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A Fuzzy Inventory Model with Shortages Using Different Fuzzy Numbers

Nabendu Sen*, Sanjukta Malakar

Department of Mathematics, Assam University, Silchar

Abstract It is found from the literature that most of the authors have considered inventory problems without shortage in fuzzy environment and they also considered different costs as fuzzy numbers and defuzzified by using signed distance method. In our present investigation an attempt has been made to study inventory model with shortage by considering the associated costs involved as fuzzy numbers. In the present piece of work we have referred the work of Dutta and Kumar (2012). They have considered fuzzy inventory model without shortages using trapezoidal fuzzy number and for defuzzification signed distance method was used. Following their work we have extended it for purchasing inventory model with shortages using trapezoidal fuzzy number for different costs and signed distance method for defuzzification, and then for the same purchasing inventory model, the associated costs were considered as different fuzzy numbers like triangular fuzzy number, trapezoidal fuzzy number and parabolic fuzzy number and for defuzzification we have applied Graded Mean Integration Value of defuzzification. Finally numerical illustration has been given. It is observed from this study that the optimal values are improved in fuzzy environment as compared to that of in crisp environment.

Keywords Fuzzy Inventory Model, Defuzzification, Optimal Order Quantity

1. Introduction

There are many more reasons of maintaining inventories. The proper inventory control help in growth of an organization. The problem of inventory control is broadly associated with answering two questions when to order and how much to order. The problem of making optimal decision with reference to the above two questions is called inventory problem that helps in making decision for minimizing the total cost or maximizing the profit gain. Therefore the solution of inventory problem is a set of specific values of variables under discussion that minimizes the total cost of the system or maximizes the total profit of the system. The objective of many inventory problems is to deal with minimisation of inventory carrying cost (Donalson, 1977). Thus, it is essential to determine a suitable inventory policy to meet the future demand. The basic well known square root formula for EOQ model for constant demand was first given by Haris (1915). Some of the important work done by many researchers like Buchanan (1980), Goyal (1986), Goyel et- al (1986), Hariga (1996), Teng and Thompson (1996). Sarkar and Sana (2010) have developed an inventory model with increasing demand under inflation. In all the above mentioned works, the parameters were taken in crisp environment. However we have found some of the works in inventory management have been done in imprecise environment by considering different parameters as fuzzy numbers and even the demand was also considered as fuzzy demand. Dutta and Kumar (2012) developed fuzzy inventory model without shortages using fuzzy trapezoidal number and used Signed distance method for defuzzification. Yao et al (1999a, 1999b, 2003) considered the fuzzified problem for inventory with or without back order model. Kao and Hsu (2002) consider a single period inventory model with fuzzy demand. Dey and Rawat (2011) proposed an EOQ model without shortage cost by using Triangular fuzzy number and in this case defuzzification was computed by signed distance method. Urgeletti (1983) treated EOQ model in fuzzy sense and used triangular fuzzy number. For different fuzzy numbers and methods of defuzzification, Sen et al (2014) and Dutta and Kumar (2012) were referred.

2. Assumptions and Notations for the Inventory Model in Crisp and Fuzzy Environment

2.1. Assumptions

The following assumptions have been made in this present piece of work

i. Demand is deterministic and uniform at a rate D unit of quantity per unit time. Production is instantaneous (i.e., production rate is infinite).

* Corresponding author:
nsen08@yahoo.com (Nabendu Sen)
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ii. Shortage are allowed and fully back logged.
iii. Lead time is zero.
iv. The inventory planning horizon is infinite and the inventory system involves only one item and one stocking point.
v. Only a single order will be produced at the beginning of each cycle and the entire lot is delivered in one batch.
vi. C_1 be the inventory carrying cost per unit quantity per unit time, C_2 be the shortage cost per unit quantity per unit time & C_3 be the ordering cost per order, known and constant.
vii. Q is the lot-size per cycle whereas S_1 is the initial inventory level after fulfilling the back-logged quantity of previous cycle and Q – S_1 be the maximum shortage level.
viii. T is the cycle length or scheduling period where as t_1 be the no shortage period.

2.2. Notations

The following notations are used for developing model in respective environment

C_1: carrying cost per unit quantity per unit time
C_2: shortage cost per unit quantity per unit time
C_3: set up cost per order
T: cycle length or scheduling period
D: total demand
TC: total cost
S_1: initial inventory level after fulfilling the back logged quantity of previous cycle
Q: lot size per cycle
$\widetilde{C_1}$: Fuzzy carrying cost per unit quantity per unit time
$\widetilde{C_2}$: Fuzzy shortage cost per unit quantity per unit time
$\widetilde{C_3}$: Fuzzy set up cost per order
$\widetilde{TC}$: Fuzzy total cost
Q^*: Optimal order quantity
F (S_1): defuzzified total cost
F (S_1^*): Minimum defuzzified total costetc

3. Mathematical Formulation of Inventory Problems in Different Environments

3.1. Purchasing Inventory Model in Crisp Environment

Regarding the cycle length or scheduling period of the inventory system, two cases may arise:

Case I: Cycle length or scheduling period T is constant.

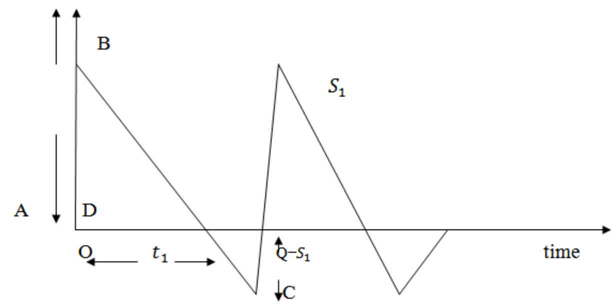
Case II: Cycle length or scheduling period T is variable.

Case I: In this case, T is constant i.e., inventory is to be replenished after every time period T. As t_1 be the no shortage period,

$$S_1 = Dt_1 \text{ or, } t_1 = {}^{S_1}/_{D}$$

Now, inventory carrying cost during period 0 to t_1 is

$$C_1 \text{ (Area of } \Delta \text{ OAB)} = \frac{1}{2}C_1S_1t_1 = \frac{1}{2}C_1\frac{S_1^2}{D}$$



Again shortage cost during the interval (t_1, T) is

$$C_2\text{(Area of } \Delta \text{ ACD)} = \frac{1}{2}C_2(Q - S_1)(T - t_1)$$
$$= \frac{1}{2}C_2\frac{(Q - S_1)^2}{D}\left[\because T - t_1 = \frac{Q - S_1}{D}\right]$$

Hence, the total average cost of the system is given by

$$\text{C} = [\tfrac{1}{2}C_1\frac{S_1^2}{D} + \tfrac{1}{2}C_2\frac{(Q-S_1)^2}{D}]/T \tag{3.1}$$

Since the set up cost C_3 & time period T are constant, the average set up cost C_3/T also being constant will not be considered in the cost expression.

