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A Comparion of Ordinary Least Squares Regression and Least Squares Ratio via Generated Data

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Abstract Regression Analysis (RA) is one of the most used tools for functional relationship. The Ordinary Least Squares (OLS) method is the basic technics of RA. In this study we introduce one of the robust regression approaches, called Least Squares Ratio (LSR), and make a comparison of OLS and LSR according to mean square errors of regression parameter estimations. In this study for certain theoratical model, we generate data for different sample sizes, error variances and number of outliers. It is found that no matter what the sample size is LSR always performs well when there is 1 or more outliers.

Keywords Ordinary Least Squares Regression, Least Squares Ratio, Estimation, Data Generation with Outliers

1. Introduction

RA is usually used to construct a functional relationship between a dependent variable and certain number of regressors. There are many methods of estimation regression parameters. OLS is used in linear regression analysis due to its simplicity. There are also some other regression models such as Robust Regression [7], Principal Component Regression [3], and Logistic Regression [2]. To choose the "best fitting" line representing the fuctional relationship between dependent and independent variables, the unknown regression coefficients $\boldsymbol{\beta}$ of the regression model must be estimated [5]. Although the estimation procedure is not unique, the easiest and the most common estimation procedures is OLS used for minimizing the square distance between the observed and the predicted values of the dependent variable. Since OLS method is only taking the distance betweeen observed and predicted into consideration and does not require any statistical assumption, using OLS can be problematic especially when there exists an outlier or nonconstant variance of the error term. In this case we propose to use one of the new robust regression methods called LSR that is removing the effect of any outlier in the data set in place of OLS.

In this paper we extend the stduy of [1] in terms of the number of regressors, the number of outliers, the number of different variances of the error term and the number of different sample sizes. Similar comparisons have been made according to mean square errors of the regression coefficients. When there exists no outlier in the data set, OLS method preforms somehow better but not much better. However in the presence of even a single outlier no matter what either the sample size or the degree of variation in the error term LSR always performs better.

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2. Ordinary Least Squares Regression (OLS)

OLS method is the simplest and presents the most basic form for regression analysis. The main idea behind it is to minimize the sum of the square of the residual values for the regression model given by (1):

$$\boldsymbol{Y} = \boldsymbol{X\beta} + \boldsymbol{\varepsilon} \tag{1}$$

where $\boldsymbol{Y}$ is a vector representing the dependent variable with n rows and a single column, $\boldsymbol{X}$ is regressor matrix with n rows and p (p-1 is the number of regressors) columns. $\boldsymbol{\beta}$ is the coefficient vector with p rows and a single column. OLS method use the formulation form as follows [8]:

$$\boldsymbol{min}\sum\left(\boldsymbol{Y} - \widehat{\boldsymbol{Y}}\right)^{\boldsymbol{2}} \tag{2}$$

Where $\boldsymbol{\beta}$ is the px1 vector of regression coefficients, and $\widehat{\boldsymbol{Y}}$ is the predicted values obtained from $\boldsymbol{X\beta}$.

Taking the partial derivatives of (2) with respect to the members of $\boldsymbol{\beta}$, and solving for the roots results the OLS estimates as $\widehat{\boldsymbol{\beta}}_{\boldsymbol{OLS}}$ in matrix form [5].

$$\widehat{\boldsymbol{\beta}}_{\boldsymbol{OLS}} = [\boldsymbol{X}'\boldsymbol{X}]^{-\boldsymbol{1}}\boldsymbol{X}'\boldsymbol{Y} \tag{3}$$

3. Least Squares Ratio (LSR)

As with other forecasting techniques, OLS aims to estimate observed values with zero error: we can indicate this goal by $\boldsymbol{Y} = \widehat{\boldsymbol{Y}}$ or $\boldsymbol{Y} - \widehat{\boldsymbol{Y}} = \boldsymbol{0}$. Hence, the ordinary least squares approach satisfies this goal by estimating the regression parameters minimizing the sum of $\left(\boldsymbol{Y}-\widehat{\boldsymbol{Y}}\right)^{2}$. LSR also starts with the same goal $\boldsymbol{Y}=\widehat{\boldsymbol{Y}}$ as in OLS. However, it proceeds by dividing through by Y and so $\widehat{\boldsymbol{Y}}/\boldsymbol{Y}=\mathbf{1}$ is obtained under an assumption of $\boldsymbol{Y}\neq\mathbf{0}$. Hence, it is obvious that, as a result, equations $1-\left(\widehat{\boldsymbol{Y}}/\boldsymbol{Y}\right)=\mathbf{0}$ and $\left(\boldsymbol{Y}-\widehat{\boldsymbol{Y}}\right)/\boldsymbol{Y}=\mathbf{0}$ are raised by basic mathematical operations. The final equation is taken into account as the origin of the LSR which minimizes the sum of $\left[\left(\boldsymbol{Y}-\widehat{\boldsymbol{Y}}\right)/\boldsymbol{Y}\right]^{2}$. Consequently the aim of LSR can be written mathematically as follows [4]:

$$\min_{\boldsymbol{\beta}}\sum_{i=1}^{n}\left(\frac{Y-\widehat{Y}}{Y}\right)^{2} \tag{4}$$

Formula (4) can also be written as in (5), by using formula (1):

$$\min_{\widehat{\boldsymbol{\beta}}_{LSR}}\sum_{i=1}^{n}\left(\frac{Y-\widehat{\beta}_{LSR}X}{Y}\right)^{2} \tag{5}$$

Taking the partial derivatives of $\sum_{i=1}^{n}\left(\frac{Y-\widehat{\beta}_{LSR}X}{Y}\right)^{2}$ with respect to the members of $\widehat{\boldsymbol{\beta}}_{LSR}$ and setting them equal to zero yields normal equations given (6a)–(6d) [1]:

$$\sum\frac{1}{Y}=\beta_0\sum\frac{1}{Y^2}+\beta_1\sum\frac{x_1}{Y^2}+\beta_2\sum\frac{x_2}{Y^2}+\cdots+\beta_n\sum\frac{x_n}{Y^2} \tag{6a}$$

$$\sum\frac{x_1}{Y}=\beta_0\sum\frac{x_1}{Y^2}+\beta_1\sum\frac{x_1^2}{Y^2}+\beta_2\sum\frac{x_1x_2}{Y^2}+\cdots+\beta_n\sum\frac{x_1x_n}{Y^2} \tag{6b}$$

$$\sum\frac{x_2}{Y}=\beta_0\sum\frac{x_2}{Y^2}+\beta_1\sum\frac{x_1x_2}{Y^2}+\beta_2\sum\frac{x_2^2}{Y^2}+\cdots+\beta_n\sum\frac{x_2x_n}{Y^2} \tag{6c}$$

$$\vdots$$

$$\sum\frac{x_n}{Y}=\beta_0\sum\frac{x_n}{Y^2}+\beta_1\sum\frac{x_1x_n}{Y^2}+\beta_2\sum\frac{x_2x_n}{Y^2}+\cdots+\beta_n\sum\frac{x_n^2}{Y^2} \tag{6d}$$

