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# Comparing Two Estimators of Reliability Function for Three Extended Rayleigh Distribution

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**Abstract** In this paper we work on transforming one scale parameter Rayleigh distribution into three parameters one's $(\alpha, \beta, \theta)$, by introducing two shapes parameters $(\alpha, \beta)$, since this distribution is important due to its many applications, in the analysis of signal and statistical error. Also it is a good model for representing remission time for (failure model) from certain disease that beat some people or anything else. The resulted distribution have three parameters, $(\theta)$ is scale parameter, $(\alpha, \beta)$ are shaped parameters obtained from exposing one parameter Rayleigh into different exponents through $(\alpha \,\&\, \beta)$, this expansion done through introducing another parameters, this allow greater flexibility on the tail of distribution and permits wide application in the fields of engineering and biological studied.

**Keywords** Three Parameters Modified Rayleigh, Moment's Estimators (MOM), Maximum Likelihood Estimators (MLE), Reliability Function Estimator

## 1. Introduction

The one scale probability distribution is one of continuous probability which represents positive valued real random variable, like failure time, remission time until death, as well as velocity of winds and representing single data, also errors data. Many researchers studied one scale parameter Rayleigh, Dyre et al (1973), Diebolt J, Robert C.P. (1994), study divergence and distance measure in econometrics, which can also be applied in biological data. The expanding of probability distribution was firstly introduced by Lehmann (1953), and then in (2006), it was studied by Nadarajah and Kotez [17], the expansion yield a new exponentiated generalized distributions, many other researchers like Gupta and Kundu (2001) [9], Nadarajah and Kotez (2006) [17], proposed exponentiated gamma ( $E\Gamma$ ). Many other researchers introduced transmuted Rayleigh like Merovci in (2013) [13] and (2014), also Khan M. Shuaib, King Robert introduced transmuted of Weibull, Merovci F. (2013) introduced transmuted Rayleigh and in (2014), Merovi F. introduced transmuted generalized Rayleigh. In (2016), Mohammed Shuaib [15] introduced three parameters transmuted Rayleigh.

Many researchers proposed different formula to extend the base line distribution [G( $x$ )] to another form called exponentiated type distribution to obtain a new formula of p.d.f which may be gives greater flexibility and extend the scope of applications in engineering and biological applications.

In our research we apply the formula given by Gauss, Ortega and Daniel (2013) [7] on one scale parameter Rayleigh to obtain new exponentiated three parameters Rayleigh. This paper deals with extending one parameter Rayleigh into three parameters one's through applying definition shown in equation (3), the resulted p.d.f have one scale parameter $(\theta)$ and two shape parameters $(\alpha, \beta)$.

## 2. Theoretical Aspects

The p.d.f of one parameter Rayleigh failure model is;

$$g_T(t,\theta) = \frac{2}{\theta} t\, e^{-\frac{t^2}{\theta}}\ t,\theta > 0 \tag{1}$$

The cumulative distribution function CDF is;

$$G_T(t,\theta) = 1 - e^{-\frac{t^2}{\theta}}\ t,\theta > 0 \tag{2}$$

The class of distribution in equation (1) can be extended to three parameters one, applying the method introduced by Gauss and Corderion (2013) [7], by applying the following definition on p.d.f given in equation (1):

$$f_T(t) = \alpha\beta\ [\{1 - F_T(t)\}^{\alpha-1}]\ [1 - \{1 - F_T(t)\}^{\alpha}]^{\beta-1}\ f(t) \tag{3}$$

$$f_T(t) = \alpha\beta(e^{-\frac{t^2}{\theta}})^{\alpha-1}[1 - e^{-\frac{\alpha t^2}{\theta}}]^{\beta-1}\frac{2}{\theta} t\, e^{-\frac{t^2}{\theta}}$$

$$f_T(t) = \frac{2\alpha\beta}{\theta} t\, e^{-\frac{\alpha t^2}{\theta}} \left[1 - e^{-\frac{\alpha t^2}{\theta}}\right]^{\beta-1} \tag{4}$$

The new cumulative distribution function corresponding to (4) is;

$$F_T(t) = \left[1 - e^{-\frac{\alpha t^2}{\theta}}\right]^{\beta} \tag{5}$$

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This exponentiated Rayleigh (ER), used in many applications in engineering for signal analysis, also in error analysis for different system.

This paper include deriving the formula for r^th^ moments about origin, as well as deriving maximum likelihood estimator for three parameters $(\alpha, \beta, \theta)$, then comparing different estimators of reliability function through simulation procedure by two methods (MLE, MOM), the comparison done using different sets of sample size and different sets of initial values of $(\alpha, \beta, \theta)$, the reselults are compared using statistical measure mean square error (MSE).