∵ T is constant, Q = DT is also constant. Hence the above expression i.e., the expression for average cost is a function of single variable S_1. So, we can minimise the above expression with respect to S_1.

$$S_1^* = \frac{C_2Q}{C_1+C_2} = \frac{C_2DT}{C_1+C_2} \tag{3.2}$$

$$\text{And } C_{min} = \frac{C_1C_2Q}{C_1+C_2} = \frac{C_1C_2DT}{C_1+C_2} \tag{3.3}$$

Case II: In this case, cycle length T is a variable, the average cost of the inventory system will be

$$C = [C_3 + \tfrac{1}{2}C_1\frac{S_1^2}{D} + \tfrac{1}{2}C_2\frac{(Q-S_1)^2}{D}]/T \tag{3.4}$$

Where Q = DT

Here the average cost is a function of two independent variables T & S_1.

Now for the optimal value of C,

$$\frac{\partial C}{\partial S_1} = 0 \ \& \ \frac{\partial C}{\partial T} = 0$$

Again $\frac{\partial C}{\partial T} = 0$ gives

$$-\frac{C_1S_1^2}{2DT^2} + C_2\frac{DT-S_1}{T} - \frac{C_2}{2D}\frac{(DT-S_1)^2}{T^2} - \frac{C_3}{T^2} = 0$$

Putting $S_1 = C_2\frac{DT}{C_1+C_2}$ in above simplifying we get,

$$\text{T} = T^* = \sqrt{\frac{2C_3(C_1+C_2)}{C_1C_2D}} \tag{3.5}$$

Then,

$$S_1 = S_1^* = \sqrt{\frac{2C_3C_2D}{C_1(C_1+C_2)}} \tag{3.6}$$

Obviously for the value of T &S_1 given

$$\frac{\partial^2 C}{\partial S_1^2} > 0, \frac{\partial^2 C}{\partial T^2} > 0 \ \& \ \frac{\partial^2 C}{\partial S_1^2}\cdot\frac{\partial^2 C}{\partial T^2} - (\frac{\partial^2 C}{\partial S_1 \partial T})^2 > 0$$

Hence, C is minimum for the value of T & S_1

∴ the optimum order quantity for minimum cost is given by

$$Q^* = DT^* = \sqrt{\frac{2C_3(C_1+C_2)}{C_1C_2D}} = \sqrt{\frac{2C_3(C_1+C_2)D}{C_1C_2}} \quad (3.7)$$

$$C_{min} = C^* = \sqrt{\frac{2C_1C_2C_3D}{C_1+C_2}} \quad (3.8)$$

3.2. Purchasing Inventory Model in Fuzzy Environment

We consider the model in fuzzy environment. Since the inventory carrying cost and shortage cost, ordering cost are in fuzzy nature, we represent them by trapezoidal fuzzy numbers.

Let,

$\widetilde{C_1}$: Fuzzy carrying cost per unit quantity per unit time

$\widetilde{C_2}$: Fuzzy shortage cost per unit quantity per unit time

$\widetilde{C_3}$: Fuzzy ordering cost per order

CASE 1:

Total demand and scheduling period T is constant.

The fuzzy total average cost is given by

$$\widetilde{TC} = \frac{1}{T}\left[\frac{\widetilde{C_1}S_1^2}{2D} + \frac{1}{2}\widetilde{C_2}\frac{(Q-S_1)^2}{D}\right] \quad (3.9)$$

Our aim is to apply signed distance method to defuzzify the total average cost and then obtain S_1^* by using simple calculus method.

Suppose,

$\widetilde{C_1}$ = (a_1, b_1, c_1, d_1) and $\widetilde{C_2}$ = (a_2, b_2, c_2, d_2) are fuzzy trapezoidal numbers in LR form and a_1 , b_1, c_1 , d_1 , a_2, b_2, c_2 , d_2 are known positive numbers.

From (7.1), we get,

$$\widetilde{TC} = \frac{1}{T}\left[(a_1, b_1, c_1, d_1) \otimes \frac{s_1^2}{2D} + \frac{1}{2}(a_2, b_2, c_2, d_2) \otimes \frac{(Q-S_1)^2}{D}\right]$$

$$= \frac{1}{T}\left[\left(\frac{a_1S_1^2}{2D}, \frac{b_1S_1^2}{2D}, \frac{c_1S_1^2}{2D}, \frac{d_1S_1^2}{2D}\right) + \left\{\frac{a_2(Q-S_1)^2}{2D}, \frac{b_2(Q-S_1)^2}{2D}, \frac{c_2(Q-S_1)^2}{2D}, \frac{d_2(Q-S_1)^2}{2D}\right\}\right]$$

$$= \frac{1}{T}\left(\frac{a_1S_1^2}{2D} + \frac{a_2(Q-S_1)^2}{2D}, \frac{b_1S_1^2}{2D} + \frac{b_2(Q-S_1)^2}{2D}, \frac{c_1S_1^2}{2D} + \frac{c_2(Q-S_1)^2}{2D}, \frac{d_1S_1^2}{2D} + \frac{d_2(Q-S_1)^2}{2D}\right.$$

$$= (a, b, c, d)$$

Now,

$$A_L(\alpha) = a + (b-a)\alpha$$

$$= \frac{a_1S_1^2}{2DT} + \frac{a_2(Q-S_1)^2}{2DT} + \left[\left\{\frac{b_1S_1^2}{2DT} + \frac{b_2(Q-S_1)^2}{2DT}\right\} - \left\{\frac{a_1S_1^2}{2DT} + \frac{a_2(Q-S_1)^2}{2DT}\right\}\right]\alpha$$

$$= \frac{a_1S_1^2}{2DT} + \frac{a_2(Q-S_1)^2}{2DT} + \left\{\frac{(b_1-a_1)s_1^2}{2DT} + \frac{(b_2-a_2)(Q-S_1)^2}{2DT}\right\}\alpha$$

$$A_R(\alpha) = d - (d-c)\alpha$$

$$= \frac{d_1S_1^2}{2DT} + \frac{d_2(Q-S_1)^2}{2DT} - \left[\frac{d_1S_1^2}{2DT} + \frac{d_2(Q-S_1)^2}{2DT} - \left\{\frac{c_1S_1^2}{2DT} + \frac{c_2(Q-S_1)^2}{2DT}\right\}\right]\alpha$$

$$= \frac{d_1S_1^2}{2DT} + \frac{d_2(Q-S_1)^2}{2DT} + \left[\frac{(c_1-d_1)s_1^2}{2DT} + \frac{(c_2-d_2)(Q-S_1)^2}{2DT}\right]\alpha$$

