American Journal of Mathematics and Statistics 2014, 4(5): 231-239
DOI: 10.5923/j.ajms.20140405.04

A Modified Sir Model to Study on Physical Behaviour among Smallpox Infective Population in Bangladesh

Abdul Kuddus[1,*], Azizur Rahman[2], M. R. Talukder[3], Ashabul Hoque[1]

[1]Department of Mathematics, University of Rajshahi, Rajshahi, Bangladesh
[2]Discipline of Mathematics and Statistics, School of Computing and Mathematics, Charles Sturt University, Wagga Wagga, Australia
[3]Department of Applied Physics and Electronic Engineering, University of Rajshahi, Rajshahi, Bangladesh

Abstract The susceptible infected removal (SIR) model is widely used in epidemiology and public health to compute number of individuals in each categories of the population, and to explain the change in the number of people needing medical attention during an epidemic as well as evaluate policies effectively during the epidemic. Some extensions of this model have been studied and findings demonstrated that there exists a large class of functions representing interacting between the susceptible and infective population for which the model has realistic behavior and preserves the essential factures of the classical model. In this study we propose a new modification of SIR epidemic model to examine the appropriateness of the policies taken to control or eradicate disease with an application to smallpox data. The solutions of the parameters for our proposed model showed a very realistic behavior. In particular, the solutions have demonstrated that the number of infective people increases significantly during an initial period of the epidemic and then decreases and remain stable over the rest of the time. The temporal behavior of infectives for smallpox disease has been considered as an example within the modified frame work and the smallpox data from 1950 to 1977 in Bangladesh have been fitted by taking care of the model coefficients which are applied to estimate prediction of the infective population. Results revealed that the infectives curve of smallpox for the modified model has systematic noticeable variation over time, and the predicted numbers of infectives remain consistent for the next five years. Infection free and most critical periods are distinguishable in the results so that the local and international health systems can act accordingly and provide emergency and regular supports during the critical period of a smallpox epidemic. This modified model can further explore underlying factors for smallpox disease as a basis to generate hypotheses on potential determinants of smallpox in Bangladesh and worldwide contexts. It can also be applicable for the prediction of other infectious diseases.

Keywords SIR model, Reproduction number, Smallpox disease, Infective population, Prediction, Bangladesh

1. Introduction

Smallpox is a seasonal worldwide viral disease that can be spread all age's people. In Bangladesh the disease like smallpox outbreak regularly, occasionally in epidemic form. The illness can be more severe if it occurs in teenagers or adults with weakened immune systems (Peters [1]). The most critical form of infection is from a mother to her child, either before or after birth. Unborn babies infected with smallpox have been known to be born with birth defects. Young children with smallpox may get pneumonia (infection of the lungs), or have problems with other internal organs, such as the brain. The smallpox virus spreads easily from person to person through coughs or sneezes, physical contact with fluid from blisters that comes in contact with body entrance cavities (Tucker [2]).

* Corresponding author:
kuddus_epi@yahoo.com (Abdul Kuddus)
Published online at http://journal.sapub.org/ajms
Copyright © 2014 Scientific & Academic Publishing. All Rights Reserved

It is an obvious reason to study epidemic disease for infected area with such a mathematical model. One of the most basic procedures in modeling of diseases is to use a compartmental model in which the population is divided into different groups. This model is also call disease with removal model i.e. a susceptible person can be infected and infected person can be remove from the disease (Kapur, [4]). The susceptible infected removal (SIR) model is used in epidemiology to compute the amount of susceptible, infected, recovered people in population (Rahman and Kuddus [5]). It is also used to explain the change in the number of people needing medical attention during an epidemic (Derouich et al. [6]). In 2009 the disease influenza was epidemic in many countries. Therefore, governments have different policies to control the spread of disease. Those policies have positive and negative social or economical influence on people and society (Rahman and Chowdhury [7], Rahman et al. [8, 9], Rahman and Biswas [10], Rahman [11], Sally et al. [12]. Therefore, it is necessary and essential to develop an appropriate model for evaluations of policies. In recent decades although micro simulation technology based models

are becoming popular useful tools for micro data simulation, analysis and policy evaluations (Rahman et al. [13, 14], Rahman and Harding [15, 16]) the SIR epidemic model can evaluate policies effectively during the epidemic (Yang and Hsu [17], Rahman and Kuddus [5]).

