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Mathematical Analogue Traffic System Model

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Abstract This paper presents the construction of a traffic system model which we call the mathematical analogue traffic system MATS. This is built from analogue wall clock with the combination of bulbs, electric wire for connection and battery to generate power to the system. The paper is basically on Traffic light, how it operates and the mathematical model behind it. The analogue traffic systems shall be applied to traffic problems of different kinds to find their solutions.

Keywords Traffic light, Mathematical model, Model, Analogue traffic Systems (MATS)

1. Introduction

Mathematical modeling can be defined as the process of creating a mathematical representation of some phenomenon in order to gain a better understanding of that phenomenon. It is a process that attempts to match observation with symbolic statements. Mathematical modeling can also be defined as the use of mathematics to describe the real-world phenomena, investigate important questions about the observed world, explain real world phenomenon, test ideas and make predictions about the real-world. The real-world refers to engineering, physics, physiology, ecology, wild-life, management, chemistry, economics, etc. There are relevant and exhaustive materials on this work such as [2, 3, 4, 6, 8, 10, 11, 12] just to mention but a few.

Traffic is the movement of motorized vehicles, unmotorized vehicles and pedestrians on roads. Traffic Flow is the pattern of the way people move through an area or road network, or a measure of the density of traffic [9]. Traffic Jam means the number of vehicles blocking one another until they can scarcely move, that is a situation when all road traffic is stationary or very slow. Traffic Congestion is a condition on any network as user increases and is characterized by slower speeds, longer trip times, and increased queuing. Also, pedestrian is a person traveling in foot, whether walking or running, in fact in some communities those travelling using roller skate boards and similar devices are considered to be pedestrians. In modern times, the term mostly refers to someone walking on road or footpath.

2. Mathematical Analogue Traffic System Model

2.1. Traffic Light

Traffic light at times can also be seen as a cluster in some cases but the only difference between cluster and traffic light is that cluster as a method which is Slow, Join (the cluster) and Go method while traffic light is Slow, Stop and Go.

Traffic light on a road is set to operate with Red, Green and Yellow phase (light) of difference phase length. In some cases the length varies, i.e Red phase is less than Green phase or vise-visa depending on the location, nature of traffic, the yellow phase or amber phase may be or not be ignored. When yellow phase is ignored in a traffic light system it is not totally ignored but incorporated with both the red phase and the green phase. This happens only when a traffic light system is computerized, where the time taking for yellow phase will be added to Red and Green phases, this is often so because yellow phase length is short and so may not be considered. In some cases, yellow phase length can be two seconds. In this research we are put into consideration the yellow phase and also incorporate it with other phases. [5, 9, 11, 12].

It is noteworthy that the common traffic systems we have in this country operate on three phases which are Red, Green and Yellow. RED Phase means STOP, GREEN Phase means GO, and YELLOW Phase means STAND FOR or SLOW-DOWN.

Traffic system can be modeled to solve traffic problems of different kinds. Here we shall derive a model and compare the model with an existing model, putting into consideration the movement of pedestrians. With the formula below it is possible to find the average delay per vehicle.

$$Average\,delay\ per\ vehicle = \frac{1}{N}\sum Delay$$

Given N = 40 and Σ delay = 1650, the average delay per vehicle $= \frac{1}{40}\sum Delay = \frac{1650}{40} = 41.25$

Where N is the total number of vehicles that arrive

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during phase cycle and the $\sum Delay$ is the sum of all vehicles delayed.

Problem 1

The flow of pedestrians at a road crossing is controlled by a set of traffic lights that operate on a fixed cycle basis. The red light is shown to pedestrians for r_p seconds the green light for, g_p seconds. We can assume that pedestrians arrive at the rate of one person every b seconds, and that all pedestrians arriving during the red phase cross the road at the start of green phase, and those pedestrians arriving during the green phase cross the road immediately.

Solution.

From this information we can find the formula for their average delay per pedestrian.

The formula can be obtained by using a similar formular:

Average Delay per pedestrians = $\frac{1}{N}\sum Delay$

N = number of pedestrians = $\frac{r_p+g_p}{b}$

The $\sum Delay$ can be expressed as the sum of an arithmetic progression:

$$s_n = \frac{1}{2}n(A+L)$$

Note in this case d will be zero because there is no time between the crossings of pedestrians.