Defuzzifying $\widetilde{TC}$ by using signed distance method, we have

$$d(\widetilde{TC}) = ½ \int_0^1 [A_L(\alpha) + A_R(\alpha)]\, d\alpha$$

$$= \int_0^1 \left[\left\{\frac{a_1S_1^2}{2DT} + \frac{a_2(Q-S_1)^2}{2DT} + \left\{\frac{(b_1-a_1)s_1^2}{2DT} + \frac{(b_2-a_2)(Q-S_1)^2}{2DT}\right\}\alpha\right\} + \frac{d_1S_1^2}{2DT} + \frac{d_2(Q-S_1)^2}{2DT} + \left[\frac{(c_1-d_1)s_1^2}{2DT} + \frac{(c_2-d_2)(Q-S_1)^2}{2DT}\right]\alpha\right] d\alpha$$

$$= \frac{a_1S_1^2 + a_2(Q-S_1)^2 + d_1S_1^2 + d_2(Q-S_1)^2}{4DT} + \left[\frac{(b_1-a_1)s_1^2 + (c_1-d_1)s_1^2 + (b_2-a_2)(Q-S_1)^2 + (c_2-d_2)(Q-S_1)^2}{8DT}\right]$$

$$= \frac{(a_1+b_1+c_1+d_1)s_1^2}{8DT} + \frac{(a_2+b_2+c_2+d_2)(Q-S_1)^2}{8DT}$$

$$= F(s_1) \quad (3.10)$$

COMPUTATION OF (S_1^*) AT WHICH F(S_1) IS MINIMUM

F (s_1) is minimum when $\frac{dF(S_1)}{dS_1} = 0$ and $\frac{d^2F(S_1)}{dS_1} > 0$

$$\frac{dF(S_1)}{dS_1} = 0$$

$$\Longrightarrow \frac{(a_1+b_1+c_1+d_1)\,2\,s_1}{8DT} + \frac{(a_2+b_2+c_2+d_2)2(Q-S_1)}{8DT}(-1) = 0$$

$$\Longrightarrow (a_1+b_1+c_1+d_1+a_2+b_2+c_2+d_2)\,s_1 = (a_2+b_2+c_2+d_2)\ Q$$

$$\Longrightarrow s_1 = \frac{(a_2+b_2+c_2+d_2)}{(a_1+b_1+c_1+d_1+a_2+b_2+c_2+d_2)}\ Q$$

Hence,

$$S1^* = \frac{(a_2+b_2+c_2+d_2)}{(a_1+b_1+c_1+d_1+a_2+b_2+c_2+d_2)}\ Q$$

Also at $s_1 = S_1^*$, we have $\frac{d^2F(S_1)}{dS_1} > 0$

This is shown that $F(s_1)$ is minimum at $S_1^* = S_1$. And from (3.10), we find:

$$F(S_1^*) = \frac{(a_1+b_1+c_1+d_1)s_1^2}{8DT} + \frac{(a_2+b_2+c_2+d_2)(Q-S_1)^2}{8DT} \qquad (3.11)$$

Case 2:

In this case, cycle length T is a variable, the average total cost of inventory system in fuzzy environment will be

$$\widetilde{TC} = \frac{1}{T}[\widetilde{C_3} + \frac{\widetilde{C_1}S_1^2}{2D} + \frac{\widetilde{C_2}(Q-S_1)^2}{2D}]$$

Suppose,

$\widetilde{C_1} = (a_1, b_1, c_1, d_1)$, $\widetilde{C_2} = (a_2, b_2, c_2, d_2)$ and $\widetilde{C_3} = (a_3, b_3, c_3, d_3)$ are fuzzy trapezoidal numbers in LR form and from (4), we get,

$$\widetilde{TC} = \frac{1}{T}[(a_3, b_3, c_3, d_3) + (a_1, b_1, c_1, d_1)\frac{S_1^2}{2D} + (a_2, b_2, c_2, d_2)\frac{(Q-S_1)^2}{2D}$$

$$= \frac{1}{T}\ [(a_3, b_3, c_3, d_3) + (\frac{a_1S_1^2}{2D}, \frac{b_1S_1^2}{2D}, \frac{c_1S_1^2}{2D}, \frac{d_1S_1^2}{2D}) + \{\frac{a_2\ (Q-S_1)^2}{2D}, \frac{b_2\ (Q-S_1)^2}{2D}, \frac{c_2\ (Q-S_1)^2}{2D}, \frac{d_2\ (Q-S_1)^2}{2D}\}\ ]$$

$$= [(\frac{a_3}{T} + \frac{a_1S_1^2}{2DT} + \frac{a_2\ (Q-S_1)^2}{2DT}, \frac{b_3}{T} + \frac{b_1S_1^2}{2DT} + \frac{b_2\ (Q-S_1)^2}{2DT}, \frac{c_3}{T} + \frac{c_1S_1^2}{2DT} + \frac{c_2\ (Q-S_1)^2}{2DT}, \frac{d_3}{T} + \frac{d_1S_1^2}{2DT} + \frac{d_2\ (Q-S_1)^2}{2DT})]$$

$$= (a, b, c, d) \qquad (*)$$

Now,

$$A_L(\alpha) = a + (b-a)\,\alpha$$

$$= \frac{a_3}{T} + \frac{a_1S_1^2}{2DT} + \frac{a_2\ (Q-S_1)^2}{2DT} + \{(\frac{b_3}{T} + \frac{b_1S_1^2}{2DT} + \frac{b_2\ (Q-S_1)^2}{2DT}) - (\frac{a_3}{T} + \frac{a_1S_1^2}{2DT} + \frac{a_2\ (Q-S_1)^2}{2DT})\}\ \alpha$$

$$= \frac{a_3}{T} + \frac{a_1S_1^2}{2DT} + \frac{a_2\ (Q-S_1)^2}{2DT} + \frac{b_3-a_3}{T}\alpha + \frac{(b_1-a_1)S_1^2}{2DT}\alpha + \frac{(b_2-a_2)(Q-S_1)^2}{2DT}\alpha$$

$$A_R(\alpha) = d - (d-c)\,\alpha$$

$$= \frac{d_3}{T} + \frac{d_1S_1^2}{2DT} + \frac{d_2\ (Q-S_1)^2}{2DT} - \{(\frac{d_3}{T} + \frac{d_1S_1^2}{2DT} + \frac{d_2\ (Q-S_1)^2}{2DT}) - (\frac{c_3}{T} + \frac{c_1S_1^2}{2DT} + \frac{c_2\ (Q-S_1)^2}{2DT})\}\alpha$$

$$= \frac{d_3}{T} + \frac{d_1S_1^2}{2DT} + \frac{d_2\ (Q-S_1)^2}{2DT} + \frac{d_3-c_3}{T}\alpha + \frac{(c_1-d_1)S_1^2}{2DT}\alpha + \frac{(c_2-d_2)(Q-S_1)^2}{2DT}\alpha$$

