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Biswas Distribution

Deapon Biswas

Transport Officer, Private Concern, Chittagong, Bangladesh

Abstract Here we get all the theoretical discrete distributions to the random experiment "M sided N dice taken V at a time". It states joint probability functions as well as marginal probability functions of which number of possible outcomes is EB numbers and number of favorable outcomes is GB numbers. In this paper I introduce 20 definitions where first is the Biswas distribution and others are the related distributions that state different joint probability functions and marginal probability functions.

Keywords Biswas distribution, SB distribution, AB distribution, SAB distribution, ASB distribution, First way B distribution, Second way B distribution, Both way B distribution, First way SB distribution, Second way SB distribution, Both way SB distribution, First way AB distribution, Second way AB distribution, Both way AB distribution, First way SAB distribution, Second way SAB distribution, Both way SAB distribution, First way ASB distribution, Second way ASB distribution, Both way ASB distribution

1. Introduction

First of all Biswas distributions are divided into four parts i.e.,

(i) both way selected B distribution (SB distribution)
(ii) both way arranged B distribution (AB distribution)
(iii) first way selected and second way arranged B distribution (SAB distribution)
(iv) first way arranged and second way selected B distribution (ASB distribution)

And then each part divided into three parts again i.e., for B distribution we get

(i) first way B distribution
(ii) second way B distribution
(iii) both way B distribution.

2. Findings

Definition 3.1 Biswas distribution: A two dimensional random variable (X, Y) is said to follow Biswas distribution if it assumes only non negative values and its joint probability mass function is given by

P(X, Y) = b(X, Y ; M, N, U, V)

$$= \frac{GB\binom{M\ N\ U}{V\ X\ Y}}{EB\binom{M\ N}{V}}; \ \min(M, U)-M+1 \le X \le \min(M, U, V)$$

$$\min(N, U)-N+V \le Y \le \min(N, U, V)$$

M ≥ U and N ≥ U for first way and second way respectively.

$$= 0; \text{otherwise.} \qquad (3.1)$$

The four independent finite constants M, N, U and V are known as the parameters of this distribution. Biswas distribution is a joint probability distribution and a theoretical discrete distribution. Here X takes only non-negative values under the interval min(M, U)−M +1 ≤ X ≤ min(M, U, V) and Y takes only non-negative values under the interval min(N, U)−N+V ≤Y ≤ min(N, U, V). Any two dimensional random variable which follows B distribution is known as B-variate and denoted by the symbol (X, Y) ~ B(M, N, U, V).

Remark: It should be noted that

$$\sum_X \sum_Y b(X, Y; M, N, U, V) = \sum_X \sum_Y \frac{GB\binom{M\ N\ U}{V\ X\ Y}}{EB\binom{M\ N}{V}} = 1$$

$$\Rightarrow EB\binom{M\ N}{V} = \sum_X \sum_Y GB\binom{M\ N\ U}{V\ X\ Y} \qquad (3.2)$$

Example 3.1: For the GB experiment $GB\begin{bmatrix} 5 & 6 & 3 \\ 4 & 2 & 1 \end{bmatrix}$ find P(X = 2, Y = 1) where the experiment is first way arranged and second way selected.

Solution: We know

$$P(X = 2, Y = 1) = b(2, 1; 5, 6, 3, 4) = \frac{GB\binom{5\ 6\ 3}{4\ 2\ 1}}{EB\binom{5\ 6}{4}} = \frac{990}{9375}$$

= 0.105.

Definition 3.2 SB distribution: A two dimensional random variable (X, Y) is said to follow SB distribution if it assumes only non-negative values and its joint probability mass function is given by

* Corresponding author:
philosclub@yahoo.com (Deapon Biswas)
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P(X, Y) = sb(X, Y; M, N, U, V)

$= \frac{GSB\binom{M\ N\ U}{V\ X\ Y}}{ESB\binom{M\ N}{V}}$; X, Y, U and V value as (3.1)

= 0; otherwise. (3.3)

Remark: It should be noted that

$$\sum_X \sum_Y sb(X, Y; M, N, U, V) = \sum_X \sum_Y \frac{GSB\binom{M\ N\ U}{V\ X\ Y}}{ESB\binom{M\ N}{V}} = 1$$

$$\Rightarrow ESB\binom{M\ N}{V} = \sum_X \sum_Y GSB\binom{M\ N\ U}{V\ X\ Y} \quad (3.4)$$

Example 3.2: For the GSB experiment $GSB\begin{bmatrix}4\ 5\ 3\\3\ X\ Y\end{bmatrix}$ find (i) P(X =2, Y=3) and (ii) P(X=3,Y=1).

Solution: We get

(i) P(X = 2, Y = 3) = sb(2, 3; 4, 5, 3, 3) = $\frac{GSB\binom{4\ 5\ 3}{3\ 2\ 3}}{ESB\binom{4\ 5}{3}} = \frac{9}{200}$ = 0.045.

(ii) P(X = 3, Y = 1) = sb(3, 1; 4, 5, 3, 3) = $\frac{GSB\binom{4\ 5\ 3}{3\ 3\ 1}}{ESB\binom{4\ 5}{3}} = \frac{3}{200}$ = 0.015.

