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Parameters and Reliability Estimation for the Fuzzy Exponential Distribution

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Abstract This paper deals in estimating fuzzy reliability function for exponential fuzzy distribution, with two parameters (λ, θ), where the scale parameter (λ) is estimated by moments, and maximum likelihood, then we work on estimating fuzzy reliability function $\{\tilde{R} = e^{-\tilde{\lambda}\tilde{k}(\tilde{t}-\hat{\theta})}\}$, where $\{t = ([0,\infty), \mu_{\tilde{t}})\}$, $[\tilde{t} = \tilde{k}t\ , 0 < \tilde{k} < 1]$, $[t \in Real]$, we introduce all the tables of estimators of $(\hat{\tilde{R}})$, by different two methods, and the results are compared using statistical measure mean square error (MSE).

Keywords Fuzzy two parameters Exponential Model (FEM), Fuzzy Reliability Function, Moment Estimator, MLE

1. Introduction

We know that human thinking and reasoning frequently involve fuzzy information which was originating from inherently in exact human concepts, since our systems are unable to answer many questions, and then fuzzy sets have been able to provide solutions to many real world problems. We know that the fuzzy set theory is an extension of classical set theory where the elements have degree of membership. The world "fuzzy" means "vagueness", this is occurs when the boundary of a piece of information is not clear – cut, Zadeh (1965, 1975) give an extension study of fuzzy sets and he introduce the concepts of fuzzy set and fuzzy theory. While (Zadeh & Sugeno [1970]) developed the probability measures of fuzzy event, Kwaker Naak, Puri and Relescu [1978, 1986] introduced fuzzy random variable, after them Thomas [1979, 1995], elaborates the likelihood semantics for fuzzy sets. In 1988 military handbook electronic reliability design handbook in this field of study. In 1993 Cai etal developed fuzzy system reliability based on fuzzy state assumption and probability assumption [21], but (Cai etal, Chen and Cai [1991, 1994, 1995]) gives the relationship between fuzzy and reliability, many other researchers worked on finding the membership of any given system reliability. Nozer D. [2004] and Sigh Puewalla & June [2006] introduced the members function and probability measures of fuzzy set, and work on developing fuzzy reliability concept for Weibull fuzzy probability distribution and other fuzzy distribution. Here we complete the work in this field to explain the fuzzy reliability models with exponential fuzzy distribution.

2. Some Basic Definitions

- The fuzzy set theory represent the extension of classical set theory where elements have degree of membership.
- The sample space (Ω), is a collection of all possible outcomes of random experiments, but the fuzzy subset (A or Ω) is defined by the membership function denoted by $[\mu_A(x)]$ which produce [0,1] for all $(x \in \Omega)$, so $[\mu_A(x)]$ is mapping function into [0,1], and [4];

$$[\mu_A(x)] = \begin{cases} 1\ if\ x \in A \\ 0\ if\ x \notin A \end{cases}$$

And the fuzzy set has the properties;

- The height of a fuzzy set denoted by $[hgt(A)]$] is the maximum of the membership grades of A, i.e [8] $hgt(A) = sup_{x\in X}\mu_A(x)$
- A fuzzy set A is normal if $hgt(A) = 1$
- The support of a set A is the crisp subs of A with non – zero membership grades [3];

$$supp\ (A) = \{x|\mu_A(x) > 0\}\ and\ Core\ (A) = \{x|\mu_A(x) = 1\}$$

- Many other notations related to fuzzy sets are [3];
  - $A = B \Leftrightarrow \mu_A(x) = \mu_B(x)\ \forall x$
  - $A \subset B \Leftrightarrow \mu_A(x) \le \mu_B(x)\ \forall x$
  - $\overline{A}\ complement\ of\ A \Leftrightarrow \mu_{\overline{A}}(x) = 1 - \mu_A(x)\ \forall x$
  - $A \cup B \Leftrightarrow \mu_{A\cup B}(x) = Max[\mu_A(x)\ , \mu_B(x)\ ]\ \forall x$
  - $A \cap B \Leftrightarrow \mu_{A\cap B}(x) = Min[\mu_A(x)\ , \mu_B(x)\ ]\ \forall x$
  - $A \oplus B \Leftrightarrow \mu_{A\oplus B}(x) = [\mu_A(x) + \mu_B(x) - \mu_A(x)\ \mu_B(x)]\ \forall x$

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Since out research deals with fuzzy probability distribution and reliability, so we need to explain some probability measures of fuzzy events, then of reliability function [16].

