

Analysis of Constant Deteriorating Inventory Management with Quadratic Demand Rate

Gobinda Chandra Panda^{1,*}, Satyajit Sahoo¹, Pravat Kumar Sukla²

¹Dept of Mathematics, Mahavir Institute of Engineering and Technology, Odisha, India

²Panchayat Samiti College, Koksara, Kalahandi, Odisha, India

Abstract This paper investigates inventory-production systems where items follow constant deterioration. The objective is to develop an optimal policy that minimizes total average cost. The quadratic demand technique is applied to control the problem in order to determine the optimal production policy, holding cost and cost of deterioration. Sensitivity analysis is conducted to study the effect of the cost parameters on the objective function.

Keywords Production, Inventory, Deterioration, Shortage, Quadratic demand

1. Introduction

The purpose of the present paper is to give a new dimension to the inventory literature on time varying demand patterns. Researchers have extensively discussed various types of inventory models with linear trend (positive or negative) in demand. The main Limitation in linear time-varying demand rate is that it implies a uniform change in the demand per unit time. This rarely happens in the case of any commodity in the market. In recent years, some models have been developed with a demand rate that changes exponentially with time. Demands for spare parts of new aeroplanes, computer chips of advanced computer machines, etc. decrease very rapidly with time. Some modellers suggest that this type of rapid change in demand can be represented by an exponential function of time. The present authors feel that an exponential rate of change in demand is extraordinarily high and the demand fluctuation of any commodity in the real market cannot be so high. A realistic approach is to think of accelerated growth (or decline) in the demand rate in the situations cited above and it can be best represented by a quadratic function of time. Thus, this paper has the scope of direct application in the very practical situations noted above.

Goods deteriorate and their value reduces with time. Electronic products may become obsolete as technology changes. Fashion tends to depreciate the value of clothing over time. Batteries die out as they age. The effect of time is even more critical for perishable goods such as foodstuff and cigarettes. The effect of deterioration and time/age is that the classical inventory model has to be readjusted K. Heng, J.

Labban, R. Linn (1)

In general, deterioration is defined as decay, damage, spoilage, evaporation, obsolescence, pilferage, loss of utility or loss of marginal value of a commodity that results in decrease of usefulness from the original one. The decrease or loss of utility due to decay is usually a function of the on-hand inventory. It is reasonable to note that a product may be understood to have lifetime, which ends when utility reaches zero.

The continuously decaying/deterioration of items is classified as age-dependent ongoing deterioration, and age-independent ongoing deterioration. Blood, fish, strawberry are some of the examples of the former while alcohol, gasoline and radioactive chemical and grain products are examples of the latter H. Wee (4).

Haiping and Wang (7) developed an economic policy model for deteriorating items with time proportional demand. Donaldson (8) derived an analytical solution to the problems of obtaining the optimal number of replenishments and the optimal replenishment times of an EOQ model with a linearly time dependent demand pattern, over a finite time horizon. Zangwill (9) developed a discrete-in-time dynamic programming algorithm to solve an inventory model by allowing the inventory levels to be negative where the demand pattern is time dependent. Following

The approach of Donaldson (8), Murdeshwar (6), Sahu and Sukla (10) has tried to derive an exact solution for a finite horizon inventory model to obtain the optimal number of replenishments, optimal replenishment times and the optimal times at which the inventory level falls to zero, assuming the demand rate to be linearly time dependent and shortages. Hamid (3) presented a heuristic model for determining the ordering schedule when inventory items are subject to deterioration and demand changes linearly over time and obtained an optimal replenishment cycle length. Goswami and Chaudhuri (1) presented an EOQ model for deteriorating

* Corresponding author:

pravatsukla2005@gmail.com (Pravat Kumar Sukla)

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items with shortage and linear trend in demand. Bradshaw and Errol (2), published a paper in which they derived unbounded control policies for a class of linear time invariant production-inventory systems.