So, A = first term, b and L = last term = r_p, where $\frac{r_p}{b}$ therefore

$$s = \frac{r_p}{2b}[b + r_p]$$

$$s = \frac{r_p}{2}[1 + \frac{r_p}{b}]$$

Average delay per pedestrian = $\frac{br_p}{2(r_p+g_p)}\left[1 + \frac{r_p}{b}\right]$

Problem 2

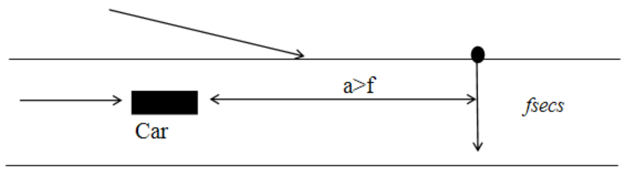
Pedestrians cross a one-way street at a point P where there are no traffic lights or other traffic controls. It takes a pedestrian f seconds to cross the road, and vehicles pass at random such that there is an average seconds between the times when successive vehicles pass point P. i.e. the number of vehicles that pass point P in one second is a Poisson random variable with mean $\frac{1}{a}$.

Solution:

It is possible to demonstrate that a deterministic model in which vehicles are assumed to pass point P at exact intervals of a second is inappropriate for determining the average delay to pedestrians.

Therefore the deterministic model is inappropriate for determining the delay to pedestrians because the cars arrive at regular intervals. In reality if $a < f$ then the pedestrians would never cross the road, which is not a realistic model.

Pedestrians can cross if and only if the times between successive vehicles passing point P is greater than the time it takes to cross the road.



We can use a stochastic model to determine the average delay to pedestrians in term of f and a.

$$X \sim P_o\left(\frac{1}{a}\right)$$

The car interval is random variable. N = waiting time

$$X \sim E_x\left(\frac{1}{a}\right)$$

$$P(X \le f) = \int_0^f \frac{1}{a}e^{-at}dt = \left(1 - e^{-\frac{f}{a}}\right) = P$$

$\int_0^f \frac{1}{a}e^{-at}dt$ ⟵ mean of exponential distribution over f.

$= a - (a+f)e^{-\frac{f}{a}} = A$ ⟵ mean waiting time of 1 vehicle.

AP is the average time one has to wait for 1 car.
AP^2 is the average time one has to wait for 2 cars.
AP^3 is the average time one has to wait for 3 cars.

Such that;

AP^n is the average time one has to wait for n cars.

This gives

$$AP + AP^2 + AP^3 + \ldots.. + AP^n$$
$$= A(P + P^2 + P^3 + \ldots. + P^n)$$

(This can be written as sum of geometric series.) That is $\frac{p}{1-P}$

Therefore the average waiting time for one person is $= \frac{AP}{1-P}$

This equation can be re-written in terms of f and a.