- The reliability of a device of a system represent the probability that it will give satisfactory performance for a specified period under specific operating conditions which is denoted by [R(t)] where;
  - $R(0) = pr(T > 0) = 1$
  - $R(\infty) = 1$
  - $0 \leq R(t) \leq 1$
  - $if\ t_1 < t_2\ then\ R(t_1) \geq R(t_2)$ (1)
- Let (T) be continuous random variable ($T > 0$), then the reliability function [$R_T(t)$] is;

$$R_T(t) = \int_t^{\infty} f_T(x)dx = 1 - \int_0^t f_T(x)dx = 1 - F_T(t) \quad (2)$$

Also the probability that the failure time (T) occurs in an interval (t_1, t_2) is;

$$pr(t_1 < T < t_2) = F_T(t_2) - F_T(t_1) = R_T(t_1) - R_T(t_2) \quad (3)$$

From the last definition we can say that the reliability function $R(T)$ or the probability of a device not failing prior to sometime (t), is given by;

$$\left.\begin{aligned} R(T) &= 1 - F(t) = \int_t^{\infty} f(t)dt \\ -\frac{dR(t)}{dt} &= f(t) \end{aligned}\right\rbrace \quad (4)$$

The probability of failure in a given time interval between (t_1, t_2) can be expressed by reliability function as;

$$\int_{t_1}^{\infty} f(t)dt - \int_{t_2}^{\infty} f(t)dt = R(t_1) - R(t_2) \quad (5)$$

Now we can say that the rate at which the failure occur in the interval ($t_1\ to\ t_2$) is denoted by [$\lambda(t)$], which is given by;

$$\lambda(t) = \frac{R(t_1) - R(t_2)}{(t_2 - t_1)R(t_1)} = \frac{R(t) - R(t + \Delta t)}{\Delta t\ R(t)} \quad (6)$$

According to above, we can say that the fuzzy reliability represent the probability of a device performing its purpose in varying degrees of success for the period of time intended under operating conditional encountered and it can denoted by ($\tilde{R}$), which is a function of a fuzzy set ($\tilde{A}_i$) [18, 19].

Let [$\mu_{\tilde{A}_i}(R)$] represent the degree of membership (R) in ($\tilde{A}_i$), then;

$$\tilde{R} = \mu_{\tilde{A}}(R).R \quad (7)$$

Where

$$R = \int_t^{\infty} f(t)dt \text{ then}$$

$$\tilde{R} = \mu_{\tilde{A}}(R) \int_t^{\infty} f(t)dt \quad (8)$$

3. Fuzzy Probability Function and its Reliability

This section introduce the probability density function of exponential distribution which is used commonly in reliability engineering and is used to model the behavior of units that have a constant failure rate (or units that do not degrade with time or wear out).

The $p.d.f$ of two parameters exponential is defined by [10];

$$f(t) = \lambda e^{-\lambda(t-\theta)}\ t > \theta, \lambda, \theta > 0 \quad (9)$$

λ is scale parameter, θ is location parameter, and its cumulative distribution is [10];

$$F_T(t) = pr(T \leq t) = 1 - e^{-\lambda(t-\theta)} \quad (10)$$

While the reliability function;

$$R_T(t) = 1 - F_T(t) = e^{-\lambda(t-\theta)} \quad (11)$$

Also the mean and variance;

$$\mu = E(T) = \frac{1}{\lambda} + \theta$$

$$\sigma^2 = v(T) = \frac{1}{\lambda^2}$$

The failure rate function (hazard function);

$$H(t) = \frac{f(t)}{R(t)} = \lambda \quad (12)$$

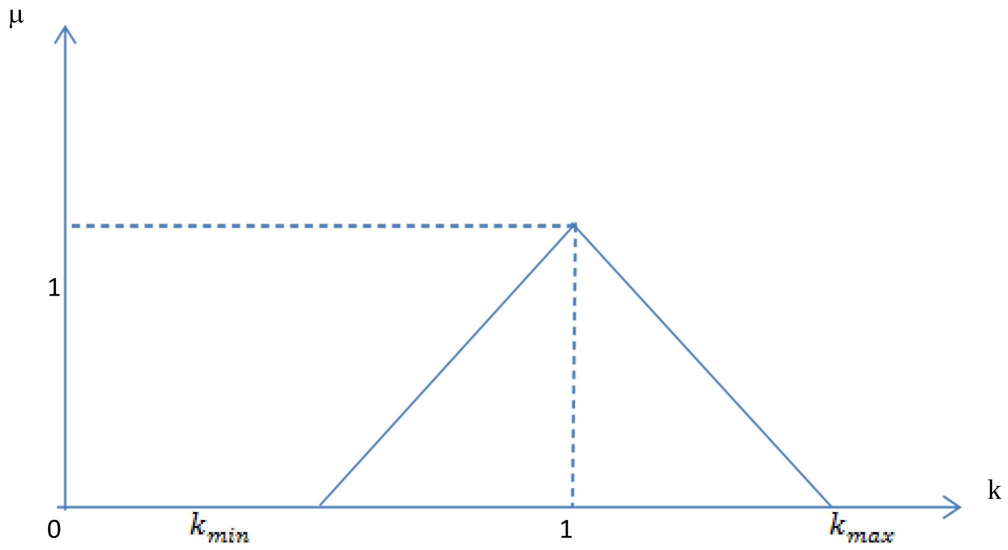
We assume the values of a fuzzy random variable ($\tilde{T}$) are fuzzy number [7];

$$\tilde{t} = \{[0, \infty), \mu_{\tilde{t}}\} \text{ and } \tilde{t} = \tilde{k}t, t \in T$$