This paper investigates inventory-production systems where items follow constant deterioration. The objective is to develop an optimal policy that minimizes the cost associated with inventory and production rate. The quadratic demand technique is applied to control the problem in order to determine the optimal production policy. Sensitivity analysis is conducted to study the effect of the cost parameters on the objective function.

2. Assumptions and Notations

The following assumptions and notations have been used in developing the model.

(i) The demand rate is assumed to be $R(t) = a + bt + ct^2$, a , b and c are constants. Such that $a > 0, b > 0, c > 0$. Here a stands for the initial demand rate and b for the positive trend in demand.

(ii) The production rate Say $k = \gamma R(t)$, where $\gamma > 1$. A fraction θ , $0 < \theta < 1$ of the on-hand inventory deteriorates per unit time.

(iii) The lead-time is zero and shortages are not allowed.

(iv) Unit holding cost C_1 per unit time and unit deterioration cost C_3 per unit time are known and constants.

(v) C is the total average cost for the production cycle and S is the stock level reached in the cycle.

(vi) The set up cost is not considered in this model because it is taken to be fixed for the whole cycle time.

(vii) Planning horizon is finite.

3. Mathematical Formulation and Solution

Let q be the inventory level at any time t ($0 \leq t \leq t_2$). The differential equations governing the system in the interval $(0, t_2)$ are

$$\frac{dq}{dt} + \theta q = k - R(t), \quad 0 \leq t \leq t_1 \tag{1}$$

$$\frac{dq}{dt} + \theta q = -R(t), \quad t_1 \leq t \leq t_2 \tag{2}$$

The stock level initially is zero. Production begins just after $t=0$, continues up to $t = t_1$ and stops as soon as the stock level becomes S . Then the inventory level decreases due to demand and deterioration both till it becomes zero at $t = t_2$. The cycle then repeats itself. Our objective is to determine the optimum values of S, C, t_1 and t_2 . The intensity of deterioration is very low initially but it increases with time. However, it remains bounded for $t \gg 1$

Using the value of $R(t)$, the two equations (1) and (2) take the form

$$\frac{dq}{dt} + \theta q = (\gamma - 1)(a + bt + ct^2), \quad 0 \leq t \leq t_1 \tag{3}$$

$$\text{and } \frac{dq}{dt} + \theta q = -(a + bt + ct^2), \quad t_1 \leq t \leq t_2 \tag{4}$$

The solution of equation (3) with initial conditions is

$$qe^{\theta t} = (\gamma - 1) \int_0^t (1 + \theta t)(a + bt + ct^2) dt$$

or

$$q = (\gamma - 1) \left(at + \frac{bt^2}{2} + \frac{ct^3}{3} - \frac{a\theta t^2}{2} - \frac{b\theta t^2}{6} - \frac{c\theta t^4}{12} \right) \tag{5}$$

Neglecting the powers of θ greater than 1.

Similarly, the solution of equation (4) also is (neglecting the powers of θ greater than 1)

$$q \exp(\theta t) = C' - \int (a + bt + ct^2) \exp(\theta t) dt$$

or

$$q = C'(1 - \theta t) + \left(-at - \frac{bt^2}{2} - \frac{ct^3}{3} + \frac{a\theta t^2}{2} + \frac{b\theta t^2}{6} + \frac{c\theta t^4}{12} \right) \tag{6}$$

For $t = t_1, q = S$

$$\therefore S = C'(1 - \theta t_1) + \left(-at_1 - \frac{bt_1^2}{2} - \frac{ct_1^3}{3} + \frac{a\theta t_1^2}{2} + \frac{b\theta t_1^2}{6} + \frac{c\theta t_1^4}{12} \right) \tag{7}$$

From (6) and (7) we get the relation

$$q = S(1 + \theta t_1 - \theta t) + a \left(t_1 - t + \frac{\theta t_1^2}{2} - \theta t_1 t + \frac{\theta t^2}{2} \right)$$

$$+ b \left(\frac{t_1^2}{2} - \frac{t^2}{2} + \frac{\theta t^3}{3} + \frac{\theta t^3}{6} - \frac{\theta t_1^2}{2} \right) + c \left(\frac{t_1^3}{3} - \frac{t^3}{3} + \frac{\theta t_1^4}{4} + \frac{\theta t^4}{12} - \frac{\theta t_1^3 t}{3} \right) \quad (8)$$

