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The Properties of Pure Diagonal Bilinear Models

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Abstract Stationarity, invertibility and covariance structure of pure diagonal bilinear models have been studied in details in this paper. We transformed the pure diagonal bilinear model into the state space form and subsequently examined the condition under which it is stationary and invertible. We also derived the covariance structure of the pure diagonal bilinear model and showed that for every pure diagonal bilinear process there exists an ARMA process with identical covariance structure.

Keywords Bilinear models, Stationarity, Invertibility and covariance structure

1. Introduction

Let $\{X_t\}$ and $\{e_t\}$ be two stochastic processes. We assume that e_t, is independent, identically, distributed with $E(e_t) = 0$ and $E(e_t^2) = \sigma^2 < \infty$. A bilinear model is one which is linear in both $\{X_t\}$ and $\{e_t\}$ but not in those variables jointly. Let $a_1, a_2, \ldots, a_p, b_1, b_2, \ldots b_q$ and b_{ij}, $1 \le i \le m$, $1 \le j \le k$, be real constants.

The general form of a bilinear model is given by Granger and Andersen(1978) as

$$X_t = \sum_{i=1}^{P} a_i X_{t-i} + \sum_{j=1}^{q} C_j e_{t-j} + \sum \sum_{i=1,j=1}^{m,k} b_{ij} X_{t-i} e_{t-j} + e_t \quad (1.1)$$

The first part on the right hand side of (1) can be identified as the autoregressive part of the process X_t, the second part as the moving average part and the third part as the 'pure' bilinear part. Following Subba Rao (1981), we denote this model by BL (p,q,m,k), where BL is the abbreviation for bilinear. On the other hand if $p = q = 0$ and $b_{ij} = 0$, for all $i \neq j$, the model is called Pure Diagonal Bilinear Model of order p[PDBL(p)] and we write it as

$$X_t = \sum_{i=1}^{P} b_i X_{t-i} e_{t-i} + e_t \quad (1.2)$$

Subba Rao (1981) obtained second order moments of the bilinear model BL(p,0,p,1), and Subba Rao and Gabr (1984) obtained third order moments of the BL(1,0,1,1) model. Sessay and SubbaRao (1988, 1991) have also shown that for the BL (p,0,p,1) model, third order moments satisfy Yule-Walker type difference equation.

It is well known that the linear autoregressive moving average model can be written in the form of first order vector difference equation (See Anderson, 1971; Priestly, 1978, 1980) and this Vector form is known as the State Space Form. It is convenient to study the properties of a model when it is in the State Space Form (Akaike, 1974). Therefore, we put the pure diagonal bilinear model in the Vectorial form and subsequently examine the conditions under which it is stationary and invertible.

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2. Vectorial Representation of the Pure Diagonal Bilinear Model

A time series $\{X_t\}$ is said to be a diagonal bilinear process if it satisfies the difference equation

$$X_t = \sum_{j=1}^{r} a_j X_{t-j} + \sum_{j=1}^{q} C_j e_{t-j} + \sum_{j=1}^{p} b_j X_{t-j} e_{e-j} + e_t \quad (2.1)$$

where $\{e_t\}$ is a sequence of independent and identically distributed random variables with zero mean and variance $\sigma^2 < \infty$. If $r = q = 0$, the model (2.1) is called Pure diagonal bilinear (PDBL) model and we write it as

$$X_t = \sum_{j=1}^{p} b_j X_{t-j} e_{e-j} + e_t \quad (2.2)$$

where $\{e_t\}$ is as defined previously. The model (2.2) is denoted by PDBL (P).

Let

$$A_{pxp} = \begin{bmatrix} 0 & 0 & 0 & \ldots & 0 & 0 \\ 1 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 1 & 0 & \ldots & 0 & 0 \\ . & . & . & \ldots & & \\ . & . & . & \ldots & & \\ 0 & 0 & 0 & \ldots & 1 & 0 \end{bmatrix} \quad (2.3)$$

$$B_{j_{pxp}} = \begin{bmatrix} b_j & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & \ldots & 0 & 0 \\ 0 & 0 & 0 & \ldots & 0 & 0 \\ . & . & . & \ldots & & \\ . & . & . & \ldots & & \\ 0 & 0 & 0 & \ldots & 0 & 0 \end{bmatrix} \quad (2.4)$$

$$(j = 1,2,\ldots,p)$$

$$C_{1xp}^T = (1, 0, 0, \ldots, 0, 0) \quad (2.5)$$

$$H^T_{1xp} = (1, 0, 0,\ldots, 0, 0) \quad (2.6)$$

$$\underline{X}^T_t = (X_t, X_{t-1},\ldots, X_{t-p+1}) \quad (2.7)$$

where T stands for operation transpose of a matrix. We now represent the model (2.2) in vectorial form.

