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Vague Congruence Relation Induced by VLI – Ideals of Lattice Implication Algebras

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Abstract In this paper, first, we investigate the further properties of VLI – ideals on lattice implication algebras. Next, we study the relation between VLI – ideal and vague congruence relation of lattice implication algebras. We show that there is a one - to - one correspondence between the set of all VLI – ideals and the set of all vague congruence relations of lattice implication algebras. We study the homomorphism theorem on lattice implication algebra induced by vague congruence.

Keywords Lattice implication algebras, VLI – ideals, Vague congruence relation

1. Introduction

In 1993, Y. XU [12] first established the lattice implication algebra by combining lattice and implication algebra. Z. M. Song and Y. Xu [11] studied the congruence relations on lattice implication algebras. Congruence relation is also one of the important tool when an algebra is studied. Lattice implication algebra is a special kind of residuated lattice. Zadeh [14] proposed the theory of fuzzy sets. In1993, W. L. Gau and D.J. Buehrer [5] introduced the concept of vague sets, which are more useful to evaluate the real life problems. The idea of vague sets is that the membership of every element can be classified into two parameters including supporting and opposing. Ranjit Biswas [4] initiated the study of vague algebra by studying vague groups. At first Ya Qin and Yi Liu [9] applied the concept of vague set theory to lattice implication algebras and introduced the notion of v- filter, and investigated some of their properties. Y. Lin et al. [8] studied the fuzzy congruence relations and properties on lattice implication algebras. Kham et al. [7] studied the vague relation and its properties on lattice implication algebras. Ch. Zhong and Y. Qin [15] studied the vague congruence relations and its properties on lattice implication algebras. Xiaoyan Qin et al. [10] studied the vague congruence and quotient lattice implication algebras. There are close correlations among ideals, congruence and quotient algebras. In [1], we introduced the concept of VLI – ideals on lattice implication algebras and studied properties of VLI – ideals.

In this paper, we investigate the further properties of VLI – ideals and the relation between VLI – ideals and vague congruence relations.

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2. Preliminaries

Definition 2.1 [12]. Let $(L, \vee, \wedge, ', 0, I)$ be a complemented lattice with the universal bounds 0, I. $\rightarrow$ is another binary operation of L. $(L, \vee, \wedge, \rightarrow, ', 0, I)$ is called a lattice implication algebra, if the following axioms hold, $\forall$ x, y, z $\in$ L,

(I_1) $x \rightarrow (y \rightarrow z) = y \rightarrow (x \rightarrow z)$;
(I_2) $x \rightarrow x = I$;
(I_3) $x \rightarrow y = y' \rightarrow x'$;
(I_4) $x \rightarrow y = y \rightarrow x = I$ implies $x = y$;
(I_5) $(x \rightarrow y) \rightarrow y = (y \rightarrow x) \rightarrow x$;
(L_1) $(x \vee y) \rightarrow z = (x \rightarrow z) \wedge (y \rightarrow z)$;
(L_2) $(x \wedge y) \rightarrow z = (x \rightarrow z) \vee (y \rightarrow z)$.

Definition 2.2 [6]. Let A be a subset of a lattice implication algebra L. A is said to be an LI - ideal of L if it satisfies the following conditions:

(1) $0 \in A$;
(2) $\forall x, y \in L$, $(x \rightarrow y)' \in A$ and $y \in A$ implies $x \in A$.

Definition 2.3 [5]. A vague set A in the universal of discourse X is characterized by two membership functions given by:

(1) A truth membership function $t_A : X \rightarrow [0,1]$ and
(2) A false membership function $f_A : X \rightarrow [0,1]$,

Where $t_A(x)$ is a lower bound of the grade of membership of x derived from the “evidence for x”, and $f_A(x)$ is a lower bound on the negation of x derived from the “evidence against x” and $t_A(x) + f_A(x) \leq 1$. The vague set $A = \{\langle x, [t_A(x), f_A(x)]\rangle / x \in X\}$. The value of x in the vague set A denoted by $V_A(x)$, defined by

$$V_A(x) = [t_A(x), 1 - f_A(x)].$$

Notation: Let I[0, 1] denote the family of all closed subintervals of [0, 1]. If $I_1 = [a_1, b_1]$, $I_2 = [a_2, b_2]$ are two elements of I[0, 1], we call $I_1 \geq I_2$ if $a_1 \geq a_2$ and $b_1 \geq b_2$. We define the term imax to mean the maximum of two interval as

$$\text{imax}\,[I_1, I_2] = [\max\{a_1, a_2\}, \max\{b_1, b_2\}].$$

Similarly, we can define the term imin of any two intervals.

