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# Improved Estimation of Population Mean Using Information on Size of the Sample

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**Abstract** In the present paper, the sample size has been used as information for improved estimation of population mean of the main variable under study. A generalized ratio type estimator of population mean has been proposed. The large sample properties the bias and the mean squared error of the proposed estimator have been obtained up to first order of approximation. The optimum value of the characterizing scalar which minimizes the mean squared error has been obtained and the minimum value of the mean squared error of the proposed estimator for this optimum value has also been obtained. A comparison of the proposed estimator has been made with mean per unit estimator and other existing estimators of population mean. A numerical study is also carried out to judge the performances of the proposed and existing estimators of the population mean.

**Keywords** Study variable, Auxiliary variable, Bias, Mean squared error, Efficiency

## 1. Introduction

Estimation of the population parameters is necessary when the size of the population is very large and we wish to get the result in very less time and with minimum cost, labor etc. To estimate any parameter the best estimator is the corresponding statistic. Thus for estimating population mean, sample mean is the most suitable estimator but it has a reasonably large sampling variance. Our aim is to search for the estimator with higher efficiency that is minimum variance or mean squared error. This aim is achieved through the use of auxiliary information supplied by the auxiliary variable. Auxiliary variable is highly positively or negatively correlated with the main variable under study. When main variable under study is positively correlated with the auxiliary variable and the line of regression passes through origin, ratio type estimators are used for improved estimation of population mean. Product type estimators are used when main and auxiliary variables are negatively correlated otherwise regression type estimators are used for the estimation of population mean.

Let the finite population under consideration consist of $N$ distinct and identifiable units and let $(x_i, y_i), i = 1,2,...,n$ be a bivariate sample of size $n$ taken from ($X$, Y) using a *SRSWOR* scheme. Let $\bar{X}$ and $\bar{Y}$ respectively be the population means of the auxiliary and the study variables, and let $\bar{x}$ and $\bar{y}$ be the corresponding sample means. It is well known and has been seen in practice that in simple random sampling scheme, sample means $\bar{x}$ and $\bar{y}$ are unbiased estimators of population means of $\bar{X}$ and $\bar{Y}$ respectively. Population mean is one of the very important measures of central tendency in almost all fields of society including field of Medical sciences, Biological sciences, Agriculture, Industry, social sciences, humanities etc. Thus the estimation of population mean is of great significance in above fields. In the present manuscript a modified ratio type estimator of population mean of the study variable using information on size of the sample has been proposed and its large sample properties have been studied up to the first order of approximation.

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## 2. Review of Existing Estimators

The usual and the most suitable estimator for estimating population mean $\bar{Y}$ is the corresponding sample mean $\bar{y}$ given by,

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \tag{1}$$

It is unbiased for population mean and its variance up to the first order of approximation is given by,

$$V(\bar{y}) = \gamma S_y^2 = \gamma \bar{Y}^2 C_y^2 \tag{2}$$

Where, $C_y = \frac{S_y}{\bar{Y}}$ , $S_y^2 = \frac{1}{N-1}\sum_{i-1}^{N}(Y_i - \bar{Y})^2$ ,

$$\gamma = \frac{N-n}{nN}.$$

Cochran (1940) used the positively correlated auxiliary variable with the study variable and proposed the following usual ratio estimator of population mean as,

$$\hat{\bar{Y}}_r = \bar{y}\frac{\bar{X}}{\bar{x}} \quad (3)$$

The above estimator is a biased estimator of population mean and its bias and mean squared error, up to the first order of approximation respectively are,

$$B(\hat{\bar{Y}}_r) = \gamma \bar{Y}[C_x^2 - \rho_{yx} C_y C_x]$$

$$MSE(\hat{\bar{Y}}_r) = \gamma \bar{Y}^2[C_y^2 + C_x^2 - 2\rho_{yx} C_y C_x] \quad (4)$$

Where, $C_x = \frac{S_x}{\bar{X}}$ , $S_x^2 = \frac{1}{N-1}\sum_{i-1}^{N}(X_i - \bar{X})^2$ ,

$$\rho_{yx} = \frac{Cov(x,y)}{S_x S_y},$$

$$Cov(x,y) = \frac{1}{N-1}\sum_{i-1}^{N}(Y_i - \bar{Y})(X_i - \bar{X}).$$

In literature various modified estimators of population mean of study variable using auxiliary variables have been given by various authors. For detailed study of the modified ratio type estimators, latest references can be made of Kadilar and Cingi (2004, 2006a, 2006b, 2009), Singh (2003), Singh and Tailor (2003, 2005), Koyuncu and Kadilar (2009), Subramani (2013), Subramani and Kumarapandiyan (2012a,b,c, 2013a,b), Tailor and Sharma (2009), Yan and Tian (2010), Yadav and Pandey (2011), Yadav and Adewara (2013), Yadav et al. (2014, 2015), Yadav et al. (2016a, 2016b, 2016c, 2016d), Abid *et al.* (2016).

