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Bipolar Fuzzy α-ideal of BP-algebra

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Abstract In this paper, the concept of bipolar fuzzy α-ideal of BP-algebra is introduced. We introduced α-ideal and fuzzy α-ideal. Several theorems are presented in this regard. The homomorphic image and inverse image of the bipolar fuzzy α-ideal are studied.

Keywords BP-algebra, α-ideal, fuzzy α-ideal, Bipolar fuzzy α-ideal

1. Introduction

Y. Imai and K. Iséki introduced two classes of abstract algebras: BCK-algebras and BCI-algebras [7,8]. It is known that the class of BCK-algebras is a proper subclass of the class of BCI-algebras. In [4,5], Q. P. Hu and X. Li introduced a wide class of abstract: BCH-algebras. They had shown that the class of BCI-algebras is a proper subclass of the class of BCH-algebras. In [27], J. Neggers, S. S. Ahn and H. S. Kim introduced Q-algebras which is a generalization of BCK / BCI-algebras and obtained several results. In 2002, Neggers and Kim [19,26,28] introduced a new notion, called a B-algebra, and obtained several results. In 2007, Walendziak [30] introduced a new notion, called a BF-algebra, which is a generalization of B-algebra. In 2013, Ahn and Han. [2] introduced a new notion, called BP-algebra which is related to several classes of algebra. In 1965, the concept of fuzzy sets, a remarkable idea in mathematics, was proposed by Zadeh [31]. In this traditional concept of fuzzy set, the membership degree expresses belongingness of an element to a fuzzy set. The membership degree of an element ranges over the interval [0, 1]. When the membership degree of an element is 1, then the element completely belongs to its corresponding fuzzy set, and the membership degree of an element is 0 means an element does not belong to the fuzzy set. Based on this tool, different fuzzy algebraic structures have been developed by many researchers, The fuzzy structures of BCK/BCI-algebras worked out by many researchers such as Jun [16,17,18,25], Liu [24], Bej and Pal [3], Jana et al. and others [10-15] have done much investigations on BCK/BCI/G/B-algebras related to these algebras. In 1994, the notion of bipolar fuzzy sets was proposed by Zhang [32,33] as a generalization of fuzzy sets [31]. Bipolar-valued fuzzy sets [22,23] are seen as an extension of fuzzy sets whose membership degree range is enlarged from the interval [0, 1] to [−1, 1]. In a bipolar fuzzy set, the membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree (0, 1] of an element indicates that the element somewhat satisfies the property, and the membership degree [−1, 0) of an element indicates that the element somewhat satisfies the implicit counter-property. Bipolar fuzzy sets have various applications in fuzzy algebras. For example, bipolar fuzzy ideals [1] in LA-semigroups, bipolar fuzzy sub-algebras and ideals [21] of BCK/BCI-algebras, bipolar fuzzy a-ideals in BCK/BCI-algebras [20] and bipolar valued fuzzy BCK/BCI-algebras [29] are some of them. The aim of this paper is to apply the notion of the bipolar fuzzy set to α-ideal of BP-algebra. The notions of α-ideal, fuzzy α-ideal and bipolar fuzzy α-ideal are defined, and a lot of properties are investigated. The homomorphic image and the inverse image of the bipolar fuzzy α-ideal are studied. Several theorems and basic properties that are related to the bipolar fuzzy α-ideal of BP-algebra are investigated. In section 5, we conclude and present some topics for future research.

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2. Preliminaries

In this section, some elementary aspects necessary for this paper are included.

Definition 2.1 [2]. An algebra $(X;*,0)$ is called BP-algebra if it satisfies the following axioms:

(B_1) $x*x=0$,

(B_2) $x*(x*y)=y$,

(B_3) $(x*z)*(y*z)=x*y$, for all $x,y,z\in X$.

In *X*, we can define a binary relation "≤" by $x\leq y$ if and only if $x*y=0$.

Example 2.2 [2]. Let $X=\{0,1,2,3,4,5\}$. Define $*$ on X as the following table:

*	0	a	b	c
0	0	c	b	a
a	a	0	c	b
b	b	a	0	c
c	c	b	a	0

Then $(X;*,0)$ is a BP-algebra.

