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Effect of Magnetic Field in an Oscillatory Flow in Particle-Fluid Suspension through an Elastic Tube: - An Application to Blood Flow in Arteries

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Abstract In the present study, we have attempted to examine various aspects of blood flow in different segments of the circulatory system in a situation where the system has been subjected to an external transverse magnetic field. The equation of motion of the tube, accounted for the elastic response of the arteries of various sizes. The investigation shows that the model considered here is also capable of taking into account the properties affecting the blood flow and hemodynamic features, which may be important for medical doctors to predict diseases for individuals on the basis of the pattern of flow for an elastic artery in the presence of a transverse magnetic field. We derived the "frequency equation" of the flow. It was observed that the roots of the frequency equation also depend on $\sigma e B_0^2$ which is the effect of the applied transverse magnetic field on the entire flow. Therefore, the results presented show that the flow of blood and pressure can be controlled by the application of an external magnetic field across the direction of blood flow.

Keywords Magnetic Field, Frequency Equation, Oscillatory flow, Particle-fluid suspension, Periodic function, Artery, Motion of the tube, Hemoglobin, Particle Phase, Fluid Phase

1. Introduction

Recently, the study of blood flow through arteries has gained serious attention of Mathematicians, Physiologists, and Clinical Persons because blood and blood vessels are substantial health risk factors and can substantially contribute to morbidity and mortality. Blood flow in the human circulatory system depends upon the pumping action of the heart which in turns produces a pressure gradient throughout the system. The rheological properties of blood and the motion of the arterial wall play an important role in the physiology of the cardiovascular system [13].

The importance to atherogenesis of arterial flow phenomena such as flow separation, recirculation and stagnation, secondary flow motion, low and oscillatory wall shear stress, and long particle residence times have become more and more evident during the past decades. The hemodynamics of flows through atherosclerotic vessels is of great interest, because these vessels present a substantial health risk and are a major cause of mortality and morbidity in the industrialized world [13]. Since blood consists of a suspension of red blood cells containing haemoglobin which contains iron oxide, it is quite apparent that blood is electrically conducting and exhibits magnetohydrodynamic flow characteristics. Many investigators, have investigated the flow of blood through arteries in the presence of magnetic field under different conditions. The Lorentz force arising out of the flow across the magnetic lines of force acts on the constituent particles of blood and alters the hemodynamic indicators of blood flow. The potential use of such MHD principles in prevention and rational therapy of arterial hypertension was explored by Vardanyan [13], who showed that for steady flow of blood in an artery of circular cross-section, a uniform transverse magnetic field alters the flow rate of blood.

Misra. J. C and Chakravarty. S [2], investigated the flow in arteries in the presence of stenosis.

The theory of non-Newtonian viscosity of red cell deformation was reported by Murata. T. [4] Flow in tubes and arteries, using a comparison, was investigated by Liepsch D.W. and McDonald D.A [5] and [6], studied blood flow in arteries. Pontrelli G [8], was able to investigate blood flow through an axisymmetric stenosis. Influence of non Newtonian behavior of blood on flow in an elastic artery model was investigated by Dutta A and Torbel J. M. [9]. On the application of a constitutive equation for whole human blood, was reported by Rodkiewicz [10].

A constitutive equation for whole human blood was the work done by [11] and [2], they studied the hydromagnetic flow of a second-grade fluid in a channel with some application to physiological systems.

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In the present study we investigated the effect of magnetic field in an oscillatory flow in particle fluid suspension through an elastic tube. The study pertains to a situation where a magnetic field is applied in a direction transverse to the direction of blood flow in an artery. Such a field has a more pronounced effect on the flow than that in the case of an axial magnetic field. Of specific interest is to determine the roots of the "frequency equation" of the flow and how the magnetic field affects these roots.