Following table-1 represents some modified estimators their constants, biases and mean squared errors.

**Table 1.** Various estimators, their constants, biases and mean squared errors

| S. No. | estimators | Constants | Bias | MSE |
|---|---|---|---|---|
| 1 | $\hat{\bar{Y}}_1 = \bar{y}\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right)$ Sisodia and Dwivedi [1981] | $\delta_1 = \left(\frac{\bar{X}}{\bar{X}+C_x}\right)$ | $\gamma\bar{Y}\left(\delta_1^2 C_x^2 - 2\delta_1\rho C_x C_y\right)$ | $\gamma\bar{Y}^2\left(C_y^2 + \delta_1^2 C_x^2 - 2\delta_1\rho C_x C_y\right)$ |
| 2 | $\hat{\bar{Y}}_2 = \bar{y}\left(\frac{\bar{X}C_x+\beta_2}{\bar{x}C_x+\beta_2}\right)$ Upadhyaya and Singh [1999] | $\delta_2 = \left(\frac{\bar{X}C_x}{\bar{X}C_x+\beta_2}\right)$ | $\gamma\bar{Y}\left(\delta_2^2 C_x^2 - 2\delta_2\rho C_x C_y\right)$ | $\gamma\bar{Y}^2\left(C_y^2 + \delta_2^2 C_x^2 - 2\delta_2\rho C_x C_y\right)$ |
| 3 | $\hat{\bar{Y}}_3 = \bar{y}\left(\frac{\bar{X}+\rho}{\bar{x}+\rho}\right)$ Singh and Tailor [2003] | $\delta_3 = \left(\frac{\bar{X}}{\bar{X}+\rho}\right)$ | $\gamma\bar{Y}\left(\delta_3^2 C_x^2 - 2\delta_3\rho C_x C_y\right)$ | $\gamma\bar{Y}^2\left(C_y^2 + \delta_3^2 C_x^2 - 2\delta_3\rho C_x C_y\right)$ |
| 4 | $\hat{\bar{Y}}_4 = \bar{y}\left(\frac{\bar{X}+\beta_2}{\bar{x}+\beta_2}\right)$ Singh et.al [2004] | $\delta_4 = \left(\frac{\bar{X}}{\bar{X}+\beta_2}\right)$ | $\gamma\bar{Y}\left(\delta_4^2 C_x^2 - 2\delta_4\rho C_x C_y\right)$ | $\gamma\bar{Y}^2\left(C_y^2 + \delta_4^2 C_x^2 - 2\delta_4\rho C_x C_y\right)$ |
| 5 | $\hat{\bar{Y}}_5 = \bar{y}\left(\frac{\bar{X}+\beta_1}{\bar{x}+\beta_1}\right)$ Yan and Tian [2010] | $\delta_5 = \left(\frac{\bar{X}}{\bar{X}+\beta_1}\right)$ | $\gamma\bar{Y}\left(\delta_5^2 C_x^2 - 2\delta_5\rho C_x C_y\right)$ | $\gamma\bar{Y}^2\left(C_y^2 + \delta_5^2 C_x^2 - 2\delta_5\rho C_x C_y\right)$ |
| 6 | $\hat{\bar{Y}}_6 = \bar{y}\left(\frac{\bar{X}+M_d}{\bar{x}+M_d}\right)$ Subramani and Kumarpandiyan [2013] | $\delta_6 = \left(\frac{\bar{X}}{\bar{X}+M_d}\right)$ | $\gamma\bar{Y}\left(\delta_6^2 C_x^2 - 2\delta_6\rho C_x C_y\right)$ | $\gamma\bar{Y}^2\left(C_y^2 + \delta_6^2 C_x^2 - 2\delta_6\rho C_x C_y\right)$ |
| 7 | $\hat{\bar{Y}}_7 = \bar{y}\left(\frac{\bar{X}+n}{\bar{x}+n}\right)$ Jerajuddin and Kishun [2016] | $\delta_7 = \left(\frac{\bar{X}}{\bar{X}+n}\right)$ | $\gamma\bar{Y}\left(\delta_7^2 C_x^2 - 2\delta_7\rho C_x C_y\right)$ | $\gamma\bar{Y}^2\left(C_y^2 + \delta_7^2 C_x^2 - 2\delta_7\rho C_x C_y\right)$ |

Thus biases and mean squared errors of above estimators may be written as,

$$B(\hat{\bar{Y}}_i) = \gamma \bar{Y}[\delta_i^2 C_x^2 - \rho_{yx} C_y C_x]$$

$$MSE(\hat{\bar{Y}}_i) = \gamma \bar{Y}^2[C_y^2 + \delta_i^2 C_x^2 - 2\delta_i \rho_{yx} C_y C_x] \quad (5)$$

$$i = 1, 2, \ldots, 7$$

## 3. Proposed Estimator

Motivated by Jerajuddin and Kishun (2016) estimator of population mean, we propose the following generalized estimator of the population mean using information on size of the sample as,

$$\tau_p = \bar{y}\left[\alpha + (1-\alpha)\left(\frac{\bar{X}+n}{\bar{x}+n}\right)\right] \quad (6)$$

where $\alpha$ is a suitably chosen constant to be defined such that the mean squared error of the proposed estimator is minimum.