Definition 3.3 AB distribution: A two dimensional random variable (X, Y) is said to follow AB distribution if it assumes only non-negative values and its joint probability mass function is given by

P(X, Y) = ab(X, Y; M, N, U, V)

$= \frac{GAB\binom{M\ N\ U}{V\ X\ Y}}{EAB\binom{M\ N}{V}}$; X, Y, U and V value as (3.1)

= 0; otherwise. (3.5)

Remark: It should be noted that

$$\sum_X \sum_Y ab(X, Y; M, N, U, V) = \sum_X \sum_Y \frac{GAB\binom{M\ N\ U}{V\ X\ Y}}{EAB\binom{M\ N}{V}} = 1$$

$$\Rightarrow EAB\binom{M\ N}{V} = \sum_X \sum_Y GAB\binom{M\ N\ U}{V\ X\ Y} \quad (3.6)$$

Example 3.3: For the GAB experiment $GAB\begin{bmatrix}5\ 6\ 3\\4\ X\ Y\end{bmatrix}$ find (i) P(X=1,Y=3) and (ii) P(X=2,Y=2).

Solution: We get

(i) P(X = 1, Y = 3) = ab(1, 3; 5, 6, 3, 4) = $\frac{GAB\binom{5\ 6\ 3}{4\ 1\ 3}}{EAB\binom{5\ 6}{4}} = \frac{14040}{225000}$ = 0.0624.

(ii) P(X = 2, Y = 2) = ab(2, 2; 5, 6, 3, 4) = $\frac{GAB\binom{5\ 6\ 3}{4\ 2\ 2}}{EAB\binom{5\ 6}{4}} = \frac{71280}{225000}$ = 0.316.

Definition 3.4 SAB distribution: A two dimensional random variable (X, Y) is said to follow SAB distribution if it assumes only non-negative values and its joint probability mass function is given by

P(X, Y) = sab(X, Y; M, N, U, V)

$= \frac{GSAB\binom{M\ N\ U}{V\ X\ Y}}{ESAB\binom{M\ N}{V}}$; X, Y, U and V value as (3.1)

= 0; otherwise. (3.7)

Remark: It should be noted that

$$\sum_X \sum_Y sab(X, Y; M, N, U, V) = \sum_X \sum_Y \frac{GSAB\binom{M\ N\ U}{V\ X\ Y}}{ESAB\binom{M\ N}{V}} = 1$$

$$\Rightarrow ESAB\binom{M\ N}{V} = \sum_X \sum_Y GSAB\binom{M\ N\ U}{V\ X\ Y} \quad (3.8)$$

Example 3.4: Given the following favorite probability distribution of the experiment $GSAB\begin{bmatrix}4\ 5\ 4\\3\ X\ Y\end{bmatrix}$.

X \ Y	2	3	4
0	0.006	0.008	0.001
1	0.091	0.122	0.015
2	0.206	0.274	0.034
3	0.091	0.122	0.015
4	0.006	0.008	0.001

Find the probability of getting (i) X = 0, Y = 2, (ii) X = 2, Y = 3 and (iii) Y = 4.

Solution:

X \ Y	2	3	4	P(X)
0	0.006	0.008	0.001	0.015
1	0.091	0.122	0.015	0.228
2	0.206	0.274	0.034	0.514
3	0.091	0.122	0.015	0.228
4	0.006	0.008	0.001	0.015
P(Y)	0.400	0.534	0.066	1

We get from the above table

(i) Probability of getting X = 0, Y = 2 i.e., P(X = 0, Y = 2) = 0.06
(ii) Probability of getting X = 2, Y = 3 i.e., P(X = 2, Y = 3) = 0.274
(iii) Probability of getting Y = 4 i.e., P(Y = 4) = 0.066.

Definition 3.5 ASB distribution: A two dimensional random variable (X, Y) is said to follow ASB distribution if it assumes only non-negative values and its joint probability mass function is given by

P(X, Y) = asb(X, Y; M, N, U, V)

$= \frac{GASB\binom{M\ N\ U}{V\ X\ Y}}{EASB\binom{M\ N}{V}}$; X, Y, U and V value as (3.1)

= 0; otherwise. (3.9)

Remark: It should be noted that

$$\sum_X \sum_Y asb(X, Y; M, N, U, V) = \sum_X \sum_Y \frac{GASB\binom{M\ N\ U}{V\ X\ Y}}{EASB\binom{M\ N}{V}} = 1$$

$$\Rightarrow EASB\binom{M\ N}{V} = \sum_X \sum_Y GASB\binom{M\ N\ U}{V\ X\ Y} \quad (3.10)$$

Example 3.5: Given the following bivariate probability distribution of the experiment GASB$\begin{bmatrix} 6\ 7\ 3 \\ 5\ X\ Y \end{bmatrix}$.

X \ Y	1	2	3
0	0.004	0.018	0.009
1	0.043	0.172	0.086
2	0.073	0.291	0.146
3	0.023	0.090	0.045

Find (i) P(X ≥ 2), (ii) P(X = 0, Y = 3), (iii) P(X ≤ 2, Y = 2) and (iv) P(X = 3, Y ≥ 2).