Then the vagueness is a real triangular fuzzy number, $\tilde{k} = \{[0, \infty), \mu_{\tilde{k}}\}$;

$$\mu_{\tilde{k}}(k) = \begin{cases} \frac{k - k_{min}}{1 - k_{min}} & k \in (k_{min}, 1) \\ \frac{k - k_{max}}{1 - k_{max}} & k \in (1, k_{max}) \\ 0 & otherwise \end{cases} \quad (13)$$

$$0 < k_{min} \leq 1 \leq k_{max}$$



When the random variable (T) have a crisp exponential probability function $\{Exp(\lambda,\theta)\}$, then the corresponding fuzzy random variable ($\tilde{T}$) with fuzzy exponential probability distribution $\{\widetilde{Exp}(\lambda,\theta)\}$, with the following characteristics;

For all $t \in [0,\infty)$, the cumulative fuzzy distribution function [10];

$$\tilde{F}(t) = 1 - e^{-\lambda\tilde{k}(t-\theta)} \tag{14}$$

Then the $\alpha - cut$ fuzzy distribution function where $\{\alpha \in [0,1]\}$, we have

$$\begin{aligned} F_\alpha(t) &= [F_{1\alpha}(t), F_{2\alpha}(t)] \\ &= \{1 - e^{-\lambda(1-k_{max})\alpha + k_{max}(t-\theta)}, \\ &\quad 1 - e^{-\lambda(1-k_{min})\alpha + k_{min}(t-\theta)}\} \end{aligned} \tag{15}$$

While the fuzzy reliability function is defined by;

$$\tilde{R}(t) = e^{-\lambda\tilde{k}(t-\theta)}$$

Then $\forall\alpha \in [0,1)$ $the\ \alpha - cuts$ of fuzzy reliability function can be written as;

$$R_\alpha(t) = [R_{1\alpha}(t), R_{2\alpha}(t)] = \left[e^{-\lambda[(1-k_{min})\alpha + k_{min}](t-\theta)}, e^{-\lambda[(1-k_{max})\alpha + k_{max}](t-\theta)}\right] \tag{16}$$

4. Estimation Methods

First of all we explain the estimator of (λ) by;

4.1. Moment Estimator

Since

$E(t) = \frac{1}{\lambda} + \theta$ then

$$\frac{1}{\hat{\lambda}} + \hat{\theta} = \frac{\sum_{i=1}^{n} x_i}{n} = \overline{x}$$

$$\frac{1}{\hat{\lambda}} = \overline{x} - \hat{\theta}$$

$$\hat{\theta}_{mom} = \overline{x} - \frac{1}{\hat{\lambda}_{mom}} \tag{18}$$

$$\because v(t) = \frac{1}{\lambda^2}$$

$$E(t^2) - [E(t)]^2 = \frac{1}{\lambda^2}$$

$$\frac{\sum_{i=1}^{n} t_i^2}{n} = \frac{1}{\hat{\lambda}^2} + \overline{x}^2$$

$$\frac{\sum_{i=1}^{n} t_i^2}{n} - \overline{x}^2 = \frac{1}{\hat{\lambda}^2}$$

$$\hat{\lambda}_{mom} = \sqrt{\frac{1}{\left[\frac{\sum_{i=1}^{n} t_i^2}{n} - \overline{x}^2\right]}} \tag{19}$$

4.2. Maximum Likelihood Estimator

Let $(t_1, t_2, \dots, t_n)$ be random sample from (9), then;

$$L = \prod_{i=1}^{n} f(t_i) = \lambda^n e^{-\lambda\sum_{i=1}^{n}(t_i-\theta)}$$

$$\log L = n\log\lambda - \lambda\sum_{i=1}^{n}(t_i - \theta)$$

$$\frac{\partial \log L}{\partial\lambda} = \frac{n}{\lambda} - \sum_{i=1}^{n}(t_i - \theta) = 0$$

$$\frac{n}{\hat{\lambda}} = \sum_{i=1}^{n}(t_i - \theta)$$

$$\hat{\lambda}_{MLE} = \frac{n}{\sum_{i=1}^{n}(t_i-\theta)}$$

Then according to different values of $\alpha - cut$ and to specified value of (θ), we can estimate fuzzy reliability function;

$$\hat{R}(t) = e^{-\hat{\lambda}\tilde{k}(t-\hat{\theta})} \tag{20}$$

5. Simulation Procedures

To find the estimator's for MOM, MLE, and fuzzy reliability function we perform simulation experiments using Monte Carlo assuming that;

k	0.3		0.5	
λ	0.2	0.17	0.13	0.1
θ	0.7	0.6	0.5	0.8

Table (1). Two parameters estimators (λ, θ), when $(k = 0.3)$

n	Method	Experiment 1		Experiment 2		Experiment 3		Experiment 4	
		$\theta = 0.7$	$\lambda = 0.2$	$\theta = 0.6$	$\lambda = 0.17$	$\theta = 0.5$	$\lambda = 0.13$	$\theta = 0.8$	$\lambda = 0.1$
15	MLE	0.6897	0.2093	0.5886	0.1770	0.4910	0.1350	0.8348	1.0092
	MOM	0.7143	0.3660	0.6083	0.3285	0.5122	0.2949	0.8372	1.4956
25	MLE	0.6932	0.2068	0.5959	0.1761	0.4943	0.1346	0.8243	1.0056
	MOM	0.7061	0.2896	0.6089	0.2655	0.5069	0.2318	0.8259	1.3004
50	MLE	0.7000	0.2055	0.5986	0.1744	0.5001	0.1339	0.8141	1.0031
	MOM	0.7053	0.2489	0.6047	0.2224	0.5068	0.1760	0.8148	1.1477
100	MLE	0.6975	0.2021	0.6002	0.1729	0.4995	0.1324	0.8064	1.0015
	MOM	0.6994	0.2255	0.6033	0.1991	0.5027	0.1538	0.8068	1.0730