To study the large sample properties of the proposed estimator, we define, $\bar{y} = \bar{Y}(1+\varepsilon_0)$ and $\bar{x} = \bar{X}(1+\varepsilon_1)$ such that $E(\varepsilon_i) = 0$ for $(i = 0,1)$ and $E(\varepsilon_0^2) = \gamma C_y^2$, $E(\varepsilon_1^2) = \gamma C_x^2$, $E(\varepsilon_0\varepsilon_1) = \gamma C_x C_y$.

Expressing (6) in terms of $\varepsilon_i 's$, we have

$$\tau_p = \bar{Y}(1+\varepsilon_0)\left[\alpha + (1-\alpha)\left(\frac{\bar{X}+n}{\bar{X}(1+\varepsilon_1)+n}\right)\right]$$

$$\tau_p = \bar{Y}(1+\varepsilon_0)\left[\alpha + (1-\alpha)\left(\frac{1}{1+\frac{\bar{X}}{\bar{X}+n}\varepsilon_1}\right)\right]$$

$$\tau_p = \bar{Y}(1+\varepsilon_0)[\alpha + (1-\alpha)(1+\delta\varepsilon_1)^{-1}] \quad (7)$$

Where $\delta = \dfrac{\bar{X}}{\bar{X}+n}$

We assume that $|\varepsilon_1| < 1$, so that $(1+\delta\varepsilon_1)^{-1}$ may be expanded. Now expanding the right-hand side of (7), we have,

$$\tau_p = \bar{Y}(1+\varepsilon_0)[\alpha + (1-\alpha)(1-\delta\varepsilon_1 + \delta^2\varepsilon_1^2)] \quad (8)$$

$$\tau_p = \bar{Y}(1+\varepsilon_0)[\alpha + 1 - \delta\varepsilon_1 + \delta^2\varepsilon_1^2 - \alpha + \alpha\delta\varepsilon_1 - \alpha\delta^2\varepsilon_1^2] \quad (9)$$

Retaining the terms up to the first order of approximation, we have

$$\tau_p = \bar{Y}[1 + \varepsilon_0 - \delta\varepsilon_1 - \delta\varepsilon_0\varepsilon_1 + \delta^2\varepsilon_1^2 + \alpha\delta\varepsilon_1 + \alpha\delta\varepsilon_0\varepsilon_1 - \alpha\delta^2\varepsilon_1^2] \quad (10)$$

Subtracting $\bar{Y}$ from both sides of (10), we get

$$\tau_p - \bar{Y} = \bar{Y}[\varepsilon_0 - \delta\varepsilon_1 - \delta\varepsilon_0\varepsilon_1 + \delta^2\varepsilon_1^2 + \alpha\delta\varepsilon_1 + \alpha\delta\varepsilon_0\varepsilon_1 - \alpha\delta^2\varepsilon_1^2] \quad (11)$$

Taking expectations on both sides of (11) and putting the values of different expectations, we get the bias of $\tau_p$ as

$$B(\tau_p) = \bar{Y}\gamma[-\delta\rho C_x C_y + \delta^2 C_x^2 + \alpha\delta\rho C_x C_y - \alpha\delta^2 C_x^2] \quad (12)$$

Squaring both sides of (11) and retaining the terms up to the first order of approximation, we have,

$$(\tau_p - \bar{Y})^2 = \bar{Y}^2[\varepsilon_0^2 + \delta^2\varepsilon_1^2 - 2\delta\varepsilon_0\varepsilon_1 + \alpha^2\delta^2\varepsilon_1^2 + 2\alpha\delta\varepsilon_0\varepsilon_1 - 2\alpha\delta^2\varepsilon_1^2] \quad (13)$$

Taking expectation both sides of (13) and putting the values of different expectations, we get the mean square error of $\tau_p$, up to the first order of approximation, as

$$MSE(\tau_p) = \gamma\bar{Y}^2[C_y^2 + \delta^2 C_x^2 - 2\delta\rho C_x C_y + \alpha^2\delta^2 C_x^2 + 2\alpha\delta\rho C_x C_y - 2\alpha\delta^2 C_x^2] \quad (14)$$

which is minimum for,

$$\alpha = \frac{\delta^2 C_x^2 - \delta\rho C_x C_y}{\delta^2 C_x^2} = \frac{A}{B}$$

Thus the minimum MSE of $\tau_p$ is,

$$MSE_{\min}(\tau_p) = \gamma\bar{Y}^2\left[C_y^2 + \delta^2 C_x^2 - 2\delta\rho C_x C_y - \frac{A^2}{B}\right] \quad (15)$$

