American Journal of Mathematics and Statistics 2015, 5(3): 144-149
DOI: 10.5923/j.ajms.20150503.06

Ratio Type Estimator of Population Mean through Efficient Linear Transformation

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Abstract In the present paper, we propose two ratio-type estimators of population mean of study variable y, using transformation based on known information of minimum (X_m) and maximum (X_M) value of the auxiliary variable X. The expressions for bias and mean square error, up to the first degree of approximation, of proposed estimators are obtained. The comparisons of the proposed estimators are made with the existing ones with respect to bias and mean square error theoretically. The theoretical results are supported numerically and graphically. It has been shown that the proposed estimators are better than the existing ones under some conditions.

Keywords Auxiliary variable, Bias, Linear transformation, Mean square error, Ratio estimator, Simple random sampling without

1. Introduction

To increase the efficiency of the proposed estimator of population mean of the study variable y, we use the information of auxiliary variable x which is highly correlated with y. Many authors viz. Sisodia and Dwivedi [1], Mohanty and Sahoo [2], Singh and Tailor [3], Khoshnevisan et al. [4], Gupta and Shabbir [5], Iqbal and Maqbool [6], Mehta and Sharma [7] etc. have adapted transformation method on auxiliary variable by using several known parameters such as C_x, $\beta_{1(x)}$, $\beta_{2(x)}$, Md, $Q.D.$, ρ_{xy} to reduce the bias and mean square error.

In literature, the classical ratio estimator for population mean $\bar{Y}$ is defined as,

$$\bar{y}_R = \frac{\bar{y}}{\bar{x}}\bar{X} \tag{1.1}$$

The bias and MSE of $\bar{y}_R$, up to first order of approximation, is

$$B(\bar{y}_R) = \left(\frac{1}{n} - \frac{1}{N}\right)\bar{Y}(C_x^2 - \rho C_y C_x) \tag{1.2}$$

$$MSE(\bar{y}_R) = \left(\frac{1}{n} - \frac{1}{N}\right)\bar{Y}^2(C_y^2 - 2\rho C_y C_x + C_x^2) \tag{1.3}$$

Sisodia and Dwivedi [1] using the information of $\bar{X}$ and C_x, defined ratio-type estimator for population mean $\bar{Y}$ as

$$\bar{y}_{sd} = \bar{y}\frac{\bar{X} + C_x}{\bar{x} + C_x} \tag{1.4}$$

where C_x is the known population coefficient of variation of the auxiliary variable x. The bias and MSE of $\bar{y}_{sd}$, up to first order of approximation, is

$$B(\bar{y}_{SD}) = \left(\frac{1}{n} - \frac{1}{N}\right)\bar{Y}(c_{sd}^2 C_x^2 - c_{sd} C_{yx}) \tag{1.5}$$

$$MSE(\bar{y}_{SD}) = \left(\frac{1}{n} - \frac{1}{N}\right)\bar{Y}^2(C_y^2 - 2c_{sd}\rho C_y c_{sd}^2 C_x^2) \tag{1.6}$$

Where $c_{sd} = \frac{\bar{X}}{\bar{X} + C_x}$ is a constant.

Use of higher order parameters in the proposed estimators restricts their applicability. To overcome such type of restrictions, Mohanty and Sahoo [2] using known minimum ($X_{\max} = X_M$) and maximum ($X_{\min} = X_m$) values besides population mean $\bar{X}$ of auxiliary variable x and then taking the transformation on auxiliary variable as,

$$v = \frac{x + X_m}{X_M + X_m} \text{ and } w = \frac{x + X_M}{X_M + X_m}$$

defined two estimators of $\bar{Y}$ as

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Published online at http://journal.sapub.org/ajms

$$t_1 = \bar{y}\frac{\bar{V}}{\bar{v}} \text{ and } t_2 = \bar{y}\frac{\bar{W}}{\bar{w}} \text{ respectively} \quad (1.7)$$

The expressions for bias and MSE of these estimators, up to first order of approximation, are

$$B(\bar{y}_{t_1}) = \left(\frac{1}{n} - \frac{1}{N}\right)\bar{Y}(t_1^2 C_x^2 - t_1 C_{yx}) \quad (1.8)$$

$$B(\bar{y}_{t_2}) = \left(\frac{1}{n} - \frac{1}{N}\right)\bar{Y}(t_2^2 C_x^2 - t_2 C_{yx}) \quad (1.9)$$

$$MSE(\bar{y}_{t_1}) = \left(\frac{1}{n} - \frac{1}{N}\right)\bar{Y}^2(C_y^2 - 2t_1\rho C_y C_x + t_1^2 C_x^2) \quad (1.10)$$

$$MSE(\bar{y}_{t_2}) = \left(\frac{1}{n} - \frac{1}{N}\right)\bar{Y}^2(C_y^2 - 2t_2\rho C_y C_x + t_2^2 C_x^2) \quad (1.11)$$

where $t_1 = \frac{\bar{X}}{\bar{X} + X_m}$ and $t_2 = \frac{\bar{X}}{\bar{X} + X_M}$ are constants respectively.