Theorem 2.3 [2]. If $(X;*,0)$ is a BP-algebra, then following conditions hold: for any $x, y \in X$,

1) $0*(0*x) = x$,
2) $0*(y*x) = x*y$,
3) $x*0 = x$
4) $x*y = 0$ implies $y*x = 0$,
5) $0*x = 0*y$ implies $x = y$,
6) $0*x = y$ implies $0*y = x$,
7) $(x*y)*x = 0*y$,
8) $0*x = x$ implies $x*y = y*x$.

Example 2.4 [2]. Let $X = \{0,1,2,3\}$ Define $*$ on X as the following table:

*	0	a	b	c
0	0	a	b	c
a	a	0	c	b
b	b	c	0	a
c	c	b	a	0

Then $(X,*;0)$ is a BP-algebra

3. α-ideal & Fuzzy α-ideal of BP-algebra

In this section, α-ideal and fuzzy α-ideal of BP-algebra are defined and some important properties are presented.

Definition 3.1. A nonempty subset S of a BP-algebra X is called a subalgebra of X if $x*y \in S$ for all $x, y \in S$.

Definition 3.2. A non-empty subset "I" of a BP-algebra $(X;*,0)$ is called a α-ideal if for all $x, y, z \in X$:

P$_1$) $0 \in I$,

P$_2$) $x*z \in I$ *and* $(x*y) \in I$ *imply* $(y*z) \in I$.

Definition 3.3. Let $(X,*,0)$ and $(Y,*',0')$ be BP-algebra. A mapping $f: X \to Y$ is said to be a homomorphism if $f(x*y) = f(x)*' f(y)$, for all $x, y \in X$.

Definition 3.4. Let $(X;*,0)$ be a BP-algebra. A fuzzy set μ in X is called a fuzzy α-ideal of X if it satisfies:

FP$_1$) $\mu(0) \geq \mu(x)$,

FP$_2$)
$\mu(y*z) \geq \min\{\mu(x*z), \mu(x*y)\}$, *for all* $x, y, z \in X$.

Example 3.5. Consider a BP-algebra X = {0, a, b, c} in which the "$*$" operation is given by example 2.4. Let n_1, $n_2, n_3 \in [0,1]$ be such that $n_1 > n_2 > n_3$. Define the mapping $\mu: X \to [0,1]$ by $\mu(0) = n_1$, $\mu(a) = n_2$ and $\mu(b) = \mu(c) = n_3$.

Then routine calculations give that μ is a fuzzy α-ideal of X.

Proposition 3.6. If μ is a fuzzy α-ideal of BP-algebra X, then $x \leq y$ implies $\mu(x*0) \geq \mu(y*0)$, for all $x, y \in X$.

Proof. Let μ be a Fuzzy α-ideal of BP-algebra X.

By definition 2.1, if $x \leq y$ then $x*y = 0$, and $\mu(0) \geq \mu(x)$, given that

$\mu(y*z) \geq \min\{\mu(x*z), \mu(x*y)\}$, for all $x, y, z \in X$. Then

$$\begin{aligned}\mu(x*0) &\geq \min\{\mu(y*0), \mu(x*y)\},\\ &= \min\{\mu(y*0), \mu(0)\}\\ &= \mu(y*0).\end{aligned}$$

4. Bipolar Fuzzy α-ideal of BP-algebra

In this section, we defined bipolar fuzzy α-ideal of BP-algebra and examined some related properties.

Definition 4.1 [22]. A bipolar fuzzy set A of X is defined as $A = \{(x, \mu_A^P(x), \mu_A^N(x)) | x \in X\}$ where $\mu_A^P(x): X \longrightarrow [0,1]$ and $\mu_A^N(x): X \longrightarrow [-1,0]$ are mappings. The positive membership degree $\mu_A^P(x)$ denotes the satisfaction degree of an element x to the property corresponding to a bipolar fuzzy set $A = \{(x, \mu_A^P(x), \mu_A^N(x)) | x \in X\}$ and the negative membership degree $\mu_A^N(x)$ denotes the satisfaction degree of an element x to some implicit counter property of $A = \{(x, \mu_A^P(x), \mu_A^N(x)) | x \in X\}$. If $\mu_A^P(x) \neq 0$ and $\mu_A^N(x) = 0$, this case is regarded as having only a positive satisfaction degree for $A = \{(x, \mu_A^P(x), \mu_A^N(x)) | x \in X\}$. If $\mu_A^P(x) = 0$ and $\mu_A^N(x) \neq 0$, x does not satisfy the property of $A = \{x, \mu_A^P(x), \mu_A^N(x): x \in X\}$, but somewhat satisfies the counter property of $A = \{(x, \mu_A^P(x), \mu_A^N(x)) | x \in X\}$. In some cases, it is possible for an element x to be $\mu_A^P(x) \neq 0$ and $\mu_A^N(x) \neq 0$ when the membership function of the property overlaps that of the counter property of its portion of the domain (Lee [26]). We shall use the symbol $A = (\mu_A^P, \mu_A^N)$ for the bipolar fuzzy set $A = \{(x, \mu_A^P(x), \mu_A^N(x)) | x \in X\}$.