If the matrix $\mathbf{X}$ has full rank p, the estimation $\boldsymbol{\beta}$ using LSR appears as in Eq. (7):

$$\widehat{\boldsymbol{\beta}}_{LSR}=\left[\left(\frac{X}{Y}\right)'\left(\frac{X}{Y}\right)\right]^{-1}\left(\frac{X}{Y^2}\right)'\boldsymbol{Y} \tag{7}$$

The matrix $\boldsymbol{X}/\boldsymbol{Y}$ is obtained by dividing each regressor by $Y_i\ for\ i=1,2,\cdots,n,$ where n is the sample size and p is the number of unknown regression parameters and $\boldsymbol{X}/\boldsymbol{Y}^{2}$ is computed in the same manner by dividing each regressor by Y_i^2 i $for\ j=1,2,\cdots,p$ [1].

4. The Simulation Study

Our simulation study evaluates linear multiple regression analysis with three independent variables extending the study of [1] as shown in (8). OLS and LSR are compared according to the MSE of $\boldsymbol{\beta}$:

$$\boldsymbol{y}=\boldsymbol{\beta_0}+\boldsymbol{\beta_1x_1}+\boldsymbol{\beta_2x_2}+\boldsymbol{\beta_3x_3}+\boldsymbol{e} \tag{8}$$

Here, $\boldsymbol{y}$ is the dependent variable, $\boldsymbol{x_1},\boldsymbol{x_2}$ and $\boldsymbol{x_3}$ are independent variables, $\boldsymbol{e}$ is the error term, and $\boldsymbol{\beta_i}$ is the ith actual regression parameter.

For OLS we have

$\widehat{\boldsymbol{\beta}}_{OLS}=\left[\widehat{\boldsymbol{\beta}}_{0,OLS}\,\widehat{\boldsymbol{\beta}}_{1,OLS}\,\widehat{\boldsymbol{\beta}}_{2,OLS}\,\widehat{\boldsymbol{\beta}}_{3,OLS}\right]$; on the other hand, using LSR we end up with

$\widehat{\boldsymbol{\beta}}_{LSR}=\left[\widehat{\boldsymbol{\beta}}_{0,LSR}\,\widehat{\boldsymbol{\beta}}_{1,LSR}\,\widehat{\boldsymbol{\beta}}_{2,LSR}\,\widehat{\boldsymbol{\beta}}_{3LSR}\right]$.

For our simulation protocol, the independent variables $\boldsymbol{x_1},\boldsymbol{x_2}$ and $\boldsymbol{x_3}$ are randomly generated from a normal distribution with $\boldsymbol{\mu}=\boldsymbol{\sigma^2}=\mathbf{100}$; $\boldsymbol{\beta_0},\boldsymbol{\beta_1},\boldsymbol{\beta_2}$ and $\boldsymbol{\beta_3}$ are set to be equal to 1. so $\boldsymbol{\beta}=[\mathbf{1}\quad\mathbf{1}\quad\mathbf{1}\quad\mathbf{1}]$.

Thus, the regression model becomes Eq. (9):

$$\boldsymbol{y}=\mathbf{1}+\boldsymbol{x_1}+\boldsymbol{x_2}+\boldsymbol{x_3}+\boldsymbol{e} \tag{9}$$

Finally, errors are randomly generated as Gaussian white noise with variance $\boldsymbol{\sigma_e^2}$. It is clear that the dependent variable has a normal distribution with $\boldsymbol{\mu}=\mathbf{301}$ and $\boldsymbol{\sigma^2}=\mathbf{300}+\boldsymbol{\sigma_e^2}$.

We performed our simulations using the statistical package Minitab 16.0 [6], using different sample sizes a

$$\boldsymbol{n}=[\mathbf{20},\mathbf{30},\mathbf{50},\mathbf{80},\mathbf{100},\mathbf{250},\mathbf{500},\mathbf{1000}]$$

and multiple error variances as

$$\boldsymbol{\sigma_e^2}=[\mathbf{1},\mathbf{4},\mathbf{9},\mathbf{16},\mathbf{25},\mathbf{100}].$$

For each generation, the dependent and independent variables are computed by the specified protocol. Estimation of regression parameters is then performed by both OLS and LSR. The two methods are compared by the mean square errors for parameter estimation, defined as $\sum(\mathbf{1}-\boldsymbol{\beta}_{OLS})/\mathbf{4}$ and $\sum(\mathbf{1}-\boldsymbol{\beta}_{LSR})/\mathbf{4}$, respectively.

The simulation has been replicated for 10 000 times for all combinations of $\boldsymbol{\sigma_e^2}=[\mathbf{1},\mathbf{4},\mathbf{9},\mathbf{16},\mathbf{25},\mathbf{100}]$ and $\boldsymbol{n}=[\mathbf{20},\mathbf{30},\mathbf{50},\mathbf{80},\mathbf{100},\mathbf{250},\mathbf{500},\mathbf{1000}]$. This generation is also repeated five times for each number of outliers as presented in Table 1.

Table 1. Number of outliers and outlier observation

Number of outliers	Outlier obsevations
0	
1	$y_{12}=700$
2	$y_{10}=700\ and\ y_{15}=700$
3	$y_7=700,\ y_{10}=700\ and\ y_{15}=700$
4	$y_7=700,\ y_{10}=700, y_{15}=700$ $and\ \ y_{20}=700$

The results with average $\boldsymbol{\beta}$ values and the number of frequency of the selected model best according to the mean square errors for parameter estimation are presented from Tables 2 to Table 9. The second column of Table 1, labeled $\boldsymbol{\sigma_e^2}$ presents different values of the error variance.

The columns $\boldsymbol{\beta_0},\boldsymbol{\beta_1},\boldsymbol{\beta_2}$ and $\boldsymbol{\beta_3}$ represent the mean estimation of $\boldsymbol{\beta_0},\boldsymbol{\beta_1},\boldsymbol{\beta_2}$ and $\boldsymbol{\beta_3}$ for the 10 000 replications, respectively.