# 3. Estimation of Parameters

We know that the estimators of parameters of probability distribution are (statistics), that explain how to use the sample data in estimating unknown parameters of population, here the exponentiated distribution (Rayleigh) have (explicit) p.d.f, also (explicit) CDF, so the estimators of three parameters $(\alpha, \beta, \theta)$ needs to solve implicit function obtained from applying moments or maximum likelihood methods, these methods of estimation need some algorithms for (MLE), also need equate sample moments $[\mu_r' = E(t^r)]$ to obtain moment's three estimators $(\hat{\alpha}_{mom}, \hat{\beta}_{mom}, \hat{\theta}_{mom})$, for more information and discussion see Al – Naqeeb and Hamed (2009) [6]. Now we explain the indicated methods for estimation.

### 3.1. Derivation of $r^{th}$ Moments about Origin

The $r^{th}$ moments formula about origin denoted by;

$$\mu_r' = E(t^r) \int_0^\infty t^r \; f(t, \alpha, \beta, \theta) dt \tag{6}$$

Since the p.d.f contain the terms;

$$\left[1 - e^{-\frac{\alpha\, t^2}{\theta}}\right]^{\beta - 1}$$

To simplify the formula for $r^{th}$ moments about origin we first apply the formula introduced by Mood, (1974) [3], which is;

Since,

$$\left[1 - e^{-\frac{\alpha\, t^2}{\theta}}\right]^{\beta - 1} = \sum_{j=0}^{\beta-1} C_j^{\beta-1} (-1)^j \left[e^{-\frac{\alpha\, t^2}{\theta}}\right]^{(\beta-1)j} \tag{7}$$

$$\mu_r' = \frac{2\alpha\beta}{\theta} \int_0^\infty t^{r+1} \; e^{-\frac{\alpha\, t^2}{\theta}} \sum_{j=0}^{\beta-1} C_j^{\beta-1} (-1)^j \left[e^{-\frac{\alpha\,(\beta-1) j t^2}{\theta}}\right] dt$$

$$= \frac{2\alpha\beta}{\theta} \sum_{j=0}^{\beta-1} C_j^{\beta-1} (-1)^j \int_0^\infty t^{r+1} \; e^{-\frac{\alpha[1 + (\beta-1) j \; t^2]}{\theta}} dt \tag{8}$$

$$\mu_r' = \frac{2\alpha\beta}{\theta} \left[\frac{\sum_{j=0}^{\beta-1} C_j^{\beta-1} (-1)^j \, \Gamma\left(\frac{r}{2} + 1\right)}{\frac{\alpha}{\theta} [1 + (\beta-1) j^{\left(\frac{r}{2}+1\right)}]}\right] \tag{9}$$

Then solving $[\mu_r' = \frac{\sum x_i^r}{n}]$ for $(r = 1,2,3)$, we obtain $(\hat{\alpha}_{mom}, \hat{\beta}_{mom}, \hat{\theta}_{mom})$.

### 3.2. Derivation of MLE

Let $(t_1, t_2, \dots, t_n)$ be a random sample from $p.d.f$ in equation (4), then;

$$L = \prod_{i=1}^{n} (t_i, \alpha, \beta, \theta)$$

$$L = 2^n \alpha^n \beta^n \theta^{-n} \prod_{i=1}^{n} t_i \, e^{-\frac{\alpha}{\theta} \sum_{i=1}^n t_i^2} \prod_{i=1}^{n} \left[1 - e^{-\frac{\alpha t_i^2}{\theta}}\right]^{\beta - 1} \tag{10}$$

Taking log for equation (10):

$$log\, L = n\, log\, 2 + n\, log\, \alpha + n\, log\, \beta - n\, log\, \theta + \sum_{i=1}^{n} log\, t_i - \frac{\alpha}{\theta} \sum_{i=1}^{n} t_i^2 + (\beta - 1) \sum_{i=1}^{n} log\left(1 - e^{-\frac{\alpha\, t_i^2}{\theta}}\right) \tag{11}$$

