American Journal of Mathematics and Statistics 2018, 8(2): 50-55
DOI: 10.5923/j.ajms.20180802.03

$N_{\alpha c}$ – Open Sets and Their Basic Properties in Topological Spaces

Nadia M. Ali Abbas[1], Shuker Mahmood Khalil[2,*]

[1]Ministry of Education, Baghdad, Iraq
[2]Department of Mathematics, College of Science, Basra University, Basra, Iraq

Abstract In this work, we first introduce the concept of $N_{\alpha c}$ -open sets and we study some of their basic properties. Furthermore, some relationships between our concepts with some classes of sets are investigated. Moreover, several examples are given to illustrate the concepts introduced in this paper.

Keywords N_α -Open sets, N_α - continuity mappings

1. Introduction

The sets play a very important role in General Topology. There are many types of these sets like β-open sets, regularopen sets, semi regularopen sets, b-open sets and others were discussed in soft setting and fuzzy setting see ([4], [5], [14], [15]). Also, some of these sets were applied to investigate new spaces, new maps and new separation axioms see ([9]-[13]). In 1968, Velicko [18], investigated the definitions of δ -open and θ -open, where A is said to be δ -open (resp., θ -open) if for any $a \in A$, there exists an open set B satisfies $a \in B \subseteq cl(\text{int}(A)) \subseteq A$ (resp., $a \in B \subseteq cl(A) \subseteq A$). Di Maio and Noiri [8] showed that $A \subseteq X$ in a space X is said to be semi-θ -open if for each $a \in A$, there exists a semi-open set B satisfies $a \in B \subseteq D \subseteq A$, where D is a semi closure of B. The family of all δ -open(resp., θ -open) is denoted by δ O(X), (resp.,θO(X)). An α-open sets are one of these sets. It was first studied in 1965 by O. Njasted. These sets form a topology on X which is finer than τ, see [17], [3]. The notion of N_α -open set was first studied in 2015 using the concept of an α – open set, it was studied by N. A. Dawood and N. M. Ali, see [6]. Here in this paper an attempt has been made to employ the notion of N_α -open set to show a new kind of sets that called $N_{\alpha c}$-Open set and investigate it's properties. In this paper (X,τ) or a space X always denote a topological space on which no separation axioms are assumed unless explicitly stated. Assume $A \subseteq X$ in a space X, cl(A) and int(A) refer to the closure and interior of A respectively.

* Corresponding author:
shuker.alsalem@gmail.com (Shuker Mahmood Khalil)
Published online at http://journal.sapub.org/ajms

2. Preliminary Definitions

Definition (2.1) [17], [16], [18]

Let A be a set in space X. We say A is an α-open (resp., preopen and regularopen) if A⊆(int(cl(int(A))),(resp., A⊆ int(cl(A))and A=int(cl(A)). Also, their complements are an α- closed (resp., preclosed and regularclosed)). The family of all α- open,(resp. preopen and regularopen) is denoted by $_\alpha$O(X),(resp., PO(X) and RO(X)).

Definition (2.2) [6]

Let A be a set in a space X. We say A is an N_α-open if there exists non-empty an α-open set B such that cl(B) ⊆ A, and it's complement is called an N_α -closed. Also, the family of all N_α- open (resp., N_α-closed) sets are denoted by $N_\alpha O(X)$, (resp., $N_\alpha C(X)$).

Remark (2.3) [6]

(i) X and ϕ are N_α – open sets in every topological space.
(ii) Every subset in discrete space is N_α –open set.
(iii) In the usual topological space every interval is N_α-Open set.
(iv) Every closed and open (resp., α-open, preopen and δ -open) is N_α-open set.
(v) If A is N_α-open set. Then int(A)≠ ϕ.

Proposition (2.4) [6]

If A is a closed set of a space X. Then A is N_α – open set if and only if int(A)≠ ϕ.

Proposition (2.5) [6]

Finite union of N_α- open sets is N_α- open.

Theorem (2.6) [1]

The following statements are equivalent:

(i) X is Alexindroff space.
(ii) For any an open (resp.,closed) sets then their intersection(resp., union) is an open(resp., closed) also.