Table 2. Comparison of OLS and LSR for sample size 20

Number of outliers	σ_e	$n = 20$	Average of coefficients from 10000 replications				frequency of the seleceted model
			β_0	β_1	β_2	β_3	$mse\ \beta$
0	1	OLS	1.02557	1.00008	0.99973	0.99993	5157
		LSR	1.00986	1.00011	0.99976	0.99996	4843
	2	OLS	0.98226	1.00023	0.99917	1.00078	5215
		LSR	0.93935	1.00031	0.99923	1.00086	4785
	3	OLS	1.27881	0.99937	0.99858	0.99915	5184
		LSR	1.22006	0.99939	0.99859	0.99922	4816
	4	OLS	0.70091	1.00040	1.00136	1.00106	5216
		LSR	0.58078	1.00057	1.00139	1.00120	4784
	5	OLS	0.87578	0.99937	1.00050	1.00148	5174
		LSR	0.69793	0.99947	1.00073	1.00160	4826
	10	OLS	0.57884	1.00243	1.00330	0.99893	5278
		LSR	-0.23676	1.00338	1.00447	0.99963	4722
1	1	OLS	44.26480	0.88511	0.93291	0.94830	146
		LSR	4.52250	0.99351	1.00662	1.00896	9854
	2	OLS	36.70820	0.95308	0.92415	0.96606	151
		LSR	2.52800	1.01142	1.00426	1.01347	9849
	3	OLS	31.32010	0.96935	0.95139	0.97606	211
		LSR	1.34020	1.01287	1.01000	1.01795	9789
	4	OLS	33.50990	0.93584	0.97420	0.96378	271
		LSR	1.73970	1.00637	1.01547	1.01422	9729
	5	OLS	35.55420	0.97940	0.93891	0.93556	327
		LSR	1.91080	1.01748	1.00764	1.00908	9673
	10	OLS	32.45560	0.97196	0.96559	0.94755	615
		LSR	1.08010	1.01807	1.01038	1.01008	9385
2	1	OLS	79.85430	0.88206	0.89337	0.83431	252
		LSR	6.47940	1.01666	1.01842	1.00325	9748
	2	OLS	73.40720	0.85824	0.90149	0.91506	267
		LSR	5.26440	1.00670	1.02076	1.02314	9733
	3	OLS	69.32950	0.90215	0.89093	0.92297	257
		LSR	3.76840	1.01947	1.02001	1.02599	9743
	4	OLS	71.03600	0.93700	0.89659	0.86410	296
		LSR	3.86130	1.03190	1.02089	1.01070	9704
	5	OLS	69.12710	0.90404	0.88233	0.93130	329
		LSR	3.71590	1.01915	1.01826	1.02754	9671
	10	OLS	69.11660	0.92642	0.90201	0.88894	481
		LSR	2.56160	1.03096	1.02180	1.01996	9519
3	1	OLS	70.51370	0.92341	0.87770	0.90158	322
		LSR	2.35120	1.03062	1.01899	1.02479	9678
	2	OLS	110.23100	0.87284	0.76354	0.86987	357
		LSR	7.88300	1.03673	1.00563	1.03580	9643
	3	OLS	114.01200	0.79423	0.80978	0.86467	368
		LSR	8.87200	1.01384	1.01709	1.03765	9632
	4	OLS	72.62970	0.86445	0.88214	0.93697	352
		LSR	2.79900	1.01600	1.02049	1.03304	9648
	5	OLS	99.41330	0.87784	0.83746	0.89901	399
		LSR	5.55170	1.03261	1.02744	1.04056	9601
	10	OLS	109.72800	0.79488	0.82025	0.89485	541
		LSR	6.68500	1.01816	1.02397	1.04313	9459
4	1	OLS	131.76800	0.86799	0.83677	0.78772	462
		LSR	7.79500	1.05925	1.04657	1.03486	9538
	2	OLS	137.44700	0.85845	0.79164	0.78467	497
		LSR	9.73900	1.04994	1.03470	1.03555	9503
	3	OLS	135.08600	0.84891	0.77871	0.83080	495
		LSR	10.03400	1.04938	1.02388	1.04504	9505
	4	OLS	137.83800	0.77217	0.81290	0.84614	518
		LSR	8.49900	1.03057	1.04736	1.05416	9482
	5	OLS	154.10400	0.77548	0.71496	0.77663	515
		LSR	13.79000	1.02869	1.01592	1.03397	9485
	10	OLS	129.89800	0.84108	0.77996	0.88635	562
		LSR	6.36900	1.05554	1.02888	1.06422	9438