Definition 2.4 [1]. Let A be a vague set of a lattice implication algebra L. A is said to be a vague LI - ideal of L if it satisfies the following conditions:

(1) $\forall\, x \in L$, $V_A(0) \geq V_A(x)$,
(2) $\forall\, x, y \in L$, $V_A(x) \geq \text{imin}\{V_A((x \to y)'), V_A(y)\}$.

Definition 2.5 [7]. Let X and Y be two universes. A vague relation of the universe X with the universe Y is a vague set of the Cartesian product X × Y.

Definition 2.6 [7]. Let X and Y be two universes. A vague subset R of discourse X × Y is characterized by two membership functions given by:

(1) A truth membership function $t_R : X \times Y \to [0,1]$

and

(2) A false membership function $f_R : X \times Y \to [0,1]$,

Where $t_R(x, y)$ is a lower bound of the grade of membership of (x, y) derived from the "evidence for (x, y)" and $f_R(x, y)$ is a lower bound on the negation of (x, y) derived from the "evidence against (x, y)" and $t_R(x, y) + f_R(x, y) \leq 1$.Thus the grade of membership of (x, y) in the vague set R is bounded by subinterval $[t_R(x, y), 1 - f_R(x, y)]$ of [0,1]. The vague relation R is written as

$$R = \{\langle x, [t_R(x, y), f_R(x, y)] \rangle / (x, y) \in X \times Y\}.$$

The value of (x, y) in the vague relation R denoted by $V_R(x, y)$, defined by

$$V_R(x, y) = [t_A(x, y), 1 - f_A(x, y)].$$

Definition 2.7 [15]. Let X be a nonempty universe. A vague relation R on X is called vague similarity relation, if R satisfies the following conditions:

(1) $\forall\, x \in X$, $V_R(x, x) = sup_{u,v \in X}\ V_R(u, v)$ (vague reflexivity);

(2) $\forall\, x,y \in X$, $V_R(x, y) = V_R(y, x)$ (vague symmetric);

(3) $\forall\, x, y, z \in X$, $\text{imin}\{V_R(x, z), V_R(z, y)\} \leq V_R(x, y)$ (vague transitivity).

Remark 2.8 [15]. For the vague transitivity,

$(\forall\, x, y, z \in X)$, $\text{imin}\{V_R(x, z), V_R(z, y)\} \leq V_R(x, y)$
$\Leftrightarrow (\forall\, x, y \in L)$, $sup_{z \in L}\, \text{imin}\{V_R(x, z), V_R(z, y)\} \leq V_R(x, y)$.

Definition 2.9 [15]. Let R be a vague relation on L. R is said to be a vague congruence relation on L, if

(1) R is a vague similarity relation on L;
(2) $V_R(x \to z, y \to z) \geq V_R(x, y)$ for any $x, y, z \in L$.

Theorem 2.10 [15]. Let R be a vague relation on L. Then for any x, y, z ϵ L, R satisfies the following conditions:

(1) $V_R(x', y') = V_R(x, y)$,
(2) $V_R(x \wedge z, y \wedge z) \geq V_R(x, y)$,
(3) $V_R(z \to x, z \to y) \geq V_R(x, y)$,
(4) $V_R(x, y) = \text{imin}\{V_R(x, x \wedge y), V_R(x \wedge y, y)\}$.

3. Some Properties of VLI – ideals

Theorem 3.1: The vague set A of L is a VLI – ideal if and only if A satisfies the following conditions:

(1) $\forall\, x \in L$, $V_A(0) \geq V_A(x)$,
(2) $\forall\, x, y, z \in L$,

$V_A((x \to z)') \geq \text{imin}\{V_A((x \to y)'), V_A((y \to z)')\}$.

Proof: suppose A is a VLI – ideal of L. Obviously A satisfies the first condition.

Let $x, y, z \in L$, then $y \to z \leq (x \to y) \to (x \to z)$

$\Rightarrow (y \to z)' \geq (x \to z)' \to (x \to y)'$
$\Rightarrow V_A((y \to z)') \leq V_A((x \to z)' \to (x \to y)')$

It follows that,

$V_A((x \to z)')$
$\geq \text{imin}\{V_A((x \to z)' \to (x \to y)'), V_A((x \to y)')\}$
$\geq \text{imin}\{V_A((y \to z)'), V_A((x \to y)')\}$.