Definition (2.7) [7]

Let X be a space. If every an open set of X is a closed. Then X is called Locally Indiscrete space.

Proposition (2.8) [2]

For any set A of a space X. Then A is clopen if and only if preopen and closed.

Proposition (2.9) [6]

Let Y be a subspace of a space X such that $A \subseteq Y \subseteq X$. Then:

(i) If $A \in N_\alpha O(X)$(resp.$N_\alpha C(X)$), then $A \in N_\alpha O(Y)$(resp. $N_\alpha C(Y)$).
(ii) If $A \in N_\alpha O(Y)$(resp., $N_\alpha C(Y)$), then $A \in N_\alpha O(X)$(resp. $N_\alpha C(X)$) where Y is clopen set in X.

Theorem (2. 10) [6]

A space X is N_α^{**} - regular, if and only if all N_α-open set A containing x there exists an open set G containing x such that $x \in G \subseteq cl(G) \subseteq A$.

Theorem (2.11) [6]

(i) If a space X is N_α^{**} - regular. Then every N_α-open set is an open set.
(ii) If a space X is N_α^{**} - regular and Alexindroof. Then A is N_α-open if and only if is clopen.

3. $N_{\alpha c}$- Open Set and Their Basic Properties

A new kind of sets called $N_{\alpha c}$-open using the concept of N_α- open set is studied in this section. Some definitions, propositions and counterexamples are given.

Definition (3.1)

An N_α-open set A of a space X is $N_{\alpha c}$- open if for all $x \in A$ there exists closed set F such that $x \in F \subseteq A$, it's complement is called $N_{\alpha c}$- closed set. The family of all $N_{\alpha c}$-open (resp.$N_{\alpha c}$- closed) is denoted by $N_{\alpha c}O(X)$(resp.,($N_{\alpha c}C(X)$)).

Proposition (3.2)

$A \in N_{\alpha c}O(X)$ if and only if $A \in N_\alpha O(X)$ as well as a union of closed sets.

Proof: Obvious

The above definition refer to every $N_{\alpha c}$- open is N_α- open set, but the converse need not be true in general. See the following example:

Example (3.3)

Consider X ={a,b,c,d,e,} and τ = {X,{a},{b,c}, {a,b,c},ϕ}. Let A={a,b,d,e} $\in N_\alpha O(X)$ but $A \notin N_{\alpha c}O(X)$ since only {b,c,d,e} is closed set and contains b. However, {b,c,d,e} is not contained in A.

Corollary (3.4)

(i) Every closed N_α-open set is $N_{\alpha c}$ –open set.

Proof: Follows from Proposition (3.2).

(ii) Every closed and α-open (resp. δ-open) set is $N_{\alpha c}$-open set.

Proof: Follows from Remark (2.3) (iv)) and Corollary (3.4(i)).

(iii) X and ϕ are $N_{\alpha c}$ –open sets in every topological space.

Proof: Follows from Remark (2.3) and Corollary (3.4(i)).

Proposition (3.5)

If $A \in N_{\alpha c}O(X)$ then, int (A) $\neq \varphi$.

Proof: If $A \in N_{\alpha c}O(X)$ then $A \in N_\alpha O(X)$ this implies int (A) $\neq \varphi$ see Remark (2.3). The following example explains that the converse of Proposition of (3. 5) is not true.

Example (3.6)

Consider X = {a, b, c} with τ ={X, {b,c}, φ}. Let A={b,c} then, int(A) $\neq \emptyset$ but $A \notin N_{\alpha c}O(X)$ because $A \notin N_\alpha O$ (X).

Proposition (3.7)

If A is a closed set of a space X. Then $A \in N_{\alpha c}O(X)$ if and only if int(A) $\neq \varphi$.

Proof: The first part follows from Proposition (3.5). The other part assume that int(A) $\neq \varphi$, implies that A is N_α-open set see Proposition(2.4). Therefore, $A \in N_{\alpha c}O(X)$ see Corollary(3.4).

Proposition (3.8)

If $A \in N_{\alpha c}O(X)$, then $cl(A) \in N_{\alpha c}O(X)$.