Table 3. Comparison of OLS and LSR for sample size 30

Number of outliers	σ_e	$n = 30$	Average of coefficients from 10000 replications				frequency of the seleceted model
			β_0	β_1	β_2	β_3	$mse\ \beta$
0	1	OLS	1.01080	0.999899	1.00006	0.999925	5273
		LSR	0.99775	0.999898	1.00010	0.999962	4727
	2	OLS	0.936578	1.00000	1.00065	1.00002	5272
		LSR	0.895107	1.00009	1.00070	1.00006	4728
	3	OLS	1.00206	0.999249	1.00082	0.999923	5275
		LSR	0.92216	0.999303	1.00096	1.000003	4725
	4	OLS	1.15046	0.999619	0.999675	0.999185	5173
		LSR	1.03316	0.999664	0.999793	0.999269	4827
	5	OLS	1.05230	0.999665	1.00028	0.999402	5317
		LSR	0.86702	0.999830	1.00038	0.999536	4683
	10	OLS	1.12767	1.00101	0.996504	1.00140	5286
		LSR	0.14205	1.00261	0.997936	1.00247	4714
1	1	OLS	23.4770	0.982313	0.953566	0.972580	142
		LSR	1.1446	1.011400	1.005234	1.009596	9858
	2	OLS	23.4552	1.00411	0.950352	0.953493	199
		LSR	1.1963	1.01636	1.004742	1.004212	9801
	3	OLS	22.8893	0.969533	0.979667	0.964898	240
		LSR	0.8210	1.009886	1.010815	1.008210	9760
	4	OLS	26.3336	0.956881	0.967479	0.955207	272
		LSR	1.5647	1.006260	1.008662	1.006179	9728
	5	OLS	25.7072	0.955610	0.969337	0.961617	370
		LSR	1.4654	1.005645	1.008492	1.007658	9630
	10	OLS	24.6146	0.943803	0.967266	0.986017	769
		LSR	0.4457	1.005067	1.009723	1.012694	9231
2	1	OLS	44.8451	0.932490	0.957458	0.936977	256
		LSR	1.2464	1.015459	1.022007	1.016931	9744
	2	OLS	46.0091	0.942126	0.928563	0.945491	267
		LSR	1.2702	1.018893	1.015613	1.019627	9733
	3	OLS	45.3040	0.953075	0.930755	0.940472	247
		LSR	0.9270	1.022459	1.016677	1.018475	9753
	4	OLS	51.6452	0.934893	0.916717	0.907810	302
		LSR	2.5154	1.017304	1.013321	1.010348	9698
	5	OLS	46.6748	0.935474	0.930944	0.943573	354
		LSR	1.3955	1.016934	1.015805	1.019145	9646
	10	OLS	53.7742	0.875383	0.939488	0.923616	560
		LSR	2.5010	1.002752	1.017135	1.016446	9440
3	1	OLS	41.0215	0.945123	0.941195	0.953069	284
		LSR	0.4899	1.017952	1.016737	1.019467	9716
	2	OLS	74.3953	0.838256	0.899473	0.927622	295
		LSR	3.0887	1.010310	1.025470	1.031356	9705
	3	OLS	60.3135	0.926476	0.928153	0.951831	346
		LSR	-0.5036	1.032816	1.032042	1.038226	9654
	4	OLS	41.5736	0.929306	0.967121	0.937709	360
		LSR	0.4253	1.014639	1.022692	1.016481	9640
	5	OLS	75.7456	0.889384	0.876775	0.885054	392
		LSR	3.3268	1.020287	1.020711	1.022498	9608
	10	OLS	68.8067	0.905548	0.921011	0.895491	513
		LSR	0.8936	1.029488	1.030605	1.024025	9487
4	1	OLS	99.8195	0.851024	0.845514	0.848350	373
		LSR	4.5910	1.028636	1.028650	1.028583	9627
	2	OLS	96.9553	0.881512	0.872552	0.819048	384
		LSR	4.1913	1.036105	1.032983	1.020494	9616
	3	OLS	89.5219	0.884966	0.885547	0.876068	401
		LSR	2.2651	1.037333	1.038004	1.033360	9599
	4	OLS	96.8365	0.888596	0.832285	0.852271	414
		LSR	3.5225	1.039420	1.024384	1.031609	9586
	5	OLS	103.393	0.854895	0.821582	0.833007	410
		LSR	5.832	1.028738	1.020388	1.023058	9590
	10	OLS	98.9328	0.819788	0.860159	0.872865	606
		LSR	3.4403	1.018351	1.034402	1.038917	9394

Table 4. Comparison of OLS and LSR for sample size 50

Number of outliers	σ_e	$n = 50$	Average of coefficients from 10000 replications				frequency of the seleceted model
			β_0	β_1	β_2	β_3	$mse\ \beta$
0	1	OLS	1.01890	0.999933	0.999816	1.00006	5278
		LSR	1.00795	0.999925	0.999852	1.00008	4722
	2	OLS	1.08812	0.999341	0.999948	0.999815	5212
		LSR	1.05516	0.999333	0.999992	0.999862	4788
	3	OLS	0.938568	1.00067	1.00011	0.999851	5289
		LSR	0.849539	1.00077	1.00029	0.999915	4711
	4	OLS	1.04962	0.998811	1.00037	1.00035	5219
		LSR	0.85943	0.999116	1.00061	1.00073	4781
	5	OLS	0.819127	1.00138	0.999392	1.00106	5263
		LSR	0.565136	1.00162	0.999719	1.00150	4737
	10	OLS	0.909599	1.00262	0.999621	0.998667	5286
		LSR	-0.132345	1.00405	1.001315	0.999824	4714
1	1	OLS	16.1272	0.973555	0.981476	0.973587	173
		LSR	1.2007	1.003994	1.005822	1.003924	9827
	2	OLS	14.1805	0.983029	0.978295	0.986724	198
		LSR	0.7953	1.005966	1.004952	1.006729	9802
	3	OLS	14.4837	0.987186	0.970926	0.986977	304
		LSR	0.6883	1.007545	1.004005	1.006846	9696
	4	OLS	15.0361	0.975288	0.985469	0.978773	431
		LSR	0.8276	1.004474	1.006546	1.005461	9569
	5	OLS	15.0636	0.977477	0.970323	0.990842	470
		LSR	0.6582	1.004954	1.004415	1.008165	9530
	10	OLS	16.2056	0.993425	0.971598	0.962317	957
		LSR	-0.1999	1.012578	1.005933	1.002955	9043
2	1	OLS	28.4607	0.943010	0.946012	0.996124	223
		LSR	0.8423	1.007386	1.007959	1.018353	9777
	2	OLS	30.5873	0.973968	0.955685	0.933632	265
		LSR	1.1846	1.014335	1.010135	1.005446	9735
	3	OLS	26.8981	0.965505	0.962510	0.972312	271
		LSR	0.5665	1.012074	1.010769	1.013009	9729
	4	OLS	30.6591	0.965685	0.940298	0.957134	348
		LSR	1.1412	1.011833	1.006795	1.010953	9652
	5	OLS	30.4045	0.959110	0.957898	0.948625	388
		LSR	0.8242	1.011742	1.010733	1.009881	9612
	10	OLS	30.4937	0.957430	0.960852	0.945790	701
		LSR	0.2212	1.011057	1.011622	1.010680	9299
3	1	OLS	25.9036	0.973822	0.965065	0.961762	331
		LSR	0.4363	1.012969	1.011365	1.010676	9669
	2	OLS	44.7582	0.929580	0.938558	0.934012	279
		LSR	1.3151	1.014089	1.016341	1.015272	9721
	3	OLS	40.3397	0.930824	0.959817	0.955002	327
		LSR	0.2993	1.014162	1.020893	1.020343	9673
	4	OLS	26.3622	0.968669	0.958801	0.968357	357
		LSR	0.3878	1.011318	1.010773	1.012399	9643
	5	OLS	43.1670	0.928489	0.948246	0.940977	437
		LSR	0.7150	1.015006	1.018487	1.016639	9563
	10	OLS	42.3875	0.929061	0.962905	0.933997	663
		LSR	-0.0086	1.015953	1.023008	1.013980	9337
4	1	OLS	57.9829	0.915854	0.918263	0.915267	383
		LSR	1.1906	1.021660	1.021799	1.021112	9617
	2	OLS	53.0495	0.930868	0.923309	0.944586	368
		LSR	0.0961	1.024361	1.022755	1.028312	9632
	3	OLS	55.3883	0.925204	0.933878	0.916317	350
		LSR	0.5666	1.023153	1.025635	1.021624	9650
	4	OLS	55.3209	0.916644	0.933323	0.925945	404
		LSR	0.5260	1.021571	1.025757	1.022947	9596
	5	OLS	52.7208	0.943172	0.915010	0.943319	403
		LSR	-0.2753	1.027263	1.022340	1.027995	9597
	10	OLS	57.3614	0.940916	0.899293	0.915389	601
		LSR	0.3371	1.026029	1.019532	1.021451	9399