Using the condition $q = 0$ for $t = t_2$ in equation (8), we get

$$S = a \left(t_2 - t_1 + \frac{\theta t_1^2}{2} + \frac{\theta t_2^2}{2} - \theta t_1 t_2 \right) + b \left(\frac{t_2^2}{2} - \frac{t_1^2}{2} - \frac{\theta t_1^3}{6} + \frac{\theta t_2^3}{3} - \frac{\theta t_1 t_2^2}{2} \right) + c \left(\frac{t_2^3}{3} - \frac{t_1^3}{3} + \frac{\theta t_1^4}{12} + \frac{\theta t_2^4}{4} - \frac{\theta t_2^3 t_1}{3} \right) \quad (9)$$

Now the average holding cost becomes

$$H(t) = \frac{C_1}{t_2} \left(\int_0^{t_1} q(t) dt + \int_{t_1}^{t_2} q(t) dt \right) = \frac{C_1}{t_2} \left(\int_0^{t_1} (\gamma - 1) \left(at + \frac{bt^2}{2} + \frac{ct^3}{3} - \frac{a\theta t^2}{2} - \frac{b\theta t^2}{6} - \frac{c\theta t^4}{12} \right) dt + \int_{t_1}^{t_2} \left(S(1 + \theta t_1 - \theta t) + a \left(t_1 - t + \frac{\theta t_1^2}{2} - \theta t_1 t + \frac{\theta t^2}{2} \right) + b \left(\frac{t_1^2}{2} - \frac{t^2}{2} + \frac{\theta t^3}{3} + \frac{\theta t^3}{6} - \frac{\theta t_1^2}{2} \right) + c \left(\frac{t_1^3}{3} - \frac{t^3}{3} + \frac{\theta t_1^4}{4} + \frac{\theta t^4}{12} - \frac{\theta t_1^3 t}{3} \right) \right) dt \right)$$

Now substituting the value of S from (9) and simplifying we get

$$H(t) = \frac{C_1}{t_2} + a \left(\left(\frac{\gamma t_1^2}{2} - \frac{\gamma \theta t_1^3}{6} + \frac{t_2^2}{2} - t_1 t_2 + \frac{\theta t_2^3}{6} - \frac{\theta t_2^2 t_1}{2} + \frac{\theta t_2^2 t_1^2}{2} \right) + \frac{b}{6} \left(\frac{\gamma t_1^3}{6} - \frac{\gamma \theta t_1^4}{24} + \frac{t_2^3}{3} - \frac{t_2^2 t_1}{2} + \frac{\theta t_2^4}{8} - \frac{\theta t_2^3 t_1}{3} + \frac{\theta t_1^3 t_2^2}{4} \right) + c \left(\frac{\gamma t_1^4}{12} - \frac{\gamma \theta t_1^5}{60} + \frac{t_2^4}{4} - \frac{\theta t_2^3 t_1}{3} + \frac{\theta t_2^5}{10} - \frac{\theta t_2^4 t_1}{4} + \frac{\theta t_2^3 t_1^2}{6} \right) \right) \quad (10)$$

The average cost due to deterioration in the total cycle time is

$$d(t) = \frac{C_3}{t_2} \left[\gamma \int_0^{t_1} (a + bt + ct^2) dt - \int_0^{t_2} (a + bt + ct^2) dt \right] = \frac{C_3}{t_2} \left[a(\gamma t_1 - t_2) + \frac{b}{2} (\gamma t_1^2 - t_2^2) + \frac{c}{3} (\gamma t_1^3 - t_2^3) \right] \quad (11)$$

