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Spectral Analysis of Biochemical Oxygen Demand in River Water: An Analytical Approach of Discrete Wavelet Transform

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Abstract Excess of biochemical oxygen demand (BOD) is the most outstanding danger at present for survival of living being in the surface water. In this paper, we analyse the BOD of surface water using wavelet analysis. Wavelet analysis is a new mathematical tool developed in last two decade. Wavelets have special ability to examine signals or data sets simultaneously in both time and frequency scale. Consequently, we get hidden information of signal in a new plane which is called time frequency plane. In the present manuscript we interpret the data of BOD of river Ramganga by wavelet method. We used 'db8' wavelet of Daubechies wavelet family. The data of BOD is taken during the period from Jun 2005 to May 2008.

Keywords Daubechies wavelet, River Ramganga, Time frequency plane, Wavelet analysis

1. Introduction

To conserve and manage lotic water resources efficiently is very critical. A holistic approach should be adopted for proper management of the aquatic resources especially running water. As a result of anthropogenic activities along the river bed results degradation of water quality and ultimately affects the aquatic ecosystem. The physical, chemical and biological components of the aquatic system should be monitored at regular basis to understand their function and relationship. Due to economic activity in river Ramganga flood plain. The water get polluted. Water quality indicators consists of physical, chemical and biological variables designed to provide clear signals about the status and changes of the system (Alegue and Gnauck, 2006) [1]. Some of the variables such as biochemical oxygen demand (BOD) act as stressor. Signal analysis method enable the extraction of information projecting in the time frequency and time scale domains (Mallat, 1998) [2]. Using discrete wavelet transformation method of Daubechies wavelet family [3, 4] in studying the variations, the structural characteristics of BOD at different time scale reveals information necessary in the diagnosing of water quality pollution and selecting an appropriate management strategy for pollution control.

In India a number of studies have been made to describe the hydrochemistry of several stream and rivers. Kulwinder Singh Parmar et al. (2012) [5] studied the water quality parameters by using Daubechies wavelet (db5). Also the impacts of hydrological conditions of the water on biological community of the water body have been documented (Pathak, 2008) [6]. In India as a developing country, industrial pollution is one of the main causes of water pollution has been investigated in several major rivers. The necessity to efficiently conserve and manage freshwater resources is becoming more and more urgent. This is as a result of growing world population and economic activities with the subsequent degradation of freshwater resources as a result of anthropogenic pollution. To sustainable management of freshwater ecosystems an understanding of the basic physical, chemical and biological components, their functions and interrelationship is necessary. Wavelet is a new tool in the emerging field of data analysis for Physicists, Engineers Mathematicians and environmentalists [7, 8, 9]. It represents an efficient computational algorithm under the interest of a broad community. Fourier sine's extracts only frequency information from a time signal, thus losing time information, while wavelet extracts both time evolution and frequency composition of a signal.

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2. Methodology

Wavelet analysis allows to isolate and manipulate specific types of patterns hidden in masses of data. Alfred Haar, a Hungarian Mathematician was the first who introduced some functions in 1909. These functions are Haar wavelets. From time to time over the next several decades, other precursors of wavelet theory arose. In 1987 Ingrid Daubechies discovered a whole new class of wavelets which is known as 'Daubechies Wavelets'. The Daubechies wavelets turn the theory into a practical tool that can be easily programed and used with a minimum of mathematical training.

2.1. Discrete Wavelet Transform (DWT) and Multiresolution Analysis (MRA)

Wavelets are a special kind of functions which exhibits oscillatory behavior for a short period of time and then die out. For any two real numbers *a* and *b*, a wavelet function is defined as:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right) \quad (1)$$

where *a* is scaling parameter which gives the dilate or compressed version of wavelet function.

And *b* is a parameter gives a translated version of wavelet function called translation parameter. If we choose $a = 2^{-j}$ and $b/a = k$, we get discrete wavelet

$$\psi_{j,k}(t) = 2^{j/2}\psi\left(2^j t - k\right) \quad (2)$$

The wavelet transform of a signal captures the localized time frequency information of the signal. A multi resolution analysis (MRA) is a radically new recursive method for performing discrete wavelet analysis. A MRA for $L^2(\mathbb{R})$ introduced by Mallat [10, 11] and extended by other researchers [12, 13, 14, 15] consists of a Sequence $V_j : j \in \mathbb{Z}$ of closed subspaces of $L^2(\mathbb{R})$, a space of square integrable functions, satisfying following properties;

1. $V_{j+1} \subset V_j : j \in \mathbb{Z}$
2. $\bigcap_{j\in\mathbb{Z}} V_j = \{0\}$, $\overline{\bigcup_{j\in\mathbb{Z}} V_j} = L^2(\mathbb{R})$
3. For every $L^2(\mathbb{R})$, $f(t) \in V_j \Rightarrow f\left(\frac{t}{2}\right) \in V_{j+1}, \forall\, j \in \mathbb{Z}$
4. There exists a function $\phi(t) \in V_0$ such that $\{\phi(t-k) : k \in \mathbb{Z}\}$ is orthonormal basis of V_0.

