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Modeling of Exchange Rate Risk

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Abstract In this work we present a mathematical modeling approach to exchange rate risk using the stochastic model of Vasicek. The risk measure used in this modeling approach is the conditional value at risk CVaR. The objective of this approach is to provide a tool of decision for the exchange market managers.

Keywords Exchange rates, Risk, CVaR, Vasicek Stochastic model

1. Introduction

The exchange rate is very important macroeconomic variable that plays a crucial role in international trade. Indeed, the change of the exchange rate affects the terms of trade, it allows modification and adjustment of prices in the different sectors of tradables and non-tradables.

Thus, exchange rate fluctuations constitute an area of research that requires considerable effort at modeling. It aims to establish a model to study the convergence of the exchange rate towards equilibrium.

In another, the modeling and the evaluation of exchange rate risk is a very important task for managers in the field of foreign trade.

The organization of this work is as follows: In Section 1 we present the stochastic model Vasick. Then, value at risk conditional of exchange rate is presented in section 3.

2. Stochastic Model of Exchange Rate Vasicek

The exchange rate can be modeled by several stochastic models, among them there are stochastic processes that tend to return to a moderate level in more or less long term that are used to model the exchange rates of some currencies.

In this context fits the stochastic process Vasick or the process of returning to the mean.

Thereby, more the fluctuations of r_t is around to its mean of long term is low, more the volatility of r_t is low.

The model of exchange rate of Vasicek is given by the following equation:

$$dr_t = \eta\left(\mu - r_t\right)dt + \sigma dz_t \tag{1}$$

Where:

- η : is the speed of exchange rate of return to the mean;
- μ : is the average exchange rate.
- σ : is the exchange rate volatility which is assumed independent
- z_t : is a Brownian motion as $dz_t = \varepsilon_t \sqrt{dt}$ with $\varepsilon_t \sim N\left(0,1\right)$

Thus we can develop this model as follows:

$$d\left(e^{\eta t} r_t\right) = e^{\eta t} dr_t + r_t \eta e^{\eta t} \Rightarrow e^{\eta t} dr_t = d\left(e^{\eta t} r_t\right) - r_t \eta e^{\eta t}$$

In other,

$$dr_t = \eta\left(\mu - r_t\right)dt + \sigma dz_t \Rightarrow$$
$$e^{\eta t} dr_t = e^{\eta t}\eta\left(\mu - r_t\right)dt + e^{\eta t}\sigma dz_t$$

So we get:

$$d\left(e^{\eta t} r_t\right) = \eta\mu e^{\eta t} dt + e^{\eta t}\sigma dz_t \Rightarrow$$
$$r_t = r_0 e^{-\eta t} + \int_0^t \eta e^{-\eta(t-s)}\mu ds + \sigma\int_0^t e^{-\eta(t-s)} dz_s$$

So the exchange rate r_t according to this model can be expressed as follows:

$$r_t = \mu + \left(r_0 - \mu\right)e^{-\eta t} + \sigma\int_0^t e^{-\eta(t-s)} dz_s \tag{2}$$

Knowing that z_t is a Brownian motion as $dz_t = \varepsilon_t \sqrt{dt}$ with $\varepsilon_t \sim N\left(0,1\right)$ then dz_t follows the normal law and thereafter variable $\left(\sigma\int_0^t \eta e^{-\eta(t-s)} dz_s\right)$ also follows the normal law.

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As the term $\mu+\left(r_0-\mu\right)e^{-\eta t}$ is not a random term then $r_t\left|r_0\right.$ is a random variable that follows the normal law.

By using Iso isometry [2] it is found that:

$$E\left[\left(\sigma\int_0^t \eta e^{-\eta(t-s)}dz_s\right)\right]=0 \text{ and } E\left[\left(\sigma\int_0^t \eta e^{-\eta(t-s)}dz_s\right)^2\right]=\int_0^t\left(\sigma e^{-\eta(t-s)}\right)ds=\frac{\sigma^2}{2\eta}\left(1-e^{-2\eta t}\right)$$

Then the mean and variance are respectively:

- $E\left(r_t\left|r_0\right.\right)=\overline{r}+\left(r_0-\overline{r}\right)e^{-\eta t}$
- $V\left(r_t\left|r_0\right.\right)=\dfrac{\sigma^2}{2\eta}\left(1-e^{-2\eta t}\right)$

Thus the random variable follows the normal distribution with mean and variance respectively:

$\mu+\left(r_0-\mu\right)e^{-\eta t}$ and $\dfrac{\sigma^2}{2\eta}\left(1-e^{-2\eta t}\right)$. i.e :

$$r_t\sim N\left(\mu+\left(r_0-\mu\right)e^{-\eta t},\sigma\sqrt{\frac{1}{2\eta}\left(1-e^{-2\eta t}\right)}\right) \tag{3}$$

3. Modeling of Exchange Rate Risk

Value at risk VaR is a risk measure proposed by JP Morgan in 1994.