Defuzzifying $\widetilde{TC}$ by using signed distance method, we have,

$$d\,(\widetilde{TC}(T, S_1)) = ½\int_0^1 [\ A_L(\alpha) + A_R(\alpha)]\ d$$

$$= ½\int_0^1 [\frac{a_3}{T} + \frac{a_1S_1^2}{2DT} + \frac{a_2\ (Q-S_1)^2}{2DT} + \frac{b_3-a_3}{T}\alpha + \frac{(b_1-a_1)S_1^2}{2DT}\alpha + \frac{(b_2-a_2)(Q-S_1)^2}{2DT}\alpha$$

$$+ \frac{d_3}{T} + \frac{d_1S_1^2}{2DT} + \frac{d_2\ (Q-S_1)^2}{2DT} + \frac{d_3-c_3}{T}\alpha + \frac{(c_1-d_1)S_1^2}{2DT}\alpha + \frac{(c_2-d_2)(Q-S_1)^2}{2DT}\alpha]\ d\alpha$$

$$= \frac{a_3+d_3}{2T} + \frac{(a_1+d_1)\,S_1^2}{4DT} + \frac{(a_2+d_2)(Q-S_1)^2}{4DT} + \frac{b_3-a_3+d_3-c_3}{4T} + \frac{(b_1-a_1+c_1-d_1)S_1^2}{8DT} + \frac{(b_2-a_2+c_2-d_2)(Q-S_1)^2}{8DT}$$

$$= \frac{a_3+b_3+c_3+d_3}{4T} + \frac{(a_1+b_1+c_1+d_1)S_1^2}{8DT} + \frac{(a_2+b_2+c_2+d_2)(DT-S_1)^2}{8DT}$$

$$= F(S_1, T)$$

COMPUTATION OF (S_1^*) AND T^* AT WHICH F(S_1,T) IS MINIMUM:

For minimum value of F(S_1, T), we must have,

$$\frac{\partial F}{\partial S_1} = 0 \text{ and } \frac{\partial F}{\partial T} = 0$$

Now,

$$\frac{\partial F}{\partial T} = 0$$

$$\Rightarrow -\left(\frac{a_3+b_3+c_3+d_3}{4T^2}\right) - \frac{(a_1+b_1+c_1+d_1)S_1^2}{8DT^2} + \frac{(a_2+b_2+c_2+d_2)(DT-S_1)}{4DT}D - \frac{(a_2+b_2+c_2+d_2)(DT-S_1)^2}{8DT^2} = 0 \ (**)$$

Putting the value of $s_1 = \frac{(a_2+b_2+c_2+d_2)\,DT}{(a_1+b_1+c_1+d_1+a_2+b_2+c_2+d_2)}$ in the above simplifying, we get

$$T=T^*=\sqrt{\frac{2(a_3+b_3+c_3+d_3)(a_1+b_1+c_1+d_1+a_2+b_2+c_2+d_2)}{(a_1+b_1+c_1+d_1)(a_2+b_2+c_2+d_2)\,D}} \quad (3.12)$$

Then,

$$S_1= S_1^*=\sqrt{\frac{(a_2+b_2+c_2+d_2)DT}{a_1+b_1+c_1+d_1+a_2+b_2+c_2+d_2}} \quad (3.13)$$

Clearly for the value of S_1 and T,

$$\frac{\partial^2}{\partial S_1^2}(F) > 0 \text{ and } \frac{\partial^2}{\partial T^2}(F) > 0 \text{ and } \frac{\partial^2 F}{\partial S_1^2}\cdot\frac{\partial^2 F}{\partial T^2} - \left(\frac{\partial^2 F}{\partial S_1 \partial T}\right)^2 > 0$$

Hence, F (S_1, T) is minimum for the value of T and S_1.

Therefore, the optimal order quantity for minimum cost is given by

$$Q^*= DT^*=\sqrt{\frac{2(a_3+b_3+c_3+d_3)(a_1+b_1+c_1+d_1+a_2+b_2+c_2+d_2)D}{(a_1+b_1+c_1+d_1)(a_2+b_2+c_2+d_2)}} \quad (3.14)$$

$$\widetilde{C_{min}} = \widetilde{C^*} = \sqrt{\frac{2(a_3+b_3+c_3+d_3)(a_1+b_1+c_1+d_1)(a_2+b_2+c_2+d_2)D}{(a_1+b_1+c_1+d_1)+(a_2+b_2+c_2+d_2)}} \quad (3.15)$$

3.3. Use of Graded Mean Integration Method for Defuzzification of Different Associated Costs which are Considered Different Fuzzy Numbers

By applying another important method of defuzzification, graded mean integration value of fuzzy numbers we can make study on the same inventory purchasing model whose costs are different fuzzy numbers.

By this method of defuzzification:

For triangular fuzzy numbers (TFN), $\tilde{A} = (a_1, a_2, a_3)$

$$P_{dG0.5}(\tilde{A}) = \frac{1}{6}(a_1+4a_2+a_3)$$

For parabolic fuzzy numbers (PFN), $\tilde{A} = (a_1, a_2, a_3)$

$$P_{dG0.5}(\tilde{A}) = \frac{1}{15}(4a_1+7a_2+4a_3)$$

For trapezoidal fuzzy numbers (T_rFN), $\tilde{A} = (a_1, a_2, a_3, a_4)$

$$P_{dG0.5}(\tilde{A}) = \frac{1}{6}(a_1+2a_2+2a_3+a_4)$$

In crisp environment:

CASE I:

Total demand and scheduling period T is constant.

$$\left.\begin{aligned} S_1^* &= \frac{C_2\,Q}{C_1+C_2} = \frac{C_2\,DT}{C_1+C_2} \text{ and} \\ C_{min} &= \frac{C_1C_2\,Q}{C_1+C_2} = \frac{C_1C_2\,DT}{C_1+C_2} \end{aligned}\right\} \quad (3.16)$$

CASE II:

In this case, cycle length T is a variable.

$$\left.\begin{aligned} S_1^* &= \sqrt{\frac{2C_3\,(C_1+C_2)D}{C_1C_2}} \\ C_{min} &= \sqrt{\frac{2C_1C_2C_3}{C_1+C_2}} \end{aligned}\right\} \quad (3.17)$$

In fuzzy environment:

For triangular fuzzy number:

Suppose, $\widetilde{C_1} = (a_1, b_1, c_1)$, $\widetilde{C_2} = (a_2, b_2, c_2)$ and $\widetilde{C_3} = (a_3, b_3, c_3)$. Then,