Definition 4.2 [32]. For every two bipolar fuzzy set $A = (\mu_A^P, \mu_A^N)$ and $B = (\mu_B^P, \mu_B^N)$ in X, we define

$(A \cup B)(x) = \{\max\{\mu_A^P(x), \mu_B^P(x)\}, \min\{\mu_A^N(x), \mu_B^N(x)\}\}$

$(A \cap B)(x) =$
$\{\min\{\mu_A^P(x), \mu_B^P(x)\}, \max\{\mu_A^N(x), \mu_B^N(x)\}\}$.

Proposition 4.3 [21]. A bipolar fuzzy set $A = (\mu_A^P, \mu_A^N)$ of X is called a bipolar fuzzy subalgebra of X if it satisfies $\mu_A^P(x*y) \geq \min\{\mu_A^P(x), \mu_A^P(y)\}$ and
$\mu_A^N(x*y) \leq \max\{\mu_A^N(x), \mu_A^N(y)\}$ for all $x, y \in X$.

Definition 4.4 [21]. A bipolar fuzzy set $A = (\mu_A^P, \mu_A^N)$ of X is called a bipolar fuzzy ideal of X if it satisfies the following conditions

(i) $\mu_A^P(0) \geq \mu_A^P(x)$ and $\mu_A^N(0) \leq \mu_A^N(x)$
(ii) $\mu_A^P(x) \geq \min\{\mu_A^P(x*y), \mu_A^P(y)\}$
(iii) $\mu_A^N(x) \leq \max\{\mu_A^N(x*y), \mu_A^N(y)\}$ for all $x, y \in X$.

Definition 4.5. A bipolar fuzzy set $\alpha = (\mu_\alpha^P, \mu_\alpha^N)$ of a BP-algebra X is called a bipolar fuzzy α-ideal of X if it satisfies the following conditions

(i) $\mu_\alpha^P(0) \geq \mu_\alpha^P(x)$ and $\mu_\alpha^N(0) \leq \mu_\alpha^N(x)$,
(ii) $\mu_\alpha^P(y*z) \geq \min\{\mu_\alpha^P(x*z), \mu_\alpha^P(x*y)\}$,
(iii) $\mu_\alpha^N(y*z) \leq \max\{\mu_\alpha^N(x*z), \mu_\alpha^N(x*y)\}$, for all $x, y, z \in X$.

Example 4.6. Consider a BP-algebra $X = \{0, a, b, c\}$ in which the "$*$" operation is given by example 2.4. Define a bipolar fuzzy set $\alpha = (\mu_\alpha^P, \mu_\alpha^N)$ by, $\mu_\alpha^P = \begin{pmatrix} 0 & a & b & c \\ 0.7 & 0.7 & 0.3 & 0.3 \end{pmatrix}$ and $\mu_\alpha^N = \begin{pmatrix} 0 & a & b & c \\ -0.8 & -0.8 & -0.6 & -0.3 \end{pmatrix}$, routine calculation gives that $\alpha = (\mu_\alpha^P, \mu_\alpha^N)$ is a bipolar fuzzy α-ideal of X.

Theorem 4.7. The intersection of any set of bipolar fuzzy α-ideals in BP-algebra X is also a bipolar fuzzy α-ideal of X.

Proof. Let $\alpha_i = \{(\mu_{\alpha_i}^P, \mu_{\alpha_i}^N) \mid i \in \omega\}$ be a family of bipolar fuzzy α-ideals in BP-algebra X. Then for any $x, y, z \in X$, $(\cap \mu_{\alpha_i}^P)(0) = \inf_{i\in\omega}(\mu_{\alpha_i}^P(0)) \geq \inf_{i\in\omega}(\mu_{\alpha_i}^P(x)) = (\cap \mu_{\alpha_i}^P)(x)$.