Deriving (11) for $(\alpha, \beta, \theta)$ yields;

$$\frac{\partial\, log\, L}{\partial \alpha} = \frac{n}{\alpha} - \frac{1}{\theta} \sum_{i=1}^{n} t_i^2 + (\beta - 1) \sum_{i=1}^{n} \frac{\left(\frac{t_i^2}{\theta}\right) e^{-\frac{\alpha\, t_i^2}{\theta}}}{\left(1 - e^{-\frac{\alpha\, t_i^2}{\theta}}\right)} = 0$$

$$\hat{\alpha}_{MLE} = \frac{n}{\left[\frac{\sum_{i=1}^n t_i^2}{\theta} - (\beta - 1) \sum_{i=1}^n \frac{\left(\frac{t_i^2}{\theta}\right) e^{-\frac{\alpha\, t_i^2}{\theta}}}{\left(1 - e^{-\frac{\alpha\, t_i^2}{\theta}}\right)}\right]} \tag{12}$$

$$\frac{\partial\, log\, L}{\partial \beta} = \frac{n}{\beta} + \sum_{i=1}^{n} log\left(1 - e^{-\frac{\alpha\, t_i^2}{\theta}}\right) = 0$$

$$\hat{\beta}_{MLE} = -\frac{n}{\sum_{i=1}^n log\left(1 - e^{-\frac{\alpha\, t_i^2}{\theta}}\right)} \tag{13}$$

$$\frac{\partial\, log\, L}{\partial \theta} = -\frac{n}{\theta} + \frac{\alpha}{\theta^2} \sum_{i=1}^{n} t_i^2 + (\beta - 1) \sum_{i=1}^{n} \frac{\left(-e^{-\frac{\alpha\, t_i^2}{\theta}}\right)\left(\frac{\alpha\, t_i^2}{\theta^2}\right)}{\left(1 - e^{-\frac{\alpha\, t_i^2}{\theta}}\right)}$$

$$\Rightarrow \frac{\alpha}{\theta^2} \sum_{i=1}^{n} t_i^2 - \frac{(\beta - 1)\alpha}{\theta^2} \sum_{i=1}^{n} \frac{\left(e^{-\frac{\alpha\, t_i^2}{\theta}}\right) t_i^2}{\left(1 - e^{-\frac{\alpha\, t_i^2}{\theta}}\right)} = \frac{n}{\theta}$$

$$\frac{\alpha \sum t_i^2}{\hat{\theta}^2} = \frac{(\beta - 1)\alpha}{\hat{\theta}^2} \sum_{i=1}^{n} \frac{\left(e^{-\frac{\alpha\, t_i^2}{\theta}}\right) t_i^2}{\left(1 - e^{-\frac{\alpha\, t_i^2}{\theta}}\right)} + \frac{n}{\theta}$$

$$\Rightarrow n\hat{\theta} = \alpha \sum t_i^2 - (\beta - 1)\alpha \sum_{i=1}^{n} \frac{\left(e^{-\frac{\alpha\, t_i^2}{\theta}}\right) t_i^2}{\left(1 - e^{-\frac{\alpha\, t_i^2}{\theta}}\right)}$$

$$\hat{\theta}_{MLE} = \frac{1}{n} \alpha \sum_{i=1}^{n} t_i^2 - \alpha(\beta - 1) \sum_{i=1}^{n} \frac{\left(e^{-\frac{\alpha\, t_i^2}{\theta}}\right) t_i^2}{\left(1 - e^{-\frac{\alpha\, t_i^2}{\theta}}\right)} \quad (14)$$

Solving equations (12, 13, 14) numerically gives MLE for $(\alpha, \beta, \theta)$ for the given values of $(t_i)$ and estimated values of $(\alpha, \beta, \theta)$.

## 4. Simulation Procedure

Let:

$$u_i = \left(1 - e^{-\frac{\alpha\, t_i^2}{\theta}}\right)^{\beta}$$

Then

$$(u_i)^{\frac{1}{\beta}} = \left(1 - e^{-\frac{\alpha\, t_i^2}{\theta}}\right)$$

$$e^{-\frac{\alpha\, t_i^2}{\theta}} = \left(1 - (u_i)^{\frac{1}{\beta}}\right)$$

$$-\frac{\alpha\, t_i^2}{\theta} = \ln\left(1 - (u_i)^{\frac{1}{\beta}}\right)$$

$$t_i^2 = -\frac{\theta}{\alpha} \ln\left(1 - (u_i)^{\frac{1}{\beta}}\right)$$

$$t_i = \left[-\frac{\theta}{\alpha} \ln\left(1 - (u_i)^{\frac{1}{\beta}}\right)\right]^{\frac{1}{2}}$$

Where $\ln\left(1 - (u_i)^{\frac{1}{\beta}}\right)$ is negative value and ti= $\sqrt{\frac{-\theta}{\alpha}(negative\ value)}$ reduced to positive.