Moreover, some possible extensions of the SIR epidemic model have been investigated by Satsuma et al. [18]. The findings demonstrated that there exists a large class of functions representing interacting between the susceptible and infective population for which the model has realistic behavior and preserves the essential factures of the classical SIR model. A study on the exact infection free periodic solution of the impulsive epidemic system proved that the infection free periodic solution is globally asymptotically stable if reproduction number, Ro<1 (Zhang et al. [19]). Additionally, a homotopy perturbation method (HPM) to solve the SIR model was considered Yildirim and Cherruault [20] that when homotopy parameter (*P)* is zero, the system of equations usually reduces to sufficiently simplified which normally admits a rather simple solution. Kaddar [21] introduced a latent period into susceptible and infectious individuals in incidence rate. The study found that if the basic reproduction number (Ro) is less than unity the disease free equilibrium is locally asymptotically stable and Ro>1, the endemic equilibrium is locally asymptotically stable.

Shulgin et al. [22] estimated that from SIR epidemic model we can show a planned pulse vaccination regime system that can eradicate epidemic if magnitudes of vaccination proportion and on the period of the pulses are adhered. John and O'Neill [23] said that by the SIR epidemic model we can calculate reproduction number Ro that can help us epidemic will occur or not. If Ro $\leq$ 1 no epidemic occur and if Ro>1 an epidemic occur. If Ro increases above 1, then a finite probability of extinction of the infection and the probability of extinction decreases.

A modified framework of the SIR model is used in the modeling of smallpox in Bangladesh by computing the amount of people in a closed population that are susceptible , infected, or recovered at a given period of time. The theoretical result has been verified by the empirical data for smallpox disease, and predicts the steady number of infectives for the five years period.

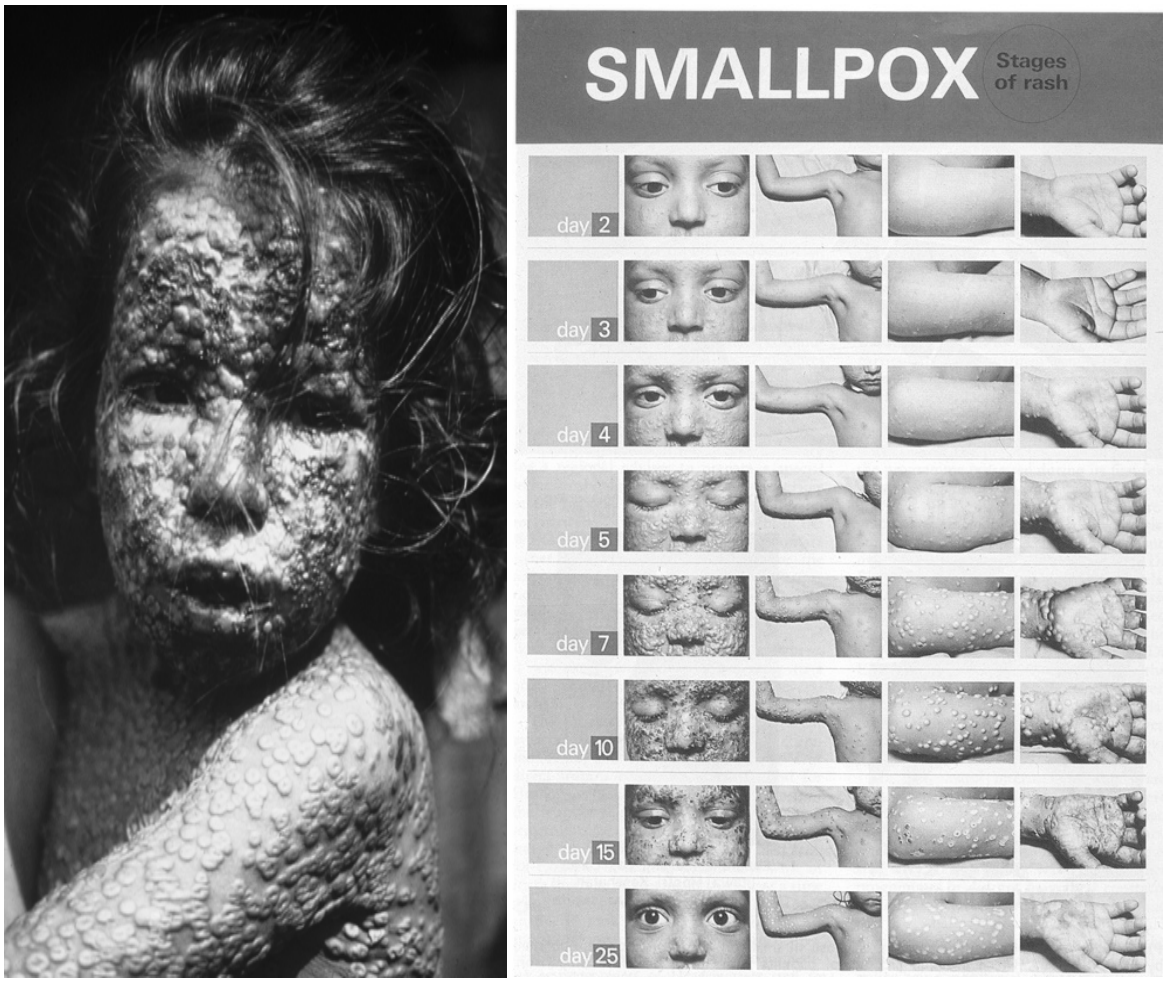

Figure 1. Smallpox affected of a child (Source: WHO [3])

The remainder of the paper is organized as follows. The theoretical representation of the model with necessary assumptions and solution are provided in section 2. Some empirical results, smallpox data and effects of the model are compared in section 3. The strategy and output of the modified model are discussed in the concluding section 4.