CASE 1:

$$\left.\begin{aligned} \widetilde{S_1^*} &= \frac{\widetilde{C_2}\,DT}{\widetilde{C_1}+\widetilde{C_2}} \\ \widetilde{C_{min}} &= \frac{\widetilde{C_1\ C_2}\ DT}{\widetilde{C_1}+\widetilde{C_2}} \end{aligned}\right\} \quad (3.18)$$

CASE 2:

$$\left.\begin{aligned} Q^* &= \sqrt{\frac{2\,\widetilde{C_3}(\widetilde{C_1}+\widetilde{C_2})DT}{\widetilde{C_1}\widetilde{C_2}}} \\ \widetilde{C_{min}} &= \sqrt{\frac{2\widetilde{C_1}\widetilde{C_2}\widetilde{C_3}\,D}{\widetilde{C_1}+\widetilde{C_2}}} \end{aligned}\right\} \quad (3.19)$$

For trapezoidal fuzzy number:

Suppose, $\widetilde{C_1} = (a_1^{/}, b_1^{/}, c_1^{/}, d_1^{/})$, $\widetilde{C_2} = (a_2^{/}, b_2^{/}, c_2^{/}, d_2^{/})$ and $\widetilde{C_3} = (a_3^{/}, b_3^{/}, c_3^{/}, d_3^{/})$. Then,

CASE 1:

$$\left.\begin{aligned} \widetilde{S_1^*} &= \frac{\widetilde{C_2}\ DT}{\widetilde{C_1}+\widetilde{C_2}} \\ \widetilde{C_{min}} &= \frac{\widetilde{C_1\ C_2}\ DT}{\widetilde{C_1}+\widetilde{C_2}} \end{aligned}\right\} \quad (3.20)$$

CASE 2:

$$\left.\begin{aligned} Q^* &= \sqrt{\frac{2\ \widetilde{C_3}(\widetilde{C_1}+\widetilde{C_2})DT}{\widetilde{C_1}\widetilde{C_2}}} \\ \widetilde{C_{min}} &= \sqrt{\frac{2\widetilde{C_1}\widetilde{C_2}\widetilde{C_3}\ D}{\widetilde{C_1}+\widetilde{C_2}}} \end{aligned}\right\} \quad (3.21)$$

For parabolic fuzzy number:

Suppose,

$\widetilde{C_1} = (a_1^{//}, b_1^{//}, c_1^{//})$, $\widetilde{C_2} = (a_2^{//}, b_2^{//}, c_2^{//})$ and $\widetilde{C_3} = (a_3^{//}, b_3^{//}, c_3^{//})$.

CASE 1:

$$\left.\begin{aligned} \widetilde{S_1^*} &= \frac{\widetilde{C_2}\ DT}{\widetilde{C_1}+\widetilde{C_2}} \\ \widetilde{C_{min}} &= \frac{\widetilde{C_1\ C_2}\ DT}{\widetilde{C_1}+\widetilde{C_2}} \end{aligned}\right\} \quad (3.22)$$

CASE 2:

$$\left.\begin{aligned} Q^* &= \sqrt{\frac{2\ \widetilde{C_3}(\widetilde{C_1}+\widetilde{C_2})DT}{\widetilde{C_1}\widetilde{C_2}}} \\ \widetilde{C_{min}} &= \sqrt{\frac{2\widetilde{C_1}\widetilde{C_2}\widetilde{C_3}\ D}{\widetilde{C_1}+\widetilde{C_2}}} \end{aligned}\right\} \quad (3.23)$$

Now, to convert the expressions (3.18), (3.19); (3.20), (3.21); (3.22), (3.23) in crisp form, we use graded mean integration method. By this method, the above mentioned expressions (3.18), (3.19); (3.20), (3.21); (3.22), (3.23) reduced to the following form:

For triangular fuzzy number:

CASE 1:

$$\left.\begin{aligned} P_{dGW}\ (\widetilde{S_1}) &= \frac{P_{dGW}\ (\widetilde{C_2}\ )DT}{P_{dGW}\ (\widetilde{C_1}\ )+P_{dGW}\ (\widetilde{C_2}\ )} \\ P_{dGW}(\widetilde{C_{min}}) &= \frac{P_{dGW}\ (\widetilde{C_1}\ )\ P_{dGW}\ (\widetilde{C_2}\ )DT}{P_{dGW}\ (\widetilde{C_1}\ )+P_{dGW}\ (\widetilde{C_2}\ )} \end{aligned}\right\} \quad (3.24)$$

CASE 2:

$$\left.\begin{aligned} P_{dGW}\ (Q^*) &= \sqrt{\frac{2P_{dGW}\ (\widetilde{C_3})\{P_{dGW}\ (\widetilde{C_1})+P_{dGW}\ (\widetilde{C_2})\}DT}{P_{dGW}\ (\widetilde{C_1})\ P_{dGW}\ (\widetilde{C_2})}} \\ P_{dGW}\ (\widetilde{C_{min}}) &= \sqrt{\frac{2P_{dGW}\ (\widetilde{C_1})\ P_{dGW}\ (\widetilde{C_2})\ P_{dGW}\ (\widetilde{C_3})D}{P_{dGW}\ (\widetilde{C_1}\ )+P_{dGW}\ (\widetilde{C_2}\ )}} \end{aligned}\right\} \quad (3.25)$$

For trapezoidal fuzzy number:

CASE 1:

$$\left.\begin{aligned} P_{dGW}\ (\widetilde{S_1}) &= \frac{P_{dGW}\ (\widetilde{C_2}\ )DT}{P_{dGW}\ (\widetilde{C_1}\ )+P_{dGW}\ (\widetilde{C_2}\ )} \\ P_{dGW}(\widetilde{C_{min}}) &= \frac{P_{dGW}\ (\widetilde{C_1}\ )\ P_{dGW}\ (\widetilde{C_2}\ )DT}{P_{dGW}\ (\widetilde{C_1}\ )+P_{dGW}\ (\widetilde{C_2}\ )} \end{aligned}\right\} \quad (3.26)$$

CASE 2:

$$\left.\begin{aligned} P_{dGW}\ (Q^*) &= \sqrt{\frac{2P_{dGW}\ (\widetilde{C_3})\{P_{dGW}\ (\widetilde{C_1})+P_{dGW}\ (\widetilde{C_2})\}DT}{P_{dGW}\ (\widetilde{C_1})\ P_{dGW}\ (\widetilde{C_2})}} \\ P_{dGW}\ (\widetilde{C_{min}}) &= \sqrt{\frac{2P_{dGW}\ (\widetilde{C_1})\ P_{dGW}\ (\widetilde{C_2})\ P_{dGW}\ (\widetilde{C_3})D}{P_{dGW}\ (\widetilde{C_1}\ )+P_{dGW}\ (\widetilde{C_2}\ )}} \end{aligned}\right\} \quad (3.27)$$