## 4. Theoretical Efficiency Comparison

From equation (15) and equation (2), proposed estimator is better than the mean per unit estimator if,

$$V(\bar{y}) - MSE_{\min}(\tau_p) = \gamma\bar{Y}^2\left[\delta^2 C_x^2 - 2\delta\rho C_x C_y - \frac{A^2}{B}\right] > 0$$

$$\text{or,} \quad \delta^2 C_x^2 - 2\delta\rho C_x C_y > \frac{A^2}{B} \quad (16)$$

From equation (15) and equation (2), proposed estimator is better than the usual ratio estimator by Cochran (1940) if,

$$MSE(\hat{\bar{Y}}_r) - MSE_{\min}(\tau_p) = \gamma \bar{Y}^2 \begin{bmatrix} (R^2-\delta^2)C_x^2 \\ -2(R-\delta)\rho C_x C_y - \dfrac{A^2}{B} \end{bmatrix} > 0$$

or, $$(R^2-\delta^2)C_x^2 - 2(R-\delta)\rho C_x C_y > \frac{A^2}{B} \tag{17}$$

From equation (15) and equation (2), proposed estimator is better than the usual ratio estimator by Cochran (1940) if,

$$MSE(\hat{\bar{Y}}_i) - MSE_{\min}(\tau_p) = \gamma \bar{Y}^2 \begin{bmatrix} (R^2-\delta_i^2)C_x^2 \\ -2(R-\delta_i)\rho C_x C_y - \dfrac{A^2}{B} \end{bmatrix} > 0$$

or,

$$(R^2-\delta_i^2)C_x^2 - 2(R-\delta_i)\rho C_x C_y > \frac{A^2}{B}, \tag{18}$$

$$i = 1, 2, \ldots, 7$$

## 5. Empirical Study

To judge the performances of the proposed estimator and the existing estimators of population mean using auxiliary variable, we have considered four natural populations from two sources. First two populations, population-1 and population-2 are from Murthy (1967) and rest two populations, population-3 and population-4 are from Mukhopadhyay (2009).

**Murthy (1967)**

**Population 1:** $Y$ = Output for 80 factories in a region and $X$ = Number of workers

$N = 80,\ n = 20,\ \bar{Y} = 51.8264,\ \bar{X} = 11.2646,$
$\rho = 0.9413,\ C_y = 0.3542,\ C_x = 0.7507,$
$\beta_1 = 1.0500,\ \beta_2 = -0.0634,\ M_d = 7.5750$

**Population 2:** $Y$ = Output for 80 factories in a region and $X$ = Fixed Capital

$N = 80,\ n = 20,\ \bar{Y} = 51.8264,\ \bar{X} = 2.8513,$
$\rho = 0.9150,\ C_y = 0.3542,\ C_x = 0.9485,$
$\beta_1 = 1.3006,$
$\beta_2 = 0.6977,\ M_d = 1.4800$

**Mukhopadhyay (2009)**

**Population 3:** $Y$ = Output for 40 factories in a region and $X$ = Number of workers

$N = 40,\ n = 8,\ \bar{Y} = 50.7858,\ \bar{X} = 2.3033,$
$\rho = 0.8006,\ C_y = 0.3295, C_x = 0.8406,$
$\beta_1 = 0.9740,\ \beta_2 = -0.5344,\ M_d = 1.250$

**Population 4:** $Y$ = Output for 40 factories in a region and $X$ = Fixed Capital

$N = 40,\ n = 8,\ \bar{Y} = 50.7858,\ \bar{X} = 9.4543,$
$\rho = 0.8349,\ C_y = 0.3295,\ C_x = 0.6756,$
$\beta_1 = 0.8799,\ \beta_2 = -0.4622,\ M_d = 7.0700$

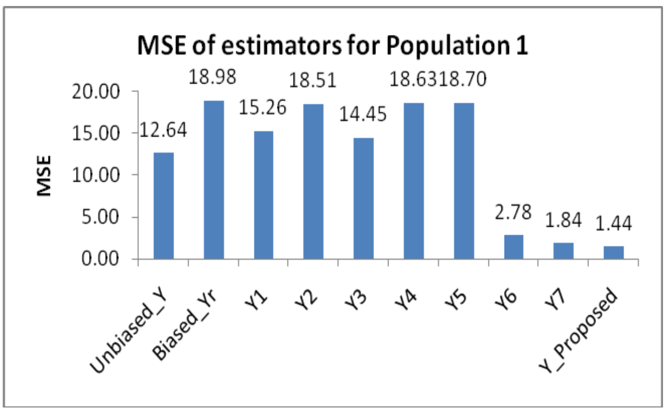
Following Table-2 and Table-3 represents the biases, mean squared errors of proposed and existing estimators of population mean. Table-4 shows the efficiencies of the proposed estimator over other existing estimators of population mean.