2. Theoretical Approaches of the Model

This section offers the theoretical representation of the proposed model and its solution by illustrating any necessary assumptions under the relevant sub-heading.

Basic assumptions

The SIR model is used in epidemiology to compute the amount of susceptible, infected, recovered people in a population. This model is an appropriate one to use under the following assumptions (Smith and Moore [24], Rahman and Kuddus, [5]).

- The population is fixed. That means, the only way a person can leave the susceptible group is to become infected and the only way a person can leave the infected group is to recover from the disease once a person has recovered the person received immunity;
- Age, sex, social status and race do not affect the probability of being infected;
- There is no inherited immunity; and
- The member of the population mix homogeneously.

The model equations

Let us consider a fixed homogeneously mixing group of N individuals consisting at any time *t* with *S(t)* susceptibles, *I(t)* infectives and *R(t)* removals. Therefore, the model can be formulated as:

$$S(t)+I(t)+R(t) = N \text{ (constant)} \quad (1)$$

In the beginning of epidemic, the value of R(*t*) is zero and $S(0)= S_0 > 0$,

$I(0) = I_0 > 0$, then we have

$$S_0+I_0 = N \quad (2)$$

Let the infected person be removed from population at a rate proportional to the current number of infective persons. If a formerly infected individual enter a new class which is not susceptible to the disease, then the model can be expressed by the following three equations:

$$\frac{dS}{dt} = -\alpha SI - ((1/w) * \exp(-(t - tc)/w)) \quad (3)$$

$$\frac{dI}{dt} = \alpha SI - \beta I = \alpha I(S - \rho) \text{ where } \rho = \frac{\beta}{\alpha} \quad (4)$$

and

$$\frac{dR}{dt} = \beta I \quad (5)$$

We use the term (1/w)*exp(-(t-tc)/w) to represent the amount of people who are migrate from closed population during the epidemic time.

Parameter Fitting

Here we used origin software to fit α parameter. An empirical data on α and α-fit values are presented in table 1. Then a comparison between α and α-fit parameter is depicted in figure 1. Results reveal that from the time period six the values of α-fit are significantly lower than its counterparts over the remainder time periods.

Table 1. Data on α and α-fit parameters

Time	α	α Fit
1	28	27.99999
2	8.5	8.49974
3	14.5	14.49541
4	10.3	10.26276
5	11	10.81235
6	10	9.30154
7	10.8	8.68809
8	10	4.51016

3. Results and Discussions

The empirical results of the model are presented and discussed in this section. It is noted that on an epidemiological graph for the SIR model, the symbol *S* represents the number of susceptibles, *I* represents the number of infectious people and *R* signifies the number of recovered. All graphs generated using MATLAB code.

A typical depiction of the variation of infected population over time (year) for a SIR model is presented in Figure 2.

Table 2. Consideration of parameters where we used in this model

No. of Step	α	β	Tc	W
I	28	2	100	300
II	8.5	2	100	300
III	14.5	2	100	300
IV	10.3	2	100	300
V	11	2	100	300
VI	9.30154	2	100	300

Here normalization of the total population size 1.412207 and the initial normalize population are S_0 = .58602, I_0 = .580811 and R_0 = .245376. The line represents the infected populations and we consider a time period from 1950 to 1980 years.

Figure-2 represents the variation of infected population and considers a time period from 1950 to 1980 years. Here the peaks obtained per five years and smallpox model represents the time varying transmission rate. When we run the SIR epidemic model then we find different pattern of peaks that are depending on the parameters α. Here α represents the infectious rate or transmission rate of the smallpox disease. Here we ignore seasonal factors such as environmental changes and immune system changes.

$$Y = 489.4 - 1047.07643\,X + 892.63583\,X^2 - 390.56556\,X^3 + 96.06875\,X^4 - 13.42764\,X^5 + 0.99542\,X^6 - 0.03038\,X^7$$