Solution:

X \ Y	1	2	3	P(X)
0	0.004	0.018	0.009	0.031
1	0.043	0.172	0.086	0.301
2	0.073	0.291	0.146	0.510
3	0.023	0.090	0.045	0.158
P(Y)	0.143	0.571	0.286	1

Solution: We get from the above table

(i) P(X ≥ 2) = P(X = 2) + P(X = 3) = 0.510 + 0.158 = 0.668
(ii) P(X = 0, Y = 3) = 0.009
(iii) P(X ≤ 2, Y = 2) = P(X = 0, Y = 2) + P(X = 1, Y = 2) + P(X= 2, Y= 2)
= 0.018 + 0.172 + 0.291 = 0.481
(iv) P(X = 3, Y ≥ 2) = P(X = 3, Y = 2) + P(X = 3, Y = 3)
= 0.090 + 0.045= 0.135

Now we describe three kinds of B distributions i.e., first way B distribution, second way B distribution and both way B distribution and their different forms.

Definition 3.6 First way B distribution: Let the joint distribution of two random variables X and Y is given then the first way B distribution can be written as

$$P(X) = b_1(X;\ M, N, U, V)$$
$$= \frac{GB_1\binom{M\ N\ U}{V\ X}}{EB\binom{M\ N}{V}}\ ;\ \text{X, U and V value as (3.1)}$$
$$= 0;\ \text{otherwise.} \quad (3.11)$$

Remark: It should be noted that

$$\sum_X P(X) = \sum_X b_1(X;\ M, N, U, V) = 1$$
$$\text{i.e., } \sum_X \frac{GB_1\binom{M\ N\ U}{V\ X}}{EB\binom{M\ N}{V}} = 1$$
$$\Rightarrow EB\binom{M\ N}{V} = \sum_X GB_1\binom{M\ N\ U}{V\ X} \quad (3.12)$$

The first way B distribution is also known as the marginal probability distribution of X.

Example 3.6: Given the following bivariate probability distribution of example 3.4 find first way B distribution of (i) X = 0 and (ii) X = 2.

Solution: From the given table we get (i) P(X = 0) = 0.015 and (ii) P(X = 2) = 0.514.

Definition 3.7 Second way B distribution: Let the joint distribution of two random variables X and Y is given then the second way B distribution can be written as

$$P(Y) = b_2(Y;\ M, N, U, V)$$
$$= \frac{GB_2\binom{M\ N\ U}{V\ Y}}{EB\binom{M\ N}{V}}\ ;\ \text{Y, U and V value as (3.1)}$$
$$= 0;\ \text{otherwise.} \quad (3.13)$$

Remark: It should be noted that

$$\sum_Y P(Y) = \sum_Y b_2(Y;\ M, N, U, V) = 1$$
$$\text{i.e., } \sum_Y \frac{GB_2\binom{M\ N\ U}{V\ Y}}{EB\binom{M\ N}{V}} = 1$$
$$\Rightarrow EB\binom{M\ N}{V} = \sum_Y GB_2\binom{M\ N\ U}{V\ Y} \quad (3.14)$$

The second way B distribution is also known as the marginal probability distribution of Y.

Example 3.7: Given the following bivariate probability distribution of example 3.4 find second way B distribution of (i) Y = 2 and (ii) Y = 4.

Solution: From the given table we get (i) P(Y = 2) = 0.400 and (ii) P(Y = 4) = 0.066.

Definition 3.8 Both way B distribution: A two dimensional random variable (X , Y) is said to follow both way B distribution if it assumes only non-negative values and its joint probability mass function is given by

$$P(X, Y) = b_{12}(X, Y;\ M, N, U, V)$$
$$= \frac{GB_{12}\binom{M\ N\ U}{V\ X\ Y}}{EB\binom{M\ N}{V}}\ ;\ \text{X, Y, U and V value as (3.1)}$$
$$= 0;\ \text{otherwise.} \quad (3.15)$$

Remark: It should be noted that

$$\sum_X \sum_Y b_{12}(X, Y;\ M, N, U, V) = 1$$
$$\text{i.e., } \sum_X \sum_Y \frac{GB_{12}\binom{M\ N\ U}{V\ X\ Y}}{EB\binom{M\ N}{V}} = 1$$
$$\Rightarrow EB\binom{M\ N}{V} = \sum_X \sum_Y GB_{12}\binom{M\ N\ U}{V\ X\ Y} \quad (3.16)$$

The both way B distribution is also known as the joint probability distribution of X and Y.

Example 3.8: Let the bivariate probability distribution of example 3.4 find both way B distribution of (i) P(X = 0, Y = 4) and (ii) P(X = 4, Y = 3).

Solution: From the given table we get (i) P(X = 0, Y= 4) = 0.001 and (ii) P(X = 4, Y = 3) = 0.008.