For parabolic fuzzy number:

CASE 1:

$$\left.\begin{aligned} P_{dGW}\ (\widetilde{S_1}) &= \frac{P_{dGW}\ (\widetilde{C_2}\ )DT}{P_{dGW}\ (\widetilde{C_1}\ )+P_{dGW}\ (\widetilde{C_2}\ )} \\ P_{dGW}(\widetilde{C_{min}}) &= \frac{P_{dGW}\ (\widetilde{C_1}\ )\ P_{dGW}\ (\widetilde{C_2}\ )DT}{P_{dGW}\ (\widetilde{C_1}\ )+P_{dGW}\ (\widetilde{C_2}\ )} \end{aligned}\right\} \quad (3.28)$$

CASE 2:

$$\left.\begin{aligned} P_{dGW}\ (Q^*) &= \sqrt{\frac{2P_{dGW}\ (\widetilde{C_3})\{P_{dGW}\ (\widetilde{C_1})+P_{dGW}\ (\widetilde{C_2})\}DT}{P_{dGW}\ (\widetilde{C_1})\ P_{dGW}\ (\widetilde{C_2})}} \\ P_{dGW}\ (\widetilde{C_{min}}) &= \sqrt{\frac{2P_{dGW}\ (\widetilde{C_1})\ P_{dGW}\ (\widetilde{C_2})\ P_{dGW}\ (\widetilde{C_3})D}{P_{dGW}\ (\widetilde{C_1}\ )+P_{dGW}\ (\widetilde{C_2}\ )}} \end{aligned}\right\} \quad (3.29)$$

Where,

For triangular fuzzy number:

$P_{dGW}\ (\widetilde{C_1}) = \frac{1}{6}\ (a_1+ b_1+c_1)$

$P_{dGW}\ (\widetilde{C_2}) = \frac{1}{6}\ (a_2+ b_2+ c_2)$ and

$P_{dGW}\ (\widetilde{C_3}) = \frac{1}{6}\ (a_3+ b_3 +c_3)$.

For trapezoidal fuzzy number:

$P_{dGW}\ (\widetilde{C_1}) = \frac{1}{15}\ (a_1^{/} +b_1^{/}+c_1^{/}+ d_1^{/})$,

$P_{dGW}\ (\widetilde{C_2}) = \frac{1}{15}\ (a_2^{/}+ b_2^{/}+ c_2^{/}+d_2^{/})$ and

$P_{dGW}\ (\widetilde{C_3}) = \frac{1}{15}\ (a_3^{/}+ b_3^{/} +c_3^{/} +d_3^{/})$.

For parabolic fuzzy number:

$P_{dGW}\ (\widetilde{C_1}) = \frac{1}{6}\ (a_1^{//}+b_1^{//}+c_1^{//})$,

$P_{dGW}\ (\widetilde{C_2}) = \frac{1}{6}\ (a_2^{//}+ +b_2^{//}+c_2^{//})$ and

$P_{dGW}\ (\widetilde{C_3}) = \frac{1}{6}\ (a_3^{//}+b_3^{//}+c_3^{//})$.

4. Algorithm for Finding Fuzzy Total Cost and Fuzzy Optimal Order Quantity

Step I: Calculate total cost (TC) for the crisp model as given in equation (3.1) and (3.4) for the crisp values of C_1, C_2, C_3, S_1, T and D.

Step II: Now, determine fuzzy total cost ($\widetilde{TC}$) using fuzzy arithmetic on fuzzy carrying cost, fuzzy shortage cost and fuzzy ordering cost, taken as fuzzy trapezoidal numbers.

Step III: Used Signed distance method for defuzzification. Then, in case I, find S_1^* which can be obtained by putting the first derivative of F(S_1) equal to zero and where second derivative of F(S_1) is positive at $S_1 = S_1^*$.

And in case II, again we have used signed distance method for defuzzification and using the value of S_1, we obtained the value of T=T^*. Then, find the Fuzzy Optimal order quantity Q^* which can be obtained by putting the first partial order derivative of F(S_1, T) equal to zero at S_1,T and second partial order derivative of F(S_1, T) is positive at $S_1 = S_1^*$ and T=T^*.

Step IV: Using Graded Mean Integration Method for defuzzification, we find the optimal order quantity Q^* and $\widetilde{C_{min}}$ for different type of fuzzy numbers, viz. triangular fuzzy number, parabolic fuzzy number and trapezoidal fuzzy number.

5. Numerical Example

To illustrate the developed model (both in crisp and Fuzzy environment), we have taken an example from Bhunia and Sahoo (2011)

In crisp environment

Let,
D= 600 units per unit per year
C_1= Rs. 10
C_2 = Rs. 1 per month
C_3 = Rs. 100 per order.

Then, optimal order quantity Q*= 148 units and the minimum cost per year C_{min}=C*= Rs.809.04

Table 1. Numerical data for fuzzy valued parameters

Fuzzy No.	Demand	$\widetilde{C_1}$	$\widetilde{C_2}$	$\widetilde{C_3}$	P_{dGW} (Q*)	P_{dGW} (C*)
TFN	600	(6,9,13)	(9,13,15)	(92,96,101)	147.3079	783.3932
PFN	600	(9,8,13)	(10,14,13)	(97,99,103)	147.8889	807.633
T_rFN	600	(7,9,11,12)	(10,11,13,15)	(96,98,101,103)	148.1758	805.7995

6. Sensitivity Analysis

Table 2. Sensitivity Analysis when associated costs are Triangular Fuzzy Number

S.No	Demand	For $\widetilde{C_1}$=(6,9,13) $\widetilde{C_2}$=(9,13,15) $\widetilde{C_3}$=(92,96,101)		For $\widetilde{C_1}$=(6,8,13) $\widetilde{C_2}$=(9,12,15) $\widetilde{C_3}$=(92,95,101)	
		P_{dGW} (C*)	P_{dGW} (Q*)	P_{dGW} (C*)	P_{dGW} (Q*)
1	600	783.3932	147.3079	755.1191	151.7641
2	625	799.5473	150.3455	770.6902	154.1191
3	650	815.3814	153.3229	785.9529	157.9611
4	675	830.9139	156.2436	800.9248	160.9702
5	700	846.1613	159.1107	815.6219	163.924