Table 5. Comparison of OLS and LSR for sample size 80

Number of outliers	σ_e	$n = 80$	Average of coefficients from 10000 replications				frequency of the seleceted model
			β_0	β_1	β_2	β_3	$mse\ \beta$
0	1	OLS	0.975528	1.00019	0.999938	1.00011	5217
		LSR	0.964815	1.00022	0.999927	1.00014	4783
	2	OLS	1.06563	0.999591	1.00000	0.999738	5230
		LSR	1.01272	0.999693	1.00012	0.999798	4770
	3	OLS	0.956438	0.999808	1.00059	0.999975	5248
		LSR	0.859111	0.999956	1.00071	1.000117	4752
	4	OLS	0.942965	0.999815	1.00057	1.00022	5254
		LSR	0.747323	1.000057	1.00089	1.00060	4746
	5	OLS	1.20622	0.998083	1.00009	0.999807	5213
		LSR	0.94801	0.998433	1.00041	1.000140	4787
	10	OLS	0.908931	0.997840	1.00152	1.00165	5377
		LSR	-0.244116	0.999457	1.00334	1.00339	4623
1	1	OLS	9.07808	0.988896	0.997943	0.982202	176
		LSR	0.74999	1.003894	1.005620	1.002504	9824
	2	OLS	10.0560	0.990453	0.987471	0.981421	273
		LSR	0.9457	1.004158	1.003672	1.002048	9727
	3	OLS	9.55548	0.989058	0.991733	0.983473	363
		LSR	0.77667	1.004012	1.004393	1.002865	9637
	4	OLS	8.38937	0.990925	0.990444	0.994394	461
		LSR	0.41288	1.004183	1.004431	1.005794	9539
	5	OLS	9.15569	0.988383	0.990469	0.989362	617
		LSR	0.70344	1.003353	1.003552	1.004004	9383
	10	OLS	10.2500	0.982684	0.991471	0.983673	1214
		LSR	-0.1450	1.002676	1.008152	1.004165	8786
2	1	OLS	18.3457	0.978115	0.972750	0.975270	263
		LSR	0.7233	1.007907	1.006773	1.007381	9737
	2	OLS	18.2520	0.971469	0.975769	0.979940	265
		LSR	0.7107	1.006375	1.008016	1.007638	9735
	3	OLS	17.5540	0.975655	0.970807	0.987932	362
		LSR	0.4397	1.007677	1.006396	1.010387	9638
	4	OLS	19.8650	0.966224	0.970140	0.974980	413
		LSR	0.7425	1.005867	1.007314	1.007819	9587
	5	OLS	16.5256	0.987480	0.982478	0.974594	457
		LSR	0.1632	1.009910	1.009373	1.006929	9543
	10	OLS	20.2976	0.972185	0.965603	0.968932	923
		LSR	0.2440	1.007422	1.006728	1.006492	9077
3	1	OLS	29.0766	0.951907	0.958413	0.958347	287
		LSR	1.0549	1.008650	1.010003	1.010100	9713
	2	OLS	24.8369	0.956802	0.987627	0.966806	310
		LSR	0.1851	1.009199	1.016158	1.012001	9690
	3	OLS	26.9878	0.967942	0.960701	0.961367	349
		LSR	0.5228	1.012367	1.010749	1.010582	9651
	4	OLS	22.8135	0.969274	0.972016	0.960225	395
		LSR	0.6740	1.008668	1.010022	1.006783	9605
	5	OLS	27.1559	0.957744	0.963513	0.966482	466
		LSR	0.3043	1.010468	1.011558	1.012625	9534
	10	OLS	27.2579	0.964626	0.955845	0.966833	740
		LSR	-0.2795	1.014257	1.009378	1.012248	9260
4	1	OLS	33.1878	0.958756	0.951378	0.967373	349
		LSR	0.0066	1.016067	1.015055	1.018407	9651
	2	OLS	35.3656	0.954962	0.941049	0.959588	343
		LSR	0.4612	1.015817	1.012231	1.016694	9657
	3	OLS	34.7502	0.952430	0.945159	0.964291	347
		LSR	0.2471	1.015325	1.013538	1.017755	9653
	4	OLS	35.8824	0.950800	0.950126	0.950115	411
		LSR	0.3423	1.015266	1.015287	1.014699	9589
	5	OLS	34.1434	0.955052	0.955148	0.958079	436
		LSR	-0.0395	1.015816	1.016067	1.016690	9564
	10	OLS	37.3095	0.941710	0.954394	0.940174	733
		LSR	-0.0924	1.012639	1.017577	1.013961	9267