2. The Governing Equations

Using [1] and [13], the equations governing conservation of mass and linear momentum for both the fluid and particle phase, and applying a continuum approach, can be expressed as:

For the fluid phase,

$$\rho_f(1-\varphi)\left[\frac{\partial u_f}{\partial t}+u_f\frac{\partial u_f}{\partial r}+w_f\frac{\partial u_f}{\partial z}\right]$$
$$=-(1-\varphi)\frac{\partial p}{\partial r}+(1-\varphi)\mu\left[\frac{\partial^2 u_f}{\partial r^2}+\frac{1}{r}\frac{\partial u_f}{\partial r}+\frac{\partial^2 u_f}{\partial z^2}-\frac{u_f}{r^2}\right]$$
$$+\ \varphi s\left(u_p-u_f\right)-\sigma eB_0^2\left(u_p-u_f\right) \quad (1)$$

$$\rho_f(1-\varphi)\left[\frac{\partial w_f}{\partial t}+u_f\frac{\partial w_f}{\partial r}+w_f\frac{\partial w_f}{\partial z}\right]$$
$$=-(1-\varphi)\frac{\partial p}{\partial z}+(1-\varphi)\mu\left[\frac{\partial^2 w_f}{\partial r^2}+\frac{1}{r}\frac{\partial w_f}{\partial r}+\frac{\partial^2 w_f}{\partial z^2}\right]$$
$$+\varphi s\left(w_p-w_f\right) \quad (2)$$

and with the equation of mass conservation at the fluid phase as

$$\frac{\partial}{\partial r}\left[(1-\varphi)u_f\right]+\frac{\partial}{\partial z}\left[(1-\varphi)w_f\right]+\frac{(1-\varphi)u_f}{r}=0 \quad (3)$$

For the particulate phase:-

$$\rho_f\varphi\left[\frac{\partial u_p}{\partial t}+u_p\frac{\partial u_p}{\partial r}+w_p\frac{\partial u_p}{\partial z}\right]$$
$$=-\varphi\frac{\partial p}{\partial r}+\varphi s\left(u_f-u_p\right)-\ \sigma eB_0^2\left(u_f-u_p\right) \quad (4)$$

$$\rho_f\varphi\left[\frac{\partial w_p}{\partial t}+u_p\frac{\partial w_p}{\partial r}+w_p\frac{\partial w_p}{\partial z}\right]=-\varphi\frac{\partial p}{\partial r}+\varphi s\left(w_f-w_p\right) \quad (5)$$

With the mass conservation equation at the particulate phase as

$$\frac{\partial}{\partial r}\left[\varphi u_p\right]+\frac{\partial}{\partial z}\left[\varphi w_p\right]+\frac{\varphi u_p}{r}=0 \quad (6)$$

The equations of motion of the tube are [14]

$$\frac{\partial^2\zeta}{\partial t^2}=\frac{\rho_f}{\rho}\frac{\angle}{hR}\left[\sigma\frac{\partial w_f}{\partial y}+R\frac{\partial u_f}{\partial z}\right]_{y=1}+\frac{B}{\rho}\left[\frac{\partial^2\zeta}{\partial z^2}+\sigma\frac{\partial\zeta}{\partial z}\right] \quad (7)$$

$$\frac{\partial^2\xi}{\partial t^2}=\frac{p}{h\rho}-\left[\sigma\frac{\partial\zeta}{\partial z}+\frac{\xi}{R^2}\right] \quad (8)$$

with boundary conditions of (7) and (8) as

$$u_f=\frac{\partial\xi}{\partial t},\ \ w_f=\frac{\partial\zeta}{\partial t}\ \ \text{at}\ y=1 \quad (9)$$

From equations (1) to (9), we have u_f, w_f as the radial and axial velocities of the fluid phase, ρ_f, ρ_p as the densities of the fluid and particle phases respectively, μ as the dynamic viscosity of the fluid, φ as the volume particle fraction, σe as the electrical conductivity, σ as poisson ratio, B_0 as the applied magnetic field, S is the drag, ζ is the axial displacement of the tube, ξ is the radial displacement of the tube, R is the radius of the tube, ρ is the density of the material, h is the thickness of the tube, and $\angle$ as the kinematic viscosity.