Proof: Suppose that $A \in N_{\alpha c}O(X)$ then, by Proposition (3.5) int (A) $\neq \emptyset$ where int(A) is an open set so it is an α –open and it's closure contain in cl(A) this implies $cl(A) \in N_\alpha O(X)$ thus by Corollary.(3.4) $cl(A) \in N_{\alpha c}O(X)$.

Proposition (3.9)

Finite union of $N_{\alpha c}$- open sets is $N_{\alpha c}$ – open.

Proof: Let A_i be $N_{\alpha c}$-open set, i=1, 2,…n. Thus A_i is N_α-open set, hence by Proposition (2.5) $\bigcup_{i=1} A_i \in N_\alpha O(X)$. Let $x \in \bigcup_{i=1} A_i$ then there exists $A_i \in N_{\alpha c}O(X)$ such that $x \in A_i$ for some i= 1,2, …n, thus there exists closed set F such that $x \in F \subseteq A_i \subseteq \bigcup_{i=1} A_i$, hence $x \in F \subseteq \bigcup_{i=1} A_i$. Thus $\bigcup_{i=1} A_i \in N_{\alpha c}O(X)$.

Definition (3.10)

Let $A \subseteq X$. The union of all $N_{\alpha c}$ - open sets contain in A is called $N_{\alpha c}$ -interior of A briefly $N_{\alpha c}$-int(A).

Proposition (3.11)

Let A be a set of a space X. Then:

(i) $N_{\alpha c}$ int(A) $\subseteq$A.
(ii) If $A \subseteq B$ then, $N_{\alpha c}$ int(A) $\subseteq N_{\alpha c}$ int(B).
(iii) If A is $N_{\alpha c}$ – open set then, A = $N_{\alpha c}$ int(A).

Proof: Obvious.

Corollary (3.12)

A finite set $A\in N_{\alpha c}O(X)$ if and only if for all $x\in A$ there exists $N_{\alpha c}$ –open set B such that $x\in B\subseteq A$.

Proof: Let $A=\{x_1,x_2,\ldots,x_n\}$, suppose there exists $B_i\in N_{\alpha c}O(X)$ such that $x_i\in B_i\subseteq A$, this implies $A=\bigcup_{i=1}^{n} B_i$, thus by Proposition (3.9) $A\in N_{\alpha c}O(X)$. Conversely; suppose $A\in N_{\alpha c}O(X)$, hence by Proposition (3.11 (iii)), $A= N_{\alpha c}\,int(A)$, thus the proof is complete.

The intersection of two $N_{\alpha c}$ –open sets need not be $N_{\alpha c}$-open set. See the next example:

Example (3.13)

Consider $X=\{a,b,c,d\}$ with $\tau = \{X,\{b\},\{d\},\{b,d\},\emptyset\}$, $N_\alpha O(X) =\{X,\{a,c,d\}, \{a,b,c\}, \phi\}$. We have $A=\{a,c,d\}$, $B=\{a,b,c\}$ are $N_{\alpha c}$-open sets, but their intersection $\{a,c\}$ is not $N_{\alpha c}$- open set since $\{a,c\}$ is not N_α- open set.

Proposition (3.14)

If the collection of all $N_\alpha O(X)$ sets is form a topology on X, then $N_{\alpha c}O(X)$ is also.

Proof: We shall prove only the finite intersection of $N_{\alpha c}O(X)$ is also. Let $A, B\in N_{\alpha c}O(X)$, then $A,B\in N_\alpha O(X)$ so, $A\cap B\in N_\alpha O(X)$. Suppose $x\in A\cap B$, this implies $x\in A$, $x\in B$, then there exist closed sets E and F such that $x\in E\subseteq A$, $x\in F\subseteq B$ this implies $x\in E\cap F\subseteq A\cap B$. Thus $A\cap B\in N_{\alpha c}O(X)$.

Definition (3.15)

Let $A\subseteq X$. The intersection of all $N_{\alpha c}$ – closed sets containing A is called $N_{\alpha c}$ – closure of A briefly $N_{\alpha c}\,cl(A)$.