Table 6. Comparison of OLS and LSR for sample size 100

Number of outliers	σ_e	$n=100$	Average of coefficients from 10000 replications				frequency of the seleceted model
			β_0	β_1	β_2	β_3	$mse\ \beta$
0	1	OLS	0.990635	1.00006	0.999977	1.00005	5250
		LSR	0.974933	1.00009	1.000009	1.00009	4750
	2	OLS	1.06900	0.999756	0.999832	0.999706	5251
		LSR	1.02196	0.999832	0.999890	0.999787	4749
	3	OLS	0.919933	1.00041	1.00015	1.00022	5314
		LSR	0.808320	1.00059	1.00032	1.00040	4686
	4	OLS	1.00591	1.00025	0.999449	1.00029	5276
		LSR	0.82801	1.00044	0.999695	1.00060	4724
	5	OLS	1.07211	0.999841	0.999866	0.999630	5182
		LSR	0.76610	1.000283	1.000484	1.000030	4818
	10	OLS	1.06135	0.999913	1.00058	0.999030	5247
		LSR	-0.11690	1.001725	1.00204	1.001120	4753
1	1	OLS	7.88573	0.987580	0.992078	0.991301	206
		LSR	0.86565	1.002381	1.003317	1.003174	9794
	2	OLS	9.39859	0.981252	0.988711	0.985939	322
		LSR	1.14623	1.001230	1.002547	1.002094	9678
	3	OLS	8.64530	0.984804	0.990809	0.987724	450
		LSR	0.94522	1.002103	1.002678	1.002734	9550
	4	OLS	7.39956	0.991017	0.991422	0.993475	543
		LSR	0.56895	1.003636	1.003767	1.003530	9457
	5	OLS	8.15408	0.987637	0.994436	0.986205	729
		LSR	0.64612	1.003221	1.003739	1.002504	9271
	10	OLS	8.39091	0.986752	0.991535	0.987736	1295
		LSR	-0.06677	1.004084	1.005121	1.002641	8705
2	1	OLS	13.9853	0.977543	0.988636	0.983780	232
		LSR	0.5997	1.005318	1.007480	1.006509	9768
	2	OLS	16.6005	0.975074	0.976118	0.972657	277
		LSR	1.0368	1.004877	1.005350	1.004517	9723
	3	OLS	15.3642	0.985088	0.977479	0.973417	329
		LSR	0.7621	1.006870	1.005528	1.004708	9671
	4	OLS	18.5423	0.980208	0.968739	0.975309	422
		LSR	0.5585	1.008302	1.006216	1.008284	9578
	5	OLS	15.0410	0.986649	0.978654	0.974028	501
		LSR	0.4364	1.007767	1.006298	1.005327	9499
	10	OLS	16.4629	0.975684	0.977204	0.972316	954
		LSR	-0.2640	1.006585	1.007103	1.007756	9046
3	1	OLS	9.14537	0.993414	0.986234	0.986753	331
		LSR	0.75667	1.004762	1.003261	1.003350	9669
	2	OLS	22.1101	0.971028	0.968024	0.969302	291
		LSR	0.6483	1.008889	1.008569	1.008943	9709
	3	OLS	21.6115	0.974612	0.965247	0.973860	353
		LSR	0.5156	1.009995	1.007703	1.009855	9647
	4	OLS	9.44953	0.988405	0.987513	0.987457	566
		LSR	0.62092	1.003677	1.003691	1.004368	9434
	5	OLS	22.6014	0.959818	0.973945	0.969854	457
		LSR	0.4472	1.007293	1.010770	1.009093	9543
	10	OLS	21.6797	0.981433	0.967168	0.964043	868
		LSR	-0.5397	1.013376	1.009731	1.008928	9132
4	1	OLS	27.2265	0.962195	0.973869	0.961090	330
		LSR	0.2212	1.012314	1.014683	1.011936	9670
	2	OLS	27.7927	0.960920	0.958549	0.972313	349
		LSR	0.3342	1.012141	1.011362	1.014176	9651
	3	OLS	27.4660	0.966863	0.968801	0.959041	352
		LSR	0.1501	1.013483	1.013908	1.011684	9648
	4	OLS	29.1845	0.962206	0.954352	0.961227	401
		LSR	0.4670	1.012881	1.010398	1.012275	9599
	5	OLS	28.8304	0.964926	0.962758	0.953632	503
		LSR	0.2834	1.013357	1.012808	1.010708	9497
	10	OLS	29.8908	0.959715	0.952134	0.958834	789
		LSR	-0.4140	1.013366	1.012930	1.012484	9211

Table 7. Comparison of OLS and LSR for sample size 250

Number of outliers	σ_e	$n = 250$	Average of coefficients from 10000 replications				frequency of the seleceted model
			β_0	β_1	β_2	β_3	$mse\ \beta$
0	1	OLS	1.01104	1.00002	0.999936	0.999939	5292
		LSR	1.00058	1.00004	0.999944	0.999953	4708
	2	OLS	0.996839	1.00011	0.999922	1.00001	5278
		LSR	0.950512	1.00018	0.999990	1.00007	4722
	3	OLS	0.981276	0.999923	0.999953	1.00031	5324
		LSR	0.865506	1.000151	1.000110	1.00049	4676
	4	OLS	0.967772	1.00024	1.00008	1.00001	5320
		LSR	0.761872	1.00055	1.00043	1.00035	4680
	5	OLS	0.953928	1.00045	1.00012	0.999926	5348
		LSR	0.625220	1.00107	1.00060	1.000462	4652
	10	OLS	0.966819	1.00035	1.00079	0.999183	5369
		LSR	-0.292996	1.00238	1.00273	1.001232	4631
1	1	OLS	3.88976	0.997399	0.995183	0.994507	233
		LSR	0.93394	1.001476	1.001125	1.000975	9767
	2	OLS	4.02009	0.995496	0.996320	0.993978	477
		LSR	0.91781	1.001377	1.001329	1.000849	9523
	3	OLS	3.38880	0.997102	0.997948	0.997027	697
		LSR	0.73799	1.001657	1.001753	1.001622	9303
	4	OLS	3.95077	0.994656	0.997190	0.994574	869
		LSR	0.79074	1.001254	1.001735	1.001019	9131
	5	OLS	3.96441	0.995246	0.994517	0.996566	1054
		LSR	0.70992	1.001695	1.001181	1.001368	8946
	10	OLS	4.11395	0.994867	0.998260	0.991665	1977
		LSR	-0.08430	1.002888	1.003253	1.001053	8023
2	1	OLS	6.61402	0.990839	0.993162	0.991762	244
		LSR	0.85013	1.002253	1.002778	1.002385	9756
	2	OLS	6.30985	0.992831	0.992146	0.993850	391
		LSR	0.78009	1.002616	1.002534	1.002778	9609
	3	OLS	6.24158	0.991107	0.992240	0.996156	502
		LSR	0.68770	1.002405	1.002803	1.003323	9498
	4	OLS	3.68164	0.995392	0.995578	0.998186	877
		LSR	0.70813	1.001289	1.001448	1.002112	9123
	5	OLS	5.99817	0.991595	0.995972	0.994306	823
		LSR	0.43176	1.002886	1.003786	1.003314	9177
	10	OLS	6.17320	0.994360	0.991367	0.994347	1566
		LSR	-0.54178	1.005723	1.004541	1.004433	8434
3	1	OLS	5.33308	0.993662	0.994598	0.992320	322
		LSR	0.91858	1.001712	1.001921	1.001571	9678
	2	OLS	9.50425	0.986414	0.993570	0.982836	352
		LSR	0.77953	1.003374	1.004940	1.002633	9648
	3	OLS	8.90333	0.987631	0.990946	0.990259	452
		LSR	0.60313	1.003643	1.004420	1.004332	9548
	4	OLS	5.11665	0.995859	0.995327	0.991587	776
		LSR	0.65856	1.002469	1.002652	1.001695	9224
	5	OLS	9.49707	0.986648	0.986873	0.989334	714
		LSR	0.46194	1.004123	1.004165	1.004433	9286
	10	OLS	9.85116	0.985361	0.986983	0.987037	1253
		LSR	-0.32443	1.004788	1.004545	1.006354	8747
4	1	OLS	12.4821	0.982065	0.983921	0.983028	339
		LSR	0.7967	1.004429	1.004885	1.004704	9661
	2	OLS	12.9213	0.986930	0.979587	0.978141	359
		LSR	0.8369	1.005467	1.003956	1.003990	9641
	3	OLS	13.0105	0.979467	0.977885	0.986407	431
		LSR	0.7666	1.004129	1.003860	1.005802	9569
	4	OLS	11.4627	0.984614	0.984572	0.990049	543
		LSR	0.3774	1.005441	1.005125	1.006692	9457
	5	OLS	11.9703	0.979150	0.990078	0.984852	607
		LSR	0.3793	1.004350	1.006875	1.005352	9393
	10	OLS	12.8512	0.979971	0.988786	0.976602	1141
		LSR	-0.3858	1.006637	1.007711	1.004968	8859