In order to study the oscillatory flow along the axis of the tube we assume [14] that the pressure wave is harmonic in time, t, having frequency n and wave velocity c. Therefore we also assume that

$$(p,u_f,\ w_f,u_p,\ w_p)=(p_1(r),u_1(r),$$
$$w_1(r),u_1(r),\ w_1(r))\ X\exp[in\left(t-\tfrac{z}{c}\right)] \quad (10)$$

Using eqn (10) and neglecting the inertia terms in eqn (1), (2), (4) and (5) we write eqn (1) as

$$\rho_f(1-\varphi)\frac{\partial u_f}{\partial t}=-(1-\varphi)\frac{\partial p}{\partial r}$$
$$+(1-\varphi)\mu\left[\frac{\partial^2 u_f}{\partial r^2}+\frac{1}{r}\frac{\partial u_f}{\partial r}+\frac{\partial^2 u_f}{\partial z^2}-\frac{u_f}{r^2}\right]$$
$$+\varphi s(u_p-u_f)-\sigma eB_0^2(u_p-u_f) \quad (11)$$

which simplifies to

$$u_1in=-\frac{\partial p_1}{\rho_f\partial r}+\frac{\mu}{\rho_f}\frac{\partial^2 u_1}{\partial r^2}+\frac{\mu}{\rho_f}\frac{\partial u_1}{\partial r}$$
$$-\frac{\mu u_1}{\rho_f}+\frac{1}{\rho_f}(u_2-u_1)\frac{(\varphi s-\sigma eB_0^2)}{(1-\varphi)} \quad (12)$$

Now, let

$$y=\frac{r}{R}, \quad (13)$$

and substitute (13) into (12) we get

$$u_1in=-\frac{\partial p_1}{\rho_f R\partial y}+\frac{\mu}{\rho_f}\frac{\partial^2 u_1}{R\partial y^2}+\frac{\mu}{\rho_f}\frac{\partial u_1}{R\partial y}$$
$$-\frac{\mu u_1}{\rho_f}+(u_2-u_1)\frac{(\varphi s-\sigma eB_0^2)}{\rho_f(1-\varphi)} \quad (14)$$

From [1],

$$\alpha=R\left(\frac{n}{\angle}\right)^{1/2} \quad (15)$$

Substitute (15) into (14), we get

$$\frac{\partial^2 u_1}{\partial y^2}+R^2\frac{\partial u_1}{\partial y}-i\alpha^2u_1-R^2u_1$$
$$=\frac{R^2}{\mu(1-\varphi)}(u_2-u_1)(\varphi s-\sigma eB_0^2)-\frac{R}{\mu}\frac{\partial p_1}{\partial y} \quad (16)$$

Similarly using eqn (10) and neglecting inertia terms in eqn (2), eqn (2) simplifies to

$$\frac{\partial^2 w_1}{\partial y^2}+\frac{1}{y}\frac{\partial w_1}{\partial y}-i\alpha^2w_1=\frac{-R^2p_1in}{\mu c}-\frac{R^2\varphi s(w_2-w_1)}{\mu(1-\varphi)} \quad (17)$$

For equations (16) and (17) the equation of mass conservation (3) now transforms to

$$\frac{1}{y}\frac{\partial}{\partial y}(u_1\, y) = \frac{inRw_1}{c} \tag{18}$$

∴ Equations of motion of the fluid (1), (2) and the equation of continuity (3) can now be written as

$$\frac{\partial^2 u_1}{\partial y^2} + \frac{1}{y}\frac{\partial u_1}{\partial y} - i\alpha^2 u_1 - R^2 u_1 = \frac{R^2}{\mu(1-\varphi)}(u_2 - u_1)(\varphi s - \sigma e B_0^2) - \frac{R}{\mu}\frac{\partial p_1}{\partial y} \tag{19}$$

$$\frac{\partial^2 w_1}{\partial y^2} + \frac{1}{y}\frac{\partial w_1}{\partial y} - i\alpha^2 w_1 = \frac{-R^2 p_1 in}{\mu c} - \frac{R^2 \varphi s (w_2 - w_1)}{\mu(1-\varphi)} \tag{20}$$