Proposition (3.16)

Let X be a space, $A\subseteq B\subseteq X$. Then;

(i) $N_{\alpha c}\,cl(A)\subseteq N_{\alpha c}\,cl(B)$.
(ii) If A is $N_{\alpha c}$ – closed set then $A= N_{\alpha c}\,cl(A)$. If X is a finite set then, A is $N_{\alpha c}$ – closed set if and only if $A = N_{\alpha c}\,cl(A)$.

Proof: Obvious.

Proposition (3.17)

Let A be a set of a space X. Then, $x\in N_{\alpha c}cl(A)$ if and only if $U_x \cap A \neq \varphi$ for each $N_{\alpha c}$- open set U_x containing x.

Proof: Suppose that $U_x \cap A = \varphi$ for some $U_x\in N_{\alpha c}O(X)$ this implies $A\subseteq U^c$ where $U^c\in N_{\alpha c}C(X)$ not containing x this implies $x \notin N_{\alpha c}cl(A)$ which is a contradiction to hypothesis. Thus $U_x \cap A \neq \varphi$ for any $U_x\in N_{\alpha c}O(X)$. Conversely is a similarly.

Proposition (3.18)

Every regular closed set is $N_{\alpha c}$ – open.

Proof: Suppose that $A\neq \phi$ is regular closed so it is closed set since $cl(int(A)) \neq \phi$ then $int(A) \neq \phi$ where int(A) is an α – open set hence $A\in N_\alpha O(X)$, thus by Corollary (3.4) $A\in N_{\alpha c}O(X)$.

Lemma (3.19) [14]

Let X_1, X_2, be topological spaces. Then A_1, A_2, are N_α -open sets in X_1, X_2 respectively if and only if $A_1 \times A_2$ is N_α -open set in $X_1\times X_2$.

Proposition (3.20)

Let X_1, X_2 be topological spaces. Then A_1, A_2, are $N_{\alpha c}$ -open sets in X_1, X_2 respectively if and only if $A_1 \times A_2$ is $N_{\alpha c}$ -open set in $X_1\times X_2$.

Proof: Assume $A_1, A_2\in N_{\alpha c}\,O(X_1)$, $N_{\alpha c}\,O(X_2)$ respectively, then by Proposition (3. 2)) $A_1, A_2\in N_\alpha\,O(X_1)$, $N_\alpha\,O(X_2)$ respectively hence $A_1\times A_2\in N_\alpha\,O(X_1\times X_2)$ see Lemma (3. 19), on the other hand for each $x_1\in A_1$, $x_2\in A_2$ there exist closed sets F_1, F_2, such that $x_1\in F_1\subseteq A_1$, $x_2\in F_2\subseteq A_2$. Thus $(x_1,x_2)\in F_1 \times F_2\subseteq A_1\times A_2$, where $F_1 \times F_2$ is closed set in $X_1 \times X_2$. Hence $A_1 \times A_2\in N_{\alpha c}\,O(X_1 \times X_2)$. Conversely; if $A_1 \times A_2\in N_{\alpha c}\,O(X_1 \times X_2)$ then, by Proposition (3.2) $A_1 \times A_2\in N_\alpha\,O(X_1 \times X_2)$, hence by Lemma (3. 19) $A_1 \in N_\alpha O(X_1)$, $A_2\in N_\alpha\,O(X_2)$. On the other hand for each $(x_1, x_2) \in A_1\times A_2$ there exists closed set $F_1\times F_2$ in $X_1 \times X_2$ such that $(x_1, x_2)\in F_1\times F_2 \subseteq A_1\times A_2$, thus $x_1\in F_1\subseteq A_1$, $x_2\in F_2 \subseteq A_2$, where F_1, F_2 are closed sets in X_1, X_2 respectively. Thus $A_1, A_2\in N_{\alpha c}\,O(X_1)$, $N_{\alpha c}\,O(X_2)$ respectively.

Proposition (3.21)

Let X_1, X_2 be topological spaces. Then $N_{\alpha c}\,cl(A_1 \times A_2) = N_{\alpha c}\,cl(A_1) \times N_{\alpha c}\,cl(A_2)$, where $A_1 \subseteq X_1$, $A_2 \subseteq X_2$.