Definition 3.9 First way SB distribution: Let the joint distribution of two random variables X and Y is given then the first way SB distribution can be written as

$$P(X) = sb_1(X;\ M, N, U, V)$$
$$= \frac{GSB_1\binom{M\ N\ U}{V\ X}}{ESB\binom{M\ N}{V}}\ ;\ \text{X, U and V value as (3.1)}$$
$$= 0;\ \text{otherwise.} \quad (3.17)$$

Remark: It should be noted that

$\sum_X P(X) = \sum_X sb_1(X;\ M, N, U, V) = 1$

i.e., $\sum_X \frac{GSB_1\binom{M\ N\ U}{V\ X}}{ESB\binom{M\ N}{V}} = 1$

$$\Rightarrow ESB\binom{M\ N}{V} = \sum_X GSB_1\binom{M\ N\ U}{V\ X} \qquad (3.18)$$

Example 3.9: Consider the GSB experiment $GSB_1\begin{bmatrix}6\ 6\ 3\\ 4\ X\end{bmatrix}$. Find the probability of getting (i) X = 1 and (ii) X = 2.

Solution: We get the probabilities as

(i) $P(X=1) = \frac{GSB_1\binom{6\ 6\ 3}{4\ 1}}{ESB\binom{6\ 6}{4}} = \frac{F\binom{6\ 3}{4\ 1}C\binom{6}{4}}{F\binom{6}{4}C\binom{6}{4}} = \frac{900}{1890} = 0.476$

(ii) $P(X=2) = \frac{GSB_1\binom{6\ 6\ 3}{4\ 2}}{ESB\binom{6\ 6}{4}} = \frac{F\binom{6\ 3}{4\ 2}C\binom{6}{4}}{F\binom{6}{4}C\binom{6}{4}} = \frac{675}{1890} = 0.357.$

Definition 3.10 Second way SB distribution: Let the joint distribution of two random variables X and Y is given then the second way SB distribution can be written as

$P(Y) = sb_2(Y;\ M, N, U, V)$

$= \frac{GSB_2\binom{M\ N\ U}{V\ Y}}{ESB\binom{M\ N}{V}}$; Y, U and V value as (3.1)

$= 0$; otherwise. (3.19)

Remark: It should be noted that

$\sum_Y P(Y) = \sum_Y sb_2(Y;\ M, N, U, V) = 1$

i.e., $\sum_Y \frac{GSB_2\binom{M\ N\ U}{V\ Y}}{ESB\binom{M\ N}{V}} = 1$

$$\Rightarrow ESB\binom{M\ N}{V} = \sum_Y GSB_2\binom{M\ N\ U}{V\ Y} \qquad (3.20)$$

Example 3.10: Consider the GSB experiment $GSB_2\begin{bmatrix}6\ 6\ 3\\ 4\ Y\end{bmatrix}$. Find the probability of getting

(i) Y = 1 and (ii) Y = 3.

Solution: We get the probabilities as

(i) $P(Y=1) = \frac{GSB_2\binom{6\ 6\ 3}{4\ 1}}{ESB\binom{6\ 6}{4}} = \frac{F\binom{6}{4}C\binom{6\ 3}{4\ 1}}{F\binom{6}{4}C\binom{6}{4}} = \frac{378}{1890} = 0.2$

(ii) $P(Y=2) = \frac{GSB_2\binom{6\ 6\ 3}{4\ 1}}{ESB\binom{6\ 6}{4}} = \frac{F\binom{6}{4}C\binom{6\ 3}{4\ 3}}{F\binom{6}{4}C\binom{6}{4}} = \frac{378}{1890} = 0.2.$

Definition 3.11 Both way SB distribution: A two dimensional random variable (X, Y) is said to follow both way SB distribution if it assumes only non-negative values and its joint probability mass function is given by

$P(X, Y) = sb_{12}(X, Y;\ M, N, U, V)$

$= \frac{GSB_{12}\binom{M\ N\ U}{V\ X\ Y}}{ESB\binom{M\ N}{V}}$; X, Y, U and V value as (3.1)

$= 0$; otherwise. (3.21)

Remark: It should be noted that

$\sum_X \sum_Y sb_{12}(X, Y;\ M, N, U, V) = 1$

i.e., $\sum_X \sum_Y \frac{GSB_{12}\binom{M\ N\ U}{V\ X\ Y}}{ESB\binom{M\ N}{V}} = 1$

$$\Rightarrow ESB\binom{M\ N}{V} = \sum_X \sum_Y GSB_{12}\binom{M\ N\ U}{V\ X\ Y} \qquad (3.22)$$

Example 3.11: Consider the GSB experiment $GSB_{12}\begin{bmatrix}6\ 6\ 3\\ 4\ X\ Y\end{bmatrix}$. Find the probability of getting (i) X = 0, Y = 2 and (ii) X = 3, Y = 3.