And $(\cap \mu_{\alpha_i}^N)(0) = \inf_{i\in\omega}(\mu_{\alpha_i}^N(0)) \leq \inf_{i\in\omega}(\mu_{\alpha_i}^N(x)) = (\cap \mu_{\alpha_i}^N)(x)$.

Also $(\cap \mu_{\alpha_i}^P)(y*z) =$

$$\inf_{i \in \omega}\left(\mu_{\alpha_i}^P(y*z)\right) \geq \inf_{i \in \omega}\langle \min\{\mu_{\alpha_i}^P(x*z), \mu_{\alpha_i}^P(x*y)\}\rangle$$
$$= \min\{\langle \inf_{i\in\omega} \mu_{\alpha_i}^P(x*z), \inf_{i\in\omega} \mu_{\alpha_i}^P(x*y)\rangle\}$$
$$= \min\{(\cap \mu_{\alpha_i}^P)(x*z), \cap \mu_{\alpha_i}^P(x*y)\}.$$

And $(\cap \mu_{\alpha_i}^N)(y*z) =$

$$\inf_{i \in \omega}\left(\mu_{\alpha_i}^N(y*z)\right) \leq \inf_{i \in \omega}\langle \max\{\mu_{\alpha_i}^N(x*z), \mu_{\alpha_i}^N(x*y)\}\rangle$$
$$= \max\{\langle \inf_{i\in\omega} \mu_{\alpha_i}^N(x*z), \inf_{i\in\omega} \mu_{\alpha_i}^N(x*y)\rangle\}$$
$$= \max\{(\cap \mu_{\alpha_i}^N)(x*z), \cap \mu_{\alpha_i}^N(x*y)\}.$$

Proposition 4.8. Let $\alpha = (\mu_\alpha^P, \mu_\alpha^N)$ be a bipolar fuzzy α-ideal of BP-algebra$X$. If $x \leq y*x$ holds in X, then $\mu_\alpha^P(x) \geq \mu_\alpha^P(x*y)$, and $\mu_\alpha^N(x) \leq \mu_\alpha^N(x*y)$.

Proof. Let $x \leq y*x$ holds in X. Then $x*(y*x) = 0$.
Since $\mu_\alpha^P(y*z) \geq \min\{\mu_\alpha^P(x*z), \mu_\alpha^P(x*y)\}$,
and $\mu_\alpha^N(y*z) \leq \max\{\mu_\alpha^N(x*z), \mu_\alpha^N(x*y)\}$.

Then $\mu_\alpha^P(y*(y*x)) \geq \min\{\mu_\alpha^P(x*(y*x)), \mu_\alpha^P(x*y)\}$
$\Rightarrow \mu_\alpha^P(x) \geq \min\{\mu_\alpha^P(0), \mu_\alpha^P(x*y)\}$
$\mu_\alpha^P(x) \geq \mu_\alpha^P(x*y)$.

And $\mu_\alpha^N(y*(y*x)) \leq \max\{\mu_\alpha^N(x*(y*x)), \mu_\alpha^N(x*y)\}$
$\Rightarrow \mu_\alpha^N(x) \leq \max\{\mu_\alpha^N(0), \mu_\alpha^N(x*y)\}$
$\mu_\alpha^N(x) \leq \mu_\alpha^N(x*y)$.

Proposition 4.9. Let $\alpha = (\mu_\alpha^P, \mu_\alpha^N)$ be a bipolar fuzzy α-ideal of BP-algebra X. If $x \leq y$ holds in X, then $\mu_\alpha^P(y*z) \geq \mu_\alpha^P(x*z)$, and $\mu_\alpha^N(y*z) \leq \mu_\alpha^N(x*z)$.

Proof. Let $x \leq y$ holds in X. Then $x*y = 0$.
Since $\mu_\alpha^P(y*z) \geq \min\{\mu_\alpha^P(x*z), \mu_\alpha^P(x*y)\}$.
Then $\mu_\alpha^P(y*z) \geq \min\{\mu_\alpha^P(x*z), \mu_\alpha^P(0)\}$,
$\mu_\alpha^P(y*z) \geq \mu_\alpha^P(x*z)$.
And $\mu_\alpha^N(y*z) \leq \max\{\mu_\alpha^N(x*z), \mu_\alpha^N(x*y)\}$.
Then $\mu_\alpha^N(y*z) \leq \max\{\mu_\alpha^N(x*z), \mu_\alpha^N(0)\}$,
$\mu_\alpha^N(y*z) \leq \mu_\alpha^N(x*z)$.