Proof: Let $(x_1, x_2)\in N_{\alpha c}\,cl(A_1 \times A_2)$, then by Proposition (3. 17)) for each $N_{\alpha c}$ - open set in $X_1\times X_2$ say $G=G_1 \times G_2$ containing (x_1, x_2) then $(A_1 \times A_2) \cap (G_1 \times G_2) \neq \emptyset$. Then $(A_1 \cap G_1) \times (A_2 \cap G_2) \neq \emptyset$, where $G_1,G_2\in N_{\alpha c}O(X_1)$, $N_{\alpha c}O(X_2)$ resp., see Proposition (3.20) which means $A_1\cap G_1\neq \varphi$ and $A_2 \cap G_2\neq \varphi$. Thus $x_1\in N_{\alpha c}\,cl(A_1)$, $x_2\in N_{\alpha c}\,cl(A_2)$. Hence $(x_1, x_2)\in N_{\alpha c}\,cl(A_1) \times N_{\alpha c}\,cl(A_2)$.

Conversely; Let $(x_1, x_2)\in N_{\alpha c}\,cl(A_1) \times N_{\alpha c}\,cl(A_2)$, then $x_1\in N_{\alpha c}\,cl(A_1)$, $x_2\in N_{\alpha c}cl(A_2)$, hence for each $N_{\alpha c}$ – open sets G_1,G_2 containing x_1, x_2 respectively, then $A_1 \cap G_1 \neq \emptyset$ and $A_2\cap G_2\neq \emptyset$, thus $(A_1 \cap G_1) \times (A_2 \cap G_2) \neq \emptyset$, this implies $(A_1 \times A_2) \cap (G_1 \times G_2) \neq \emptyset$, but $G_1 \times G_2\in N_{\alpha c}\,O(X_1 \times X_2)$ see Proposition (3. 20). Thus $(x_1, x_2)\in N_{\alpha c}\,cl(A_1 \times A_2)$.

Proposition (3.22)

Let Y be a subspace of a space X such that $A\subseteq Y\subseteq X$. Then:-

(i) If $A\in N_{\alpha c}O(X)$ then, $A\in N_{\alpha c}O(Y)$.
(ii) If $A\in N_{\alpha c}O(Y)$ then, $A\in N_{\alpha c}O(X)$, where Y is clopen set in X.

Proof:

(i) Suppose that $A\in N_{\alpha c}O(X)$, hence $A\in N_\alpha O(X)$, thus by Proposition (2.9) $A\in N_\alpha O(Y)$, let $x\in A$, hence there ∃ closed set F in X s.t $x\in F\subseteq A$ for each $x\in A$, hence F is closed in Y, thus $A\in N_{\alpha c}O(Y)$.

(ii) Suppose that $A \in N_{\alpha c}O(Y)$ hence $A \in N_{\alpha}O(Y)$ since Y is clopen set in X then by Proposition (2.9) $A \in N_{\alpha}O(X)$. Let $x \in A$, thus $\exists$ closed set F in Y s.t $x \in F \subseteq A$ so F is closed set in X. Thus $A \in N_{\alpha c}O(X)$.

Corollary (3.23)

Let Y be a subspace of a space X where $A \subseteq Y \subseteq X$ such that Y is an α - closed set in X then $A \in N_{\alpha c}C(X)$ if and only if $A \in N_{\alpha c}C(Y)$

Proof: Follows from Proposition (2.9).

Now, we shall discuss the relationships between our concept with some other classes of sets.

Remarks (3.24)

The following sets are independent with $N_{\alpha c}$-open sets

(i) Open sets are incomparable with $N_{\alpha c}O(X)$

Open $\nrightarrow$ / $\nleftarrow$ $N_{\alpha c}O(X)$

Example (1): Let (R, τ_u) be the usual topology on real number R, we observe $[a,b] \in N_{\alpha c}O(R)$ but $[a,b] \notin \tau_u$. On the other hand every an open set is incomparable with $N_{\alpha}O(X)$, see [14].