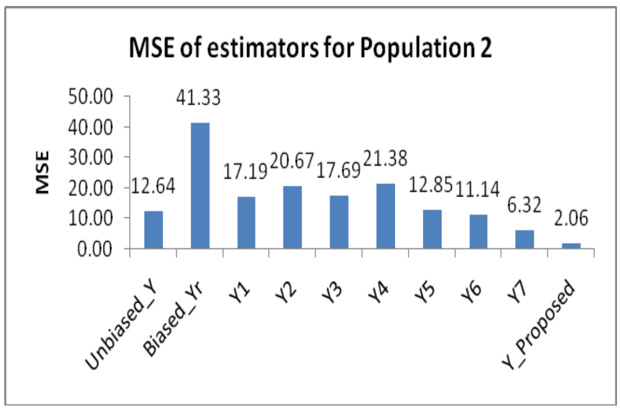
**Table 2.** Biases of the Existing and proposed modified ratio estimators for four natural populations

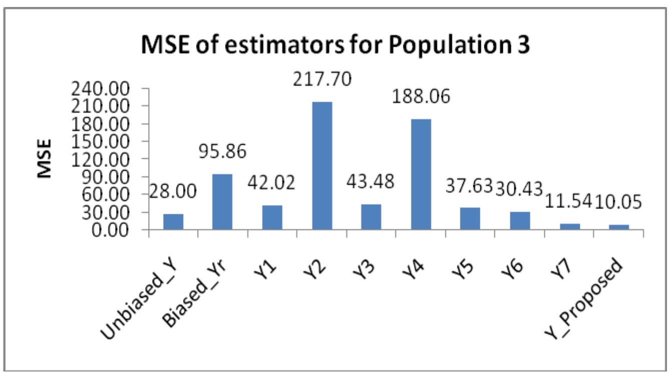
| Estimator | Population I | Population II | Population III | Population IV |
|---|---|---|---|---|
| $\bar{y}$ | 0 | 0 | 0 | 0 |
| $\hat{\bar{Y}}_r$ | 0.6088 | 1.1510 | 2.4624 | 1.3742 |
| $\hat{\bar{Y}}_1$ | 0.0506 | 0.0879 | 0.2760 | 0.2573 |
| $\hat{\bar{Y}}_2$ | 0.1134 | 0.1550 | 3.7353 | 0.6588 |
| $\hat{\bar{Y}}_3$ | 0.0350 | 0.0975 | 0.3047 | 0.2225 |
| $\hat{\bar{Y}}_4$ | 0.1156 | 0.1686 | 3.1516 | 0.5777 |
| $\hat{\bar{Y}}_5$ | 0.1170 | 0.0040 | 0.1896 | 0.2131 |
| $\hat{\bar{Y}}_6$ | -0.1901 | -0.0289 | 0.0479 | -0.3213 |
| $\hat{\bar{Y}}_7$ | -0.2083 | -0.1219 | -0.3242 | -0.3424 |
| $\tau_p$ (Proposed) | **-0.0408** | **-0.1296** | **-0.1017** | **0.1269** |

**Table 3.** Variance/Mean squared errors of the Existing and proposed modified ratio estimators for four natural populations

| Estimator | Population I | Population II | Population III | Population IV |
|---|---|---|---|---|
| $\bar{y}$ | 12.6367 | 12.6367 | 28.0024 | 28.0024 |
| $\hat{\bar{Y}}_r$ | 18.9793 | 41.3276 | 95.8641 | 49.8536 |
| $\hat{\bar{Y}}_1$ | 15.2581 | 17.1925 | 42.0198 | 41.0685 |
| $\hat{\bar{Y}}_2$ | 18.5128 | 20.6715 | 217.7034 | 61.4577 |
| $\hat{\bar{Y}}_3$ | 14.4503 | 17.6909 | 43.47728 | 39.3031 |
| $\hat{\bar{Y}}_4$ | 18.6279 | 21.3750 | 188.0582 | 57.3389 |
| $\hat{\bar{Y}}_5$ | 18.7015 | 12.8461 | 37.6296 | 38.8230 |
| $\hat{\bar{Y}}_6$ | 2.7825 | 11.1406 | 30.4328 | 11.6863 |
| $\hat{\bar{Y}}_7$ | 1.8289 | 6.3206 | 11.5391 | 10.6117 |
| $\tau_p$ (Proposed) | **1.4400** | **2.0569** | **10.0540** | **8.4831** |