Definition 4.10. Let $\alpha = (\mu_\alpha^P, \mu_\alpha^N)$ be a bipolar fuzzy α-ideal of BP-algebra X and $(\theta, \tau) \in [-1,0] \times [1,0]$. We define $\cup(\mu_\alpha^P, \mu_\alpha^N; \theta, \tau) = \{x \in X \mid \mu_\alpha^P(x) \geq \theta \text{ } and \text{ } \mu_\alpha^N \leq \tau\}$ is called upper θ-level cut of μ_α^P and lower τ-level cut of μ_α^N of the bipolar fuzzy α-ideal.

Theorem 4.11. Let $\alpha = (\mu_\alpha^P, \mu_\alpha^N)$ be a bipolar fuzzy α-ideal of BP-algebra X. Then for every $(\theta, \tau) \in [-1,0] \times [1,0]$, $\cup(\mu_\alpha^P, \mu_\alpha^N; \theta, \tau)$ is α-ideal of BP-algebra.

Proof. Assume that $\alpha = (\mu_\alpha^P, \mu_\alpha^N)$ is a bipolar fuzzy α-ideal of X. For $x \in \cup(\mu_\alpha^P, \mu_\alpha^N; \theta, \tau)$ so $\mu_\alpha^P(x) \geq \theta$ and $\mu_\alpha^N(x) \leq \tau$ where $(\theta, \tau) \in [-1,0] \times [1,0]$. Now we get that

$\mu_\alpha^P(0) \geq \mu_\alpha^P(x) \geq \theta$ and $\mu_\alpha^N(0) \leq \mu_\alpha^N(x) \leq \tau$ this implies that $0 \in \cup(\mu_\alpha^P, \mu_\alpha^N; \theta, \tau)$.

Next, let $(x*z), (x*y) \in \cup(\mu_\alpha^P, \mu_\alpha^N; \theta, \tau)$. This means $\mu_\alpha^P(x*z) \geq \theta$ and $\mu_\alpha^P(x*y) \geq \theta$ also, $\mu_\alpha^N(x*z) \leq \tau$, and $\mu_\alpha^N(x*y) \leq \tau$.

Then $\mu_\alpha^P(y*z) \geq \min\{\mu_\alpha^P(x*z), \mu_\alpha^P(x*y)\} \geq \theta$, and $\mu_\alpha^N(y*z) \leq \max\{\mu_\alpha^N(x*z), \mu_\alpha^N(x*y)\} \leq \tau$.

This implies that $(y*z) \in \cup(\mu_\alpha^P, \mu_\alpha^N; \theta, \tau)$.

Hence $\cup(\mu_\alpha^P, \mu_\alpha^N; \theta, \tau)$ is a α-ideal of BP-algebra.

Definition 4.12. Let $(X,*,0)$ and $(Y,*,0)$ be BP-algebras, and let f be a mapping from the set X and the set Y. If $\alpha = (\mu_\alpha^P, \mu_\alpha^N)$ and $\delta = (\beta_\delta^P, \beta_\delta^N)$ are bipolar fuzzy sets of X and Y respectively. Then

$$\mu_\alpha^P(f^{-1}(y)) = \beta_\delta^P(y)$$
$$= \begin{cases} \sup_{x \in f^{-1}(y)} \mu_\alpha^P(x), & f^{-1}(y) = \{x \in X, f(x) = y\} \neq \emptyset, \\ 1, Otherwise, \end{cases}$$

And

$$\mu_\alpha^N(f^{-1}(y)) = \beta_\delta^N(y)$$
$$= \begin{cases} \inf_{x \in f^{-1}(y)} \mu_\alpha^N(x), & f^{-1}(y) = \{x \in X, f(x) = y\} \neq \emptyset, \\ -1, Otherwise, \end{cases}$$

For all $y \in Y$ is called the image of $\alpha = (\mu_\alpha^P, \mu_\alpha^N)$ under f. Similarly, the inverse image $\alpha = \delta \circ f$ in X defined as, $\beta_\delta^P(f(x)) = \mu_\alpha^P(x)$, and $\beta_\delta^N(f(x)) = \mu_\alpha^N(x)$ for all $x \in X$.