Solution: We get the probabilities as

(i) $P(X=0, Y=2) = \frac{GSB_{12}\binom{6\ 6\ 3}{4\ 0\ 2}}{ESB\binom{6\ 6}{4}} = \frac{F\binom{6\ 3}{4\ 0}C\binom{6\ 3}{4\ 2}}{F\binom{6}{4}C\binom{6}{4}} = \frac{135}{1890} = 0.071$

(ii) $P(X=3, Y=3) = \frac{GSB_{12}\binom{6\ 6\ 3}{4\ 3\ 3}}{ESB\binom{6\ 6}{4}} = \frac{F\binom{6\ 3}{4\ 3}C\binom{6\ 3}{4\ 3}}{F\binom{6}{4}C\binom{6}{4}} = \frac{18}{1890} = 0.009.$

Definition 3.12 First way AB distribution: Let the joint distribution of two random variables X and Y is given then the first way AB distribution can be written as

$P(X) = ab_1(X;\ M, N, U, V)$

$= \frac{GAB_1\binom{M\ N\ U}{V\ X}}{EAB\binom{M\ N}{V}}$; X, U and V value as (3.1)

$= 0$; otherwise. (3.23)

Remark: It should be noted that

$\sum_X P(X) = \sum_X ab_1(X;\ M, N, U, V) = 1$

i.e., $\sum_X \frac{GAB_1\binom{M\ N\ U}{V\ X}}{EAB\binom{M\ N}{V}} = 1$

$$\Rightarrow EAB\binom{M\ N}{V} = \sum_X GAB_1\binom{M\ N\ U}{V\ X} \qquad (3.24)$$

Example 3.12: Consider the GAB experiment $GAB_1\begin{bmatrix}4\ 5\ 4\\ 3\ X\end{bmatrix}$. Find the probability of getting (i) X = 1 and (ii) X = 3.

Solution: We get the probabilities as

(i) $P(X=1) = \frac{GAB_1\binom{4\ 5\ 4}{3\ 1}}{EAB\binom{4\ 5}{3}} = \frac{H\binom{4\ 4}{3\ 1}P\binom{5}{3}}{H\binom{4}{3}P\binom{5}{3}} = \frac{240}{3840} = 0.062$

(ii) $P(X=3) = \frac{GAB_1\binom{4\ 5\ 4}{3\ 3}}{EAB\binom{4\ 5}{3}} = \frac{H\binom{4\ 4}{3\ 3}P\binom{5}{3}}{H\binom{4}{3}P\binom{5}{3}} = \frac{1440}{3840} = 0.375.$

Definition 3.13 Second way AB distribution: Let the joint distribution of two random variables X and Y is given then the second way AB distribution can be written as

$P(Y) = ab_2(Y;\ M, N, U, V)$

$= \frac{GAB_2\binom{M\ N\ U}{V\ Y}}{EAB\binom{M\ N}{V}}$; Y, U and V value as (3.1)

$= 0$; otherwise. (3.25)

Remark: It should be noted that

$\sum_Y P(Y) = \sum_Y ab_2(Y;\ M, N, U, V) = 1$

i.e., $\sum_Y \frac{GAB_2\binom{M\ N\ U}{V\ Y}}{EAB\binom{M\ N}{V}} = 1$

$$\Rightarrow EAB\binom{M\ N}{V} = \sum_Y GAB_2\binom{M\ N\ U}{V\ Y} \qquad (3.26)$$

Example 3.13: Consider the GAB experiment $GAB_2\begin{bmatrix}4\ 5\ 4\\ 3\ Y\end{bmatrix}$. Find the probability of getting (i) Y = 2 and (ii)

$Y = 3$.

Solution: We get the probabilities as

(i) $P(Y=2) = \frac{GAB_2\binom{4\,5\,4}{3\,2}}{EAB\binom{4\,5}{3}} = \frac{H\binom{4}{3}P\binom{5\,4}{3\,2}}{H\binom{4}{3}P\binom{5}{3}} = \frac{2304}{3840} = 0.6.$

(ii) $P(Y=3) = \frac{GAB_2\binom{4\,5\,4}{3\,3}}{EAB\binom{4\,5}{3}} = \frac{H\binom{4}{3}P\binom{5\,4}{3\,3}}{H\binom{4}{3}P\binom{5}{3}} = \frac{1536}{3840} = 0.4.$

Definition 3.14 Both way AB distribution: A two dimensional random variable (X, Y) is said to follow both way AB distribution if it assumes only non-negative values and its joint probability mass function is given by

$$P(X, Y) = ab_{12}(X, Y; M, N, U, V)$$

$$= \frac{GAB_{12}\binom{M\,N\,U}{V\,X\,Y}}{EAB\binom{M\,N}{V}} \text{ ; X, Y, U and V value as (3.1)}$$

$$= 0\text{; otherwise.} \tag{3.27}$$

Remark: It should be noted that

$$\sum_X \sum_Y ab_{12}(X, Y;\ M, N, U, V) = 1$$

i.e., $\sum_X \sum_Y \frac{GAB_{12}\binom{M\,N\,U}{V\,X\,Y}}{EAB\binom{M\,N}{V}} = 1$

$$\Rightarrow EAB\binom{M\,N}{V} = \sum_X \sum_Y GAB_{12}\binom{M\,N\,U}{V\,X\,Y} \tag{3.28}$$

Example 3.14: Consider the GAB experiment $GAB_{12}\begin{bmatrix}4\,5\,4\\3\,X\,Y\end{bmatrix}$. Find the probability of getting (i) X = 1, Y = 2 and (ii) X = 3, Y = 3.