From (10) and (11) the total average cost of the inventory I

$$C = \frac{C_1}{t_2} \left(\frac{a}{6} (3\gamma t_1^2 - \gamma \theta t_1^3 + 3t_2^2 - 6t_1 t_2 + \theta t_2^3 - 3\theta t_2^2 t_1 + 3\theta t_2 t_1^2) + \frac{b}{24} (4\gamma t_1^3 - \gamma \theta t_1^4 + 8t_2^3 - 12t_2^2 t_1 + 3\theta t_2^4 - 8\theta t_2^3 t_1 + 6\theta t_1^2 t_2^2) + \frac{c}{12} (5\gamma t_1^4 - \gamma \theta t_1^5 + 15t_2^4 - 20\theta t_2^3 t_1 + 6\theta t_2^5 - 15\theta t_2^4 t_1 + 10\theta t_2^3 t_1^2) \right)$$

$$+ \frac{C_3}{t_2} \left[a(\gamma t_1 - t_2) + \frac{b}{2}(\gamma t_1^2 - t_2^2) + \frac{c}{3}(\gamma t_1^3 - t_2^3) \right] \tag{12}$$

By putting $t_1 = z t_2$ (where $0 < z < 1$) in equation (12), we get

$$\begin{aligned} C = & \frac{C_1}{t_2} \left(\frac{a}{6}(3\gamma z^2 t_2^2 - \gamma \theta z^3 t_2^3 + 3t_2^2 - 6t_2^2 z + \theta t_2^3 - 3\theta t_2^3 z + 3\theta t_2^3 z^2) \right. \\ & + \frac{b}{24}(4\gamma z^3 t_2^3 - \gamma \theta z^4 t_2^4 + 8t_2^3 - 12t_2^3 z + 3\theta t_2^4 - 8\theta t_2^4 z + 6\theta t_2^4 z^2) \\ & \left. + \frac{c}{12}(5\gamma z^4 t_2^4 - \gamma \theta z^5 t_2^5 + 15t_2^4 - 20\theta t_2^4 z + 6\theta t_2^5 - 15\theta t_2^5 z + 10\theta t_2^5 z^2) \right) \\ & + \frac{C_3}{t_2} \left[a(\gamma z t_2 - t_2) + \frac{b}{2}(\gamma z^2 t_2^2 - t_2^2) + \frac{c}{3}(\gamma z^3 t_2^3 - t_2^3) \right] \end{aligned} \tag{13}$$

For calculating the optimum value of C we differentiate it partially with respect to t_2 and equate them to zero. Thus we get the following equation:-

$$\begin{aligned} \frac{dC}{dt_2} = & C_1 \left(\frac{a}{6}(3\gamma z^2 - 2\gamma z t_2 + 3 - 6z + 2\theta t_2 - 6\theta t_2 z + 6\theta t_2 z^2) \right. \\ & + \frac{b}{24}(8\gamma z^3 t_2 - 3\gamma \theta z^4 t_2^2 + 16t_2 - 24t_2 z + 9\theta t_2^2 - 24\theta t_2^2 z + 18\theta z^2 t_2^2) \\ & \left. + \frac{c}{60}(15\gamma z^4 t_2^2 - 4\gamma \theta z^5 t_2^3 + 45t_2^2 - 60\theta t_2^2 z + 24\theta t_2^3 - 60\theta t_2^3 z + 40\theta t_2^3 z^2) \right) \\ & + C_3 \left[\frac{b}{2}(\gamma z^2 - 1) + \frac{2c}{3}(\gamma z^3 - 1)t_2 \right] \end{aligned} \tag{14}$$

This equation gives us the optimum value of t_2 which, when substituted equation (13), give the total average cost, provided $\frac{d^2 C}{dt_2^2} > 0$. Equation (14) is highly non-linear in t_2 and cannot be solved analytically. This equation, therefore, can be solved by some suitable numerical method like Newton-Raphson, and optimal value of t_2 can be obtained. This optimal value of t_2 gives the minimum cost of the system in question. We have solved this equation on computer for a set of values of the parameters with the help of Newton-Raphson method. A numerical example is given below as an illustration.