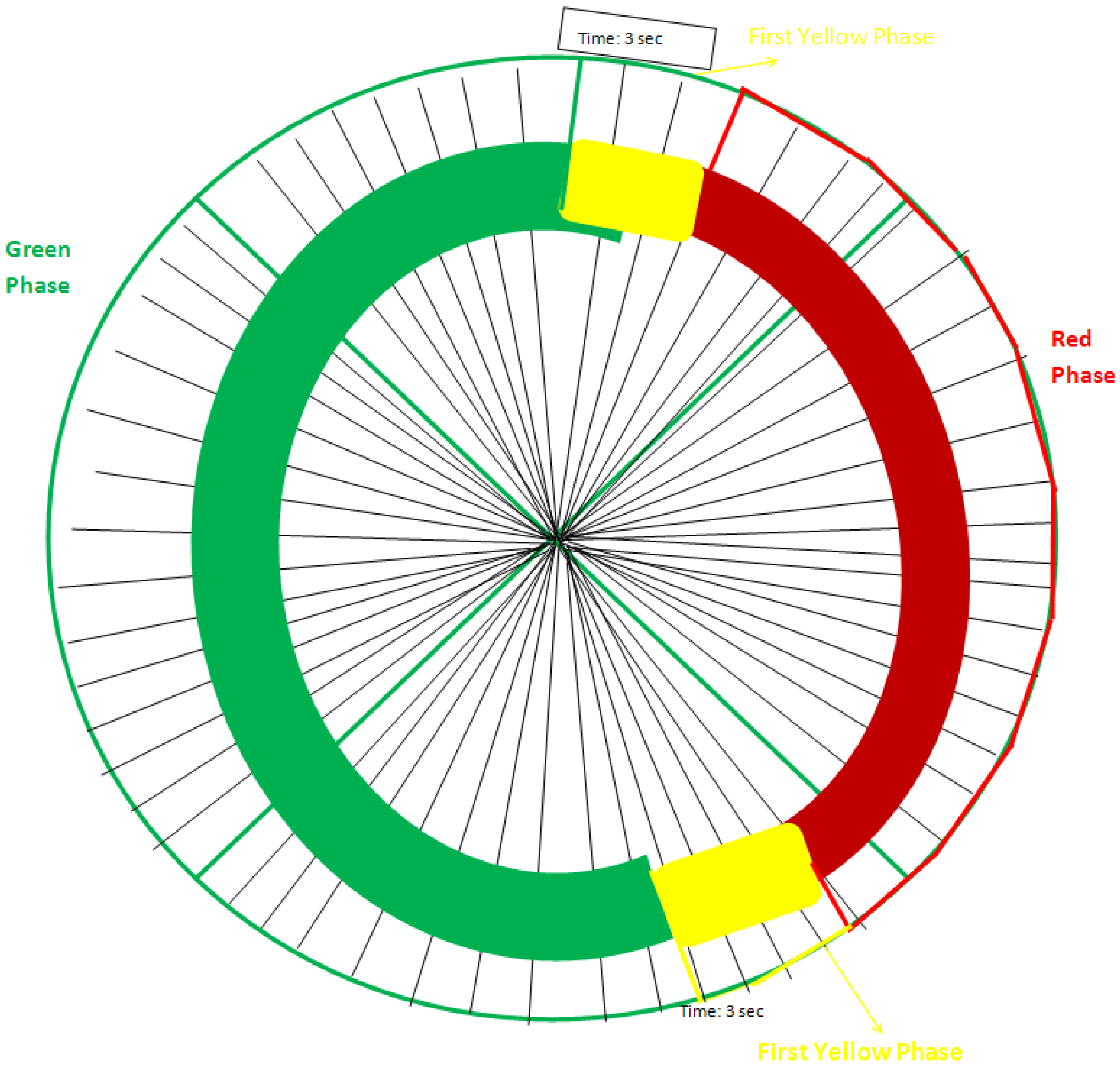
$$= \frac{\left[\left(a-(a+f)e^{-\frac{f}{a}}\right)\left(1-e^{-\frac{f}{a}}\right)\right]}{1-\left(1-e^{-\frac{f}{a}}\right)}$$

$$\frac{a-(a+f)e^{-\frac{f}{a}}-ae^{-\frac{f}{a}}+(a+f+e^{-\frac{2f}{a}}}{e^{-\frac{f}{a}}}$$