Table (2). MSE values for (λ, θ)

n	Method	Experiment 1		Experiment 2		Experiment 3		Experiment 4	
		$\theta = 0.7$	$\lambda = 0.2$	$\theta = 0.6$	$\lambda = 0.17$	$\theta = 0.5$	$\lambda = 0.13$	$\theta = 0.8$	$\lambda = 0.1$
15	MLE	0.0193	0.0023	0.0150	0.0017	0.0067	0.0006	0.0406	0.0012
	MOM	0.0257	0.0891	0.0196	0.0783	0.0108	0.0698	0.0434	0.9731
best		mle	mom	mle	mle	mle	mle	mle	mle
25	MLE	0.0127	0.0016	0.0088	0.0010	0.0045	0.0004	0.0204	0.0005
	MOM	0.0158	0.0450	0.0111	0.0428	0.0061	0.0359	0.0215	0.5286
best		mle	mom	mle	mle	mle	mle	mle	mle
50	MLE	0.0081	0.0010	0.0062	0.0007	0.0022	0.0003	0.0106	0.0002
	MOM	0.0092	0.0251	0.0072	0.0215	0.0029	0.0156	0.0111	0.2049
best		mle	mle	mle	mom	mle	mom	mle	mle
100	MLE	0.0051	0.0006	0.0017	0.0002	0.0012	0.0001	0.0051	0.0001
	MOM	0.0056	0.0136	0.0020	0.0113	0.0014	0.0097	0.0053	0.1047
best		mle	mle	mle	mle	mle	mle	mle	mle

Table (3). Two parameters estimators (λ, θ), when $(k = 0.3)$

n	Method	Experiment 1		Experiment 2		Experiment 3		Experiment 4	
		$\theta = 0.7$	$\lambda = 0.2$	$\theta = 0.6$	$\lambda = 0.17$	$\theta = 0.5$	$\lambda = 0.13$	$\theta = 0.8$	$\lambda = 0.1$
15	MLE	0.5786	0.1883	0.4776	0.1650	0.3810	0.1240	0.8238	1.0052
	MOM	0.6032	0.2540	0.5873	0.3175	0.4222	0.2839	0.8262	1.4846
25	MLE	0.5821	0.1858	0.4859	0.1641	0.3843	0.1236	0.8133	1.0045
	MOM	0.6050	0.1786	0.5979	0.2535	0.4969	0.2208	0.8149	1.3003
50	MLE	0.6800	0.1745	0.4866	0.1634	0.4801	0.1319	0.8031	1.0021
	MOM	0.6753	0.1379	0.5937	0.1344	0.4868	0.1650	0.8038	1.1367
100	MLE	0.5865	0.1821	0.5982	0.1618	0.3795	0.1214	0.8854	1.0012
	MOM	0.6874	0.1143	0.5923	0.1871	0.4727	0.1428	0.8758	1.0620

Table (4). MSE values for (λ, θ)

n	Method	Experiment 1		Experiment 2		Experiment 3		Experiment 4	
		$\theta = 0.7$	$\lambda = 0.2$	$\theta = 0.6$	$\lambda = 0.17$	$\theta = 0.5$	$\lambda = 0.13$	$\theta = 0.8$	$\lambda = 0.1$
15	MLE	0.0182	0.0012	0.0130	0.0016	0.0057	0.0005	0.0306	0.0011
	MOM	0.0246	0.0781	0.0185	0.0673	0.0106	0.0578	0.0324	0.8631
best		mle	mom	mle	mle	mle	mle	mle	mle
25	MLE	0.0117	0.0015	0.0076	0.0010	0.0034	0.0003	0.0203	0.0004
	MOM	0.0148	0.0340	0.0101	0.0318	0.0052	0.0249	0.0224	0.4175
best		mle	mom	mle	mle	mle	mle	mle	mle
50	MLE	0.0072	0.0010	0.0052	0.0006	0.0021	0.0003	0.0105	0.0002
	MOM	0.0082	0.0141	0.0073	0.0223	0.0027	0.0145	0.0101	0.2038
best		mle	mle	mle	mom	mle	mom	mle	mle
100	MLE	0.0041	0.0005	0.0016	0.0002	0.0012	0.0001	0.0041	0.0001
	MOM	0.0053	0.0126	0.0020	0.0112	0.0013	0.0087	0.0053	0.1037
best		mle	mle	mle	mle	mle	mle	mle	mle