THEOREM 2.1

If $\{X_t\}$ satisfies (2.2) then

$$\underline{X}_t = A\ \underline{X}_{t-1} + \sum_{j=1}^{p} B_j \underline{X}_{t-j} e_{t-j} + Ce_t \quad (2.8)$$

is the vectorial form of (2.2)

Proof. By direct Verification.

2.1. Stationarity

We want to examine the conditions under which a process $\{X_t\}$ satisfying (2.8) exists. This type of problem has been tackled by Bhaskara Rao et al (1983) for the special class of models satisfying

$$X_t = \sum_{j=1}^{P} a_j X_{t-j} + \sum_{j=1}^{q} b_j X_{t-j} e_{t-j} + e_t \quad (2.9)$$

After putting this model in vectorial form, they gave a sufficient condition for the existence of a strictly stationary process $\{X_t\}$ satisfying (2.9). Earlier, Subba Rao and Gabr (1981) gave a sufficient condition for the existence of a second order stationary process $\{X_t\}$ satisfying (2.9) with P=q. The sufficient conditions in both cases were the same.

Subba Rao and Gabr (1981) also obtained the same sufficient conditions for the existence of a second order stationary process $\{X_t\}$ satisfying

$$X_t = e_t + \sum_{j=1}^{P} a_j X_{t-j} + \sum\sum_{i=1,j=1}^{P,P} b_{ij} X_{t-j} e_{t-j} \quad (2.10)$$

Under some conditions involving a,b, and σ^2. Akamanam et al (1986) have shown that under some conditions on the spectral radius of a matrix, the process $\{X_t\}$ satisfying (2.1) do exist, are stationary, ergodic and unique.

2.1.1. Vectorial Representaion Method

We now give a set of sufficient conditions under which there is a strictly stationary and ergodic process X_t, t ε Z, satisfying (2.8). We use the methods of Akamanam et al (1986).

THEOREM 2.2 (AKAMANAM, BHASKARA RAO, SUBRAMANYAM, 1986)

Let $\{e_t\}$ be sequence of independent and identically distributed random variables with zero mean and variance $\sigma^2 < \infty$. Let A_{pxp}, $B_{j\,pxp}$, j = 1, 2, …,p, C^T_{1xp},

H^T_{1xp} and $\underline{X}^T$ be the matrices given by (2.3) to (2.7) respectively.

$$\Gamma_1 = A \otimes A + (B_1 \otimes B_1)\sigma^2$$

$$\Gamma_j = \sigma^2[\ B_j \otimes (A^{j-1}B_1 + \ldots + AB_{j-1} + B_j) + (A^{j-1}B_1 + \ldots + AB_{j-1}) \otimes + B_j\ ] \ \text{(for j = 2,…, P)}$$

where ⊗ is the symbol for Kronecker product of matrices. Suppose all the eigenvalues of the block companion matrix

$$\Gamma_{p^3\times p^3} = \begin{bmatrix} \Gamma_1 & \Gamma_2 & \ldots & \Gamma_{P-1} & \Gamma_P \\ IP^2 & 0 & \ldots & 0 & 0 \\ \cdot & \cdot & & & \\ \cdot & \cdot & & & \\ \cdot & \cdot & & & \\ 0 & 0 & \ldots & IP^2 & 0 \end{bmatrix} \quad (2.11)$$

have moduli less than unity, where I_n stands for the identity matrix of order n x n.

Then there exists a strictly stationary and ergodic process $\{X_t\}$ conforming to the model (2.8).

Further, if a process $\{U_t\}$ conforms to the above bilinear model (2.8), then $U_t = \underline{X_t}$

Proof. See Akamanam et al (1986).

2.1.2. Characteristic Equation Method

We use the above theorem to show that a sufficient condition for the existence of the strictly stationary process $\{X_t\}$ satisfying (2.2) is that the roots (in modulus) of the characteristic equation

$$\left|\Gamma_p + \beta\Gamma_{p-1} + \beta^2\Gamma_{p-2} + \ldots + \beta^{p-1}\Gamma_1 - \beta^p I_{pXp}\right| = 0 \quad (2.12)$$

are in absolute value less than unity.