With the equation of mass conservation as

$$\frac{1}{y}\frac{\partial}{\partial y}(u_1\, y) = \frac{inRw_1}{c} \tag{21}$$

Again equation (4), using (10) gives

$$u_2 = \frac{(\varphi s - \sigma e B_0^2)_{u_1}}{(\rho_p in + \varphi s - \sigma e B_0^2)} - \frac{\varphi}{R(\rho_p in + \varphi s - \sigma e B_0^2)}\frac{\partial p_1}{\partial y} \tag{22}$$

Using (10) and (5), we get

$$w_2 = \frac{\varphi p_1}{c(\rho_p in + \varphi s)} - \frac{\varphi s w_1}{(\rho_p in + \varphi s)} \tag{23}$$

With the mass conservation equation for the particulate phase as

$$\frac{1}{y}\frac{\partial}{\partial y}(u_2, y) = \frac{inRw_2}{c} \tag{24}$$

From equations (22) and (23) we can simplify equation (19) and (20). Using (22) and (23), equation (19) becomes

$$\frac{\partial^2 u_1}{\partial y^2} + \frac{1}{y}\frac{\partial u_1}{\partial y} + \left[i^3\alpha^2 - R^2 - \frac{R^2}{\mu(1-\varphi)}(\varphi s - \sigma e B_0^2)\right] \text{x} \left[\frac{(\varphi s - \sigma e B_0^2)}{(\rho_p in + \varphi s - \sigma e B_0^2)} - 1\right] u_1 = \frac{-R}{\mu}\left[1 + \frac{\varphi(\varphi s - \sigma e B_0^2)}{(1-\varphi)(\rho_p in + \varphi s - \sigma e B_0^2)}\right]\frac{\partial p_1}{\partial y} \tag{25}$$

Let

$$i^3\alpha^2 - \frac{R^2}{\mu(1-\varphi)}[(\varphi s - \sigma e B_0^2)]\left[\frac{\varphi s - \sigma e B_0^2}{\rho_p in + \varphi s - \sigma e B_0^2} - 1\right] = i^3\alpha^3\beta^2 \tag{25a}$$

$$\therefore\ \beta^2 = \frac{-R^2}{i^3\alpha^2}\left[\frac{(\varphi s - \sigma e B_0^2)}{\mu(1-\varphi)}\left(\frac{\varphi s - \sigma e B_0^2}{\rho_p in + \varphi s - \sigma e B_0^2}\right) - 1\right] + 1 \tag{26}$$

∴ equation (25) can be written as

$$\frac{\partial^2 u_1}{\partial y^2} + \frac{1}{y}\frac{\partial u_1}{\partial y} - R^2 u_1 + i^3\alpha^2\beta^2 u_1 = \frac{-R}{\mu}\left[1 + \frac{(\varphi s - \sigma e B_0^2)(\varphi s - \sigma e B_0^2)}{(1-\varphi)(\rho_p in + \varphi s - \sigma e B_0^2)}\right]\frac{\partial p_1}{\partial y} \tag{27}$$

Equation (27) becomes

$$\frac{\partial^2 u_1}{\partial y^2} + \frac{1}{y}\frac{\partial u_1}{\partial y} - R^2 u_1 + i^3\gamma^2 u_1 = \frac{-R}{\mu} M \frac{\partial p_1}{\partial y} \tag{28}$$

where

$$\gamma^2 = \alpha^2\beta^2,\ M = 1 + \frac{(\varphi s - \sigma e B_0^2)(\varphi s - \sigma e B_0^2)}{(1-\varphi)(\rho_p in + \varphi s - \sigma e B_0^2)} \tag{29}$$