Table 3. Sensitivity Analysis when associated costs are Trapezoidal Fuzzy Number

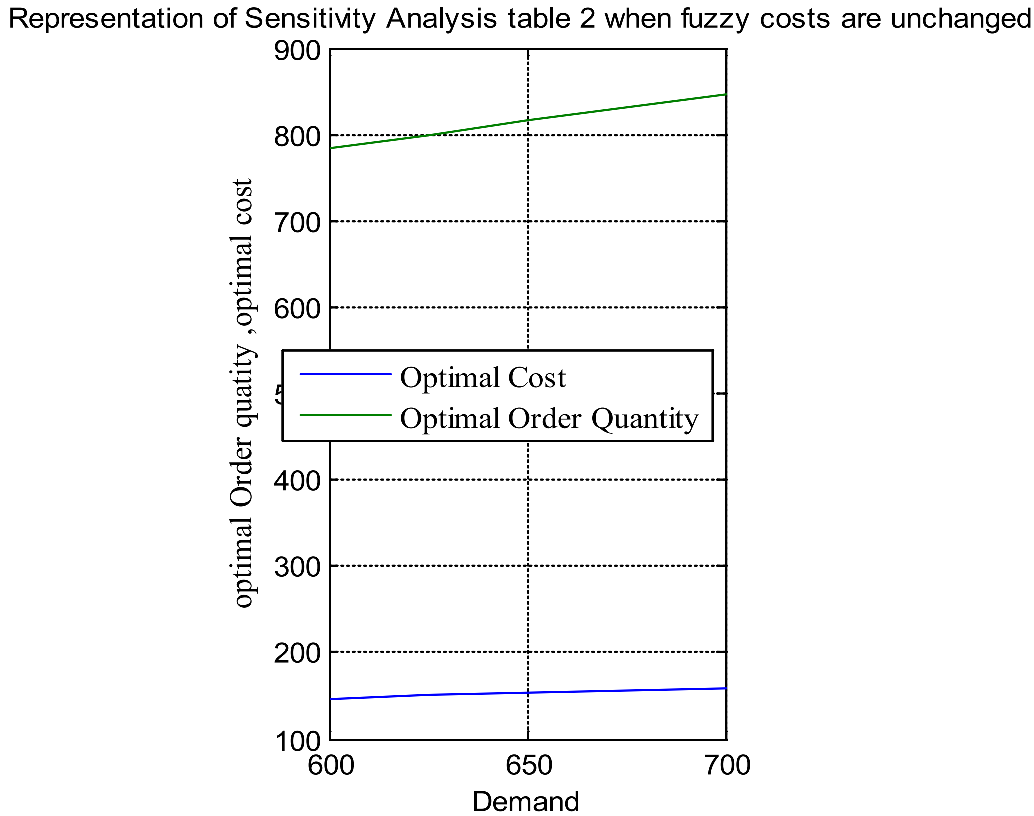
S.No	Demand	For $\widetilde{C_1}$=(7,9,11,12) $\widetilde{C_2}$=(10,11,13,15) $\widetilde{C_3}$=(96,98,101,103)		For $\widetilde{C_1}$=(7,8,10,12) $\widetilde{C_2}$=(10,11,12,15) $\widetilde{C_3}$=(96,97,102,103)	
		P_{dGW} (C*)	P_{dGW} (Q*)	P_{dGW} (C*)	P_{dGW} (Q*)
1	600	805.7995	148.1758	785.3293	152.0381
2	625	822.4177	151.2313	801.5233	155.1733
3	650	838.7027	154.2263	817.3966	158.2463
4	675	854.6795	157.1642	832.9675	161.2608
5	700	870.363	160.0482	848.2526	164.22

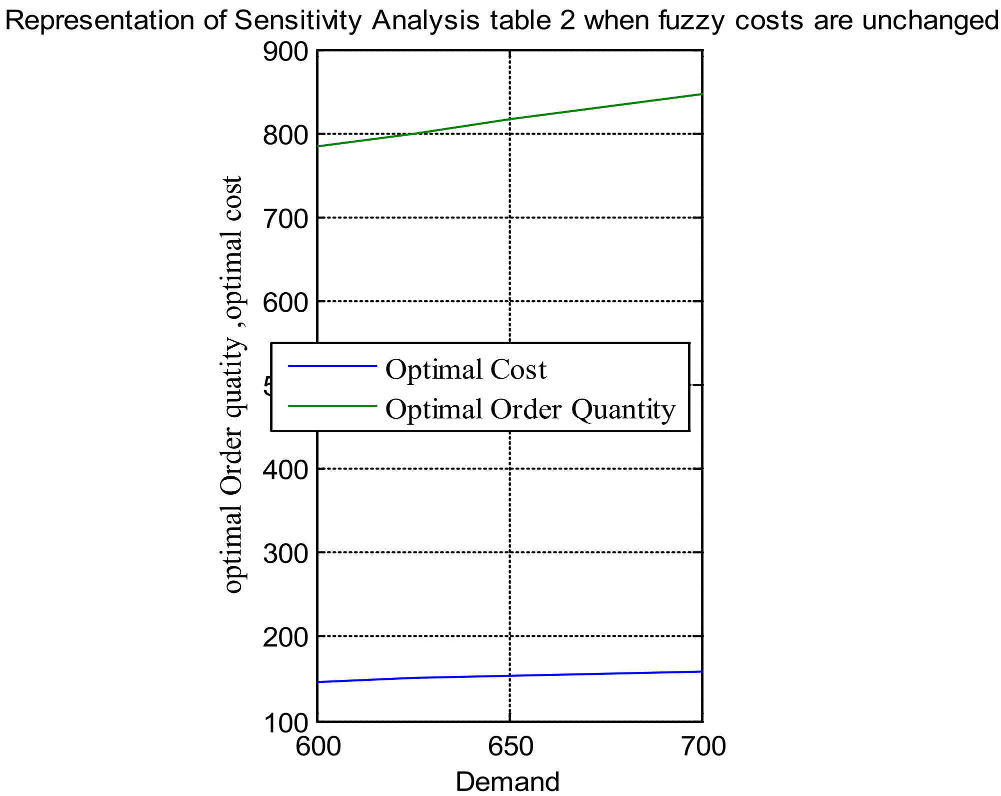
Table 4. Sensitivity Analysis when associated costs are Parabolic Fuzzy Number