It represents the maximum loss that may arise in a portfolio over $\left[0,t\right]$ for a given level of probability α . i.e :

$$P\left[-\Delta V\left(t\right)\leq VaR\right]=1-\alpha \tag{4}$$

Where $\Delta V\left(t\right)=V\left(t\right)-V\left(0\right)$ with:

- $V\left(0\right)$: Portfolio value at beginning of the period.
- $V\left(t\right)$: Portfolio value by the end of the period.

Value at Risk depends on three parameters:

- The distribution of the change in portfolio value
- The probability that the losses are less than the VaR
- The horizon for which VaR is calculated.

VaR offers several advantages such as ease of comparison and interpretation.

However, studies such as Szergõ [9] showed that VaR does not take into account the amount of losses exceeding the VaR. Thus VaR is not sub-additive, it means that diversification does not imply a reduced risk.

To overcome the limitations of VaR, a new risk measure called the conditional VaR (CVaR), defined as the expected loss exceeding the VaR can be adopted.

This is the average value of losses that exceed VaR. CVaR is expressed as follows:

$$CVaR_\alpha\left(X\right)=\frac{1}{1-\alpha}\int_\alpha^1 VaR_\theta\left(X\right)d\theta \tag{5}$$

The nominal exchange rate (NER) is the amount of domestic currency per unit of currency of a foreign country. If TCN = 10.5 DH / EUR then it means that the buyer must pay to have a 10.5 DH /EURO. Thus it should be noted that exchange offices must display two NER: one for the purchase of currency and the other for sales.

Exchange rate risk or currency risk occurs when a change in exchange rates negatively affect the cash flows of a company following a trade with the outside. The exchange rate risk measurement can be performed by VaR.

The calculation of VaR to the exchange rate time horizon is given by the following equation:

$$P\left(r_0-r_T\leq VaR_\alpha\right)=\alpha$$

as $r_t \sim N\left(\mu + \left(r_0 - \mu\right)e^{-\eta t}, \sigma\sqrt{\frac{1}{2\eta}\left(1 - e^{-2\eta t}\right)}\right)$ Then

$$\Rightarrow \quad P\left(\frac{r_0 - r_T - \left(r_0 - \mu - \left(r_0 - \mu\right)e^{-\eta T}\right)}{\sigma\sqrt{\frac{1}{2\eta}\left(1 - e^{-2\eta T}\right)}} \le \frac{VaR_\alpha - \left(r_0 - \mu - \left(r_0 - \mu\right)e^{-\eta T}\right)}{\sigma\sqrt{\frac{1}{2\eta}\left(1 - e^{-2\eta T}\right)}}\right) = \alpha$$

$$\Rightarrow \quad \frac{VaR_\alpha - \left(r_0 - \mu\right)\left(1 - e^{-\eta T}\right)}{\sigma\sqrt{\frac{1}{2\eta}\left(1 - e^{-2\eta T}\right)}} = \tau_\alpha$$

Then

$$VaR_\alpha = \left(r_0 - \mu\right)\left(1 - e^{-\eta T}\right) + \tau_\alpha \sigma\sqrt{\frac{1}{2\eta}\left(1 - e^{-2\eta T}\right)} \tag{6}$$

Thus, in the case of normal distribution was in:

$$CVaR_\alpha = E\left[\left(\left(r_0 - r_T\right) - VaR_\alpha\right)\middle|\left(r_0 - r_T\right) > VaR_\alpha\right] = E\left(\left(r_0 - r_T\right)\middle|\left(r_0 - r_T\right)X > VaR_\alpha\right) - VaR_\alpha$$

Put $X = \left(r_0 - r_T\right)$ then

$$CVaR_\alpha = E\left[\left(X - VaR_{1-\alpha}\right)\middle|X > VaR_{1-\alpha}\right] = E\left(X\middle|X > VaR_{1-\alpha}\right) - VaR_{1-\alpha}$$

Gold $E\left(X\middle|X > VaR_\alpha\right) = \frac{1}{1-\alpha}\int_{VaR_\alpha}^{\infty} x dF_X\left(x\right) = \frac{1}{1-\alpha}\int_{VaR_\alpha}^{\infty} x f_X dx$,

where $f_X\left(x\right) = \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-m)^2}{2\sigma^2}}$ with m_X and σ_X^2 are the mean and variance respectively X.