Recall that $y = \frac{r}{R}$

∴ $R^2 = \frac{r^2}{y^2}$ and $r = 1$,

$R^2 = \frac{1}{y^2}$, replace R^2 with $\frac{1}{y^2}$ in LHS of equation (28), we get

$$\frac{\partial^2 u_1}{\partial y^2} + \frac{1}{y}\frac{\partial u_1}{\partial y} - \frac{u_1}{y^2} + i^3\gamma^2 u_1 = \frac{-R^2}{\mu R} M \frac{\partial p_1}{\partial y} \tag{30}$$

To solve equation (30), let $p_1 = A_{10}(ky)$, where k is to be determined.

$$\frac{\partial p_1}{\partial y} = A_1 k_1 (ky) \tag{31}$$

Therefore, equation (30) can be written as

$$\frac{\partial^2 u_1}{\partial y^2} + \frac{1}{y}\frac{\partial u_1}{\partial y} - \frac{u_1}{y^2} + i^3\gamma^2 u_1 = \frac{-R^2}{\mu}\frac{M}{R} A_1 k_1 (ky) \tag{32}$$

Let the particular solution, c_r, of (32), then

$$u_1 = u_c + u_r \tag{33}$$

where u_c is the complementary solution,

$$\text{i.e } u_c = A_{10}\left(i^{3/2}\gamma y\right) = A_{10}\left(i^{3/2}\gamma\right)\frac{{}_0\left(i^{3/2}\gamma y\right)}{{}_0\left(i^{3/2}\gamma\right)} = \frac{c_{10}\left(i^{3/2}\gamma y\right)}{{}_0\left(i^{3/2}\gamma\right)}$$

$$\therefore\ u_1 = \frac{c_{10}\left(i^{3/2}\gamma y\right)}{{}_0\left(i^{3/2}\gamma\right)} + u_r \tag{34}$$

As in [1], where $u_r = \frac{B_1}{c} A_1 k_1 (ky)$, and c will be determined, we have

$c = i^3\gamma^2 - k^2 \Rightarrow k^2 = i^3\gamma^2 - c$,

$$\text{and } u_1 = \frac{c_{10}\left(i^{3/2}\gamma y\right)}{{}_0\left(i^{3/2}\gamma\right)} + \frac{1}{R}\frac{R^2}{\mu}\frac{M A_1 k_1 (ky)}{i^3\gamma^2 - k^2} \tag{35}$$

Substitute $w_2 = \frac{\varphi p_{in}}{c(\rho_p in + \varphi s)} - \frac{\varphi s w_1}{(\rho_p in + \varphi s)}$ into eqn (20) and simplify, we get

$$\frac{\partial^2 w_1}{\partial y^2} + \frac{1}{y}\frac{\partial w_1}{\partial y} - i^3\gamma^2 w_1 = \frac{-in R^2 N p_1}{\mu c},\ N = M \tag{36}$$

Equation (36) can be written as

$$\frac{1}{y}\left[\frac{\partial}{\partial y}\left(y\frac{\partial w_1}{\partial y}\right)\right] + i^3\gamma^2 w_1 = B_1 A_1 k_0 (ky)$$
$$p_1 = A_{10}(ky),\ B_1 = \frac{-iR^2 nN}{\mu c}. \tag{37}$$

The general solution of (37) is

$w_1 = w_c + w_r$ where w_c and w_r are the complementary and particular solutions respectively.

The solution of (32) is

$$w_1 = \frac{c_{20}\left(i^{3/2}\gamma y\right)}{{}_0\left(i^{3/2}\gamma\right)} + \frac{B_1 A_{10}(ky)}{(i^3\gamma^2 - k^2)} \tag{38}$$

where $c_2 = A_{10}(ky)$. Therefore we have

$$u_1 = \frac{c_{10}\left(i^{3/2}\gamma y\right)}{{}_0\left(i^{3/2}\gamma\right)} + \frac{1}{R}\frac{R^2}{\mu}\frac{kA_{11}(ky)}{i^3\gamma^2 - k^2} \tag{39}$$

and

$$w_1 = \frac{c_{20}\left(i^{3/2}\gamma y\right)}{{}_0\left(i^{3/2}\gamma\right)} + \frac{inR^2}{\mu c}\frac{A_{10}(ky)}{i^3\gamma^2 - k^2} \tag{40}$$