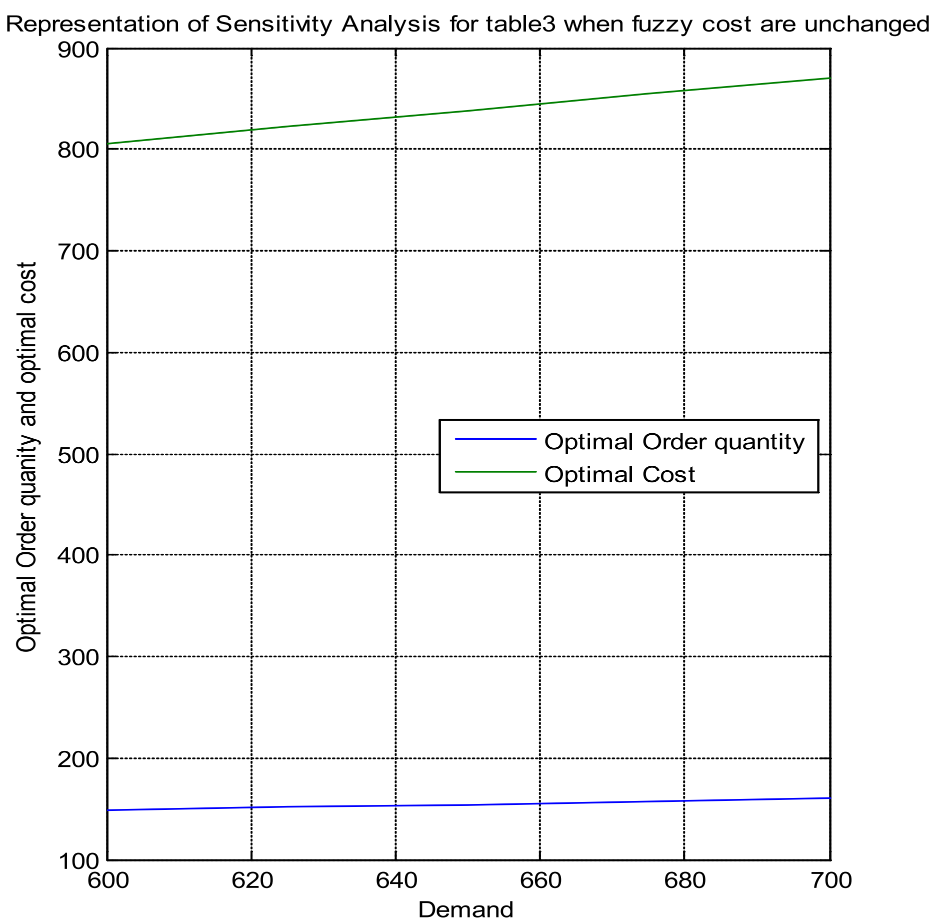
S.No	Demand	For $\widetilde{C_1}$=(6,9,13) $\widetilde{C_2}$=(10,14,13) $\widetilde{C_3}$=(97,99,103)		For $\widetilde{C_1}$=(9,7,13) $\widetilde{C_2}$=(10,12,13) $\widetilde{C_3}$=(97,98,103)	
		P_{dGW} (C*)	P_{dGW} (Q*)	P_{dGW} (C*)	P_{dGW} (Q*)
1	600	783.3932	147.3079	755.1191	151.7641
2	625	799.5473	150.3455	770.6902	154.1191
3	650	815.3814	153.3229	785.9529	157.9611
4	675	830.9139	156.2436	800.9248	160.9702
5	700	846.1613	159.1107	815.6219	163.924

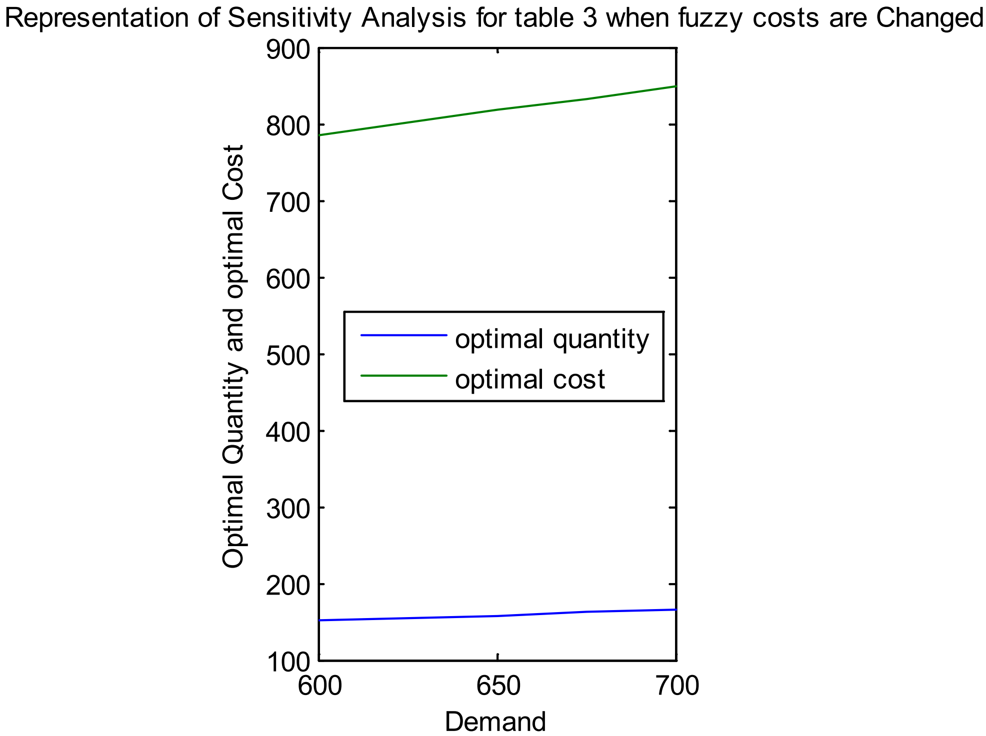
7. Graphical Representation of Different Results

Similarly the representation of different parameters in table 4 can be made.




Representation of Sensitivity Analysis table 2 when fuzzy costs are unchanged
optimal Order quatity ,optimal cost
Demand
Optimal Cost
Optimal Order Quantity
900
800
700
600
400
300
200
100
600
650
700


Representation of Sensitivity Analysis for table3 when fuzzy cost are unchanged
Optimal Order quanity and optimal cost
Demand
Optimal Order quantity
Optimal Cost
900
800
700
600
500
400
300
200
100
600
620
640
660
680
700



8. Conclusions

Through this investigation, it may be concluded that study of inventory control in fuzzy environment is essential to overcome the imprecision arises in different stages. In reality the cost and demand are not fixed in nature. Different authors have drawn conclusions by considering these as fuzzy numbers. In this study, by considering different costs namely carrying cost, shortage cost and set up cost as different fuzzy numbers and defuzzified by two different defuzzification methods. It is observed that the optimal values are improved as compared to the values in crisp environment. Sensitivity analysis of the results obtained in fuzzy environment helps in making better decision out of number of alternatives. This can further be extended by considering demand as fuzzy numbers and applying fuzzy differential equation. Also, one can apply interval arithmetic for the objective as well as for the parameters involved in different inventory models with shortages. The present study is helpful for business organizations where customers' demands are not fulfilled instantly.

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