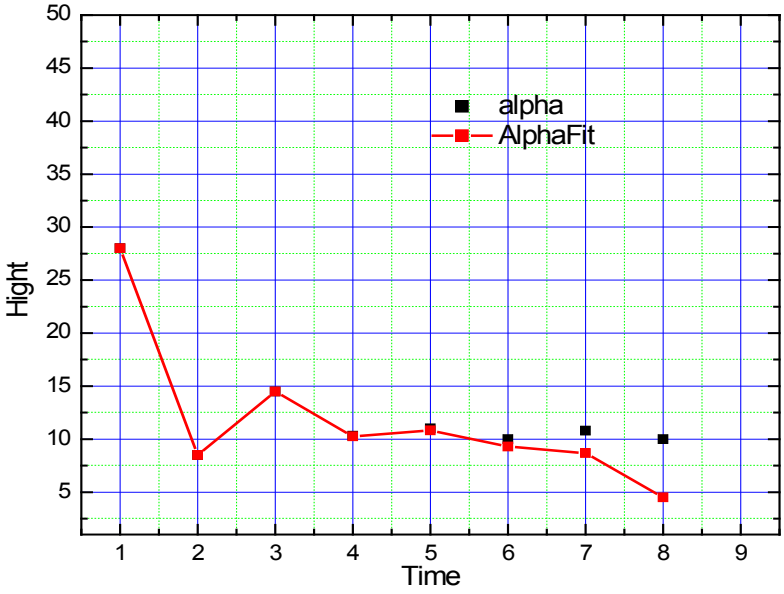

Figure 1. Comparison between α and α fit parameter curve

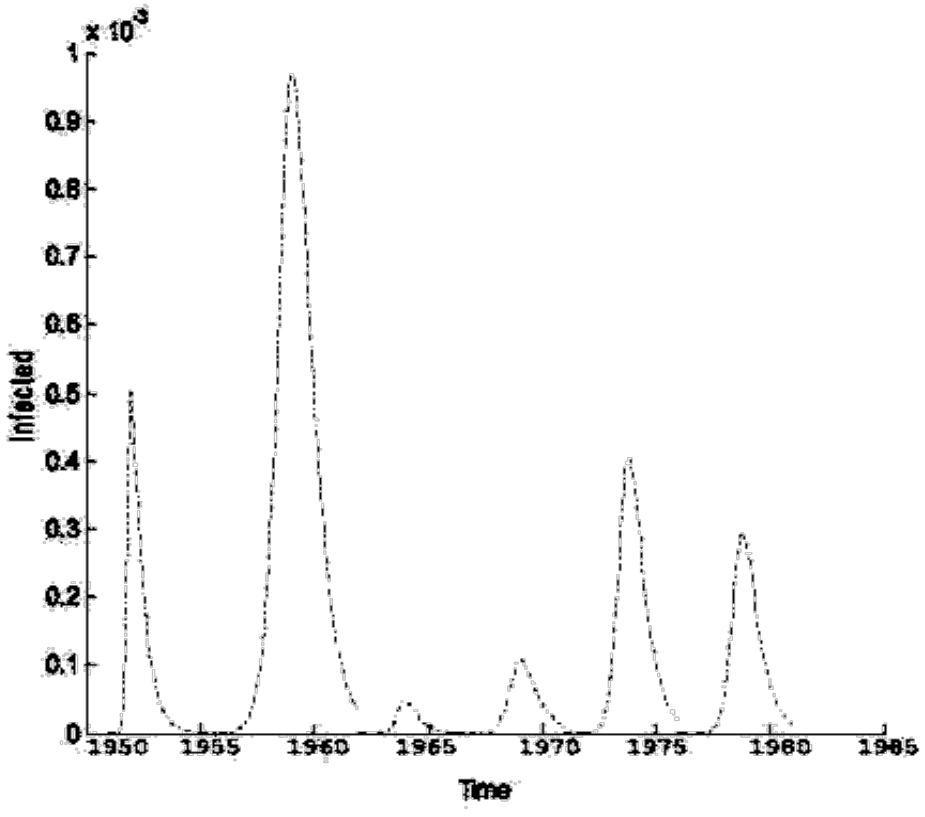

Figure 2. The infectivity curve shows the variation of infectives of smallpox with time (year). The graph is plotted against SIR model

The graph represents the number of infected population in Bangladesh between the years 1950 to 1980. It can be clear seen that in 1950 the number of infected population was 0 and it was dramatically increases in 1952 and then sharply fell in 1955. After 1955 the number of infected population shot up in the highest point in 1959 and then rapidly drop in 1962. Next in the year, the numbers of infected population gradually go up in 1964 and then minimally drop in 1965. From 1965 to 1967 the numbers of infected population remain stable and then move upward in 1969 and reduce in 1970. In 1971 the numbers of infected population erratically increase and then collapse in 1975. From 1975 over the next 5 years, initially the numbers of infected population gradually rise in 1978 and then substantially drop in 1980.