Conversely suppose that the vague set A of L satisfies the inequalities (1) and (2).

Taking z = 0 in (2), we get

$V_A((x \to 0)') \geq \text{imin}\{V_A((x \to y)'), V_A((y \to 0)')\}$
$\Rightarrow V_A(x) \geq \text{imin}\{V_A((x \to y)'), V_A(y)\}$.

Hence A is a VLI – ideal of L.

Theorem 3.2: The vague set A of L is a VLI – ideal of L if and only if for any x, y, z ϵ L,

$(x' \to (y' \to z')) = I$ implies $V_A(z) \geq \text{imin}\{V_A(x), V_A(y)\}$.

Proof: Suppose A is a VLI – ideal of L.

Let $x, y, z \in L$.

Suppose $(x' \to (y' \to z')) = I$
$\Rightarrow (z \to y)' \to x = I$
$\Rightarrow (z \to y)' \leq x$
$\Rightarrow V_A(z \to y)' \geq V_A(x)$.

It follows that, $V_A(z) \geq \text{imin}\{V_A((z \to y)'), V_A(y)\}$
$\geq \text{imin}\{V_A(x), V_A(y)\}$.

Conversely suppose that the vague set A of L satisfies the condition

$(x' \to (y' \to z')) = I$ implies $V_A(z) \geq \text{imin}\{V_A(x), V_A(y)\}$.

Then, we have

$(z \to y)' \to x = I$ implies $V_A(z) \geq \text{imin}\{V_A(x), V_A(y)\}$.

Let $x, y \in L$, then $(x \to y)' \to (x \to y)' = I$.
It follows that $V_A(x) \geq \text{imin}\{V_A((x \to y)'), V_A(y)\}$.
So A is a VLI – ideal of L.

Corrolary 3.3: The vague set A of L is a VLI – ideal of L if and only if A satisfies the condition

$V_A(z) \geq \mathrm{imin}\{V_A(a_1), V_A(a_2), \ldots\ldots, V_A(a_n)\}$, where $a_n' \to (\ldots\ldots \to (a_1' \to x')\ldots) = I$ for $a_1, a_2, \ldots\ldots, a_n \in L$.

Definition 3.4: Let A be a VLI – ideal of L. Then for any $a \in L$, the set A[a] defined by

$A[a] = \{x \in L \ / \ V_A(x) \geq V_A(a)\}$.

Theorem 3.5: Let A be a VLI – ideal of L. Then A[a] is an ideal of L.

Proof: Let A be a VLI – ideal of L.
So $V_A(0) \geq V_A(x)$ for all $x \in L$.
Obviously $0 \in A[a]$.
Let $x, y \in L$ such that $(x \to y)' \in A[a]$, $y \in A[a]$.

Then $V_A(x) \geq \mathrm{imin}\{V_A((x \to y)'), V_A(y)$
$\geq V_A(a)$.

So $x \in A[a]$.
Therefore A[a] is an ideal of L.

Theorem 3.6: Let A be a vague set of L and A[a] be an ideal of L for any $a \in L$, then

$V_A(z) \leq \mathrm{imin}\{V_A((x \to y)'), V_A(y)\}$ implies $V_A(z) \leq V_A(x)$ for all $x, y, z \in L$.

Proof: Let A[a] be an ideal of L for any $a \in L$.
Let $x, y, z \in L$.
Suppose $V_A(z) \leq \mathrm{imin}\{V_A((x \to y)'), V_A(y)\}$.
Then $(x \to y)' \in A[z]$, $y \in A[z]$
$\Rightarrow x \in A[z]$.(since A[a] is an ideal of L)
Hence $V_A(z) \leq V_A(x)$.

Theorem 3.7: Let A be a vague set of L and A satisfies

(1) $\forall x \in L$, $V_A(0) \geq V_A(x)$,
(2) $\forall x, y, z \in L$,
$V_A(z) \leq \mathrm{imin}\{V_A((x \to y)'), V_A(y)\}$ implies $V_A(z) \leq V_A(x)$.

Then A[a] is an ideal of L for any $a \in L$.

Proof: Let A be a vague set of L and A satisfies the above conditions.
Let $a \in L$.

Then $V_A(0) \geq V_A(a)$.
$\Rightarrow 0 \in A[a]$.