It is to be noted that the transformations in v and w depend upon both X_M and X_m but estimators generated through these transformations depend only upon X_M and X_m respectively. To utilize the information on both X_M and X_m, in this paper, we propose a modified ratio-type estimator of population mean in simple random sampling using transformation on auxiliary variable in different forms. The expressions for bias and mean square error have been obtained. The comparison has been made with Classical ratio estimator, Mohanty and Sahoo [2] and Sisodia and Dwivedi [1]. The results have also been illustrated numerically and graphically.

2. Sampling Design and Terminology

A simple random sample of size n is drawn from a finite population of size N and both auxiliary variable x and study variable y are measured on it.

Let Y_i and X_i denote the respective values of variables y and x on the i[th] (i=1, 2,…,N) unit of the population. Corresponding small letters denote the values in the sample.

Taking,

$$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i \qquad \bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$$

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \qquad \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2 \qquad \rho_{yx} = \frac{S_{yx}}{S_y S_x}$$

$$S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2 \qquad K = \rho\frac{C_y}{C_x}$$

$$S_{yx} = \frac{1}{N-1}\sum_{i=1}^{N}\left(Y_i - \bar{Y}\right)\left(X_i - \bar{X}\right) \qquad C_x = \frac{S_x}{\bar{X}}$$

$$C_y = \frac{S_y}{\bar{Y}} \qquad C_z = \frac{S_z}{\bar{Z}} = \frac{S_x}{\bar{X} + \frac{X_M}{X_m}} \qquad S_z^2 = S_x^2$$

Defining,

$$\varepsilon_0 = \frac{\bar{y}}{\bar{Y}} - 1 \qquad \varepsilon_1 = \frac{\bar{x}}{\bar{X}} - 1$$

So we have,

$$E(\varepsilon_0) = E(\varepsilon_1) = 0 \qquad E(\varepsilon_0^2) = \left(\frac{1}{n} - \frac{1}{N}\right)C_y^2$$

$$E(\varepsilon_1^2) = \left(\frac{1}{n} - \frac{1}{N}\right)C_x^2 \qquad E(\varepsilon_0\varepsilon_1) = \left(\frac{1}{n} - \frac{1}{N}\right)C_{yx}$$

3. Proposed Estimator

Using known minimum ($X_{\max} = X_M$) and maximum ($X_{\min} = X_m$) values besides population mean $\bar{X}$ of auxiliary variable x and then taking the transformation on auxiliary variable as $z = x + \frac{X_M}{X_m}$, we propose an estimator of $\bar{Y}$ as

$$\bar{y}_{hm1} = \bar{y}\frac{\bar{Z}}{\bar{z}} \quad (3.1)$$

When S_x^2 is also known, we propose an another estimator of population mean $\bar{Y}$ as,

$$\bar{y}_{hm2} = \bar{y}\frac{\bar{Z} + C_z}{\bar{z} + C_z} \quad (3.2)$$

To find the bias and the MSE of the estimator $\bar{y}_{hm1}$, we expand $\bar{y}_{hm1}$ in terms of ε's, up to the second degree of approximation, as

$$\bar{y}_{hm1} = \bar{Y}\{1 + \varepsilon_0 - c_1\varepsilon_1 - c_1\varepsilon_0\varepsilon_1 + c_1^2\varepsilon_1^2\} \quad (3.3)$$

where $c_1 = \frac{\bar{X}}{\bar{X} + \frac{X_M}{X_m}}$

Taking the expectation in (3.3), we obtain

$$E\left(\bar{y}_{hm1}\right) = \bar{Y} + \left(\frac{1}{n} - \frac{1}{N}\right)\bar{Y}(c_1^2 C_x^2 - c_1 \rho C_y C_x)$$

$$\Rightarrow B(\bar{y}_{hm1}) = \left(\frac{1}{n} - \frac{1}{N}\right)\bar{Y}(c_1^2 C_x^2 - c_1 \rho C_y C_x) \quad (3.4)$$