Table 8. Comparison of OLS and LSR for sample size 500

Number of outliers	σ_e	$n = 500$	Average of coefficients from 10000 replications				frequency of the seleceted model
			β_0	β_1	β_2	β_3	$mse\ \beta$
0	1	OLS	1.00464	1.00003	0.999921	1.00002	5269
		LSR	0.98980	1.00005	0.999949	1.00004	4731
	2	OLS	1.00716	0.999950	1.00001	0.999967	5271
		LSR	0.95383	1.000033	1.00011	1.000055	4729
	3	OLS	1.03391	0.999797	0.999972	0.999913	5239
		LSR	0.91730	0.999985	1.000165	1.000102	4761
	4	OLS	1.02559	0.999789	1.00006	0.999881	5268
		LSR	0.81187	1.000137	1.00042	1.000257	4732
	5	OLS	0.967256	1.00004	1.00004	1.00023	5262
		LSR	0.625061	1.00065	1.00064	1.00079	4738
	10	OLS	0.912005	1.00021	0.999832	1.00088	5395
		LSR	-0.392906	1.00236	1.001962	1.00301	4605
1	1	OLS	2.57972	0.997266	0.996192	0.998721	339
		LSR	0.98340	1.000471	1.000360	1.000742	9661
	2	OLS	2.40734	0.997155	0.998554	0.998209	632
		LSR	0.91808	1.000545	1.000837	1.000654	9368
	3	OLS	2.39591	0.997360	0.997619	0.999030	932
		LSR	0.82994	1.000715	1.000719	1.001147	9068
	4	OLS	2.45083	0.996989	0.998643	0.997830	1164
		LSR	0.75887	1.000660	1.001155	1.001009	8836
	5	OLS	2.41170	0.998157	0.998880	0.996840	1452
		LSR	0.64341	1.001546	1.001074	1.000782	8548
	10	OLS	2.55352	0.995791	0.998630	0.997992	2552
		LSR	-0.26927	1.002200	1.002700	1.002611	7448
2	1	OLS	3.81752	0.995998	0.996024	0.995771	286
		LSR	0.93571	1.001196	1.001144	1.001210	9714
	2	OLS	3.63128	0.997620	0.995681	0.996326	467
		LSR	0.86611	1.001627	1.001145	1.001267	9533
	3	OLS	3.69716	0.995196	0.996455	0.997320	687
		LSR	0.79369	1.001301	1.001531	1.001587	9313
	4	OLS	2.49020	0.996790	0.997710	0.998595	1196
		LSR	0.80738	1.000470	1.000913	1.000977	8804
	5	OLS	3.95822	0.996596	0.996363	0.993434	1129
		LSR	0.66268	1.001929	1.001862	1.000910	8871
	10	OLS	3.58160	0.997091	0.995126	0.997847	1984
		LSR	-0.40799	1.003522	1.002684	1.004132	8016
3	1	OLS	5.28314	0.992396	0.995900	0.992816	315
		LSR	0.89776	1.001515	1.002281	1.001621	9685
	2	OLS	5.15704	0.993767	0.994291	0.994298	447
		LSR	0.85172	1.001715	1.001919	1.002050	9553
	3	OLS	5.04630	0.994675	0.994317	0.994490	586
		LSR	0.73309	1.002255	1.002025	1.002257	9414
	4	OLS	2.87617	0.998114	0.997430	0.997679	1068
		LSR	0.69787	1.001338	1.001378	1.001483	8932
	5	OLS	5.35788	0.992049	0.995737	0.992611	878
		LSR	0.64331	1.001920	1.002615	1.001862	9122
	10	OLS	5.28870	0.994671	0.993068	0.993272	1744
		LSR	-0.34713	1.004381	1.003543	1.003325	8256
4	1	OLS	6.74705	0.992581	0.992337	0.989537	367
		LSR	0.88180	1.002603	1.002520	1.001949	9633
	2	OLS	6.17746	0.995592	0.990729	0.993804	439
		LSR	0.72323	1.003295	1.002303	1.002859	9561
	3	OLS	6.83543	0.987501	0.993892	0.992196	494
		LSR	0.80385	1.001525	1.002929	1.002881	9506
	4	OLS	6.31119	0.992709	0.993824	0.992237	668
		LSR	0.54366	1.003165	1.003418	1.002853	9332
	5	OLS	7.10799	0.990903	0.990217	0.989769	851
		LSR	0.63963	1.002613	1.002576	1.002746	9149
	10	OLS	6.84854	0.989571	0.990579	0.993353	1601
		LSR	-0.44550	1.004692	1.004418	1.004711	8399