We proceed by considering the following cases.

CASE 1 P=2

$$\Gamma = \begin{bmatrix} \Gamma_1 & \Gamma_2 \\ I_{pxp} & \underline{0} \end{bmatrix}$$

The eigenvalues of Γ are obtained as follows:

$$\left|\ \Gamma - \beta I_{pxp}\ \right| = \left| \frac{\Gamma_1 - \beta I_{pxp} \ \vdots\ \Gamma_2}{I_{pxp}\ \vdots \beta I_{pxp}} \right| = 0$$

Applying the procedure for obtaining the determinant of a partitioned matrix, we can show that

$$\left|\ \Gamma - \beta I_{pxp}\ \right| = \beta^{pxp}$$

$$\left|(\Gamma_1 - \beta I_{pxp}) - (\Gamma_2\,(1/\beta) I_{pxp}\ .I_{pxp}\ \right| = 0$$

(See Morrison 1976). Simplifying the left-hand side, we have

$$\beta^{pxp-1}\left|\Gamma_2 + \beta\Gamma_1 - \beta^2\ I_{pxp}\ \right| = 0$$

This implies $\left|\ \Gamma_2 + \beta\Gamma_1 - \beta^2\ I_{pxp}\ \right| = 0$, since $\beta^{pxp-1} \neq 0$

CASE 2 P=3

$$\Gamma = \begin{vmatrix} \Gamma_1 & \Gamma_2 & \Gamma_2 \\ I_{pxp} & \underline{0} & \underline{0} \\ \underline{0} & I_{pxp} & \underline{0} \end{vmatrix}$$

Proceeding as in P = 2, we have

$$\left|\Gamma - \beta\, I_{pxp}\right| = \begin{bmatrix} \Gamma_1 - \beta I_{pxp} & \Gamma_2 & \vdots & \Gamma_3 \\ I_{pxp} & -\beta I_{pxp} & \vdots & \underline{0} \\ \ldots\ldots\ldots\ldots & \ldots\ldots\ldots & \vdots & \ldots\ldots \\ \underline{0} & I_{pxp} & \vdots & -\beta I_{pxp} \end{bmatrix}$$

$$\left|\Gamma - \beta I_{pxp}\right| = \beta^{pxp} \left|\Gamma_1 - \beta I_{pxp} + (1/\beta)\Gamma_2 + (1/\beta^2)\Gamma_3\right| = 0$$

Simplifying, we have

$$\left|\Gamma - \beta I_{pxp}\right| = \beta^{pxp-2} \left|\Gamma_3 + \beta\Gamma_2 + \beta^2\Gamma_1 - \beta^3\Gamma_{pxp}\right| = 0$$

This implies that,

$$\left|\Gamma_3 - \beta\Gamma_2 + \beta^2\Gamma_1 - \beta^3\Gamma_{pxp}\right| = 0 \text{ since } \beta^{pxp-2} \neq 0$$

Thus, based on the behavior of the above two cases considered, it can be shown in general that

$$\left|\Gamma_p - \beta\Gamma_{p-1} + \beta^2\Gamma_{p-2} + \ldots + \beta^{p-1}\Gamma_1 - \beta^p I_{pxp}\right| = 0$$

Therefore, a sufficient condition for the existence of a strictly stationary process $\{X_t\}$ satisfying (2.8) is that the roots (in modulus) of the characteristic equation (2.12) are in absolute value less than unity.

The following example will illustrate further the work of this section.

$$\Gamma_2\ 4x4 = \begin{bmatrix} b_1^2\sigma^2 & 0 & 0 & 0 \\ b_1b_2\sigma^2 & 0 & 0 & 0 \\ b_1b_2\sigma^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Let $\beta_j = b_j\sigma$

Therefore,

$$\Gamma_1 = \begin{bmatrix} \beta_1^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

$$\Gamma_2 = \begin{bmatrix} \beta_2^2 & 0 & 0 & 0 \\ \beta_1\beta_2 & 0 & 0 & 0 \\ \beta_1\beta_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

From case one,

$$\left|\Gamma_2 + Y\Gamma_1 - Y^2 I_{2\times 2}\right| = 0$$

Therefore,

$$\left|\begin{bmatrix} \beta_1^2 & 0 & 0 & 0 \\ \beta_1\beta_2 & 0 & 0 & 0 \\ \beta_1\beta_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} Y\beta_1^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ Y & 0 & 0 & 0 \end{bmatrix} - \begin{bmatrix} -Y^2 & 0 & 0 & 0 \\ 0 & -Y^2 & 0 & 0 \\ 0 & 0 & -Y^2 & 0 \\ 0 & 0 & 0 & -Y^2 \end{bmatrix}\right| = 0$$