Table 3. Smallpox victims and deaths, 1950-1977

Year	Cases	Deaths	Year	Cases	Deaths
1950	21273	9445	1964	69	42
1951	38871	30341	1965	316	142
1952	10490	8053	1966	3207	1491
1953	1102	788	1967	6648	3455
1954	445	188	1968	9039	4024
1955	1926	979	1969	1925	597
1956	4962	3170	1970	1473	502
1957	24920	18149	1971	not available	not available
1958	79060	58891	1972	10754	3262
1959	15048	9508	1973	32711	9354
1960	1905	1271	1974	16485	2780
1961	660	469	1975	13798	2492
1962	610	379	1976	0	0
1963	3735	2847	1977	0	0

(Source: Banglapedia [25])

The susceptible, infected and recovered populations data used in this study are given in Table 4. The dataset is made up of mix homogeneously. The disease smallpox is highly infectious and everyone will be eventually becomes infected. So, using such a closed population we can observe how many people will be in each stage at a given period of time (years) and how long it takes until everyone that will get the disease, recovered from it. We also consider a period that lasts about 25 years.

The graph shows the number of cases of smallpox disease in Bangladesh between the years 1950 to 1975. It is clear seen that the number of cases of the disease increases from 1950 to 1953 and then decreases until 1955. After 1955 the number of cases of smallpox disease dramatically rose and it was highest in 1959 and then drop in 1961. Next in the year, the number of cases of this disease slowly increases and then decline in 1965. During 1966 the number of cases of disease moves upward in 1968 and then slump in 1971. After 1971 the number of cases of disease rocketed and then rapidly decline in the next following years.

The double line graph represents two time series data of smallpox infected population in Bangladesh. Here the solid line shows the graph from data and the dashed line shows the graph from SIR epidemic model.

At the onset, it is clear seen that the data of the smallpox infected population is completely fitted with the SIR model infected population from 1950 to 1975 and the SIR epidemic model predict in the next five years.

We examine numerically the behavior of the SIR epidemic model in a neighborhood of the corresponding parameters (Table-1**)** and the data set (Table-5). Results reveal that the curves of the data are completely fitted and/or consistent with the modified SIR model.

The SIR epidemic model represents a finite number of graphs and is burning through a bounded supply of removal populations. Removal population can never be re-infected and therefore it must come to an end after a relatively small number of steps.

Table 4. Year wise total population and Smallpox victims and deaths, 1950-1977

Year	Total Population	Cases	Deaths
1950	42284000	21273	9445
1951	42423000	38871	30341
1952	43233000	10490	8053
1953	44059000	1102	788
1954	44900000	445	188
1955	45758000	1926	979
1956	46632000	4962	3170
1957	47522000	24920	18149
1958	48430000	79060	58891
1959	49355000	15048	9508
1960	51585000	1905	1271
1961	55222700	660	469
1962	55961000	610	379
1963	57429000	3735	2847
1964	58936000	69	42
1965	60482000	316	142
1966	61937000	3207	1491
1967	63428000	6648	3455
1968	64954000	9039	4024
1969	66517000	1925	597
1970	66671000	1473	502
1971	69774000	not available	not available
1972	71471000	10754	3262
1973	73210000	32711	9354
1974	76398000	16485	2780
1975	76617000	13798	2492
1976	80815000	0	0
1977	82560000	0	0

(Source: Lahmeyer [26])

Table 5. Year wise total, susceptible, infected and removal population

Year	Total Population	Susceptible	Infected	Removal
1950	0.55188796220160	0.55148703290392	0.00027765378441	0.00012327551326729
1951	0.55370218097811	0.55279883054675	0.00050734171267	0.00039600871869167
1952	0.56427424722973	0.56403222522417	0.00013691478393	0.00010510722163489
1953	0.57505514441965	0.57503047626506	0.00001438323088	0.00001028492371145
1954	0.58603182061422	0.58602355874023	0.00000580811047	0.00000245376352507
1955	0.59723037968075	0.59719246381351	0.00002513802420	0.00001277784303745
1956	0.60863776968558	0.60853163136119	0.00006476369474	0.00004137462965138
1957	0.62025399062871	0.61969185689860	0.00032525418641	0.00023687954370440
1958	0.63210514637744	0.63030461907932	0.00103188587389	0.00076864142422700
1959	0.64417818499811	0.64385768171555	0.00019640549747	0.00012409778508686
1960	0.67328399702416	0.67324254408291	0.00002486393359	0.00001658900766149
1961	0.72076301604083	0.72074828040774	0.00000861427621	0.00000612135687902
1962	0.73039925865017	0.73038635028780	0.00000796167952	0.00000494668285106
1963	0.74955949723952	0.74947358941227	0.00004874897216	0.00003715885508438
1964	0.76922876124098	0.76922731247634	0.00000090058342	0.00000054818121305
1965	0.78940705065455	0.78940107286895	0.00000412441103	0.00000185337457744
1966	0.80839761410653	0.80833629612227	0.00004185755120	0.00001946043306316
1967	0.82785804716969	0.82772618348408	0.00008676925487	0.00004509443073991
1968	0.84777529791039	0.84760480050119	0.00011797642821	0.00005252098098333
1969	0.86817547019591	0.86814255321926	0.00002512497226	0.00000779200438545
1970	0.87018546797708	0.87015969040813	0.00001922549826	0.00000655207068927
1971	0.910685618	.8900000000	.00002000	.00000700
1972	0.93283474946813	0.93265181356618	0.00014036049441	0.0425754075466
1973	0.95553206207500	0.95498303248626	0.00042694180143	0.00012208778730569
1974	0.99714162653197	0.99692911494838	0.00021516112612	0.00003628437553023
1975	1.00000000000000	0.99978738400094	0.00018009058042	0.00003252541864077
1976	80815000	0		0
1977	82560000	0		0