$$ae^{\frac{f}{a}}+(a+f)e^{-\frac{f}{a}}-f-2a$$

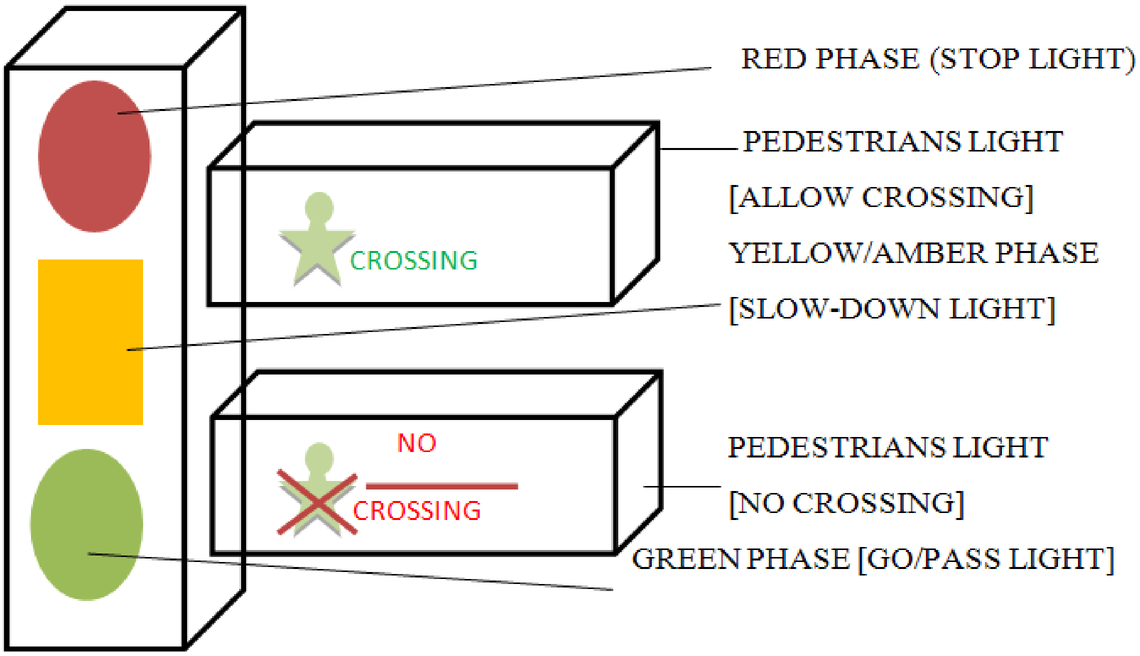
2.2. Model of Traffic System Construction (MATS)

For understanding and clarity of the derived model above, we construct a traffic system which we call Mathematical Analogue Traffic System, MATS is built from analogue wall clock with the combination of bulbs, electric wire for connection and battery to generate power to the system, MATS operate in three phase, these phases are model on the clock with four segment, the phases are Red, Green and Yellow or Amber Phase. Diagram below illustrate the model "MATS".



TRAFFIC CONTROL SYSTEM

Diagram of Mathematical Analogue Traffic System (MATS)



The relationship between the light is that anytime the light is changing to any phase it passes through yellow phase. The traffic light changes from Yellow to Red and likewise Yellow to Green, for a complete cycle it operate from Yellow to red to Yellow to Green.

The crossing response to the Red and Green phase only, ***when the Red phase is on, crossing occurs,*** *when* ***turn Green the pedestrians stop crossing***

Using a wall clock for illustration, the clock is divided into four parts, the first part of the clock is the yellow or amber phase, follow by the Red phase and next to it is another yellow phase and to end of the cycle is the green phase, and this can be simply stated as Yellow-Red-Yellow-Green for a complete cycle.

MATS phase lengths are different with a fixed or constant length with the total length of the phases is 60 sec which are divided into the phases. RED phases remaining 6 seconds are divided into two, for the two yellow phases.

MATS operate in seconds with normal second hand of a clock i.e. the operation depend on the clock seconds hand. As have said early that the operation start from yellow and end up in green phase, the seconds hand move through the time (sec) part, starting from the initial value t_0, on the normal time is equivalent to 0/60 and through the initial value which is yellow and change to Red to Yellow to Green after n second which is 60/0; i.e. the end of the final value of first iteration is the initial value of another iteration. Note that MATS can only be used on one-way /lane traffic.

Mathematically, $t_0 = 0$, $t_n = 60$, the complete time taken can be calculated by:

$T_c = t_n - t_0$ while t_c is the complete time taken (sec). Considering the three phases.

Phase	Length	
Yellow	$TY_k = Y_n - Y_0$	Where $Y_0 = T_0$
Red	$TR_k = R_n - R_0$	Where $R_0 = Y_n$
Yellow	$TYY_k = YY_n - YY_0$	Where $YY_0 = R_n$
Green	$TG_k = G_n - G_0$	Where $G_0 = YY_n$

For a complete cycle phase, i.e. moving from 0(sec) to 60 (sec), where $G_n = T_0$ for another iteration. If the yellow phases were incorporated then we are left with two phases, the Red and the Green phases.