Implies

$$\begin{bmatrix} \beta_2^2 + Y\beta_1^2 Y^2 & 0 & 0 & 0 \\ \beta_1\beta_2 & -Y^2 & 0 & 0 \\ \beta_1\beta_2 & 0 & -Y^2 & 0 \\ Y & 0 & 0 & -Y^2 \end{bmatrix} = 0$$

Implies

$$\left|Y^4\right| \left|\begin{bmatrix} \beta_2^2 + Y\beta_1^2 - Y^2 & 0 \\ \beta_1\beta_2 & -Y^2 \end{bmatrix} - \underline{0}\right| = 0$$

$$\left|Y^4\right| \left|(-Y^2)(\beta_2^2 + Y\beta_1^2 - Y^2) - \underline{0}\right| = 0$$

$$Y^6 \left|Y^2\ \beta_2^1 Y - \beta_2^2\right| = 0$$

This implies that

$$Y^2 - Y\beta_1^2 - \beta_2^2 = 0 \text{ since } Y^6 \neq 0$$

We have shown that

$$\left|\Gamma_2 + Y\Gamma_1 - Y^2 I_{2\times 2}\right| = 0 \text{ implies}$$

$$\left|Y^2\ Y\beta_1^2 - \beta_2^2\right| = 0$$

It is also easy to show that

$$\left|\Gamma_3 + Y\Gamma_2 - Y^2\Gamma_3 - Y^3 I_{p\times p}\right| = 0 \text{ implies}$$
$$Y^3 - Y^2\beta_1^2 - Y\beta_2^{2-}\beta_3^2 = 0$$

In general

$$\left|\Gamma_p + \beta\Gamma_{p-1} + \beta^2\Gamma_{p-2} + \ldots + \beta^{p-1}\Gamma_1 - \beta^p I_{p\times p}\right| = 0$$

Implies

$$Y^p - Y^{p-1}\beta_1^2 - Y^{P-2}\beta_2^2 - Y^{P-3}\beta_3^2 \ldots \beta_{P-1}^2 Y - \beta_P^2 = 0$$

An alternative approach for obtaining the characteristic equation is given below

THEOREM 2.3

Let $\{e_t\}$ be sequence of independent and identically distributed random variable with zero mean and E $e_t^2 = \sigma^2 < \infty$. Suppose there exist a stationary and ergodic process $\{X_t\}$ Satisfying

$$X_t = e_t + \sum_{J=1}^{P} b_j X_{t-j} e_{t-j} \quad (2.13)$$

Let

$$H_{pxp} = \begin{bmatrix} \beta_1^2 & \beta_1^2 & \ldots & \beta_{p-1}^2 & \beta_p^2 \\ 1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & \ldots & 0 & 0 \\ \vdots & & & & \\ 0 & 0 & \ldots & 0 & 0 \end{bmatrix} \quad (2.14)$$

Where $\beta_j = b_j\,\sigma$

Then e (H) < 1

Where e(H) is the spectral radius of the matrix H.

Proof: Squaring both sides of (2.13) and taking expectations, we obtain

$$EX_t^2 = e_t + \sum_{J=1}^{P}\beta_1^2\, EX_{t-j}^2 + C \quad (2.15)$$

Where

$$C = \sigma^2(1 + 2\sum_{J=1}^{P}\beta_1^2 + 2\sum\sum_{i<j}\beta_i\ \beta_j)$$

Let

$$W^T = (EX_t^2, EX_{t-1}^2, \ldots EX_{t-p+1}^2)$$

R^T = (C, 0, 0, …, 0, 0)and H is defined in (2.14). With this notation, we can write (2.13) as the first order difference equation.

$$W_t = HW_{t-1} + R, \quad t = 1,2,\ldots.$$

Because of stationarity of $\{X_t\}$ we have $W_t = W_{t-1}$ for all t. consequently e(H) < 1.

This therefore implies that the root (in modulus) of the equation

$$Y^p - \beta_1^2 Y^{p-1} - \beta_2^2 Y^{P-2} - \ldots \beta_{P-1}^2 Y - \beta_P^2 = 0 \quad (2.16)$$

Lies inside the unit circle.