Theorem 4.13. An into homomorphic inverse image of a bipolar fuzzy α-ideal of BP-algebra is also bipolar fuzzy α-ideal.

Proof. Let $f : (X,*,0) \longrightarrow (Y,*',0')$ be an into homomorphism of BP-algebras. Assume that $\delta = (\beta_\delta^P, \beta_\delta^N)$ is a bipolar fuzzy α-ideal in Y, and $\alpha = (\mu_\alpha^P, \mu_\alpha^N)$ is a bipolar fuzzy α-ideal in X. Then for all $x \in X$,

$\mu_\alpha^P(0) = \beta_\delta^P(f(0)) \geq \beta_\delta^P(f(x)) = \mu_\alpha^P(x)$, and $\mu_\alpha^N(0) = \beta_\delta^N(f(0)) \geq \beta_\delta^N(f(x)) = \mu_\alpha^N(x)$. Now, let $x, y, z \in X$. Then

$$\mu_\alpha^P(y * z) = \beta_\delta^P(f(y * z)) = \beta_\delta^P(f(y) *' f(z))$$
$$\geq \min\{\beta_\delta^P(f(x) *' f(z)), \beta_\delta^P(f(x) *' f(y))\}$$
$$= \min\{\beta_\delta^P(f(x * z)), \beta_\delta^P(f(x * y))\}$$
$$= \min\{\mu_\alpha^P(f(x * z)), \mu_\alpha^P(f(x * y))\}.$$

And,

$$\mu_\alpha^N(y * z) = \beta_\delta^N(f(y * z)) = \beta_\delta^N(f(y) *' f(z))$$
$$\leq \max\{\beta_\delta^N(f(x) *' f(z)), \beta_\delta^N(f(x) *' f(y))\}$$
$$= \max\{\beta_\delta^N(f(x * z)), \beta_\delta^N(f(x * y))\}$$
$$= \max\{\mu_\alpha^N(f(x * z)), \mu_\alpha^N(f(x * y))\}.$$

Hence the inverse image of a bipolar fuzzy α-ideal of BP-algebra is also bipolar fuzzy α-ideal.

Definition 4.14. A bipolar fuzzy subset $\alpha = (\mu_\alpha^P, \mu_\alpha^N)$ has "sup" and "inf" properties if for any subset S of X, there exist $c, b \in S$ such that $\mu_\alpha^P(c) = \sup_{c \in S} \mu_\alpha^P(S)$, and $\mu_\alpha^N(b) = \inf_{b \in S} \mu_\alpha^N(S)$.

Theorem 4.15. An onto homomorphic image of a bipolar fuzzy α-ideal of BP-algebra with "sup" and "inf" properties is a bipolar fuzzy α-ideal.

Proof. Let $f : (X, *, 0) \longrightarrow (Y, *', 0')$ be an onto homomorphism of BP-algebras and $\delta = (\beta_\delta^P, \beta_\delta^N)$ is a bipolar fuzzy α-ideal in Y. Let $\alpha = (\mu_\alpha^P, \mu_\alpha^N)$ is a bipolar fuzzy α-ideal in X with "sup" and "inf" properties. Then for all $x' \in Y$, we get $\beta_\delta^P(x') = \mu_\alpha^P(f^{-1}(x')) = \sup_{x \in f^{-1}(x')} \mu_\alpha^P(x)$ and $\beta_\delta^N(x') = \mu_\alpha^N(f^{-1}(x')) = \inf_{x \in f^{-1}(x')} \mu_\alpha^N(x)$. Since $\alpha = (\mu_\alpha^P, \mu_\alpha^N)$ is a bipolar fuzzy α-ideal in X.

We have $\mu_\alpha^P(0) \geq \mu_\alpha^P(x)$ and $\mu_\alpha^N(0) \leq \mu_\alpha^N(x)$. Note that $0 \in f^{-1}(0')$, such that 0 and $0'$are the zero elements of X and Y respectively.