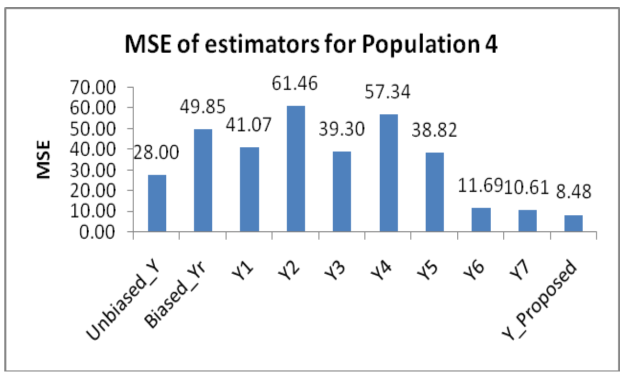
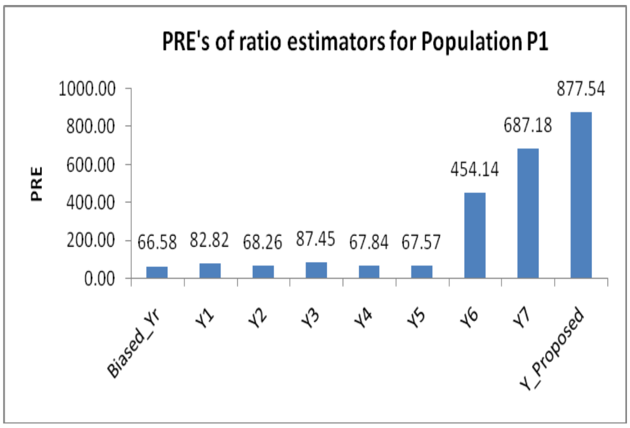
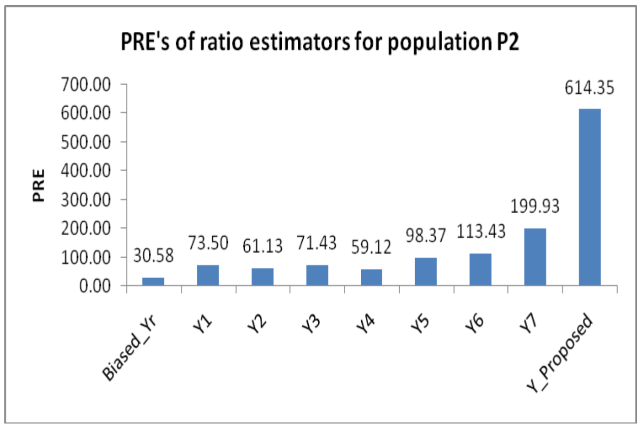
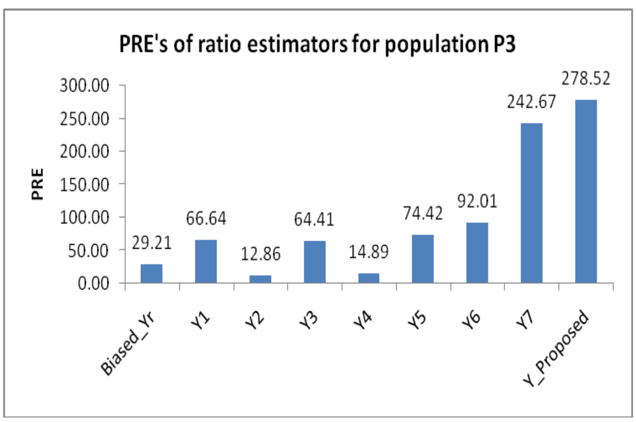



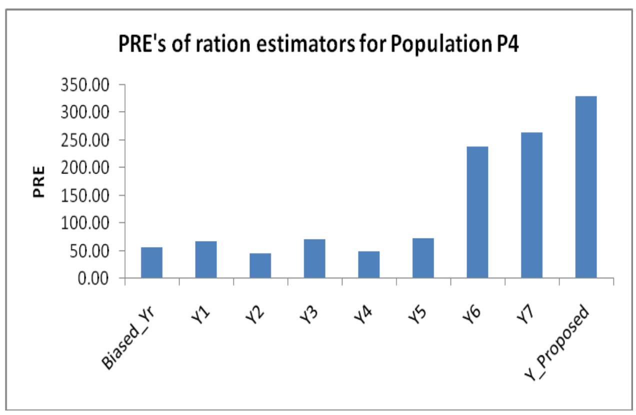
**Table 4.** Percentage relative efficiency of the proposed estimator over other existing modified ratio estimators compare to estimator $\bar{y}$ for four natural populations

| Estimator | Population I | Population II | Population III | Population IV |
|---|---|---|---|---|
| $\bar{y}$ | NA | NA | NA | NA |
| $\hat{\bar{Y}}_r$ | 66.68 | 30.58 | 29.21 | 56.12 |
| $\hat{\bar{Y}}_1$ | 82.82 | 73.50 | 66.64 | 68.18 |
| $\hat{\bar{Y}}_2$ | 68.26 | 61.13 | 12.86 | 45.56 |
| $\hat{\bar{Y}}_3$ | 87.45 | 71.43 | 64.41 | 71.25 |
| $\hat{\bar{Y}}_4$ | 67.84 | 59.12 | 14.89 | 48.84 |
| $\hat{\bar{Y}}_5$ | 67.57 | 98.37 | 74.72 | 72.13 |
| $\hat{\bar{Y}}_6$ | 454.14 | 113.43 | 92.01 | 239.62 |
| $\hat{\bar{Y}}_7$ | 687.18 | 199.93 | 242.67 | 263.88 |
| $\tau_p$ (Proposed) | 877.54 | 614.35 | 278.52 | 330.10 |