(ii) Closed sets are incomparable with $N_{\alpha c}O(X)$ i. e

Closed $\nrightarrow$ / $\nleftarrow$ $N_{\alpha c}O(X)$

Example (2): In previous example (R, τ_U). Assume that $A=(0,1]$ then $A \in N_{\alpha c}O(R)$ because $A \in N_{\alpha}O(X)$ see Remark(2.3) so, $A=\bigcup_{n=1} [1/n,1]$ hence $A \in N_{\alpha c}O(X)$ see Proposition(3.2) but A is not closed set in (R, τ_U). On the other hand consider X={a,b,c,d} with τ={X,{b},{d},{b, d}, ∅}, let A={a,c}, then A is closed set in X but $A \notin N_{\alpha c}O(X)$ because $A \notin N_{\alpha}O(X)$.

(iii) $_{\alpha}$Open sets are incomparable with $N_{\alpha c}O(X)$

$_{\alpha}O(X)$ $\nrightarrow$ / $\nleftarrow$ $N_{\alpha c}O(X)$

Example (3): See Ex. (2) in (X, τ) let A={a,b,d} then $A \in {}_{\alpha}O(X)$ but $A \notin N_{\alpha c}O(X)$ because $A \notin N_{\alpha}O(X)$, also if A =[a,b] in (R, τ_U), then $A \in N_{\alpha c}O(X)$ but $A \notin {}_{\alpha}O(X)$.

(iv) Preopen sets are incomparable with $N_{\alpha c}O(X)$

PO(X) $\nrightarrow$ / $\nleftarrow$ $N_{\alpha c}O(X)$

Example (4): See Ex. 2. in (X, τ). Assume that A={b,c,d}, then $A \in PO(X)$ but $A \notin N_{\alpha c}O(X)$. Also we have A=[a,b] in (R, τ_U) where $A \in N_{\alpha c}O(X)$ but $A \notin PO(X)$.

(v) δ-Open sets are incomparable with $N_{\alpha c}O(X)$

δ-Open $\nrightarrow$ / $\nleftarrow$ $N_{\alpha c}O(X)$

Example (5): In Ex. 2. We notice in (X,τ), if A= {b}, then $A \in \delta O(X)$ but $A \notin N_{\alpha c}O(X)$. Also if A={a,c,d}, then $A \in N_{\alpha c}O(X)$ but $A \notin \delta O(X)$.

Proposition (3.25)

Let $A \subseteq X$. Then:

(i) If $A \in \theta O(X)$, then $A \in N_{\alpha c}O(X)$.

Proof: Follows from Definition of θ – open sets and $\tau \subseteq {}_{\alpha}O(X)$.

(ii) If A is clopen set, then $A \in N_{\alpha c}O(X)$.

Proof: Direct by Remark (2.3) and Corollary (3.4(i)). The converse of (i), (ii) is not true. See the following example:

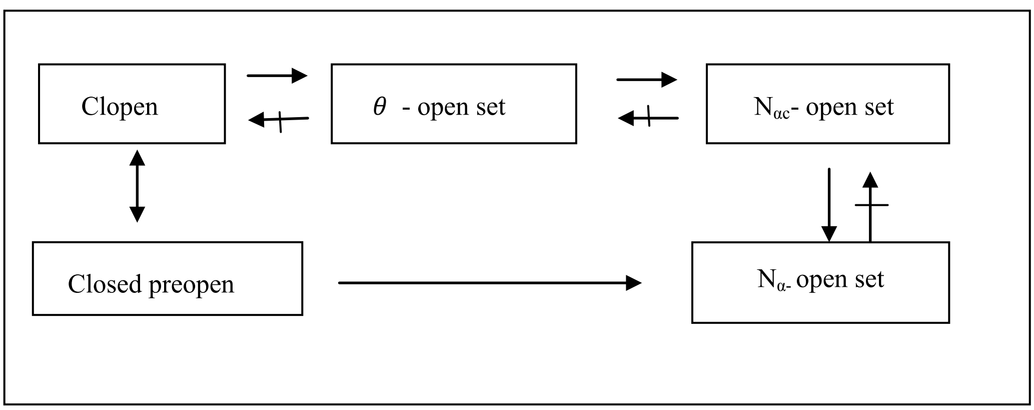
Example 6: See example (2) in Remarks (3.24) of a space X. Suppose that A={a,c,d}. Then $A \in N_{\alpha c}O(X)$ but it is neither θ- open nor clopen set.