Substitute u_1 and w_1 into (18) and u_2 and w_2 into (23) then

$$\gamma i^{3/2} c_1 = \frac{inR\,c_2}{c} \tag{41}$$

and

$$\frac{i^2 n^2 R^2 N A_1}{c^2 \mu} = \frac{Rk^2 A_1}{\mu} \tag{42}$$

$\Rightarrow k = \frac{inR\sqrt{N}}{c}$, so that using

$${}_0(ky) = I_0\left(\frac{inR\sqrt{N}}{c}\right) = 1$$

and

$${}_1(ky) = i_1\left(\frac{nRy\sqrt{N}}{c}\right) = \frac{inRy\sqrt{N}}{2c}$$

In the expression for the velocity components we get

$$u_1 = \frac{inR}{2c}\left[\frac{c_2 2_1\left(i^{3/2}\gamma y\right)}{c\gamma i^{3/2}{}_0\left(i^{3/2}\gamma\right)} + \frac{NA_1 y}{\rho_f c\beta^2}\right] \tag{42a}$$

$$w_2 = \frac{c_{20}\left(i^{3/2}\gamma y\right)}{{}_0\left(i^{3/2}\gamma\right)} + \frac{iR^2 NA_1}{\mu c(i^3\gamma^2 - k^2)} \tag{43}$$

$i^3\gamma^2 - k^2 = i^3\gamma^2$, we have

$$w_2 = \frac{c_{20}\left(i^{3/2}\gamma\right)}{{}_0\left(i^{3/2}\gamma\right)} + \frac{NA_1}{\rho_f c\beta^2} \tag{44}$$

At the inner surface of the tube, equation (42a) can be written as

$$u_1 = \frac{inR}{2c}c_2 F_{10}(\gamma) + \frac{1}{2c}inR\frac{NA_1}{\rho_f c\beta^2} \tag{45}$$

And eqn (44) as

$$w_1 = c_2 + \frac{NA_1}{\rho_f c\beta^2} \tag{46}$$

$$\frac{\partial w_1}{\partial y}\Big/ y = 1 = \frac{-c_2}{2}i^3\gamma F_{10}(\gamma) + \frac{1}{2}\frac{n^2 RN^2 A_1}{\rho_f c^3\beta^2} \tag{47}$$

From eqn (7) to (9), let

$$\xi = D_1 \exp\left[in\left(t - \frac{z}{c}\right)\right] \tag{48}$$

$$\zeta = E_1 \exp\left[in\left(t - \frac{z}{c}\right)\right] \tag{49}$$

where in (48) and (49) D_1 and E_1 are arbitrary constants respectively, the boundary conditions u_1 and w_1 becomes

$$\frac{\partial\xi}{\partial t} = D_1 in \exp\left[in\left(t - \frac{z}{c}\right)\right] = u_f = u_1 \exp\left[in\left(t - \frac{z}{c}\right)\right]$$

$$\therefore\ u_1 = D_1 in \tag{50}$$

$$\Rightarrow\ D_1 in = \frac{1}{2}\frac{inR}{c}F_{10}(\gamma)c_2 + \frac{NA_1}{\rho_f c\beta^2} \tag{51}$$

and

$$\frac{\partial\zeta}{\partial t} = E_1 \exp\left[in\left(t - \frac{z}{c}\right)\right] = w_f$$

$$= w_1 \exp\left[in\left(t - \frac{z}{c}\right)\right]$$

$$\therefore\ inE_1 = c_2 + \frac{NA_1}{\rho_f c\beta^2} \tag{52}$$

Using eqns (51) and (52), the equations of motion of the tube can be written as

$$-D_1 n^2 = \frac{A_1}{\rho h} - \frac{B}{\rho}\left[\frac{\sigma}{R}\left(\frac{-inE_1}{c}\right) + \frac{D_1}{R^2}\right] \tag{53}$$

and

$$-n^2 E_1 = \frac{\rho_f v}{\rho hR}\left[-\frac{1}{2}i^3\gamma^2 c_2 F_{10}(\gamma)\right] + \left[\frac{n^2 RN}{2c^3\rho_f\beta^2} + \frac{n^2 R^3}{\rho_f 2c^3\beta^2}\right]A_1 + \frac{B}{\rho}\left[\frac{-n^2 E_1}{c^2} + \frac{\sigma}{R}\left(\frac{-in}{c}\right)D_1\right] \tag{54}$$