Table (5). Comparison between real and estimated fuzzy reliability by two methods

n	t_i	Real	ML	Mom	Mse_ML	Mse_Mom	best
15	1.20	0.83333	0.90582	0.80477	0.0691e-004	0.0012	ML
	2.08	0.48077	0.47001	0.37564	0.2502e-004	0.0007	ML
	2.96	0.33784	0.31647	0.23112	0.4044e-004	0.0006	ML
	3.84	0.26042	0.23883	0.16355	0.5203e-004	0.0005	ML
	4.72	0.21186	0.19211	0.12477	0.6068e-004	0.0004	ML
	5.60	0.17857	0.16094	0.09991	0.6725e-004	0.0004	ML
	6.48	0.15432	0.13868	0.08277	0.7234e-004	0.0003	ML
	7.36	0.13587	0.12198	0.07030	0.7637e-004	0.0003	ML
	8.24	0.12136	0.10900	0.06087	0.7962e-004	0.0003	ML
	9.12	0.10965	0.09860	0.05351	0.8229e-004	0.0003	ML
25	1.20	0.83333	0.85165	0.79982	0.0199e-004	0.3168e-003	ML
	2.08	0.48077	0.47362	0.39847	0.0904e-004	0.1952e-003	ML
	2.96	0.33784	0.32711	0.25600	0.1542e-004	0.1393e-003	ML
	3.84	0.26042	0.24969	0.18510	0.2036e-004	0.1093e-003	ML
	4.72	0.21186	0.20193	0.14332	0.2411e-004	0.0913e-003	ML
	5.60	0.17857	0.16954	0.11603	0.2701e-004	0.0797e-003	ML
	6.48	0.15432	0.14616	0.09694	0.2930e-004	0.0717e-003	ML
	7.36	0.13587	0.12848	0.08291	0.3112e-004	0.0660e-003	ML
	8.24	0.12136	0.11465	0.07219	0.3261e-004	0.0617e-003	ML
	9.12	0.10965	0.10354	0.06377	0.3385e-004	0.0584e-003	ML
50	1.20	0.83333	0.84445	0.80273	0.7656e-005	0.1334e-003	ML
	2.08	0.48077	0.47750	0.40865	0.4506e-005	0.0936e-003	ML
	2.96	0.33784	0.33249	0.26576	0.2904e-005	0.0734e-003	ML
	3.84	0.26042	0.25504	0.19374	0.2006e-005	0.0617e-003	ML
	4.72	0.21186	0.20691	0.15092	0.1460e-005	0.0544e-003	ML
	5.60	0.17857	0.17412	0.12277	0.1106e-005	0.0494e-003	ML
	6.48	0.15432	0.15034	0.10298	0.0865e-005	0.0459e-003	ML
	7.36	0.13587	0.13231	0.08835	0.0693e-005	0.0433e-003	ML
	8.24	0.12136	0.11818	0.07715	0.0567e-005	0.0413e-003	ML
	9.12	0.10965	0.10679	0.06831	0.0472e-005	0.0397e-003	ML
100	1.20	0.83333	0.83868	0.80566	0.0054e-005	0.1373e-004	ML
	2.08	0.48077	0.47975	0.41851	0.0648e-005	0.0578 e-004	ML
	2.96	0.33784	0.33587	0.27537	0.1334e-005	0.0270e-004	ML
	3.84	0.26042	0.25842	0.20236	0.1928e-005	0.0135e-004	ML
	4.72	0.21186	0.21003	0.15859	0.1016e-005	0.0069e-004	ML
	5.60	0.17857	0.17695	0.12963	0.0815e-005	0.0036e-004	ML
	6.48	0.15432	0.15289	0.10915	0.0144 e-005	0.0018e-004	ML
	7.36	0.13587	0.13462	0.09396	0.0417e-005	0.0008e-004	ML
	8.24	0.12136	0.12026	0.08227	0.0648e-005	0.0003e-004	ML
	9.12	0.10965	0.10869	0.07303	0.0845e-005	0.0001e-004	ML

Table (6). Comparison between real and estimated fuzzy reliability by two methods