Proposition (3.26)

Every closed preopen is $N_{\alpha c}$-open set.

Proof: It follows by Proposition (2.8) and Proposition. (3.25(ii)).

From above discussions, we have the following Diagram:



Now, we can prove the converse of previous results such as (3. 2) and (3. 25) are true if we add some conditions. See the next results:-

Proposition (3.27)

$N_{\alpha c}O(X) = N_\alpha O(X)$ where a space X is T_1-space.

Proof: We show only $N_\alpha O(X) \subseteq N_{\alpha c}O(X)$. Assume that A is N_α -open set, let x∈ A then {x} is closed set, thus x∈{x}⊆ A. Thus A∈ $N_{\alpha c}O(X)$.

Proposition (3.28)

If a space X is N_α^{**} - regular space. Then we have the following results:-

(i) $N_\alpha O(X) = N_{\alpha c}O(X)$.
(ii) $\theta O(X) = N_{\alpha c}O(X)$.
(iii) $CO(X) = N_{\alpha c}O(X)$ where X is Alexindroff space.
(iv) $\tau = N_{\alpha c}O(X)$ where X is Locally Indiscrete space.

Proof: (i) We have $N_{\alpha c}O(X)) \subseteq N_\alpha O(X)$. To prove $N_\alpha O(X) \subseteq N_{\alpha c}O(X)$. Let A be N_α-open set, since X is N_α^{**}-regular space then for all x∈ A ∃ an open set G s.t x∈ G⊆ cl(G) ⊆ A see Theorem (2.10). Hence A∈$N_{\alpha c}O(X)$ see Proposition (3.2). Thus $N_\alpha O(X) = N_{\alpha c}O(X)$.

(ii) We have $\theta O(X)) \subseteq N_{\alpha c}O(X)$ by Proposition (3.25). Assume A∈ $N_{\alpha c}O(X)$, then A∈ N_α O(X). Since X is N_α^{**}-regular space, then for all x∈ A there exists an open set G such that x∈ G ⊆ cl(G) ⊆ A this implies A∈ $\theta O(X)$. Thus $\theta O(X) = N_{\alpha c}O(X)$.

(iii) We have every clopen set is $N_{\alpha c}$-Open see Proposition (3.25). We shall prove $N_{\alpha c}O(X) \subseteq CO(X)$. Since A∈ $N_{\alpha c}O(X)$, then A∈ $N_\alpha O(X)$. Since X is Alexindroof space then by Theorem (2.11) A∈ CO(X). Thus $CO(X) = N_{\alpha c}O(X)$.

(v) Suppose that A is nonempty an open set. Since X is Locally Indiscrete space, then A is closed, thus A is clopen set so it is N_α – open set see Remark (2. 3). Thus A∈ $N_{\alpha c}O$ (X) see Corollary (3.4). Let A∈ $N_{\alpha c}O$ (X), then A∈ $N_\alpha O$ (X). Since X is N_α** - regular space, then A is an open set see Theorem (2.11). Hence $N_{\alpha c}$ O(X) = τ.

Proposition (3.29)

Each δ-Open set is $N_{\alpha c}$ – Open where a space X is Locally Indiscrete.

Proof: Suppose that A∈ δ O(X) so it is an open set. We have X is Locally Indiscrete hence A is closed, this implies A is clopen set then by Remark (2. 3) A is N_α - open set, thus by Corollary (3.4) A∈ $N_{\alpha c}O(X)$.

Proposition (3.30)

$N_{\alpha c}O$ (X) ⊆ $\delta O(X)$ where X is a discrete space.