4. Conclusions

The SIR model is used in the modeling of infectious diseases by computing the amount of infected population at a given period of time. The model is also used by researchers and health workers to explain the increase and decrease in people needing medical care for infectious diseases in the period of epidemic and pandemic levels. From numbers that generated by the SIR model researchers and health workers can calculate different numbers that will help to develop health policies and disease monitoring plan. Thus predictability of the next season's hight of the infectious disease peak on the basis of SIR epidemic model would be of major practical use. It is to be understood as the general health awareness among the people and also because of the measure taken by the authority's in-regards to the epidemic spread of the diseases.

In addition to an increased incidence rate, the etiology of smallpox shows how the disease's negative impacts are greater in children and pregnant women. For example a bacterial infection of the skin can occur after scratching the smallpox rash and in the most severe cases to death. Pregnant

women, newborn infants, children whose age thirteen years and younger, and people who have weakened immune systems are more likely to have a severe case of smallpox and suffer more complications (Dorset and Somerset [27]). This model can be used to develop health policy and smallpox monitoring plan by disseminating the estimates of infection free and most critical periods. It can be also used to determine the intensive and continuous vaccination programme for reducing the number of infective population throughout the epidemic and pandemic levels. In conclusion, the trend for the smallpox disease as obtained in this model is interesting and showed significant variation over time. As a results health policies must be adjusted with it real needs and essential implication should also very over time. Further research should focus on this issue for an extended period of time and/or apply the model to other epidemics data from the developing countries.

Table 6. Normalization total, susceptible, infected and removal population

Year	Total Population	Susceptible	Infected	Removal
1950	42284000	42253282	21273	9445
1951	42423000	42353788	38871	30341
1952	43233000	43214457	10490	8053
1953	44059000	44057110	1102	788
1954	44900000	44899367	445	188
1955	45758000	45755095	1926	979
1956	46632000	46623868	4962	3170
1957	47522000	47478931	24920	18149
1958	48430000	48292049	79060	58891
1959	49355000	49330444	15048	9508
1960	51585000	51581824	1905	1271
1961	55222700	55221571	660	469
1962	55961000	55960011	610	379
1963	57429000	57422418	3735	2847
1964	58936000	58935889	69	42
1965	60482000	60481542	316	142
1966	61937000	61932302	3207	1491
1967	63428000	63417897	6648	3455
1968	64954000	64940937	9039	4024
1969	66517000	66514478	1925	597
1970	66671000	66669025	1473	502
1971	69774000	Not available	Not available	Not available
1972	71471000	71456984	10754	3262
1973	73210000	73167935	32711	9354
1974	76398000	76381718	16485	2780
1975	76617000	76600710	13798	2492
1976	80815000	0	0	0
1977	82560000	0	0	0