Suppose $x, y \in L$, $(x \to y)' \in A[a]$, $y \in A[a]$

Then $V_A(a) \leq \mathrm{imin}\{V_A((x \to y)'), V_A(y)\}$
$\Rightarrow V_A(a) \leq V_A(x)$
$\Rightarrow x \in A[a]$.

Therefore A[a] is an ideal of L.

4. Vague Congruence Relation Induced by VLI – ideals

Theorem 4.1: Let R be a vague congruence relation on L. Then, for all $x, y, z \in L$,

$V_R((x \to y)', (y \to x)')$
$= \mathrm{imin}\{V_R(0, (x \to y)'), V_R(0, (y \to x)')\}$.

Proof: Let $x, y \in L$, then $(x \to y) \vee (y \to x) = I$
$\Rightarrow (x \to y)' \wedge (y \to x)' = 0$.

Since R is a vague congruence relation,

$V_R((x \to y)', (y \to x)')$
$= \mathrm{imin}\{V_R((x \to y)', (x \to y)' \wedge (y \to x)'),$
$V_R((x \to y)' \wedge (y \to x)', (y \to x)')\}$
$= \mathrm{imin}\{V_R((x \to y)', 0), V_R(0, (y \to x)')\}$
$= \mathrm{imin}\{V_R(0, (x \to y)'), V_R(0, (y \to x)')\}$.

Theorem 4.2: Let R be a vague congruence relation on L. Then the vague set

$A_R = \{< x, [t_R(x, 0), f_R(x, 0)] > / x \in L\}$ is a VLI – ideal of L.

Proof: (1) Let $x \in L$, then

$V_{A_R}(x) = [t_R(x,0), 1- f_R(x,0)]$
$= V_R(x, 0)$
$\leq V_R(x \wedge 0, 0 \wedge 0)$
$= V_R(0, 0) = V_{A_R}(0)$.

Therefore $V_{A_R}(0) \geq V_{A_R}(x)$, for all $x \in L$.
(2) Let $x, y \in L$, then

$V_{A_R}(x) = V_R(0, x)$
$\geq sup_{z \in L}\{\mathrm{imin}\{V_R(0, z), V_R(z, x)\}\}$
$\geq \mathrm{imin}\{V_R(0, (x \to y)'), V_R((x \to y)', x)\}$
$= \mathrm{imin}\{V_R(0, (x \to y)'), V_R(x \to y, x')\}$
$= \mathrm{imin}\{V_R(0, (x \to y)'), V_R(x \to y, x \to 0)\}$
$\geq \mathrm{imin}\{V_R((x \to y)', 0), V_R(y, 0)\}$
$= \mathrm{imin}\{V_{A_R}((x \to y)'), V_{A_R}(y)\}$

Therefore A_R is a vague LI – ideal of L. A_R is called a vague LI – ideal induced by a vague congruence relation R.

Definition 4.3: Let R be a similarity relation on L. For each $a \in L$, we define a vague subset

$R^a = \{(x, [t_{R^a}(x), f_{R^a}(x)]) / x \in L\}$ on L, where
$V_{R^a}(x) = V_R(a, x)$, for all $x \in L$.

Theorem 4.4: Let R be a vague congruence relation on L. Then, R^0 is vague LI – ideal of L.

Proof: It is obvious from the theorem 4.2.

Theorem 4.5: Let A be a vague LI – ideal of L and R_A be a vague relation on L defined by

$R_A = \{(x, y), [t_{R_A}(x, y), f_{R_A}(x, y)] / (x, y) \in L \times L\}$,

Where
$V_{R_A}(x, y) = \mathrm{imin}\{V_R((x \to y)'), V_R((y \to x)')\}$. Then R_A is a vague congruence relation on L.

Proof: Let R_A be a vague relation on L defined by

$R_A = \{(x, y), [t_{R_A}(x, y), f_{R_A}(x, y)] / (x, y) \in L \times L\}$,

Where

$V_{R_A}(x, y) = \mathrm{imin}\{V_R((x \to y)'), V_R((y \to x)')\}$.

It is clear R_A is reflexive and symmetric.
For any $x, y, z \in L$, we have
$V_{R_A}(x, z) = \mathrm{imin}\{V_R((x \to z)'), V_R((z \to x)')\}$
$\geq \mathrm{imin}\{\mathrm{imin}\{V_R((y \to z)'), V_R((x \to y)')\},$
$\mathrm{imin}\{V_R((y \to x)'), V_R((z \to y)')\}\}$

$= \text{imin}\{\text{imin}\{V_R((y \to z)'), V_R((z \to y)')\}, \text{imin}\{V_R((y \to x)'), V_R((x \to y)')\}\}$
$= \text{imin}\{V_{R_A}(y, z), V_{R_A}(x, y)\}$
$= \text{imin}\{V_{R_A}(x, y), V_{R_A}(y, z)\}$.