Solution: We get the probabilities as

(i) $P(X = 1, Y = 2) = \frac{GAB_{12}\binom{4\,5\,4}{3\,1\,2}}{EAB\binom{4\,5}{3}} = \frac{F\binom{4\,4}{3\,1}C\binom{5\,4}{3\,2}}{H\binom{4}{3}P\binom{5}{3}} = \frac{144}{3840} = 0.037$

(ii) $P(X = 3, Y = 3) = \frac{GAB_{12}\binom{4\,5\,4}{3\,3\,3}}{EAB\binom{4\,5}{3}} = \frac{F\binom{4\,4}{3\,3}C\binom{5\,4}{3\,3}}{H\binom{4}{3}P\binom{5}{3}} = \frac{576}{3840} = 0.15.$

Definition 3.15 First way SAB distribution: Let the joint distribution of two random variables X and Y is given then the first way SAB distribution can be written as

$$P(X) = sab_1(X; M, N, U, V)$$

$$= \frac{GSAB_1\binom{M\,N\,U}{V\,X}}{ESAB\binom{M\,N}{V}} \text{ ; X, U and V value as (3.1)}$$

$$= 0\text{; otherwise.} \tag{3.29}$$

Remark: It should be noted that

$$\sum_X P(X) = \sum_X sab_1(X; M, N, U, V) = 1$$

i.e., $\sum_X \frac{GSAB_1\binom{M\,N\,U}{V\,X}}{ESAB\binom{M\,N}{V}} = 1$

$$\Rightarrow ESAB\binom{M\,N}{V} = \sum_X GSAB_1\binom{M\,N\,U}{V\,X} \tag{3.30}$$

Example 3.15: Consider the GSAB experiment $GSAB_1\begin{bmatrix}4\,5\,4\\3\,X\end{bmatrix}$. Find the probability of getting (i) $X \geq 2$ and (ii) $X \leq 1$.

Solution: The table of example 27.3.4 represents bivatrate probability distribution of the experiment $GSAB_1\begin{bmatrix}4\,5\,4\\3\,X\end{bmatrix}$. Now we get from the table

(i) $P(X \geq 2) = P(X = 2) + P(X = 3) + P(X = 4) = 0.514 + 0.228 + 0.015 = 0.757$

(ii) $P(X \leq 1) = P(X = 0) + P(X = 1) = 0.015 + 0.228 = 0.243$.

Definition 3.16 Second way SAB distribution: Let the joint distribution of two random variables X and Y is given then the second way SAB distribution can be written as

$$P(Y) = sab_2(Y; M, N, U, V)$$

$$= \frac{GSAB_2\binom{M\,N\,U}{V\,Y}}{ESAB\binom{M\,N}{V}} \text{ ; Y, U and V value as (3.1)}$$

$$= 0\text{; otherwise.} \tag{3.31}$$

Remark: It should be noted that

$$\sum_Y P(Y) = \sum_Y sab_2(Y; M, N, U, V) = 1$$

i.e., $\sum_Y \frac{GSAB_2\binom{M\,N\,U}{V\,Y}}{ESAB\binom{M\,N}{V}} = 1$

$$\Rightarrow ESAB\binom{M\,N}{V} = \sum_Y GSAB_2\binom{M\,N\,U}{V\,Y} \tag{3.32}$$

Example 3.16: Consider the GSAB experiment $GSAB_2\begin{bmatrix}4\,5\,4\\3\,Y\end{bmatrix}$. Find the probability of (i) Y = 3 and (ii) $Y \leq 3$.

Solution: The table of example 27.3.4 represents bi-variate probability distribution of the experiment $GSAB_2\begin{bmatrix}4\,5\,4\\3\,Y\end{bmatrix}$. Now we get from the table

(i) $P(Y = 3) = 0.534$

(ii) $P(Y \leq 3) = P(Y = 2) + P(Y = 3) = 0.400 + 0.534 = 0.934$.

Definition 3.17 Both way SAB distribution: A two dimensional random variable (X, Y) is said to follow both way SAB distribution if it assumes only non- negative values and its joint probability mass function is given by

$$P(X, Y) = sab_{12}(X, Y; M, N, U, V)$$

$$= \frac{GSAB_{12}\binom{M\,N\,U}{V\,X\,Y}}{ESAB\binom{M\,N}{V}} \text{ ; X, Y, U and V value as (3.1)}$$

$$= 0\text{; otherwise.} \tag{3.33}$$

Remark: It should be noted that

$$\sum_X \sum_Y sab_{12}(X, Y;\ M, N, U, V) = 1$$

i.e., $\sum_X \sum_Y \frac{GSAB_{12}\binom{M\,N\,U}{V\,X\,Y}}{ESAB\binom{M\,N}{V}} = 1$

$$\Rightarrow ESAB\binom{M\,N}{V} = \sum_X \sum_Y GSAB_{12}\binom{M\,N\,U}{V\,X\,Y} \tag{3.34}$$

Example 3.17: Consider the GSAB experiment $GSAB_{12}\begin{bmatrix}4\,5\,4\\3\,X\,Y\end{bmatrix}$. Find the probability of getting

(i) X = 1, Y = 3 and (ii) X = 2, Y = 2.

Solution: The table of example 27.3.4 represents bi-variate probability distribution of the experiment $GSAB_{12}\begin{bmatrix}4\,5\,4\\3\,x\,y\end{bmatrix}$. Now we get from the table

(i) $P(X = 1, Y = 3) = 0.122$

(ii) P(X = 2, Y = 2) = 0.206.