Eqns (51), (52), (53) and (54) are four homogeneous equations in the arbitrary constants A_1, c_2, D_1 and E_1

From these four equations we have

$$\begin{vmatrix} \frac{N}{\rho_f c\beta^2} & 1 & 0 & -in \\ \frac{inRN}{2c^2\rho_f\beta^2} & inRF_{10} & -in & 0 \\ \frac{1}{h\rho} & 0 & n^2 - \frac{B}{\rho R^2} & \frac{in\sigma B}{\rho Rc} \\ \frac{\rho_{fv} n^2 R^2 N + n^2 \rho_{fv} N^2 R}{2c^2\rho_f\beta^2 hR} & -i\rho_f Rn\beta^2 F_{10} & \frac{-in\sigma\beta}{\rho Rc} & n^2\left(1 - \frac{B}{\rho c}\right) \end{vmatrix} = \mathbf{0} \tag{55}$$

Operating on the columns and row of equation (55) several times we obtain

$$\begin{vmatrix} N & 1 & 0 & 1 \\ N & F_{10}(\gamma) & -2 & 0 \\ \beta^2 & 0 & -x & -\sigma x \\ 0 & \frac{-F_{10}\beta^2}{2} & -\sigma x & (k-x) \end{vmatrix} = 0 \quad (56)$$

where

$$k = \frac{h\rho}{\rho_f R}, \quad x = \frac{h\rho}{\rho_f R}.\frac{B}{\rho_f c^2} = \frac{kB}{\rho_f c^2} \quad (57)$$

The solution of (56) is

$$(1-\sigma^2)(1-F_{10})x^2 - \left[2 + k(1-F_{10}) + F_{10}\left(\frac{1}{2} - 2\sigma\right)\right]x + F_{10}(\gamma) = 0 \quad (58)$$

The roots of (58) are given by

$$x = P \pm [P^2 - (1-\sigma^2)B]^{1/2}$$

where

$$P = \frac{\left(\frac{\beta^2}{N} + \frac{1}{4} - \sigma\right)}{(1-F_{10})} + \frac{\frac{k}{2} + \sigma\left[1 - \frac{F_{10}\beta^2}{2}\left(H - \frac{1}{N}\right)\right]}{(1-F_{10})} - \frac{-\frac{1}{4}(1-F_{10}\beta^2)}{(1-F_{10})} \quad (59)$$

and

$$\beta^2 = \frac{-R^2}{i^3\alpha^2}\left[\frac{(\varphi s - \sigma e B_0^2)}{\mu(1-\varphi)}\left(\frac{\varphi s - \sigma e B_0^2}{\rho_p in + \varphi s - \sigma e B_0^2}\right) - 1\right] + 1 \quad (60)$$

which shows that the roots of the frequency equation, depend on $\sigma e B_0^2$, the magnetic field of the applied transverse magnetic field across the direction of the blood flow.

3. Conclusions

From the solution, the frequency equation, where the roots are determined by quantities that are also determined by $\sigma e B_0^2$, which is the effect of the introduced transverse magnetic field. This result clearly demonstrates that the magnetic field has an effect on the flow under consideration. The investigation therefore shows that the instantaneous flow characteristics, u_1, u_1 and w_1, w_2 are significantly affected by the magnetic parameter, $\sigma e B_0^2$. The result presented, show that the flow of blood and pressure can be controlled by the application of an external magnetic field. The investigation therefore shows light towards the therapeutic use of the application of external transverse magnetic field in the clinical treatment of homodynamic disease, for example, like hypertension.

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