n	t_i	Real	ML	Mom	Mse_ML	Mse_Mom	best
15	1.70	0.88235	0.97296	0.87366	0.1062 e-004	0.8156e-003	ML
	2.13	0.70423	0.73671	0.63345	0.3050e-004	0.4897e-003	ML
	2.56	0.58594	0.59146	0.48961	0.4742e-004	0.3394e-003	ML
	2.66	0.50167	0.49358	0.39505	0.6048e-004	0.2589e-003	ML
	3.42	0.43860	0.42333	0.32873	0.7047e-004	0.2110e-003	ML
	3.85	0.38961	0.37056	0.27996	0.7822e-004	0.1800e-003	ML
	4.28	0.35047	0.32951	0.24277	0.8434e-004	0.1589e-003	ML
	4.71	0.31847	0.29669	0.21359	0.8927e-004	0.1437e-003	ML
	5.14	0.29183	0.26986	0.19015	0.9330e-004	0.1324e-003	ML
	5.57	0.26930	0.24754	0.17096	0.9664e-004	0.1238e-003	ML
25	1.70	0.88235	0.90583	0.86032	0.0001e-004	0.3993e-003	ML
	2.13	0.70423	0.71073	0.64562	0.0114e-004	0.2725e-003	ML
	2.56	0.58594	0.58408	0.51143	0.0319e-004	0.2094 e-003	ML
	2.99	0.50167	0.49540	0.42041	0.0510e-004	0.1738e-003	ML
	3.42	0.43860	0.42994	0.35502	0.0669e-004	0.1516e-003	ML
	3.85	0.38961	0.37966	0.30600	0.0798e-004	0.1369e-003	ML
	4.28	0.35047	0.33987	0.26802	0.0903e-004	0.1266e-003	ML
	4.71	0.31847	0.30761	0.23781	0.0988e-004	0.1190e-003	ML
	5.14	0.29183	0.28093	0.21327	0.1059e-004	0.1133e-003	ML
	5.57	0.26930	0.25850	0.19298	0.1119e-004	0.1088e-003	ML
50	1.70	0.88235	0.89637	0.86134	0.1962e-005	0.1307e-003	ML
	2.13	0.70423	0.70870	0.65250	0.0557e-005	0.0895e-003	ML
	2.56	0.58594	0.58568	0.52064	0.0101e-005	0.0684e-003	ML
	2.99	0.50167	0.49890	0.43048	0.0001e-005	0.0563e-003	ML
	3.42	0.43860	0.43446	0.36529	0.0038e-005	0.0487e-003	ML
	3.85	0.38961	0.38473	0.31615	0.0127e-005	0.0436e-003	ML
	4.28	0.35047	0.34520	0.27789	0.0233e-005	0.0400e-003	ML
	4.71	0.31847	0.31304	0.24735	0.0340e-005	0.0373e-003	ML
	5.14	0.29183	0.28637	0.22244	0.0442e-005	0.0353e-003	ML
	5.57	0.26930	0.26389	0.20178	0.0538e-005	0.0337e-003	ML
100	1.70	0.88235	0.88897	0.86254	0.8062e-006	0.3769 e-004	ML
	2.13	0.70423	0.70664	0.65912	0.4416e-006	0.2707e-004	ML
	2.56	0.58594	0.58626	0.52950	0.2809e-006	0.2170e-004	ML
	2.66	0.50167	0.50088	0.44022	0.1994e-006	0.1865e-004	ML
	3.42	0.43860	0.43719	0.37528	0.1533e-006	0.1676e-004	ML
	3.85	0.38961	0.38786	0.32608	0.1249e-006	0.1551e-004	ML
	4.28	0.35047	0.34854	0.28761	0.1063e-006	0.1464e-004	ML
	4.71	0.31847	0.31647	0.25677	0.0934e-006	0.1400e-004	ML
	5.14	0.29183	0.28980	0.23154	0.0841e-006	0.1353e-004	ML
	5.57	0.26930	0.26729	0.21055	0.0772e-006	0.1316e-004	ML

Table (7). Comparison between real and estimated fuzzy reliability by two methods

n	t_i	Real	ML	Mom	Mse_ML	Mse_Mom	best
15	2.27	0.88106	0.97117	0.87181	0.1355e-004	0.8835e-003	ML
	2.64	0.75672	0.80412	0.70132	0.2915e-004	0.5804e-003	ML
	3.02	0.66313	0.68520	0.58203	0.4117e-004	0.4318e-003	ML
	3.39	0.59014	0.59647	0.49451	0.4990e-004	0.3489e-003	ML
	3.76	0.53163	0.52785	0.42790	0.5624e-004	0.2981e-003	ML
	4.14	0.48368	0.47327	0.37573	0.6093e-004	0.2647e-003	ML
	4.51	0.44366	0.42887	0.33391	0.6448e-004	0.2415e-003	ML
	4.88	0.40975	0.39207	0.29972	0.6721e-004	0.2248e-003	ML
	5.25	0.38066	0.36109	0.27132	0.6935e-004	0.2124e-003	ML
	5.63	0.35543	0.33465	0.24740	0.7106e-004	0.2028e-003	ML
25	2.27	0.88106	0.90440	0.85871	0.0407e-004	0.3312 e-003	ML
	2.64	0.75672	0.76770	0.70739	0.1023e-004	0.2108e-003	ML
	3.02	0.66313	0.66645	0.59819	0.1518e-004	0.1531e-003	ML
	3.39	0.59014	0.58854	0.51618	0.1883e-004	0.1215e-003	ML
	3.76	0.53163	0.52678	0.45231	0.2152e-004	0.1023e-003	ML
	4.14	0.48368	0.47664	0.40150	0.2352e-004	0.0898e-003	ML
	4.51	0.44366	0.43516	0.36017	0.2504e-004	0.0813e-003	ML
	4.88	0.40975	0.40027	0.32596	0.2621e-004	0.0751e-003	ML
	5.25	0.38066	0.37054	0.29722	0.2714e-004	0.0706e-003	ML
	5.63	0.35543	0.34490	0.27277	0.2788e-004	0.0671e-003	ML
50	2.27	0.88106	0.89500	0.85978	0.1388e-005	0.1157e-003	ML
	2.64	0.75672	0.76372	0.71283	0.0716e-005	0.0907e-003	ML
	3.02	0.66313	0.66581	0.60603	0.0407e-005	0.0769e-003	ML
	3.39	0.59014	0.59003	0.52522	0.0251e-005	0.0686e-003	ML
	3.76	0.53163	0.52967	0.46215	0.0166e-005	0.0633e-003	ML
	4.14	0.48368	0.48047	0.41167	0.0166e-005	0.0596e-003	ML
	4.51	0.44366	0.43961	0.37044	0.0085e-005	0.0570e-003	ML
	4.88	0.40975	0.40514	0.33619	0.0065e-005	0.0551e-003	ML
	5.25	0.38066	0.37568	0.30732	0.0051e-005	0.0536e-003	ML
	5.63	0.35543	0.35020	0.28269	0.0041e-005	0.0525e-003	ML
100	2.27	0.88106	0.88764	0.86103	0.0925e-006	0.2372e-004	ML
	2.64	0.75672	0.76025	0.71810	0.0021e-006	0.1784e-004	ML
	3.02	0.66313	0.66475	0.61356	0.0114e-006	0.1459e-004	ML
	3.39	0.59014	0.59053	0.53402	0.0417e-006	0.1264e-004	ML
	3.76	0.53163	0.53120	0.47165	0.0726e-006	0.1139e-004	ML
	4.14	0.48368	0.48269	0.42152	0.0995e-006	0.1054e-004	ML
	4.51	0.44366	0.44229	0.38043	0.1217e-006	0.0993e-004	ML
	4.88	0.40975	0.40813	0.34617	0.1398e-006	0.0949e-004	ML
	5.25	0.38066	0.37887	0.31721	0.1547e-006	0.0915e-004	ML
	5.63	0.35543	0.35352	0.29244	0.1668e-006	0.0889e-004	ML