2.2. Invertibility

For a time series to be useful for forecasting purposes, it is necessary that it should be invertible. We do not know of any nice conditions under which the general bilinear autoregressive moving average model is invertible. The invertibility of special cases of (2.1) have been studied by Granger and Anderson (1978), Subba Rao (1981), Pham and Tran (1981), Quinn (1982) and Iwueze (1988).

THEOREM 2.4 (IWUEZE 1988)

Let $\{e_t\}$ be sequence of independent and identically distributed random variables with $E(e_t) = 0$ and $E(e_t^4) < \infty$. Then the second order strictly stationary and ergodic process $\{X_t\}$ satisfying

$$X_t = \sum_{j=1}^{r} a_j X_{t-j} + (b + \sum_{j=1}^{r} X_{t-q-j}) e_{t-q} \quad (2.17)$$

for every t is invertible if

$$E \log\left| b + \sum_{j=0}^{m} b_j X_{t-j}\right| < 0$$

Iwueze (1988) established that the presence of autoregressive part makes no impact on the invertibility of his special case (2.17).

A sufficient condition for invertibility of diagonal bilinear models (2.9) have been derived by Guegan and Pham (1987). It follows that our pure diagonal bilinear model (2.2) is invertible.

2.3. Covariance Structure

The second order properties of various forms of the bilinear model have been shown in the literature to be similar to those of some linear time series models. In particular, the second order covariance structure of the bilinear model BL(P,0,P,1) studied by Subba Rao (1981) is similar to that of an ARMA(P,1) model. Pham (1985) also arrived at the same conclusion after obtaining a Markovian representation of bilinear models. Akamanam (1983) showed that for a special case of the bilinear process, there exists an ARMA process with identical covariance structures.

In this section, we show that for every pure diagonal bilinear process (2.2), there exists an ARMA process with identical covariance structure. We give the covariance function in section 2.5.

3. Autocovariances of PDBL(P) Model

It can be shown that for the model (2.2), the following are true

$$\mathrm{E}(X_t) = \sigma^2 \textstyle\sum_{j=1}^{P} b_j = \mu \tag{2.18}$$

$$\mathrm{E}(X_t e_t) = \sigma^2 \tag{2.19}$$

$$\mathrm{E}\,(X_t^2 e_t) = 2 \textstyle\sum_{j=1}^{P} b_j \sigma^4 = 2\,\mu\,\sigma^2 \tag{2.20}$$

$$\mathrm{E}\,(X_t^2 e_t^2) = \sigma^2 \textstyle\sum_{j=1}^{P} b_j^2 E\left(X_{t-j}^2 e_{t-j}^2\right) + 2\sigma^2 \sum\sum_{i<j} b_i b_j + 3\,\sigma^4 = \frac{3\,\sigma^4 + 2\,\sigma^6 \sum\sum_{i<j} b_i b_j}{1-\sigma^2 \sum_{j=1}^{P} b_j^2},\quad \sigma^2 \textstyle\sum_{j=1}^{P} b_j^2 < 1 \tag{2.21}$$

$$\mathrm{E}(X_t^2) = \textstyle\sum_{j=1}^{P} b_j^2 E\left(X_{t-j}^2 e_{t-j}^2\right) + 2\sigma^4 \sum\sum_{i<j} b_i b_j + \sigma^2 = \frac{\sigma^2 + 2\sigma^4 \sum\sum_{i<j} b_i b_j + 2\,\sigma^4 \sum_{j=1}^{P} b_j^2}{1-\sigma^2 \sum_{j=1}^{P} b_j^2},\ \sigma^2 \textstyle\sum_{j=1}^{P} b_j^2 < 1 \tag{2.22}$$

We have that the autocovariance function of a stationary process $\{X_t\}$ is given by

$$\mathrm{R(K)} = \mathrm{E}(X_t - \mu)(X_{t+k} - \mu) = \mathrm{E}\, X_t X_{t+k} - \mu^2 \tag{2.23}$$

Substituting (2.18) and (2.22) into (2.23), we obtain

$$\mathrm{R}\,(0) = \frac{\sigma^2 + 2\sigma^4 \sum\sum_{i<j}^{P} b_i b_j + 2\,\sigma^4 \sum_{j=1}^{P} b_j^2}{1-\sigma^2 \sum_{j=1}^{P} b_j^2} - \left(\textstyle\sum b_j\right)^2 \sigma^4 \tag{2.24}$$