(α,Θ) are scale parameters and (β) is shape parameter for given values of (ti), and estimated values of (Θ,α,β), the following tables gives the results of simulation of:

$\hat{R}(ti) = 1 - (1 - e^{-\frac{\hat{\alpha} t_i^2}{\hat{\theta}}})^{\hat{\beta}}$ And also MSE of $\hat{R}(ti)$.

**Table (1).** Moment and Maximum Likelihood Estimators for Reliability Function with MSE

| $n$ | α | $\theta$ | β | $t_i$ | $\hat{R}_{MOM}$ | $\hat{R}_{MLE}$ | Best |
|---|---|---|---|---|---|---|---|
| 40 | 1.5 | 1.5 | 2 | 1.5 | 0.9638 | 0.9822 | MLE |
| | | | | 2 | 0.9584 | 0.9643 | MLE |
| | | | | 2.5 | 0.9648 | 0.9582 | MOM |
| | | | | 3 | 0.8873 | 0.88941 | MLE |
| | | | | 3.5 | 0.8755 | 0.88763 | MLE |
| | | | | 4 | 0.8932 | 0.8745 | MOM |
| | | | | 4.5 | 0.8542 | 0.8632 | MLE |
| | | | | 5 | 0.8443 | 0.8554 | MLE |
| 40 | 1.5 | 1.5 | 3 | 1.5 | 0.9864 | 0.9821 | MOM |
| | | | | 2 | 0.9692 | 0.9628 | MOM |
| | | | | 2.5 | 0.9424 | 0.9409 | MOM |
| | | | | 3 | 0.8995 | 0.9372 | MLE |
| | | | | 3.5 | 0.8863 | 0.9335 | MLE |
| | | | | 4 | 0.8506 | 0.9224 | MLE |
| | | | | 4.5 | 0.9412 | 0.9112 | MOM |
| | | | | 5 | 0.8301 | 0.8831 | MLE |
| 40 | 1.5 | 2 | 2 | 1.5 | 0.9724 | 0.9904 | MLE |
| | | | | 2 | 0.9321 | 0.9893 | MLE |
| | | | | 2.5 | 0.9041 | 0.9874 | MLE |
| | | | | 3 | 0.8936 | 0.8667 | MOM |
| | | | | 3.5 | 0.8852 | 0.8774 | MOM |
| | | | | 4 | 0.8871 | 0.8834 | MOM |
| | | | | 4.5 | 0.8736 | 0.8984 | MLE |
| | | | | 5 | 0.8702 | 0.8852 | MLE |

| $n$ | α | $\theta$ | β | $t_i$ | $\hat{R}_{MOM}$ | $\hat{R}_{MLE}$ | Best |
|---|---|---|---|---|---|---|---|
| 40 | 1.8 | 2 | 3 | 1.5 | 0.9856 | 0.9772 | MOM |
| | | | | 2 | 0.9736 | 0.9092 | MOM |
| | | | | 2.5 | 0.8946 | 0.8758 | MOM |
| | | | | 3 | 0.8664 | 0.8691 | MLE |
| | | | | 3.5 | 0.8936 | 0.8822 | MOM |
| | | | | 4 | 0.8992 | 0.9036 | MLE |
| | | | | 4.5 | 0.8867 | 0.8979 | MLE |
| | | | | 5 | 0.8976 | 0.8993 | MLE |
| 80 | 1.5 | 1.5 | 2 | 1.5 | 0.8156 | 0.8658 | MLE |
| | | | | 2 | 0.8631 | 0.8423 | MOM |
| | | | | 2.5 | 0.8421 | 0.8531 | MLE |
| | | | | 3 | 0.8449 | 0.8973 | MLE |
| | | | | 3.5 | 0.8432 | 0.8664 | MLE |
| | | | | 4 | 0.8632 | 0.8752 | MLE |
| | | | | 4.5 | 0.8532 | 0.8766 | MLE |
| | | | | 5 | 0.6732 | 0.6641 | MOM |
| 80 | 1.5 | 1.5 | 3 | 1.5 | 0.7792 | 0.7973 | MLE |
| | | | | 2 | 0.8492 | 0.8832 | MLE |
| | | | | 2.5 | 0.8351 | 0.8994 | MLE |
| | | | | 3 | 0.8568 | 0.8732 | MLE |
| | | | | 3.5 | 0.8875 | 0.8982 | MLE |
| | | | | 4 | 0.7863 | 0.7792 | MOM |
| | | | | 4.5 | 0.7031 | 0.7582 | MLE |
| | | | | 5 | | | |
| 80 | 1.5 | 2 | 2 | 1.5 | 0.7606 | 0.76554 | MLE |
| | | | | 2 | 0.8268 | 0.83820 | MLE |
| | | | | 2.5 | 0.8386 | 0.8906 | MLE |
| | | | | 3 | 0.8424 | 0.8853 | MLE |
| | | | | 3.5 | 0.8393 | 0.8344 | MOM |
| | | | | 4 | 0.8417 | 0.8435 | MLE |
| | | | | 4.5 | 0.8468 | 0.84336 | MOM |
| | | | | 5 | 0.8752 | 0.8754 | MOM |
| 80 | 1.8 | 2 | 3 | 1.5 | 0.8453 | 0.8493 | MLE |
| | | | | 2 | 0.84336 | 0.8433 | MOM |
| | | | | 2.5 | 0.84337 | 0.8392 | MOM |
| | | | | 3 | 0.6056 | 0.6054 | MLE |
| | | | | 3.5 | 0.6044 | 0.6032 | MOM |
| | | | | 4 | 0.5410 | 0.5431 | MLE |
| | | | | 4.5 | 0.5031 | 0.5004 | MOM |
| | | | | 5 | 0.5006 | 0.5005 | MOM |