Definition 3.18 First way ASB distribution: Let the joint distribution of two random variables X and Y is given then the first way ASB distribution can be written as

$$P(X) = asb_1(X; M, N, U, V)$$
$$= \frac{GASB_1\binom{M\ N\ U}{V\ X}}{EASB\binom{M\ N}{V}}\ ; \text{X, U and V value as (3.1)}$$
$$= 0;\ \text{otherwise.} \quad (3.35)$$

Remark: It should be noted that

$$\sum_X P(X) = \sum_X asb_1(X; M, N, U, V) = 1$$

i.e., $\sum_X \frac{GASB_1\binom{M\ N\ U}{V\ X}}{EASB\binom{M\ N}{V}} = 1$

$$\Rightarrow EASB\binom{M\ N}{V} = \sum_X GASB_1\binom{M\ N\ U}{V\ X} \quad (3.36)$$

Example 3.18: Consider the GASB experiment $GASB_1\begin{bmatrix}6\ 7\ 3\\ 5\ X\end{bmatrix}$. Find the probability of getting (i) X = 0 and (ii) X = 3.

Solution: The table of example 27.3.5 represents bi-variate probability distribution of the experiment $GASB_1\begin{bmatrix}6\ 7\ 3\\ 5\ X\end{bmatrix}$. Now we get from the table

(i) P(X = 0) = 0.031
(ii) P(X = 3) = 0.158.

Definition 3.19 Second way ASB distribution: Let the joint distribution of two random variables X and Y is given then the second way ASB distribution can be written as

$$P(Y) = asb_2(Y; M, N, U, V)$$
$$= \frac{GASB_2\binom{M\ N\ U}{V\ Y}}{EASB\binom{M\ N}{V}}\ ; \text{Y, U and V value as (3.1)}$$
$$= 0;\ \text{otherwise.} \quad (3.37)$$

Remark: It should be noted that

$$\sum_Y P(Y) = \sum_Y asb_2(Y; M, N, U, V) = 1$$

i.e., $\sum_Y \frac{GASB_2\binom{M\ N\ U}{V\ Y}}{EASB\binom{M\ N}{V}} = 1$

$$\Rightarrow EASB\binom{M\ N}{V} = \sum_Y GASB_2\binom{M\ N\ U}{V\ Y} \quad (3.38)$$

Example 3.19: Consider the GASB experiment $GASB_2\begin{bmatrix}6\ 7\ 3\\ 5\ Y\end{bmatrix}$. Find the probability of getting (i) Y = 2 and (ii) Y ≤ 2.

Solution: The table of example 27.3.5 represents bi-vatrate probability distribution of the experiment $GASB_2\begin{bmatrix}6\ 7\ 3\\ 5\ Y\end{bmatrix}$. Now we get from the table

(i) P(Y = 2) = 0.571
(ii) P(Y ≤ 2) = P(Y = 1) + P(Y = 2) = 0.143 + 0.571= 0.714.

Definition 3.20 Both way ASB distribution: A two dimensional random variable (X, Y) is said to follow both way ASB distribution if it assumes only non negative values and its joint probability mass function is given by

$$P(X, Y) = asb_{12}(X, Y; M, N, U, V)$$
$$= \frac{GASB_{12}\binom{M\ N\ U}{V\ X\ Y}}{EASB\binom{M\ N}{V}}\ ; \text{X, Y, U and V value as (3.1)}$$
$$= 0;\ \text{otherwise.} \quad (3.39)$$

Remark: It should be noted that

$$\sum_X \sum_Y asb_{12}(X, Y; M, N, U, V) = 1$$

i.e., $\sum_X \sum_Y \frac{GASB_{12}\binom{M\ N\ U}{V\ X\ Y}}{EASB\binom{M\ N}{V}} = 1$

$$\Rightarrow EASB\binom{M\ N}{V} = \sum_X \sum_Y GASB_{12}\binom{M\ N\ U}{V\ X\ Y} \quad (3.40)$$

Example 3.20: Consider the GASB experiment $GASB_{12}\begin{bmatrix}6\ 7\ 3\\ 5\ X\ Y\end{bmatrix}$. Find the probability of getting (i) X = 0, Y = 2 and (ii) X = 2, Y = 3.

Solution: The table of example 27.3.5 represents bi-variate probability distribution of the experiment $GASB_{12}\begin{bmatrix}6\ 7\ 3\\ 5\ X\ Y\end{bmatrix}$. Now we get from the table

(i) P(X = 0, Y = 2) = 0.018
(ii) P(X = 2, Y = 3) = 0.146.

3. Applications

From the definitions stated in this paper we can find various joint probability functions, marginal probability functions and conditional probability functions.

4. Main Results at a Glance

The following is a list of probability laws developed in this paper.