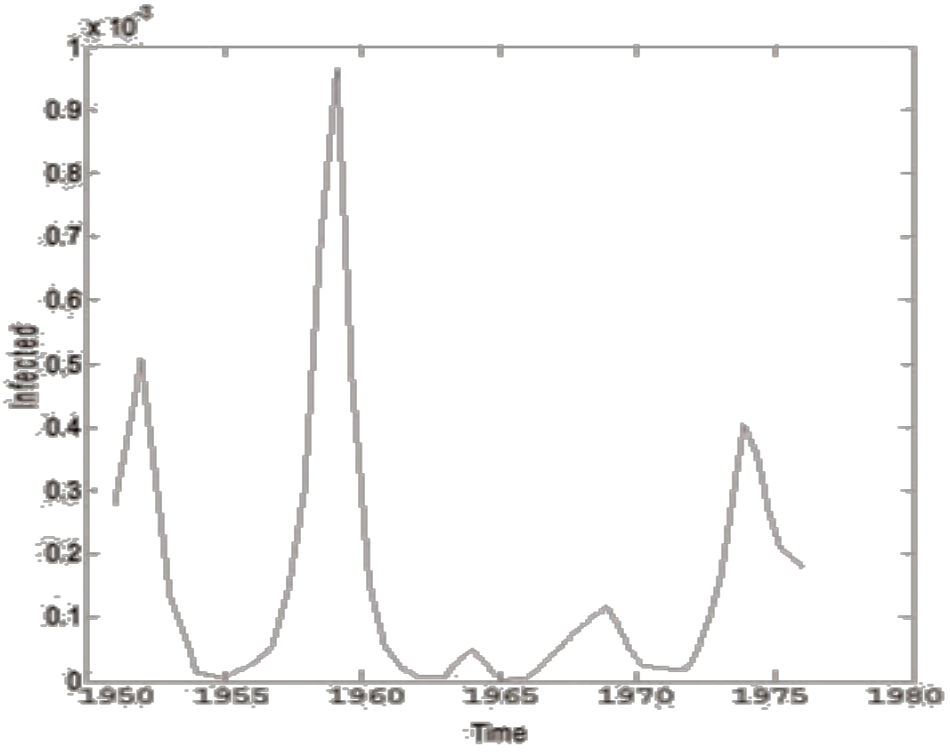

Figure 3. The infectivity curve shows the variation of infectives of smallpox with time (year).The graph is plotted against data

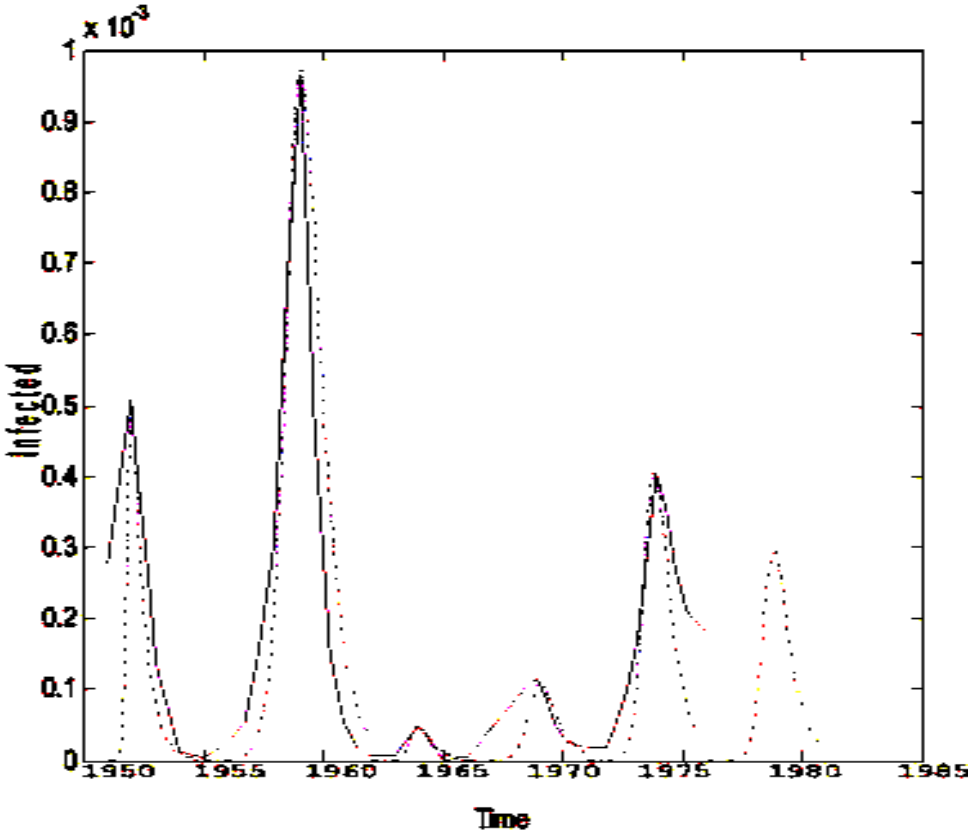

Figure 4. Comparison of infectively curves showing the variation of infectivity of smallpox with the time (year) as obtained from data (solid line) and from modified SIR model (dashed line)

REFERENCES

[1] Peters, Epidemic! Smallpox in the new world. Benchmark books, Marshall Cavendish Publishing, Tarytown, NY, 2005.

[2] Tucker, The once and future threat of smallpox, Atlantic monthly press, New York, NY, 2001.