Thus

$\beta_\delta^P(0') = \mu_\alpha^P(f^{-1}(0')) = \sup_{h \in f^{-1}(0')} \mu_\alpha^P(h) = \mu_\alpha^P(0) \geq \mu_\alpha^P(x)$ and $\beta_\delta^N(0') = \mu_\alpha^N(f^{-1}(0')) = \inf_{h \in f^{-1}(0')} \mu_\alpha^N(h) = \mu_\alpha^N(0) \leq \mu_\alpha^N(x)$. This implies that $\beta_\delta^P(0') \geq \sup_{h \in f^{-1}(x')} \mu_\alpha^P(h) = \beta_\delta^P(x')$, and $\beta_\delta^N(0') \leq \inf_{h \in f^{-1}(x')} \mu_\alpha^N(h) = \beta_\delta^N(x')$ for all $x' \in Y$. For any $x', y', z' \in Y$, let $x_o \in f^{-1}(x')$,

$y_o \in f^{-1}(y')$, $z_o \in f^{-1}(z')$ be such that $\mu_\alpha^P(y_0 * z_0) = \sup_{h \in f^{-1}(y' *' z')} \mu_\alpha^P(h)$ and $\mu_\alpha^P(x_0 * z_0) = \sup_{h \in f^{-1}(x' *' z')} \mu_\alpha^P(h)$ and $\mu_\alpha^P(x_0 * y_0) = \sup_{h \in f^{-1}(x' *' y')} \mu_\alpha^P(h)$.

Then

$$\beta_\delta^P(y' *' z') = \sup_{h \in f^{-1}(y' *' z')} \mu_\alpha^P(h)$$
$$= \mu_\alpha^P(y_0 * z_0) \geq \min\{\mu_\alpha^P(x_0 * z_0), \mu_\alpha^P(x_0 * y_0)\}$$
$$= \min\left\{\sup_{h \in f^{-1}(x' *' z')} \mu_\alpha^P(h), \sup_{h \in f^{-1}(x' *' y')} \mu_\alpha^P(h)\right\}$$
$$= \min\{\beta_\delta^P(x' *' z'), \beta_\delta^P(x' *' y')\}.$$

Similarly, we have $\mu_\alpha^N(y_0 * z_0) = \inf_{h \in f^{-1}(y' *' z')} \mu_\alpha^N(h)$ and $\mu_\alpha^N(x_0 * z_0) = \inf_{h \in f^{-1}(x' *' z')} \mu_\alpha^N(h)$ and $\mu_\alpha^N(x_0 * y_0) = \inf_{h \in f^{-1}(x' *' y')} \mu_\alpha^N(h)$.

Then

$$\beta_\delta^N(y' *' z') = \inf_{h \in f^{-1}(y' *' z')} \mu_\alpha^N(h)$$
$$= \mu_\alpha^N(y_0 * z_0) \leq \max\{\mu_\alpha^N(x_0 * z_0), \mu_\alpha^N(x_0 * y_0)\}$$
$$= \max\left\{\inf_{h \in f^{-1}(x' *' z')} \mu_\alpha^N(h), \inf_{h \in f^{-1}(x' *' y')} \mu_\alpha^N(h)\right\}$$
$$= \max\{\beta_\delta^N(x' *' z'), \beta_\delta^N(x' *' y')\}.$$

Hence the onto homomorphic image of a bipolar fuzzy α-ideal of BP-algebra is also bipolar fuzzy α-ideal.

5. Conclusions and Future Research

To investigate the structure of an algebraic system, it is clear that α-ideal with special properties plays an important role. In the present paper, we have applied the notion of the bipolar fuzzy set theory to α-ideal of BP-algebra and investigated some of their useful properties. In the future, these definitions and fundamental results can be applied to some different algebraic structures. There are more topics that could take advantage of α-ideal. Like for example cubic intuitionistic α-ideal of BP-algebra, cubic fuzzy α-ideal of BP-algebra, and fuzzy soft α-ideal in BP-algebra. There are many other aspects which should be explored and studied in the area of BP-algebra such as anti-fuzzy α-ideal of BP-algebra, interval-valued fuzzy α-ideal of BP-algebra, intuitionistic fuzzy α-ideal of BP-algebra, doubt intuitionistic fuzzy α-ideal of BP-algebra, fuzzy derivations α-ideal of BP-algebra, and interval-valued intuitionistic fuzzy α-ideal of BP-algebra. It is our hope that this work would other foundations for further study of the theory of BP-algebra.

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