## 6. Results and Conclusions

In the present manuscript we have proposed a generalized ratio type estimator of the study variable by making use of information on the size of the sample. The expressions for the bias and mean squared errors of the proposed estimator have been derived up to the first order of approximation. The optimum value of the characterizing scalar which minimizes the mean squared of the proposed estimator has been obtained and the minimum value of the mean squared error for this optimum value of the characterizing scalar has also been obtained. Table-2 and Table-3 represents the biases and the mean squared errors of the proposed and the existing estimators. Table-4 represents the percentage relative efficiencies of the proposed estimator over other estimators. From Table-2 and Table-3 we see that the proposed estimator has minimum biases and mean squared errors in all four natural populations. The relative efficiency of the proposed estimator has been given in Table-4. It can be seen from Table-4, the proposed estimator has least mean squared error for all four populations. Thus it is best for all populations among all other estimator of population mean. Thus the proposed estimator should be used for improved estimation of population mean.

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## Appendix

**R Code to calculate bias and mse's.**

```
################Calculation of Bias and MSE based for
Population1, 2, 3 and 4#############
N<-c(80)
n<-c(20)
Y_bar<-(51.8264)
X_bar<-(11.2646)
Rho<-(0.9413)
CV_Y<-(0.3542)
CV_X<-(0.7507)
Beta_1<-(0.05)
Beta_2<-(0.0634)
Mean_deviation<-(7.575)

######################################################
###############################
pop1<-data.frame(N,n,Y_bar,X_bar,Rho,CV_Y,CV_X,Beta_1,B
eta_2,Mean_deviation)

pop1
pop1$gamma<-(pop1$N-pop1$n)/(pop1$N*pop1$n)
pop1$CVxsq<-pop1$CV_X*pop1$CV_X
pop1$CVysq<-pop1$CV_Y*pop1$CV_Y

#####Calculation of variance of existing unbiased estimator
###########################

pop1$bias_yu = 0
pop1$variance_yu<-
(pop1$gamma*(pop1$Y_bar*pop1$Y_bar))*(pop1$CVysq)

#####Calculation of Bias & MSE of existing ratio estimator
proposed by Cochran ##########

pop1$bias_y<-
(pop1$gamma*pop1$Y_bar)*(pop1$CVxsq-pop1$Rho*pop1$C
V_X*pop1$CV_Y)
pop1$mse_y<-
(pop1$gamma*(pop1$Y_bar*pop1$Y_bar))*(pop1$CVysq+po
p1$CVxsq-2*pop1$Rho*pop1$CV_X*pop1$CV_Y)

#################Calculation of Bias and MSE of first
estimator####################
pop1$cons_delta1<-pop1$X_bar/(pop1$X_bar+pop1$CV_X)
pop1$deltasq_1<-(pop1$X_bar/(pop1$X_bar+pop1$CV_X))*(
pop1$X_bar/(pop1$X_bar+pop1$CV_X))
pop1$bias1<-
(pop1$gamma*pop1$Y_bar)*(pop1$deltasq_1*(pop1$CV_X*p
op1$CV_X)-2*pop1$cons_delta1*pop1$Rho*pop1$CV_X*pop
1$CV_Y)
pop1$mse1<-
(pop1$gamma*(pop1$Y_bar*pop1$Y_bar))*(pop1$CVysq+(po
p1$deltasq_1*pop1$CVxsq)-2*pop1$cons_delta1*pop1$Rho*p
op1$CV_X*pop1$CV_Y)

##############Calculation of Bias and MSE of Second
estimator####################
pop1$delta2<-(pop1$X_bar*pop1$CV_X)/((pop1$X_bar*pop1
$CV_X)+pop1$Beta_2)
pop1$delta2sq<-pop1$delta2*pop1$delta2
pop1$bias2<-(pop1$gamma*pop1$Y_bar)*((pop1$delta2sq*p
op1$CVxsq)-(2*pop1$delta2*pop1$Rho*pop1$CV_X*pop1$C
V_Y))
pop1$mse2<-
(pop1$gamma*(pop1$Y_bar*pop1$Y_bar))*(pop1$CVysq+(po
p1$delta2sq*pop1$CVxsq)-2*pop1$delta2*pop1$Rho*pop1$C