3.1. Example-1

Let $\theta=0.02, \gamma=2.0, z=0.7, C_1=5.0, C_3=60, a=200, b=40, c=10$ in suitable units.

The solution for optimal values of t_1 and t_2 is $t_1^* = 1.5534, t_2^* = 14.0345$, which gives minimum average cost $C^* = 10395.3646$

Following are a number of tables representing the optimal values of t_1, t_2 and C as also the no-production interval $t_2 - t_1$.

4. Sensitivity Analysis

We have discussed the effects of the different parameters.

- (i) Increase in the value of γ decreases the value of t_1, t_2 and C .
- (ii) Increase in the value of the parameter θ , decrease the values of t_1, t_2 and C .
- (iii) Increase in the value of holding cost C_1 increases the value of the cost t_1, t_2 and decreases the value of C .
- (iv) Increase in the value of deterioration cost C_3 increases the value of the cost t_2 and C . However the values of t_1 decrease. Increase in the value of a , decreases the value of C, t_1, t_2 . Increase in the value of b , increases the values of t_1, t_2 and C .
- (v) Increase in the value of c , decreases the value of C, t_1, t_2 .
- (vi) Keeping these variations in mind of the decision maker of the inventory system can control the parameters so as to optimize the objective function. The decision maker may control particularly the holding cost and the cost of deterioration for minimizing the total average cost.

Table 1. Variations in parameters

	Parameter	t_1	t_2	C	t_2-t_1
θ	0.005	1.5880	14.8152	10733.5044	13.2272
	0.010	1.5828	14.7280	10677.6608	13.1452
	0.015	1.5785	14.6057	10534.0876	13.0272
	0.020	1.5534	13.8345	10395.3646	12.2811
	0.025	1.5732	12.3273	10165.5343	11.7541
γ	1.3	13.0202	26.2543	20546.0034	13.2341
	1.5	9.7654	21.3454	17861.1234	11.5800
	1.7	5.4560	16.7234	15332.2341	11.2674
	1.9	2.8776	13.5431	12546.2341	10.6655
	2.1	0.8834	11.2396	08537.7582	10.3562
a	190	2.8854	16.4725	12164.0551	13.5871
	200	1.6043	14.7843	10633.6035	13.1800
	210	1.4872	14.3172	09558.8551	12.8300
	220	0.6623	12.9956	07440.6742	12.3333
	230	0.2547	12.5427	04786.2316	12.2880
b	35	3.1437	4.4910	6689.7036	1.3473
	40	3.2827	4.6896	6706.7988	1.4069
	45	3.4539	4.9342	6726.3891	1.4803
	50	3.6927	5.2754	6749.9902	1.5827
	55	4.1103	5.8719	6782.2915	1.7616
c	8	3.6306	5.1867	6868.5703	1.5561
	10	3.2827	4.6896	6706.7988	1.4069
	12	2.3296	3.3280	6662.1054	0.9984
	14	1.8543	2.6490	6600.9335	0.7947
	16	1.5458	2.2083	6543.3886	0.6625
C_1	5	1.4480	14.2350	10722.5432	12.7870
	6	2.3365	14.3956	10621.2031	12.0591
	7	2.7798	13.6065	10543.1232	10.8267
	8	2.8134	12.8939	10327.1212	10.0805
C_3	40	2.7257	12.8938	6576.6791	10.1681
	50	2.2827	13.6896	6788.7988	11.4069
	60	1.9006	13.8152	8200.9125	11.9146
	70	1.7526	14.218	8558.0664	13.4654
	80	1.0350	15.0500	1078.3515	14.0150

5. Conclusions

In this article, a deterministic inventory model has been proposed for deteriorating item with quadratic demand rate, where shortages are not allowed. The goal of the paper is to incorporate the deterioration phenomenon together into an inventory model over a finite planning horizon. This paper investigates inventory-production systems where items follow constant deterioration. The objective is to develop an optimal policy that minimizes total average cost. The

quadratic demand technique is applied to control the problem in order to determine the optimal production policy, holding cost and cost of deterioration.

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