Using the expression (3.3), the mean square error of the proposed estimator $\bar{y}_{hm1}$, up to the first order of approximation, is

$$MSE(\bar{y}_{hm1}) = E(\bar{y}_{hm1} - \bar{Y})^2$$

$$= \left(\frac{1}{n} - \frac{1}{N}\right)(C_y^2 - 2c_1 \rho C_y C_x + c_1^2 C_x^2) \quad (3.5)$$

Similarly, up to the first order of approximation, we obtain the bias and MSE of $\bar{y}_{hm2}$ as

$$B(\bar{y}_{hm2}) = \left(\frac{1}{n} - \frac{1}{N}\right)\bar{Y}(c_2^2 C_x^2 - c_2 \rho C_y C_x) \quad (3.6)$$

and

$$MSE(\bar{y}_{hm2}) = \left(\frac{1}{n} - \frac{1}{N}\right)(C_y^2 - 2c_2 \rho C_y C_x + c_2^2 C_x^2) \quad (3.7)$$

where $c_2 = \dfrac{\bar{X}(\bar{X} + \frac{X_M}{X_m})}{(\bar{X} + \frac{X_M}{X_m})^2 + S_x}$

We have proved the following theorems:

Theorem 3.1. Up to the first order of approximation, the bias of estimator $\bar{y}_{hm1}$ is

$$B(\bar{y}_{hm1}) = \left(\frac{1}{n} - \frac{1}{N}\right)\bar{Y}(c_1^2 C_x^2 - c_1 \rho C_y C_x)$$

and its MSE is

$$MSE(\bar{y}_{hm1}) = \left(\frac{1}{n} - \frac{1}{N}\right)(C_y^2 - 2c_1 \rho C_y C_x + c_1^2 C_x^2)$$

Theorem 3.2. Up to the first order of approximation, the bias of estimator $\bar{y}_{hm2}$ is

$$B(\bar{y}_{hm1}) = \left(\frac{1}{n} - \frac{1}{N}\right)\bar{Y}(c_2^2 C_x^2 - c_2 \rho C_y C_x)$$

and its MSE is

$$MSE(\bar{y}_{hm2}) = \left(\frac{1}{n} - \frac{1}{N}\right)(C_y^2 - 2c_2 \rho C_y C_x + c_2^2 C_x^2)$$

4. Comparison

First, we compare the bias of the proposed estimators $\bar{y}_{hm1}$ and $\bar{y}_{hm2}$ one by one with Classical ratio estimator ($\bar{y}_R$), Mohanty and Sahoo [2] estimators ($\bar{y}_{t_1}$ and $\bar{y}_{t_2}$), and Sisodia and Dwivedi [1] estimator $(\bar{y}_{SD})$.

Up to first order of approximation, we have

I. $\left|B(\bar{y}_{hm1})\right| < \left|B(\bar{y}_R)\right|$

iff

$[B(\bar{y}_{hm1})]^2 < [B(\bar{y}_R)]^2$

when $K < \dfrac{c_1^2 + 1}{c_1 + 1}$ or $K > c_1 + 1$

II. $[B(\bar{y}_{hm1})]^2 < [B(\bar{y}_{t_1})]^2$

when $K < \dfrac{c_1^2 + t_1^2}{c_1 + t_1}$ or $K > c_1 + t_1$

III. $[B(\bar{y}_{hm1})]^2 < [B(\bar{y}_{t_2})]^2$

when $K < \dfrac{c_1^2 + t_2^2}{c_1 + t_2}$ or $K > c_1 + t_2$

IV. $[B(\bar{y}_{hm1})]^2 < [B(\bar{y}_{SD})]^2$

when $K < \dfrac{c_1^2 + c_{sd}^2}{c_1 + c_{sd}}$ or $K > c_1 + c_{sd}$

V. $\left[B(\bar{y}_{hm2})\right]^2 < \left[B(\bar{y}_R)\right]^2$

when $K < \dfrac{c_2^2 + 1}{c_2 + 1}$ or $K > c_2 + 1$

VI. $[B(\bar{y}_{hm2})]^2 < [B(\bar{y}_{t_1})]^2$

when $K < \dfrac{c_2^2 + t_1^2}{c_2 + t_1}$ or $K > c_2 + t_1$

VII. $[B(\bar{y}_{hm2})]^2 < [B(\bar{y}_{t_2})]^2$

when $K < \dfrac{c_2^2 + t_2^2}{c_2 + t_2}$ or $K > c_2 + t_2$

VIII. $[B(\bar{y}_{hm2})]^2 < [B(\bar{y}_{SD})]^2$

when $K < \dfrac{c_2^2 + c_{sd}^2}{c_2 + c_{sd}}$ or $K > c_2 + c_{sd}$

Second, we compare MSE of the proposed estimators

$\bar{y}_{hm1}$ and $\bar{y}_{hm2}$ one by one with Classical ratio estimator ($\bar{y}_R$), Mohanty and Sahoo [2] estimators ($\bar{y}_{t_1}$ and $\bar{y}_{t_2}$), and Sisodia and Dwivedi [1] estimator $(\bar{y}_{SD})$.