Table (8). Comparison between real and estimated fuzzy reliability by two methods

n	t_i	Real	ML	Mom	Mse_ML	Mse_Mom	best
15	1.10	0.82645	0.89652	0.81100	0.0010	0.0032	ML
	1.59	0.39555	0.37688	0.34476	0.0011	0.0027	ML
	2.08	0.23114	0.21047	0.19245	0.0012	0.0023	ML
	2.57	0.15140	0.13603	0.12390	0.0012	0.0021	ML
	3.06	0.10680	0.09608	0.08707	0.0012	0.0020	ML
	3.55	0.07935	0.07203	0.06492	0.0012	0.0018	ML
	4.04	0.06127	0.05635	0.05051	0.0012	0.0018	ML
	4.53	0.04873	0.04552	0.04058	0.0012	0.0017	ML
	5.02	0.03968	0.03769	0.03343	0.0012	0.0017	ML
	5.51	0.03294	0.03184	0.02809	0.0012	0.0011	MOM
25	1.10	0.82645	0.84406	0.81474	0.4764e-003	0.0013	ML
	1.59	0.39555	0.38574	0.36936	0.5309e-003	0.0011	ML
	2.08	0.23114	0.22081	0.21031	0.5560e-003	0.0010	ML
	2.57	0.15140	0.14335	0.13601	0.5689e-003	0.0010	ML
	3.06	0.10680	0.10084	0.09541	0.5763e-003	0.0009	ML
	3.55	0.07935	0.07498	0.07079	0.5808e-003	0.0009	ML
	4.04	0.06127	0.05806	0.05473	0.5837e-003	0.0008	ML
	4.53	0.04873	0.04638	0.04366	0.5856e-003	0.0008	ML
	5.02	0.03968	0.03796	0.03570	0.5870e-003	0.0008	ML
	5.51	0.03294	0.03169	0.02978	0.5879e-003	0.0008	ML
50	1.10	0.82645	0.83718	0.81860	0.1574e-003	0.3978e-003	ML
	1.59	0.39555	0.39075	0.37858	0.1762e-003	0.3551e-003	ML
	2.08	0.23114	0.22598	0.21772	0.1850e-003	0.3227e-003	ML
	2.57	0.15140	0.14749	0.14157	0.1897e-003	0.2997e-003	ML
	3.06	0.10680	0.10402	0.09960	0.1924e-003	0.2834e-003	ML
	3.55	0.07935	0.07744	0.07402	0.1941e-003	0.2715e-003	ML
	4.04	0.06127	0.05998	0.05726	0.1952e-003	0.2626e-003	ML
	4.53	0.04873	0.04789	0.04568	0.1960e-003	0.2557e-003	ML
	5.02	0.03968	0.03917	0.03734	0.1965e-003	0.1504e-003	MOM
	5.51	0.03294	0.03267	0.03113	0.1969e-003	0.2461e-003	ML
100	1.10	0.82645	0.83162	0.82142	0.3327e-004	0.8532e-004	ML
	1.59	0.39555	0.39384	0.38576	0.3715e-004	0.7677e-004	ML
	2.08	0.23114	0.22922	0.22342	0.3893e-004	0.6968e-004	ML
	2.57	0.15140	0.15000	0.14575	0.3986e-004	0.6454e-004	ML
	3.06	0.10680	0.10587	0.10267	0.4038e-004	0.3082e-004	MOM
	3.55	0.07935	0.07878	0.07630	0.4070e-004	0.5809e-004	ML
	4.04	0.06127	0.06095	0.05899	0.4090e-004	0.5604e-004	ML
	4.53	0.04873	0.04859	0.04701	0.4104e-004	0.5446e-004	ML
	5.02	0.03968	0.03967	0.03837	0.4114e-004	0.5323e-004	ML
	5.51	0.03294	0.03302	0.03194	0.4121e-004	0.5224e-004	ML