The function $\phi(t)$ is called the scaling function of the given MRA and property 3 implies a dilation equation

$$\phi(t) = \sqrt{2}\sum_{k\in\mathbb{Z}} h_k\, \phi(2t-k)$$

where h_k is a low pass filter and defined as

$$h_k = \left(\frac{1}{\sqrt{2}}\right)\int_{-\infty}^{\infty} \phi(t)\overline{\phi(2t-k)}\,dt$$

Now consider W_0 be the orthogonal compliment of V_0 in V_1 i.e.,

$$V_1 = V_0 \oplus W_0 \quad (3)$$

If $\psi \in W_0$ be any function then

$$\psi(t) = \sqrt{2}\sum_{k\in\mathbb{Z}} g_k\, \phi(2t-k)$$

where $g_k = (-1)^{k+1} h_{1-k}$ are high pass filters.

We can express a signal in terms of bases of V_0 space and W_0 space. If we combine the bases of V_0 and W_0 space, we can express any signal in V_1 space.

Using the same argument, we can write

$$V_2 = V_1 \oplus W_1$$

In general

$$V_j = V_{j-1} \oplus W_{j-1}$$

But $V_{j-1} = V_{j-2} \oplus W_{j-2}$

Therefore

$$V_j = W_{j-1} \oplus W_{j-2} \oplus V_{j-2}$$

.. ..

$$V_j = W_{j-1} \oplus W_{j-2} \oplus W_{j-3} \oplus \ldots\ldots \oplus W_0 \oplus V_0 \quad (4)$$

Let $S = \{S_n : n \in \mathbb{Z}\}$ be a function sampled at regular time interval $\Delta t = \tau$, where $\mathbb{Z}$ is an integer . S is split into a "blurred" version a_1 at the coarser interval $\Delta t = 2\tau$ and "detail" d_1 at scale $\Delta t = \tau$. This process is repeated and gives a sequence S, a_1, a_2, a_3, a_4,…..of more and more blurred versions together with the details d_1, d_2, d_3, …….removed at every scale ($\Delta t = 2^m \tau$ in a_m and d_{m-1}). Here $a_{m'S}$ and $d_{m'S}$ are approximation and details of original signal. After N iteration S can be reconstructed as $S=a_N+d_1+d_2+d_3+d_4+d_5+\ldots+d_N$. The approximations are the high-scale, low-frequency components of the signal. The details are the low-scale, high-frequency components. Thus the original signal, S, passes through two complementary filters in which one is low pass filter and second one is high pass filter.

2.2. Daubechies Wavelet 8

The Daubechies wavelets are compactly supported orthogonal wavelets. The Daubechies family is named after Ingrid Daubechies who invented the compactly supported orthonormal wavelet, making wavelet analysis in discrete time possible. The order of the Daubechies functions denotes the number of vanishing moments. Higher order Daubechies functions are not easy to describe with an analytical expression. A vanishing moment limits the wavelet's ability to represent polynomial behavior or information in a signal. The wavelet and scaling functions for the Daubechies functions with order 8 are shown in figure 1 and figure 2.

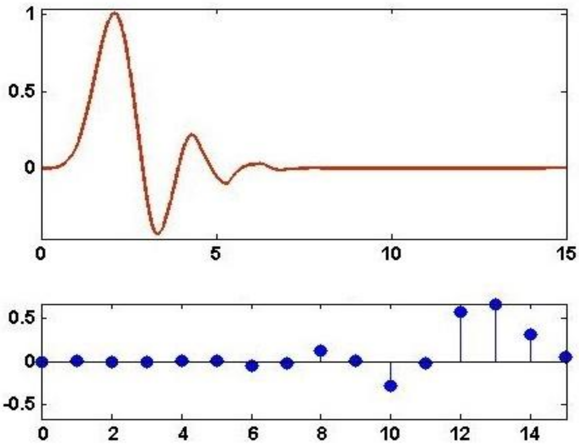

Figure 1. Scaling function (upper) and decomposition low pass filter (lower) of Daubechies wavelet of order 8 (“db8”)