Theorem 4.1. Up to the terms of order n^{-1}, we have

$MSE(\bar{y}_{hm1}) < MSE(\bar{y}_R)$ if $\frac{X_M}{X_m} > 0$, always true.

Theorem 4.2. Up to the terms of order n^{-1}, we have

$$MSE(\bar{y}_{hm1}) < MSE(t_1)$$

iff $$(c_1 + t_1) = \begin{cases} < 2K & for \frac{X_M}{X_m} < X_m \\ > 2K & for \frac{X_M}{X_m} > X_m \end{cases}$$

Theorem 4.3. Up to the terms of order n^{-1}, we have

$MSE(\bar{y}_{hm1}) < MSE(t_2)$ if $\frac{X_M}{X_m} < X_M$,always true.

Theorem 4.4. Up to the terms of order n^{-1}, we have

$$MSE(\bar{y}_{hm1}) < MSE(\bar{y}_{SD})$$

iff $$(c_1 + c_{sd}) = \begin{cases} < 2K & for \frac{X_M}{X_m} < C_x \\ > 2K & for \frac{X_M}{X_m} > C_x \end{cases}$$

Theorem 4.5. Up to the terms of order n^{-1}, we have

$$MSE(\bar{y}_{hm2}) < MSE(\bar{y}_R)$$

if $\frac{X_M}{X_m} + \frac{S_x}{\bar{X} + \frac{X_M}{X_m}} > 0$, which is always true.

Theorem 4.6. Up to the terms of order n^{-1}, we have

$$MSE(\bar{y}_{hm2}) < MSE(t_1)$$

iff

$$(c_2 + t_1) = \begin{cases} < 2K & for \frac{X_M}{X_m} + \frac{S_x}{\bar{X} + \frac{X_M}{X_m}} < X_m \\ > 2K & for \frac{X_M}{X_m} + \frac{S_x}{\bar{X} + \frac{X_M}{X_m}} > X_m \end{cases}$$

Theorem 4.7. Up to the terms of order n^{-1}, we have

$$MSE(\bar{y}_{hm2}) < MSE(t_2)$$

if $\frac{X_M}{X_m} + \frac{S_x}{\bar{X} + \frac{X_M}{X_m}} < X_M$, which is always true.

Theorem 4.8. Up to the terms of order n^{-1}, we have

$$MSE(\bar{y}_{hm2}) < MSE(\bar{y}_{SD})$$

if $\frac{X_M}{X_m} + \frac{S_x}{\bar{X} + \frac{X_M}{X_m}} > C_x$, which is always true.

In the last theorem, we compare the proposed estimator $\bar{y}_{hm2}$ with $\bar{y}_{hm1}$ as

Theorem 4.9. Up to the terms of order n^{-1}, we have

$$MSE(\bar{y}_{hm_2}) < MSE(\bar{y}_{hm_1})$$

If $\frac{X_M}{X_m} + \frac{S_x}{\bar{X} + \frac{X_M}{X_m}} > \frac{X_M}{X_m}$, which is always true.

5. Numerical

To have the rough idea about the performance of the proposed estimators over the existing one, we consider a real finite population presented in Cochran [8] on page 34. This data set concerns food cost (as study variable) and weekly income (as auxiliary variable). Note that we omitted the data of 1 family because it was an outlier. The values of the population parameters are given in Table 1. The Bias, MSE and relative efficiency of proposed estimators with respect to Classical ratio estimator ($\bar{y}_R$), Mohanty and Sahoo [2] estimators ($\bar{y}_{t_1}$ and $\bar{y}_{t_2}$), and Sisodia and Dwivedi [1] estimator $(\bar{y}_{SD})$ are given in Table 2 and Table 3 respectively. The results are also illustrated graphically.