6. Conclusions

The best estimator for (θ) is found to be MLE, also for fuzzy reliability $(\hat{R}_{MLE})$ when $(\tilde{k} = 0.3)$. We conclude that the fuzzy estimator of (R) is best than ordinary estimator since it takes all the variation under consideration.

REFERENCES

[1] A.J. Nicholson, (2007), "Road network unreliability: impact assessment and mitigation", nternational Journal of Critical Infrastructures, vol. 3, issue 3/4, pages 346-375.

[2] Bellman R.E, Zadeh L.A. (1970),"decision making a fuzzy environment", management Science, 17, 141 – 164.

[3] Castillo, Oscar, Melin, Patricia, (2008), "Type-2 Fuzzy Logic: Theory and Applications", Studies in Fuzziness and Soft Computing. Springer.

[4] Dalalah D, Magableh S. (2008). "A remote fuzzy multicriteria diagnosis of sore throat", Telemed J E Health.

[5] Didier Dubois, Walenty Ostasiewicz, Henri Prade, (2000)," Fuzzy Sets: History and Basic Notions", Fundamentals of Fuzzy Sets Volume 7 of the series The Handbooks of Fuzzy Sets Series pp 21-124. Springer.

[6] H. J. Zimmermann, (1992), "Methods of application of fuzzy mathematical programming in R.R".

[7] Harish Garg, S.P. Sharma, Monica Rani, (2013), "Weibull fuzzy probability distribution for analysing the behaviour of pulping unit in a paper industry", International Journal of Industrial and Systems Engineering ,Volume 14, Issue 4.

[8] Hsien-Chung Wu, (2004), "Fuzzy reliability estimation using Bayesian approach", Computers & Industrial Engineering Volume 46, Issue 3, Pages 467–493.

[9] Hsien-Chung Wu, (2006), "Fuzzy Bayesian system reliability assessment based on exponential distribution", Applied Mathematical Modelling Volume 30, Issue 6, Pages 509–530.

[10] Kailash C. Kapur. Michael Pecht, (2014), "Reliability Engineering", Wiley Publishing.

[11] M. A. El-Damcese, Dina. A. Ramadan, (2015), "Analyzing System Reliability Using Fuzzy Mixture Generalized Linear Failure Rate Distribution", American Journal of Mathematics and Statistics, 5(2): 43-51.

[12] M. A. Hussian, Essam A. Amin, (2017), "Fuzzy reliability estimation based on exponential ranked set samples "International Journal of Contemporary Mathematical Sciences, Vol. 12, no. 1, 31-42.

[13] Military Handbook Electronic Reliability design handbook, MIL – HDBK – 338B, 1 October 1998, super sending Mil – HDBK 338A, 12 October 1988.

[14] Nayereh Bagheri Khoolenjani, Fateme Shahsanaie, (2016), "Estimating the parameter of Exponential distribution under Type-II censoring from fuzzy data", Journal of Statistical Theory and Applications, Vol. 15, No. 2, 181-195.

[15] Nozer D. Singpurwalla and Jane Booker, (2004), "membership function and probability measures of fuzzy sets", Journal of the American Statistical Association, Vol.99, No. 467.

[16] R. Seising, (2006), "Proposals for Future Developments in Fuzzy Set Technology", Fuzzy Systems, 2006 IEEE International Conference.

[17] Shafiq, Muhammad, (2017)," Classical and Bayesian Inference of Pareto Distribution and Fuzzy Life Times", Pakistan Journal of Statistics, Vol. 33 Issue 1, p15-25.

[18] Sunil S. Bhamare, Om Prakash Yadav, Ajay Rathore, (2007), "Evolution of reliability engineering discipline over the last six decades: a comprehensive review", International Journal of Reliability and Safety, Volume 1, Issue 4.

[19] Way Kuo, Wei-Ting Kary Chien, Taeho Kim, (1998), "Reliability, Yield, and Stress Burn-In", Springer.

[20] Yager and L.A. Zadeh, (1992), "Introduction to fuzzy logic in intelligent systems", Klumer Academic publishers Bosten, 97 – 120.

[21] Zadeh L. A., (1968), "Measures of fuzzy event", dept. of electrical engineering and computer sciences university of California, Berkeley.

[22] Zhaojun Li, Kailash C. Kapur, (2013), "Some Perspectives to Define and Model Reliability Using Fuzzy Sets", Quality Engineering Volume 25, Issue 2.