Table 9. Comparison of OLS and LSR for sample size 1000

Number of outliers	σ_e	$n = 1000$	Average of coefficients from 10000 replications				frequency of the seleceted model
			β_0	β_1	β_2	β_3	$mse\ \beta$
0	1	OLS	1.00360	1.00002	0.999944	0.99999	5196
		LSR	0.98894	1.00005	0.999971	1.00002	4804
	2	OLS	0.993244	1.00008	0.999935	1.00005	5348
		LSR	0.940190	1.00016	1.000025	1.00013	4652
	3	OLS	0.995436	1.00004	0.999857	1.00014	5267
		LSR	0.880155	1.00022	1.000051	1.00032	4733
	4	OLS	1.00227	0.999930	0.999870	1.00017	5329
		LSR	0.78656	1.000286	1.000239	1.00054	4671
	5	OLS	1.01980	0.999805	1.00008	0.999947	5344
		LSR	0.68765	1.000381	1.00063	1.000476	4656
	10	OLS	1.11963	0.999393	0.999425	0.999961	5433
		LSR	-0.20277	1.001602	1.001665	1.002073	4567
1	1	OLS	1.70096	0.999422	0.998503	0.999058	435
		LSR	0.97491	1.000380	1.000241	1.000306	9565
	2	OLS	1.62388	0.999472	0.999143	0.999138	846
		LSR	0.91045	1.000463	1.000453	1.000456	9154
	3	OLS	1.69958	0.999208	0.999247	0.998543	1233
		LSR	0.85426	1.000559	1.000551	1.000492	8767
	4	OLS	1.57924	0.999550	0.998764	0.999877	1663
		LSR	0.74238	1.000697	1.000634	1.000919	8337
	5	OLS	1.70040	0.998907	0.999331	0.998732	1973
		LSR	0.61533	1.001029	1.000959	1.000920	8027
	10	OLS	1.85041	0.998582	0.998576	0.998410	3261
		LSR	-0.31573	1.002589	1.002408	1.002317	6739
2	1	OLS	2.41410	0.997783	0.998408	0.997646	368
		LSR	0.94696	1.000620	1.000732	1.000587	9632
	2	OLS	2.47769	0.998175	0.997121	0.997911	671
		LSR	0.91578	1.000754	1.000619	1.000686	9329
	3	OLS	2.38608	0.998406	0.997519	0.998169	885
		LSR	0.81304	1.000970	1.000790	1.000962	9115
	4	OLS	2.33998	0.998041	0.998834	0.997695	1163
		LSR	0.76381	1.000828	1.001090	1.000847	8837
	5	OLS	2.47840	0.996708	0.998667	0.997825	1456
		LSR	0.67257	1.000736	1.001437	1.000919	8544
	10	OLS	2.33905	0.998282	0.998228	0.998090	2756
		LSR	-0.42348	1.003043	1.003160	1.002853	7244
3	1	OLS	3.31033	0.995991	0.996585	0.996296	344
		LSR	0.97348	1.000702	1.000880	1.000832	9656
	2	OLS	3.05294	0.997754	0.996684	0.996992	579
		LSR	0.90119	1.001158	1.000869	1.000908	9421
	3	OLS	3.28030	0.997302	0.996594	0.995278	809
		LSR	0.83332	1.001261	1.001028	1.000994	9191
	4	OLS	105.920	0.860337	0.833995	0.854725	338
		LSR	6.984	1.033801	1.019011	1.034964	9662
	5	OLS	3.15317	0.997601	0.997000	0.995849	1253
		LSR	0.68954	1.001366	1.001353	1.000952	8747
	10	OLS	3.11611	0.997667	0.997105	0.996011	2359
		LSR	-0.35327	1.003317	1.003204	1.002540	7641
4	1	OLS	3.94153	0.995480	0.994485	0.996584	346
		LSR	0.94115	1.001164	1.000979	1.001339	9654
	2	OLS	3.61244	0.995552	0.997047	0.997232	531
		LSR	0.82696	1.001233	1.001623	1.001573	9469
	3	OLS	3.82751	0.996712	0.995248	0.995738	779
		LSR	0.78050	1.001744	1.001344	1.001472	9221
	4	OLS	3.46799	0.996829	0.997378	0.997061	883
		LSR	0.63548	1.001822	1.001912	1.001811	9117
	5	OLS	3.70053	0.996087	0.996921	0.995936	1105
		LSR	0.58999	1.001536	1.002024	1.001833	8895
	10	OLS	3.73128	0.995877	0.996600	0.996188	2076
		LSR	-0.40478	1.003482	1.003634	1.003246	7924

The results in Table 2 is a representation of the smallest sample size 20. From Table 2 it is clearly observed that when there no outlier is presented OLS method is always a little better, but the priority is not that high for any error variance. This can be viewed from the frequency that is higher for the selected method. Addition of any outlier makes LSR always superior no matter what the degree of the error variance. Increasing the sample size from 20 to 30, we obtain the results presented in Table 3. Reading the results from that table, we end up with same conclusion of the previous table. In fact no matter what the sample size is, existence of an outlier makes LSR always superior. This can be observed starting from Table 4 to Table 9. OLS is getting better for large sample sizes but that does never let OLS be better according to the criteria of MSE of the coefficients.

This study may be performed in terms of other criteria as well, for instance AIC or BIC. However we do not think that for the presence of an outlier, OLS will get the priority from LSR. This judgement can be justified from the study of (Deniz, Akbilgiç, and Howe, 2011).

5. Discussion and Results

In this study we introduce one of the robust regression approaches, called Least Squares Ratio (LSR), and make a comparison of OLS and LSR according to mean square errors of regression parameter estimations. For a linear model having three regressors, we generate data for different sample sizes, error variances and number of outliers. It is found that no matter what the sample size is LSR always performs well when there is 1, 2, 3 or 4 outliers.

$$\boldsymbol{n} = [\mathbf{20}, \mathbf{30}, \mathbf{50}, \mathbf{80}, \mathbf{100}, \mathbf{250}, \mathbf{500}, \mathbf{1000}]$$

and multiple error variances as

$$\boldsymbol{\sigma_e^2} = [\mathbf{1}, \mathbf{4}, \mathbf{9}, \mathbf{16}, \mathbf{25}, \mathbf{100}].$$

We performed our simulations with 10 000 replications using 8 different sample sizes as

$$\boldsymbol{n} = [\mathbf{20}, \mathbf{30}, \mathbf{50}, \mathbf{80}, \mathbf{100}, \mathbf{250}, \mathbf{500}, \mathbf{1000}]$$

and 6 different error variances as

$$\boldsymbol{\sigma_e^2} = [\mathbf{1}, \mathbf{4}, \mathbf{9}, \mathbf{16}, \mathbf{25}, \mathbf{100}].$$

For each generation, the dependent and independent variables are computed by the specified protocol. Estimation of regression parameters is then performed by both OLS and LSR. The two methods are compared by the mean square errors for parameter estimation.

According to the simulation results obtained it is found that when no effect of an outlier is presented the classical OLS performs well. However the proposed LSR robust regression method always has the priority against OLS in MSE point of view. Even if there is no outlier it has been proven that LSR is not much worse for any sample sizes and error variances.

The study may be extended for more number of explanatory variables. But it is already an extention of (Akbilgic and Deniz, 2009) study in terms of regressors. So no different results is expected. In the meantime since the LSR method is a robust method we suggest that it may be applied for nonlinear regression models using some other criteria.

When there is an outlier in the data set we definetely propose LSR method to be used in place of OLS method no matter what the number of regressors or what the sample size is. However one should be careful that LSR can not be used for any zero value of the dependent variable.

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