| $n$ | α | $\theta$ | β | $t_i$ | $\hat{R}_{MOM}$ | $\hat{R}_{MLE}$ | Best |
|---|---|---|---|---|---|---|---|
| 100 | 1.5 | 1.5 | 2 | 1.5 | 0.8720 | 0.8449 | MOM |
| | | | | 2 | 0.8421 | 0.8201 | MOM |
| | | | | 2.5 | 0.8543 | 0.8602 | MLE |
| | | | | 3 | 0.7496 | 0.7588 | MLE |
| | | | | 3.5 | 0.7036 | 0.7114 | MLE |
| | | | | 4 | 0.7022 | 0.7335 | MLE |
| | | | | 4.5 | 0.7775 | 0.7781 | MLE |
| | | | | 5 | 0.74695 | 0.8724 | MLE |
| 100 | 1.5 | 1.5 | 3 | 1.5 | 0.8433 | 0.8436 | MLE |
| | | | | 2 | 0.8221 | 0.8305 | MLE |
| | | | | 2.5 | 0.8246 | 0.8212 | MOM |
| | | | | 3 | 0.7753 | 0.7784 | MLE |
| | | | | 3.5 | 0.7932 | 0.7646 | MOM |
| | | | | 4 | 0.7512 | 0.7541 | MLE |
| | | | | 4.5 | 0.7748 | 0.7739 | MOM |
| | | | | 5 | 0.6705 | 0.6421 | MOM |
| 100 | 1.5 | 2 | 2 | 1.5 | 0.8532 | 0.5894 | MLE |
| | | | | 2 | 0.59131 | 0.5662 | MOM |
| | | | | 2.5 | 059021 | 0.5732 | MOM |
| | | | | 3 | 0.5842 | 0.5536 | MOM |
| | | | | 3.5 | 0.5773 | 0.5641 | MOM |
| | | | | 4 | 0.5562 | 0.5582 | MLE |
| | | | | 4.5 | 0.5531 | 0.5466 | MLE |
| | | | | 5 | 0.5421 | 0.5362 | MOM |
| 100 | 1.8 | 2 | 3 | 1.5 | 0.6240 | 0.6032 | MOM |
| | | | | 2 | 0.6653 | 0.6462 | MOM |
| | | | | 2.5 | 0.6664 | 0.6533 | MOM |
| | | | | 3 | 0.6432 | 0.6652 | MLE |
| | | | | 3.5 | 0.6235 | 0.6774 | MLE |
| | | | | 4 | 0.6225 | 0.6263 | MLE |
| | | | | 4.5 | 0.54631 | 0.5508 | MLE |
| | | | | 5 | 0.53212 | 0.5450 | MOM |

# 5. Conclusions

1. The studied distribution is one of continuous probability distribution that has applications for analysis signal data and data for statistical error and remission time for patients.
2. The exponentiated Rayleigh have many applications so the extending formula through adding parameters make it more flexible and applicable for different set of data (signal data, failure data, ….).
3. We apply methods of moments and maximum likelihood to estimate (scale parameter $\theta$ and two shapes parameters (α, β), and then estimate reliability function, we find that $\hat{R}_{MLE}$ is best with percentage (54/96) × 100%; while $\hat{R}_{MOM}$ is best with (42/96)×100%; i.e ($\hat{R}_{MLE}$) is best one due to best performance of maximum likelihood estimator as compared with another method of estimation, according to its properties which are invariant and have minimum variance and efficient and also consistent.

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