Proof: Assume that A∈ $N_{\alpha c}$ O (X), hence A∈ $N_\alpha O$ (X) and for all x∈ A there exists closed set F such that x∈ F⊆ A. Since X is discrete space, then every closed set is an open, this implies x∈ F = cl(F) = int (cl(F) ⊆A. Hence A∈ $\delta O(X)$.

Proposition (3.31)

Each an open set in a regular space X is $N_{\alpha c}$-open set.

Proof: Let A∈ τ. Since X is a regular space then there exists an open set G such that x∈ G ⊆ cl(G) ⊆A, for each x∈A. We get cl(G) ⊆ A, where G is an open set so it is an α - open thus A∈ $N_\alpha O$ (X) and A= ⋃ cl(G) for each x∈A. Thus by Proposition (3. 2)) A∈ $N_{\alpha c}O(X)$.

4. Conclusions

In this paper we have introduced the notion of $N_{\alpha c}$ -open sets. Next, some of their basic properties are discussed and studied. Moreover, some relationships between our concepts with some classes of sets are investigated. This new notion will help us to study a new kind of $N_{\alpha c}$ – continuous mappings and to study a new kind of $N_{\alpha c}$ – separation axioms.

REFERENCES

[1] P. S. Alexandroff Piskrete Raame, Math-sb. (N. S.), (1937), 501-518.

[2] B. Alias, S. M. Mershkhan, P_P-Open Sets and P_p-Continuaus Functions, Gen. Math. 20, (2014), 43-51.

[3] N. M. Ali AL-Tabatabai. On Some Types of Weakly Open Sets, M. Sc. Thesis University of Baghdad (2004).

[4] D. Andrijevic, On b-open sets, Math. Vesnik, 48(1996), 59-64.

[5] I. Arockiarani, A. A. Lancy, Generalized soft g β-closed sets and soft gsβ-closed sets in soft topological spaces, International Journal of Mathematical Archive, 4(2013), 1-7.

[6] N. A. Dawood, N. M. Ali, "N_α - Open Set and N_α -Regularity In Topological Spaces" International J. of Advanced Scientific and Technical Research 5(3), (2015) 87-96.

[7] J. Dontchev, Survey on preopen sets, The proceedings of the yatsushiro Topological Conference, (1998), 1-18.

[8] G. Di Maio and T. Noiri, On s-Closed Spaces, Indian Journal of Pure and Applied Mathematics, 18(3), (1987), 226-233.

[9] S. Mahmood, Fuzzy Lindelof Closed Spaces and Their Operatios, Journal of General Mathematics Notes, 21(1), (2014), 28-136.

[10] S. Mahmood, Soft Sequentially Absolutely Closed Spaces, International Journal of Applications of Fuzzy Sets and Artificial Intelligence, 7 (2017), 73-92.

[11] S. Mahmood, Tychonoff Spaces in Soft Setting and their Basic Properties, International Journal of Applications of Fuzzy Sets and Artificial Intelligence, 7, (2017), 93-112.

[12] S. Mahmood and M. Abd Ulrazaq, New Category of Soft Topological Spaces, American Journal of Computational and Applied Mathematics, 8(1), (2018), 20-25.

[13] S. Mahmood, M. Abud Alradha, Extension Permutation Spaces with Separation Axioms in Topological Groups, American Journal of Mathematics and Statistics, 7(5), (2017), 183-198.

[14] S. Mahmood, On intuitionistic fuzzy soft b-closed sets in intuitionistic fuzzy soft topological spaces, Annals of Fuzzy

Mathematics and Informatics, 10(2), (2015) 221-233.

[15] S. Mahmood, Soft Regular Generalized b-Closed Sets in Soft Topological Spaces, Journal of Linear and Topological Algebra, 3(4), (2014), 195-204.

[16] A. S. Mashhour, M. E. A. El-Monsf and S. N. El-Deeb, "On Precontinuous and Week Precontinuous Mappings, proc. Math. phys Soc., Egypt, 53(1982), 47-53.

[17] O. Njastad, On Some Classes of Nearly Open Sets, pacific J. Math, 15(3) (1965), 961-970.

[18] N. V. Velicko H-Closed Topological Spaces, Amer. Math. Soc. Transl., 78(2) (1968), 103-118.