Therefore R_A is transitive.

For any x, y, z ϵ L, we have $(y \to (x \vee z))' \leq (y \to x)'$, $(x \to (y \vee z))' \leq (x \to y)'$.

Since A is order reversing, we have

$$V_A((y \to (x \vee z))') \geq V_A((y \to x)'),$$
$$V_A((x \to (y \vee z))') \geq V_A((x \to y)').$$

It follows that,

$V_{R_A}(x \to z, y \to z)$
$= \text{imin}\{V_A((x \to z) \to (y \to z)), V_A((y \to z) \to (x \to z))\}$
$= \text{imin}\{V_A(((x \to z) \to (y \to z))'), V_A(((y \to z) \to (x \to z))')\}$
$= \text{imin}\{V_A((y \to (x \vee z))'), V_A((x \to (y \vee z))')\}$
$\geq \text{imin}\{V_A((y \to x)') \to V_A((x \to y)')\}$
$= V_{R_A}(x, y)$

Therefore R_A is a vague congruence relation on L. R_A is called a vague congruence relation induced by a vague LI – ideal A of L.

Theorem 4.6: Let the set of all vague LI – ideals of L denoted by $I_V(L)$ and the set of all vague congruence relations of L denoted by $C_V(L)$. Then $I_V(L)$ is isomorphic to $C_V(L)$.

Proof: We define the mappings f and g as follows:

f: $I_V(L) \to C_V(L)$ by $f(A) = R_A$ and
g : $C_V(L) \to I_V(L)$ by $g(R) = A_R$, for any A ϵ $I_V(L)$ and R ϵ $C_V(L)$.

Obviously these mappings are well defined.
For any A ϵ $I_V(L)$ and R ϵ $C_V(L)$,

$$(g \text{ o } f)(A) = g[f(A)] = g[R_A] = A_{R_A}.$$

For any x ϵ L, we have

$V_{A_{R_A}}(x) = V_{R_A}(x, 0)$
$= \text{imin}\{V_A((x \to 0)'), V_A((0 \to x)')\}$
$= \text{imin}\{V_A(x), V_A(0)\}$
$= V_A(x)$.

So (g o f) (A) = A.

That is (g o f) is an identical mapping on $I_V(L)$, which implies that f is injective.

Let R ϵ $C_V(L)$, then A_R is a vague LI – ideal of L.
Then R_{A_R} is a congruence relation on L.
For any R ϵ $C_V(L)$,

$$(f \text{ o } g)(R) = f(g(R)) = f(A_R) = R_{A_R}.$$

For any x, y ϵ L, we have

$V_{R_{A_R}}(x, y) = \text{imin}\{V_{A_R}((x \to y)'), V_{A_R}((y \to x)')\}$
$= \text{imin}\{V_R((x \to y)', 0), V_R((y \to x)', 0)\}$
$= V_R((x \to y)', (y \to x)')$
$= V_R(x, y)$

That implies (f o g) (R) = R.

That is f o g is an identical mapping on $C_V(L)$ which implies that f is surjective.

So f is a bijection from $I_V(L)$ to $C_V(L)$.

Therefore $I_V(L)$ is isomorphic to $C_V(L)$.

Theorem 4.7: Let A be a vague LI – ideal of L and R_A be a vague congruence relation induced by A. Then for any a, b ϵ L, $(R_A)^a = (R_A)^b$ if and only if

$$t_A((a \to b)') = t_A((b \to a)') = t_A(0) \text{ and}$$
$$f_A((a \to b)') = f_A((b \to a)') = f_A(0).$$

Proof: Let $(R_A)^a = (R_A)^b$, we have $V_{(RA)}{}^a(a) = V_{(RA)}{}^b(a)$ for all a, b ϵ L. Let a, b ϵ L, then

$V_{(RA)}{}^a(a) = V_{(RA)}{}^a(a, a)$
$= \text{imin}\{V_A((a \to a)'), V_A((a \to a)')\}$
$= V_A(0)$
$= [t_A(0), 1- f_A(0)]$.