Red	$TR_c = TY_k + TR_k$
Green	$TG_c = TYY_k - TG_k$

And the complete time taken for the whole phase will be

$$TT_c = TR_c + TG_c$$

MATS Length

MATS as a complete time for 60 sec. with
Red Phase Length = TR_k = 20 Sec
Green phase length = TG_k = 34 Sec.
And the yellow phase length = $TY_k = TYY_k$ = 3 3 sec.
Therefore the complete time for MATS can be written as

$MAT = TR_K + TG_K + 2TY_K$ or

$TR_K + TG_K + TY_K + TYY_K$

If yellow phase is incorporated then we have

Red phase length $= TR_C = TR_K + TY_K = 23\sec$

Green phase length $TG_C = TYY_K + TG_K = 37\sec$

MATS $TT_C = TR_C + TG_C = 60\sec$.

In the first section we have our Red phase length to be r_v, and Green phase length to g_v, using the above data to work with the MATS replacing r_v and g_v with TR_c and TG_c respectively.

Recalling the formula

$$s_n = \frac{a(r-1)}{2(r_v + g_v)}\left[1 + \frac{(r_v - 1)}{(a+d)}\right]$$

Taking the value of a and d to be 4 and 1, we have

$$s_n = \left(\frac{11}{15}\right)^* \left[1 + \frac{22}{3}\right]$$

$$= 0.733^* 8.333$$

The total time delay = 6.108

Calculating for N we have

$$N = \frac{r_v + g_v}{a}$$

$$N = \frac{23+37}{4}$$

$$N = 15$$

Therefore we put the values into the formula $\frac{1}{N}\sum Delay$, we obtain average delay per vehicle (A_V)

$$A_v = (1/15)^* 6.108$$

$$= 0.4072$$

Second iteration

Taking the value of and d to be 3 and 1, we obtain value S_n (total time delay) and A_v (average delay per vehicle) as

$S_n = 8.796$ and

$A_v = 0.5864$

Third iteration

a = 2 and d = 1, we obtain

S_n = 16.859 and

A_v = 1.1239

3. Discussion of Results

The table below gives a clear explanation of the results obtained. Dividing table A by a number of cycles i.e. table A was divided into three using MATS method, on table A we have 160sec as the total traffic length, with Red and Green Phase of 100 and 60 length respectively, but in our own case we take the total traffic length to be 60sec with red and Green phase of 23 and 37 using incorporated method.

Considering the left – hand side of table A we see that 33 cars goes through the light at 132 seconds resulting in no delay for the following 7 vehicles.

But the case is different in table B, 7 goes through the lights at 28, 88 and 148 seconds respectively resulting in no delay for the following 8 vehicles in each part. We can calculate the assume total number of cars that goes through the light and the number of cars that goes through without delaying as follows

Delaying = no of cycle* no of cars delay for a phase
= 3*7
= 21 (we have 21 cars)

No Delay = no of cycle no of cars without delay for a phase
= 24 (24 cars without delay)

Note: if the no of cars that goes through the light either delay or no delay are not equal for each cycle then someone can easily calculate the number of cars for each part and sum it up, depending on the number of cycle. The total average of cars (A_{tv}) can be calculated for *x* and *y* –cycle, where *x* and *y* stand for delay and no delayby:

$$X_1 + X_2 + \text{- - -} + X_n = X_{tv}$$

We therefore divide total number of car by total number of cycle

i.e. $A_{tv} = X_{tv} / n$

FOR NO DELAY:

We replace x by y, we then obtain Y_{tv} and for $A_{tv} = Y_{tv} / n$.