Simplifying, we can show that

$$\mathrm{R}\,(0) = \frac{\sigma^2 + (1+2\sigma^4 \sum_{j=1}^{P} b_j^2 \ + 2\,\sigma^2 \sum\sum_{i<j}^{P} b_i b_j) + \sum_{j=1}^{P} b_j^2{}_{\sigma^2}}{1-\sigma^2 \sum_{j=1}^{P} b_j^2} \tag{2.25}$$

Where $\sigma^2 \sum_{j=1}^{P} b_j^2 < 1$

Now

$$X_{t+k} = \textstyle\sum_{j=1}^{P} b_j X_{t+k-j}\, e_{t+k-j} + e_{t+k} \tag{2.26}$$

Therefore,

$$\mathrm{E}(X_t X_{t+k}) = \textstyle\sum_{j=1}^{P} b_j \mathrm{E}(X_{t+k-j}\, e_{t+k-j} X_t) + \mathrm{E}(X_t\, e_{t+k}) \tag{2.27}$$

Now, when $k \le p$

$$\mathrm{E}(X_t X_{t+k}) = b_k \mathrm{E}(X_t^2 e_t) + \textstyle\sum_{j=1, j\neq k} b_j \mathrm{E}(X_{t+k-j}\, e_{t+k-j} X_t) \tag{2.28}$$

But

$$(X_{t+k-j}\, e_{t+k-j} X_t) = \textstyle\sum_{j=1}^{P} b_j\, \sigma^4 \tag{2.29}$$

Therefore,

$$\mathrm{E}(X_t X_{t+k}) = 2\textstyle\sum_{j=1}^{P} b_j\, b_k \sigma^4 + \left(\sum_{j=1, j\neq k}^{p} b_j\right)\left(\sum_{j=1}^{p} b_j\right)\sigma^4 \tag{2.30}$$

Hence,

$$\mathrm{R(k)} = 2\textstyle\sum_{j=1}^{P} b_j\, b_k \sigma^4 + \left(\sum_{j=1, j\neq k}^{p} b_j\right)\left(\sum_{j=1}^{p} b_j\right)\sigma^4 - \sum_{j=1}^{P} b_j^2\, \sigma^4 - \sum\sum_{i<j}^{P} b_i b_j\, \sigma^4,\ k \le p \tag{2.31}$$

Now, for $k > p$, we have

$$\mathrm{E}(X_t X_{t+k}) = \left(\textstyle\sum_{j=1}^{p} b_j\right) E(\mathrm{Xt + k - j\ et + k - jXt}) = \sum_{j=1}^{P} b_j^2\, \sigma^4 \ - 2 \sum\sum_{i<j}^{P} b_i b_j\, \sigma^4 \tag{2.32}$$

Therefore,

$$R(K) = \sum_{j=1}^{P} b_j^2\,\sigma^4 - 2\sum\sum_{i<j}^{P} b_i b_j\,\sigma^4 - \left(\sum_{j=1}^{P} b_j^2\,\sigma^4 - 2\sum\sum_{i<j}^{P} b_i b_j\,\sigma^4\right) = 0,\ K > p \tag{2.33}$$

Thus,

$$R(k) = \frac{\sigma^2 + (1+\sigma^2\sum_{j=1}^{P} b_j^2 + 2\sigma^2\sum\sum_{i<j}^{P} b_i b_j)\sum_{j=1}^{P} b_j^2\sigma^4}{1-\sigma^2\sum_{j=1}^{P} b_j^2}\quad k = 0$$

$$R(K) = \begin{cases} 2\sum_{j=1}^{P} b_j b_k \sigma^4 + \left(\sum_{j=1, j\neq k}^{p} b_j\right)\left(\sum_{j=1}^{p} b_j\right)\sigma^4 - \sum_{j=1}^{P} b_j^2\sigma^4 - 2\sum\sum_{i<j}^{P} b_i b_j\,\sigma^4, & k \le p \\ 0 & K > P \end{cases} \tag{2.34}$$

This is similar to the autocovariance function of an ARMA(0,P) = MA(P) model. We see that the autocovariance function of a PDBL(P) process like that of a MA(P) process is zero beyond the order P of the process. In other words, the autocovariance function of a PDBL(P) process has a cut off at lag P.

4. Conclusions

In this paper we reviewed the properties of pure diagonal bilinear model. We looked at the conditions under which the pure diagonal bilinear model will be stationary and invertible. We derived the autocovariance function of the pure diagonal bilinear model and observed that it was similar to that of a moving model of order p. This implies that to distinguish between a bilinear model and an ARMA model we have to calculate the autocovariance of the squares of the data.

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