[3] WHO, What is smallpox and how does the clinical presentation differ from chickenpox, 1977. http://www.examiner.com/article/what-is-smallpox-and-how-does-the-clinical-presentation-differ-from-chickenpox (accessed on 05/04/2014).

[4] J. N. Kapur, Mathematical models in biology and medicine, Affiliated East- West Press (P) Ltd., New Delhi, India, 1979.

[5] A. Rahman and A. Kuddus, A new model to study on physical behavior among Susceptible Infective Removal population, Far East Journal of Theoretical Statistics, 46:(2) (2014), 115-135.

[6] M. Derouich, A. Boutayeb and E. H. Twizell, A model of dengue fever, BioMedical Engineering Online 2(4) (2003), 1-10. http://www.biomedcentral.com/content/pdf/1475-925x-2-4.pdf

[7] A. Rahman and S. Chowdhury, Determinants of chronic malnutrition among preschool children in Bangladesh, Journal of Biosocial Science, 39(2) (2007), 161-173.

[8] A. Rahman, S. Chowdhury, A. Karim and S. Ahmed, Factors associated with nutritional status of children in Bangladesh: A multivariate analysis. Demography India, 37(1) (2008). 95-109.

[9] A. Rahman, S. Chowdhury and D. Hossain, Acute malnutrition in Bangladeshi children: levels and determinants. Asia-Pacific Journal of Public Health, 21(3) (2009), 294-302.

[10] A. Rahman and S.C. Biswas, Nutritional status of under-5 children in Bangladesh. South Asian Journal of Population and Health, 2(1) (2009), 1-11.

[11] A. Rahman, Small Area Housing Stress Estimation in Australia: Microsimulation Modelling and Statistical Reliability. PhD thesis, University of Canberra, (2011).

[12] B. Sally, A. Gormez, A. Rahman, and colleagues, Jurisdictional, socioeconomic and gender inequalities in child health and development: Analysis of a national census of 5 year olds in Australia. BMJ Open, 2(5,e001075) (2012), 1 – 15.

[13] A. Rahman, A. Harding, R. Tanton and S. Liu, Methodological issues in spatial microsimulation modelling for small area estimation. International Journal of Microsimulation, 3(2) (2010), 3-22.

[14] A. Rahman, A. Harding, R. Tanton and S. Liu, Simulating the characteristics of populations at the small area level: New validation techniques for a spatial microsimulation model in Australia. Computational Statistics and Data Analysis, 57(1) (2013), 149-165.

[15] A. Rahman and A. Harding, Social and health costs of tobacco smoking in Australia: Level, trend and determinants. International Journal of Statistics and Systems, 6(4) (2011), 375-387.

[16] A. Rahman and A. Harding, Prevalence of overweight and obesity epidemic in Australia: some causes and consequences. JP Journal of Biostatistics, 10(1) (2013), 31-48.

[17] K. H. Yang and J. Y. Hsu, A new SIR based model for influenza epidemic. World Academic of Science, Engineering and Technology 67 (4) (2012), 374-379.

[18] J. Satsuma, R. willox, A. Ramani, B. Grammaticos and A. S. Carstea, Extending the SIR epidemic model, Physics A: Statistical and Theoretical Physics, 336 (4) (2004), 369-375.

[19] Z. Zhang, Z. Jin and J. Pan, An SIR epidemic model with nonlinear birth pulses, dynamics of continuous, discrete and impulsive systems series B: Applications and Algorithms 14(2) (2008), 111-128.

[20] A. Yildrim and Y. Cherruault, Analytical approximate solution of a SIR epidemic model with constant vaccination strategy by homotopy perturbation method, Kybernets 38 (9) (2009), 1566-1575.

[21] A. Kaddar, Stability analysis in a delayed SIR epidemic model with a satarated incidence rate, Nonlinear Analysis: Modeling and Control 15 (3) (2010), 299-306.

[22] B. Shulgin, L. Stone and Z. Agur, Pulse vaccination strategy in the SIR epidemic model, Bulletin of Mathematical Biology, 60(6) (1998), 1123-1148.

[23] A. John and P. O'Neill, Reproduction number and thresholds in stochastic epidemic model, Mathematical Bioscience 107 (2) (1991), 161-186.

[24] D. Smith and L. Moore, The SIR model for spread of disease, Journal of Online Mathematics and its Applications 1 (3) (2001), 1-9.

[25] Banglapedia, Smallpox victims and deaths from 1950 to 1977 (2012). http://www.banglapedia.org/HT/P_0244.HTM (accessed on 13/05/2014).

[26] J. Lahmeyer, Historical demographical data of the whole country of Bangladesh.http://www.populstat.info/Asia/bangladc.htm.

[27] Dorset and Somerset Health Protection Unit, Information leaflets for common childhood infection conditions (2010).