And $V_{(RA)}{}^b(a) = \text{imin}\{V_A((b \to a)'), V_A((a \to b)')\}$
$= [\min[t_A((b \to a)'), t_A((a \to b)'), \min[1-f_A((b \to a)'), 1- f_A((a \to b)')]]$.

It follows that, $\min[t_A((b \to a)'), t_A((a \to b)') = t_A(0)$

and

$$\min[1 - f_A((b \to a)'), 1 - f_A((a \to b)')] = 1- f_A(0).$$

That implies, $t_A((a \to b)') = t_A((b \to a)') = t_A(0)$ and $f_A((a \to b)') = f_A((b \to a)') = f_A(0)$.

Conversely suppose that

$t_A((a \to b)') = t_A((b \to a)') = t_A(0)$ and
$f_A((a \to b)') = f_A((b \to a)') = f_A(0)$.

For any x ϵ L,

$V_{(RA)}{}^a(x) = V_{(RA)}{}^a(a, x)$
$= \text{imin}\{V_A((a \to x)'), V_A((x \to a)')\}$
$\geq \text{imin}\{\text{imin}\{V_A((a \to b)'), V_A((b \to x)')\}, \text{imin}\{V_A((x \to b)'), V_A((b \to a)')\}\}$
$\geq \text{imin}\{[\min\{t_A((a \to b)'), t_A((b \to x)')\}, \min\{1 - f_A((a \to b)'), 1 - f_A((b \to x)')\}], [\min\{t_A((x \to b)'), t_A((b \to a)')\}, \min\{1 - f_A((x \to b)'), 1 - f_A((b \to a)')\}]\}$
$= \text{imin}\{[t_A((a \to b)'), 1 - f_A((b \to x)')], [t_A((x \to b)'), 1 - f_A((x \to b)')]\}$
$= \text{imin}\{V_A((a \to b)'), V_A((x \to b)')\}$
$= V_{(RA)}{}^b(x)$.

Similarly, we can prove that $V_{(RA)}{}^b(x) \geq V_{(RA)}{}^a(x)$.

Therefore $V_{(RA)}{}^a(x) = V_{(RA)}{}^b(x)$ for all x ϵ L, hence $(R_A)^a = (R_A)^b$.

Theorem 4.8: Let A be a VLI – ideal of L. Define $a \equiv_A b \Leftrightarrow (R_A)^a = (R_A)^b$. Then, $\equiv_A$ is a congruence relation on L.

Proof: The proof can be obtained from theorem 4.7.

Theorem 4.9: Let A be a VLI – ideal of L and let L/ R_A be the corresponding quotient algebra. Then, the map f: L → L/ R_A defined by $f(a) = (R_A)^a$, for any a ϵ L, is a lattice implication homomorphism and

$$\ker f = U(t_A, t_A(0)) \cap L(f_A, f_A(0)).$$

Proof: Since A is a VLI – ideal of L, R_A is a vague congruence relation induced by A.

By proposition 24 (10), f: L $\rightarrow$ L/ R_A is a lattice implication homomorphism.

Let ker f = {x ϵ L / f(x) = $(R_A)^0$ }.

Let x ϵ ker f $\Leftrightarrow$ f(x) = $(R_A)^0$

$\Leftrightarrow$ $(R_A)^x = (R_A)^0$

$\Leftrightarrow$ $t_A((x \rightarrow 0)') = t_A((0 \rightarrow x)') = t_A(0)$, $f_A((x \rightarrow 0)') = f_A((0 \rightarrow x)') = f_A(0)$

$\Leftrightarrow$ $t_A(x) = t_A(0)$ and $f_A(x) = f_A(0)$

$\Leftrightarrow$ $x \in U(t_A, t_A(0)) \cap L(f_A, f_A(0))$.

Hence ker f = $U(t_{A,} t_A(0)) \cap L(f_A, f_A(0))$.

5. Conclusions

Congruence theory and ideal theory play very important role in research of logic algebra. Through which we can much information such as quotient structure and homomorphic image of the logic algebra. The aim of this paper is to develop the vague congruence relation induced by VLI – ideals on lattice implication algebras. We study the concept of vague congruence induced by VLI – ideals. First, some properties of VLI – ideals are investigated. Next, some properties of vague congruence relation induced by VLI – ideals are discussed. We obtain one - to - one correspondence between the set of all VLI – ideals and the set of all vague congruence relations of lattice implication algebras. Lastly we obtain the homomorphism theorem on lattice implication algebra induced by vague congruence. We hope that it will be of great use to provide theoretical foundation to design for enriching corresponding many – valued logical system.

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