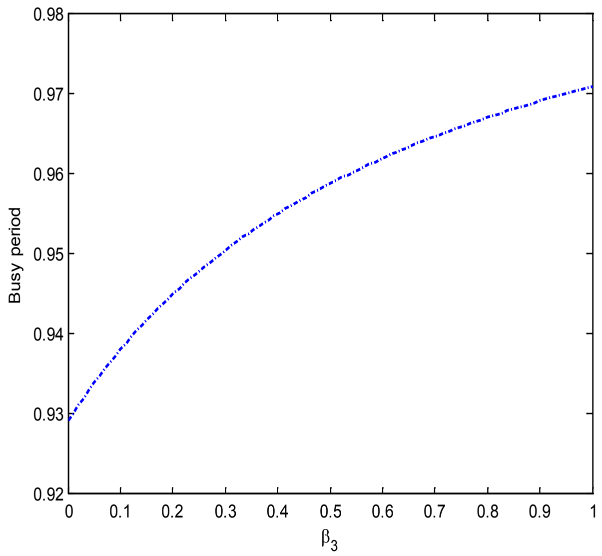

Figure 14. Effect of β_3 on Busy period

4.2. Conclusions

In this paper, we considered a system exposes to three types of failures to study the availability, generated profit and mean time to system failure. Explicit expressions of mean time to system failure, busy period, steady-state availability and profit function were derived. We performed numerical investigation to see the effect of failure and repair rates on the system availability, busy period, mean time to system failure and generated profit. It is evident from the results obtained that repair rate increase the system availability, mean time to system failure and profit generated and decreases the busy period while failure rate decreases the system availability, mean time to system failure and generated profit and increases the busy period of repairman. It is evident from that the results obtained from this paper make a tremendous effect on measure of system effectiveness studied.

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