(i) $b(X, Y; M, N, U, V) = \frac{GB\binom{M\ N\ U}{V\ X\ Y}}{EB\binom{M\ N}{V}}$

(ii) $sb(X, Y; M, N, U, V) = \frac{GSB\binom{M\ N\ U}{V\ X\ Y}}{ESB\binom{M\ N}{V}}$

(iii) $ab(X, Y; M, N, U, V) = \frac{GAB\binom{M\ N\ U}{V\ X\ Y}}{EAB\binom{M\ N}{V}}$

(iv) $sab(X, Y; M, N, U, V) = \frac{GSAB\binom{M\ N\ U}{V\ X\ Y}}{ESAB\binom{M\ N}{V}}$

(v) $asb(X, Y; M, N, U, V) = \frac{GASB\binom{M\ N\ U}{V\ X\ Y}}{EASB\binom{M\ N}{V}}$

(vi) $b_1(X; M, N, U, V) = \frac{GB_1\binom{M\ N\ U}{V\ X}}{EB\binom{M\ N}{V}}$

(vii) $b_2(Y; M, N, U, V) = \frac{GB_2\binom{M\ N\ U}{V\ Y}}{EB\binom{M\ N}{V}}$

(viii) $b_{12}(X, Y; M, N, U, V) = \frac{GB_{12}\binom{M\ N\ U}{V\ X\ Y}}{EB\binom{M\ N}{V}}$

(ix) $sb_1(X; M, N, U, V) = \frac{GSB_1\binom{M\ N\ U}{V\ X}}{ESB\binom{M\ N}{V}}$

(x) $sb_2(Y; M, N, U, V) = \frac{GSB_2\binom{M\ N\ U}{V\ Y}}{ESB\binom{M\ N}{V}}$

(xi) $sb_{12}(X,Y;\ M,N,U,V) = \frac{GSB_{12}\binom{M\ N\ U}{V\ X\ Y}}{ESB\binom{M\ N}{V}}$

(xii) $ab_{1}(X;\ M,N,U,V) = \frac{GAB_{1}\binom{M\ N\ U}{V\ X}}{EAB\binom{M\ N}{V}}$

(xiii) $ab_{2}(Y;\ M,N,U,V) = \frac{GAB_{2}\binom{M\ N\ U}{V\ Y}}{EAB\binom{M\ N}{V}}$

(xiv) $ab_{12}(X,Y;\ M,N,U,V) = \frac{GAB_{12}\binom{M\ N\ U}{V\ X\ Y}}{EAB\binom{M\ N}{V}}$

(xv) $sab_{1}(X;\ M,N,U,V) = \frac{GSAB_{1}\binom{M\ N\ U}{V\ X}}{ESAB\binom{M\ N}{V}}$

(xvi) $sab_{2}(Y;\ M,N,U,V) = \frac{GSAB_{2}\binom{M\ N\ U}{V\ Y}}{ESAB\binom{M\ N}{V}}$

(xvii) $sab_{12}(X,Y;\ M,N,U,V) = \frac{GSAB_{12}\binom{M\ N\ U}{V\ X\ Y}}{ESAB\binom{M\ N}{V}}$

(xviii) $asb_{1}(X;\ M,N,U,V) = \frac{GASB_{1}\binom{M\ N\ U}{V\ X}}{EASB\binom{M\ N}{V}}$

(xix) $asb_{2}(Y;\ M,N,U,V) = \frac{GASB_{2}\binom{M\ N\ U}{V\ Y}}{EASB\binom{M\ N}{V}}$

(xx) $asb_{12}(X,Y;\ M,N,U,V) = \frac{GASB_{12}\binom{M\ N\ U}{V\ X\ Y}}{EASB\binom{M\ N}{V}}$

5. Glossary

b(X, Y; M, N, U, V) Biswas distribution
sb(X, Y; M, N, U, V) SB distribution
ab(X, Y; M, N, U, V) AB distribution
sab(X, Y; M, N, U, V) SAB distribution
asb(X, Y ; M, N, U, V) ASB distribution
$b_1(X;\ M,N,U,V)$ First way B distribution
$b_2(Y;\ M,N,U,V)$ Second way B distribution
$b_{12}(X,Y;\ M,N,U,V)$ Both way B distribution
$sb_1(X;\ M,N,U,V)$ First way SB distribution
$sb_2(Y;\ M,N,U,V)$ Second way SB distribution
$sb_{12}(X,Y;\ M,N,U,V)$ Both way SB distribution
$ab_1(X;\ M,N,U,V)$ First way AB distribution
$ab_2(Y;\ M,N,U,V)$ Second way AB distribution
$ab_{12}(X,Y;\ M,N,U,V)$ Both way AB distribution
$sab_1(X;\ M,N,U,V)$ First way SAB distribution
$sab_2(Y;\ M,N,U,V)$ Second way SAB distribution
$sab_{12}(X,Y;\ M,N,U,V)$ Both way SAB distribution
$asb_1(X;\ M,N,U,V)$ First ay ASB distribution
$asb_2(Y;\ M,N,U,V)$ Second way ASB distribution
$asb_{12}(X,Y;\ M,N,U,V)$ Both way ASB distribution

6. Conclusions

The paper finds an attractive Biswas distribution and related distributions. We get various joint probability functions, marginal probability functions and conditional probability functions from Biswas distribution. Combination distribution, permutation distribution, formation distribution and homogenation distribution are also found here.

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