V_X*pop1$CV_Y)

#################Calculation of Bias and MSE of 3rd
estimator####################
pop1$delta3<-pop1$X_bar/(pop1$X_bar+pop1$Rho)
pop1$delta3sq<-pop1$delta3*pop1$delta3
pop1$bias3<-(pop1$gamma*pop1$Y_bar)*((pop1$delta3sq*p
op1$CVxsq)-(2*pop1$delta3*pop1$Rho*pop1$CV_X*pop1$C
V_Y))
pop1$mse3<-
(pop1$gamma*(pop1$Y_bar*pop1$Y_bar))*(pop1$CVysq+(po
```

```
p1$delta3sq*pop1$CVxsq)-2*pop1$delta3*pop1$Rho*pop1$C
V_X*pop1$CV_Y)

##################Calculation of Bias and MSE of 4th
estimator#####################
pop1$delta4<-pop1$X_bar/(pop1$X_bar+pop1$Beta_2)
pop1$delta4sq<-pop1$delta4*pop1$delta4
pop1$bias4<-(pop1$gamma*pop1$Y_bar)*((pop1$delta4sq*p
op1$CVxsq)-(2*pop1$delta4*pop1$Rho*pop1$CV_X*pop1$C
V_Y))
pop1$mse4<-
(pop1$gamma*(pop1$Y_bar*pop1$Y_bar))*(pop1$CVysq+(po
p1$delta4sq*pop1$CVxsq)-2*pop1$delta4*pop1$Rho*pop1$C
V_X*pop1$CV_Y)

##################Calculation of Bias and MSE of 5th
estimator#####################
pop1$delta5<-pop1$X_bar/(pop1$X_bar+pop1$Beta_1)
pop1$delta5sq<-pop1$delta5*pop1$delta5
pop1$bias5<-(pop1$gamma*pop1$Y_bar)*((pop1$delta5sq*p
op1$CVxsq)-(2*pop1$delta5*pop1$Rho*pop1$CV_X*pop1$C
V_Y))
pop1$mse5<-
(pop1$gamma*(pop1$Y_bar*pop1$Y_bar))*(pop1$CVysq+(po
p1$delta5sq*pop1$CVxsq)-2*pop1$delta5*pop1$Rho*pop1$C
V_X*pop1$CV_Y)

###################Calculation of Bias and MSE of 6th
estimator#####################
pop1$delta6<-pop1$X_bar/(pop1$X_bar+pop1$Mean_deviati
on)
pop1$delta6sq<-pop1$delta6*pop1$delta6
pop1$bias6<-(pop1$gamma*pop1$Y_bar)*((pop1$delta6sq*p
op1$CVxsq)-(2*pop1$delta6*pop1$Rho*pop1$CV_X*pop1$C
V_Y))
pop1$mse6<-
(pop1$gamma*(pop1$Y_bar*pop1$Y_bar))*(pop1$CVysq+(po
p1$delta6sq*pop1$CVxsq)-2*pop1$delta6*pop1$Rho*pop1$C
V_X*pop1$CV_Y)

##################Calculation of Bias and MSE of 7th
estimator#####################
pop1$delta7<-pop1$X_bar/(pop1$X_bar+pop1$n)
pop1$delta7sq<-pop1$delta7*pop1$delta7
pop1$bias7<-(pop1$gamma*pop1$Y_bar)*((pop1$delta7sq*p
op1$CVxsq)-(2*pop1$delta7*pop1$Rho*pop1$CV_X*pop1$C
V_Y))
pop1$mse7<-
(pop1$gamma*(pop1$Y_bar*pop1$Y_bar))*(pop1$CVysq+(po
p1$delta7sq*pop1$CVxsq)-2*pop1$delta7*pop1$Rho*pop1$C
V_X*pop1$CV_Y)

##############Calculation of Bias and MSE of Proposed
estimator#####################
pop1$delta_p<-pop1$X_bar/(pop1$X_bar+pop1$n)
pop1$delta_psq<-pop1$delta_p*pop1$delta_p
pop1$A<-(pop1$delta_psq*pop1$CVxsq)-(pop1$delta_p*pop1
$Rho*pop1$CV_X*pop1$CV_Y)
pop1$Asq<-pop1$A*pop1$A
pop1$B<-pop1$delta_psq*pop1$CVxsq
pop1$alpha<-(pop1$A)/(pop1$B)

pop1$bias_p<-(pop1$gamma*pop1$Y_bar)*((-pop1$delta_p*
pop1$Rho*pop1$CV_X*pop1$CV_Y)+(pop1$delta_psq*pop1$
CVxsq)+(pop1$alpha*pop1$delta_p

*pop1$Rho*pop1$CV_X*pop1$CV_Y)-pop1$alpha*pop1$delt
a_psq*pop1$CVxsq)
pop1$mse_p<-
(pop1$gamma*(pop1$Y_bar*pop1$Y_bar))*(pop1$CVysq+(po
p1$delta_psq*pop1$CVxsq)-2*pop1$delta_p*pop1$Rho*pop1
$CV_X*pop1$CV_Y

-(pop1$Asq/pop1$B))
#########Combining bias and
mse's############################################
bias_pop1<-pop1[,c('bias_yu','bias_y','bias1','bias2','bias3','bia
s4','bias5','bias6','bias7','bias_p')]
mse_pop1<-pop1[,c('variance_yu','mse_y','mse1','mse2','mse3','
mse4','mse5','mse6','mse7','mse_p')]
bias_p1<-t(bias_pop1)
mse_p1<-t(mse_pop1)
data_f<-cbind.data.frame(bias_p1,mse_p1)
```

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