Let $y = \frac{x - m_X}{\sigma_X}$ it comes :

$$\int_{VaR_\alpha}^{\infty} x f_X dx = \frac{1}{\sqrt{2\pi}}\int_{\frac{VaR_\alpha - m_X}{\sigma_X}}^{\infty}\left(\sigma_X y + m_X\right)e^{\frac{-y^2}{2}} dy = \frac{1}{\sqrt{2\pi}}\int_{\frac{VaR_\alpha - m_X}{\sigma_X}}^{\infty} y e^{\frac{-y^2}{2}} dy + \frac{1}{\sqrt{2\pi}} m_X \int_{\frac{VaR_\alpha - m_X}{\sigma_X}}^{\infty} e^{\frac{-y^2}{2}} dy$$

$$= \frac{1}{\sqrt{2\pi}}\sigma_X \, I_1 + m_X\left(1 - \Phi\left(\frac{VaR_\alpha - m_X}{\sigma_X}\right)\right)$$

where $I_1 = \int_{\frac{VaR_\alpha - m_X}{\sigma_X}}^{\infty} y e^{\frac{-y^2}{2}} dy$.

For calculating I_1, we know successively:

$$\int_z^{\infty} y e^{\frac{-y^2}{2}} dy = \int_z^{\infty} e^{\frac{-y^2}{2}} d\left(\frac{y^2}{2}\right) = \int_{\frac{z^2}{2}}^{\infty} e^{-u} du = e^{\frac{-Z^2}{2}},$$

It follows $I_1 = e^{-\frac{(v - m_X)^2}{2\sigma_X^2}}$ then

$$\int_{VaR_\alpha}^{\infty} xf_X dx = \frac{1}{\sqrt{2\pi}}\sigma_X \, e^{\frac{-(VaR_\alpha - m_X)^2}{2\sigma_X^2}} + m_X\left(1-\Phi\left(\frac{VaR_\alpha - m_X}{\sigma_X}\right)\right)$$

$$= \sigma \ \phi\left(\frac{VaR_\alpha - m_X}{\sigma_X}\right) + m_X\left(1-\Phi\left(\frac{VaR_\alpha - m_X}{\sigma_X}\right)\right),$$

where $\phi(x) = \Phi'(x) = \frac{1}{\sqrt{2\pi}} e^{\frac{-x^2}{2}}$, hence the following result:

$$CVaR_\alpha = \frac{1}{1-\alpha}\left[\sigma_X \varphi\left(\frac{VaR_\alpha - m_X}{\sigma_X}\right) + m_X \Phi\left(\frac{VaR_\alpha - m_X}{\sigma_X}\right)\right] - VaR_\alpha$$

Replace m_X and σ_X^2 expressions in formula previous. So we get:

$$CVaR_\alpha = \frac{1}{1-\alpha}\left[\sigma\sqrt{\frac{1}{2\eta}\left(1-e^{-2\eta T}\right)}\varphi(\tau_\alpha) + \left((r_0-\mu)\right)\Phi(\tau_\alpha)\right] - (r_0-\mu)\left(1-e^{-\eta T}\right) + \tau_\alpha\sigma\sqrt{\frac{1}{2\eta}\left(1-e^{-2\eta T}\right)}$$

Thus

$$CVaR_\alpha = \frac{\sqrt{\frac{1}{2\eta}\left(1-e^{-2\eta T}\right)}}{1-\alpha}\left(\left[\sigma\varphi(\tau_\alpha) + \frac{\left((r_0-\mu)\sqrt{\left(1-e^{-\eta T}\right)}\right)\Phi(\tau_\alpha)}{\sqrt{\frac{1}{2\eta}}}\right] - \frac{(r_0-\mu)\sqrt{\left(1-e^{-\eta T}\right)}}{\sqrt{\frac{1}{2\eta}}} + \tau_\alpha\sigma\right) \quad (7)$$

4. Numerical Application

In this part we will present an example of a numerical application of the previous formula of CVaR.

t	r_t	CVaR
1	10,289	0.0127
2	10,294	0.0133
.	.	.
.	.	.
.	.	.
27	10,283	0.0131
28	10,275	0.0105

Consider a sample of 28 exchange rate DH / EUR of February month 2015 from which we calculate the CVaR The results obtained are given in the following table:

5. Conclusions

In this paper, we developed a mathematical formula to express the conditional value at risk CVaR exchange rate using the Vasick stochastic model.

This formula allows to assess the risk of exchange rate, thereby providing a tool for decision support to the exchange market managers.

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