Car arrival times in secs (s)	Delay (s)	Red
2	98	
6	95	
10	92	
14	89	
18	86	
22	83	
26	80	
30	77	
34	74	
38	71	
42	68	
46	65	
50	62	
54	59	
58	56	
62	53	Green
66	50	Phase
70	47	
74	44	
78	41	
82	38	
86	35	
90	32	
94	29	
98	26	
102	23	
106	20	
110	17	
114	14	
118	11	
122	8	
126	5	
130	2	
134	0	
138	0	
142	0	
146	0	
150	0	
154	0	Red
158	0	

TABLE A

TABLE B

Car arrival times secs (s)	Delay (s)		Red	
2	98			
6	95	18		
10	92	15		
14	89	12		
18	86	9	Green	
22	83	6	Phase	
26	80	3		1st cycle of 60 sec
30	77	0		
34	74	0		
38	71	0		
42	68	0		
46	65	0		
50	62	0		
54	59	0		
58	56	0		
62	53	21	Red	
66	50	18		
70	47	15		
74	44	12		
78	41	9	Green	
82	38	6	Phase	2nd cycle of 120 sec
86	35	3		
90	32	0		
94	29	0		
98	26	0		
102	23	0		
106	20	0		
110	17	0		
114	14	0		
118	11	0		
122	8	21	Red	
126	5	18		
130	2	15		
134	0	12		
138	0	9	Green	
142	0	6	Phase	3rd cycle of 180 sec
146	0	3		
150	0	0		
154	0	0		
158	0	0		
162	98	0		
166	95	0		
170	92	0		
174	89	0	Red	
178	86	0		

4. Conclusions

We have been are able to derive a model to construct a mathematical analog Traffic System MATS which we use to determine

1. The number of cars that goes through lights either delay or no delay
2. The total average time of cars, subject to delay or no delay in traffic.

The constructed MATS is recommended for Local Area usage, like the sub settlement on highways, this can also be used by churches, schools and organization for crossing or for joining other cars on high way lane.

REFERENCES

[1] E. Ben-Naim and P. L. krapivsky, Phys. Rev. E59,88, 1999.

[2] D. N. Burghes, et al (1980), "Mathematical Models in the Social Management and Life Sciences", Ellis Horwood Limited.

[3] M. Brackstone and M. McDonald. "*Modeling of motorway operations"*–Transportation Research Records 1485. (1998).

[4] R.E. Chandler, R. Herman, and E.W. Montroll (1958), "*Traffic Dynamics: Studies in car following"*. - Operations Research 6, pp. 165-184.

[5] P. Eykhoff (1974), *"System Identification, Parameter and State estimation"* Wiley, New York.

[6] H. Ez-Zahraouy, K. Jetto, and A. Benyoussef (March 6, 2006), Laboratoire de Magn ́etisme et de la Physique des Hautes Energies Universit ́e Mohammed V, Facult ́e des Sciences, Avenue Ibn Batouta, B.P. 1014 Rabat, Morocco. *"New Topologies in the Fundamental Diagram of a Cellular Automata Traffic Flow Model"*.

[7] A. Galstyan and K. Lerman, (March 26, 2001), Information Sciences Institute, Univ. of Southern California, Marina del Rey, CA 90292-6695,*" A Stochastic Model of Platoon Formation in Traffic Flow"*.

[8] R. Herman, E.W. Montroll, R. Potts, and R.W. Rothery (1959). *"Traffic Dynamics: Analysis of Stability in Car-Following"* - Operations *Research* 1(7), 86-106.

[9] K. Lerman and A. Galstyan, Information Sciences Institute, Univ. of Southern Califonia, and M. Mataric and D. Goldberg (April 17-19 2001), TASK PI Meeting, Santa Fe, NM,*" A Stochastic Model of Platoon Formation in Traffic Flow"*.

[10] R. E. Mickens, (1994), "Non-Standard Finite-Difference Models of Differential Equations", World Scientific, Singapore.

[11] P. Serge Hoogendoorn and H.L. Piet Bovy (2000), Delft University of Technology, Faculty of Civil Engineering and Geosciences Transportation and Traffic Engineering Section. *"State-of-the-art of Vehicular Traffic Flow Modelling"* - Special Issue on Road Traffic Modelling and Control of the Journal of Systems and Control Engineering.

[12] Transportation Research Board *"Providing a Highway System with Reliable Travel Times, Future Strategic Highway Research Plan Area 3" http://wwwcf.fhwa.dot.gov/exit.cfm?link=http://www4.trb.org/trb/newshrp.nsf/web/progress_reports?OpenDocument. September 2003*