Table 1. Parameters and Constants of the Population

$N = 32$	$\bar{Y} = 27.0625$	$c_{sd} = 0.9980$
$n = 10$	$\bar{X} = 73.0000$	t_1=0.5573
$\rho = 0.3320$	$S_y = 9.9846$	$t_2 = 0.4345$
$K = 0.8587$	$S_x = 10.4140$	$c_1 = 0.9781$
$X_m = 58$	$X_M = 95$	$c_2 = 0.9762$

Table 2. Bias and Relative Efficiencies of Existing and Proposed Estimators

Estimators	*Bias*	*Efficiency*
Classical ratio estimator ($\overline{y}_R$)	0.0053	100.0000
Sisodia and Dwivedi $(\overline{y}_{SD})$	0.0053	101.5982
Mohanty and Sahoo ($\overline{y}_{t_1}$)	0.0064	82.8125
Mohanty and Sahoo ($\overline{y}_{t_2}$)	0.0070	75.7143
Proposed Estimator ($\overline{y}_{hm1}$)	0.0044	121.0493
Proposed Estimator ($\overline{y}_{hm2}$)	0.0043	123.1593

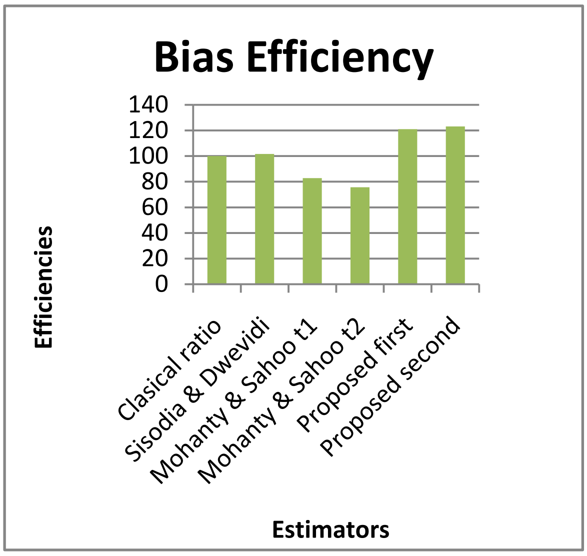

Figure 1. Bias and Relative Efficiencies of Existing and Proposed Estimators

Table 3. MSE and Relative Efficiencies of Existing and Proposed Estimators

Estimators	*MSE*	*Efficiency*
Classical ratio estimator ($\overline{y}_R$)	6.1186	100.0000
Sisodia and Dwivedi $(\overline{y}_{SD})$	6.1180	100.0092
Mohanty and Sahoo ($\overline{y}_{t_1}$)	6.1913	98.8258
Mohanty and Sahoo ($\overline{y}_{t_2}$)	6.2825	97.3902
Proposed Estimator ($\overline{y}_{hm1}$)	6.1127	100.0959
Proposed Estimator ($\overline{y}_{hm2}$)	6.1123	100.1031

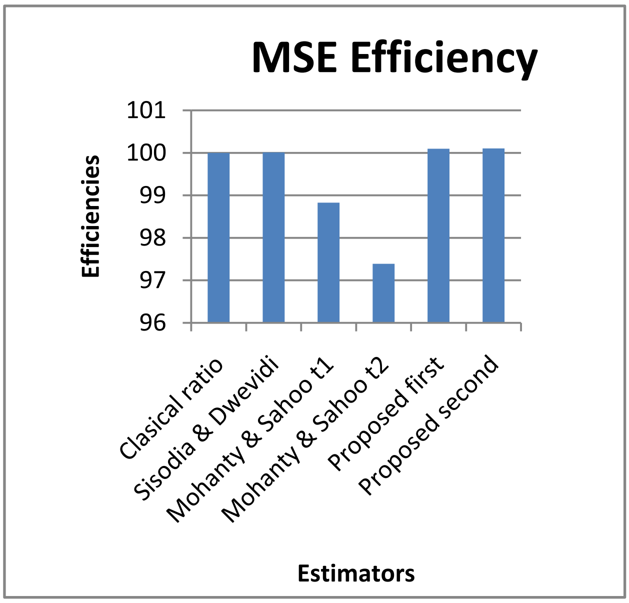

Figure 2. MSE and Relative Efficiencies of Existing and Proposed Estimators

6. Conclusions

From the Table 2 and Table 3, we observe that although there is small increase in efficiency of the proposed estimators w. r. t. the MSE yet there is a significant increase in efficiency w. r. t. the bias. So we conclude that where we have to give the more weightage to precision as compared to the accuracy, then the proposed estimators with new linear transformation are recommended to use.

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