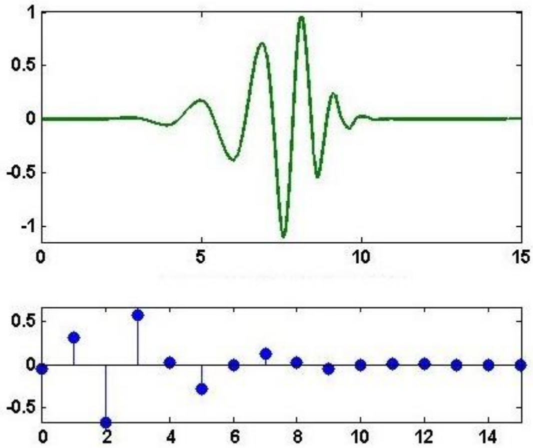

Figure 2. Wavelet function (upper) and decomposition high pass filter (lower) of Daubechies wavelet of order 8 (“db8”)

3. Study Area

River Ramganga, is the most important tributary of holy river Ganga, is spring fed originated from the southern slopes of Dudhatoli (3,110 masl) of middle Himalaya of Uttrakhand state. The river enters the plains at Kalagarh where a famous hydroelectric earthen dam has been constructed in 1975. The river traverses near about 158 km before it meets the reservoir and continues to downstream for about 370 km before joining river Ganga at Kannauj of Uttar Pradesh state. The study area of the river catchment lies between north latitude 29°29’42” and 28°49’32” and east longitude 78°45’37” and 78°47’53. The river navigates through varied catchments covered with forestland, agriculture field and human settlements with various types of industrial setups. The catchment area in downstream is inhabited on alluvial deposits of Quartz energy period. The alluvial deposits has been drained and transported from the Himalayas ranges by the Ganga river system and specially the river Ramganga. In order to carry out in depth investigation, nine sampling stations at different segments of river Ramganga were selected on the basis of varied topographical conditions, agricultural, social pattern and on the locations of various large and small-scale industries and also on the basis of human settlement.

Table 1. Sampling Spots in downstream river Ramganga

1	SS1	Kalagargh
2	SS2	Bhutpuri
3	SS3	Seohara
4	SS4	Mishripur
5	SS5	Agwanpur
6	SS6	Jigar colony
7	SS7	Lalbhagah
8	SS8	Jamamasjid
9	SS9	Kathgargh

4. Result and Discussion

In order to perform a more detailed investigation we decompose the time series of BOD at different scale by using discrete wavelet transform. The original time series of BOD is shown in figure 3. A full decomposition of BOD time series is shown in figure 4(a) and figure 4(b). In decomposition, figures a_1, a_2, a_3, a_4, a_5, a_6, a_7 and a_8 are approximations and d_1, d_2, d_3, d_4, d_5, d_6, d_7, and d_8 are details of the of BOD time series at different time mode.

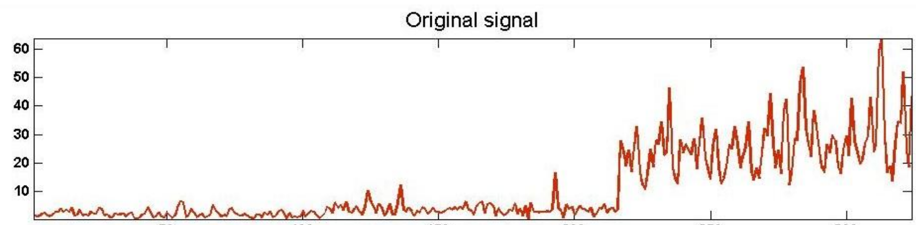

Figure 3. Wavelet decomposition of BOD time series. Original time series **s** and its approximations

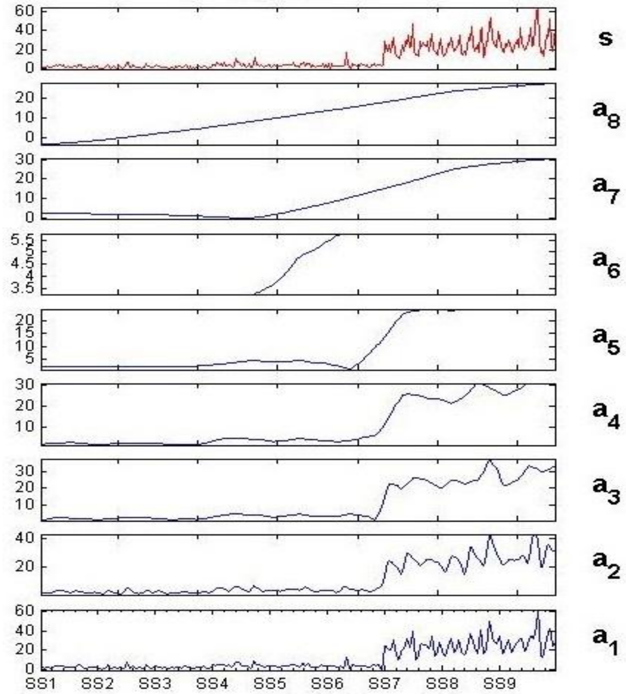

Figure 4(a). Wavelet decomposition of BOD time series. Original time series **s** and its approximations

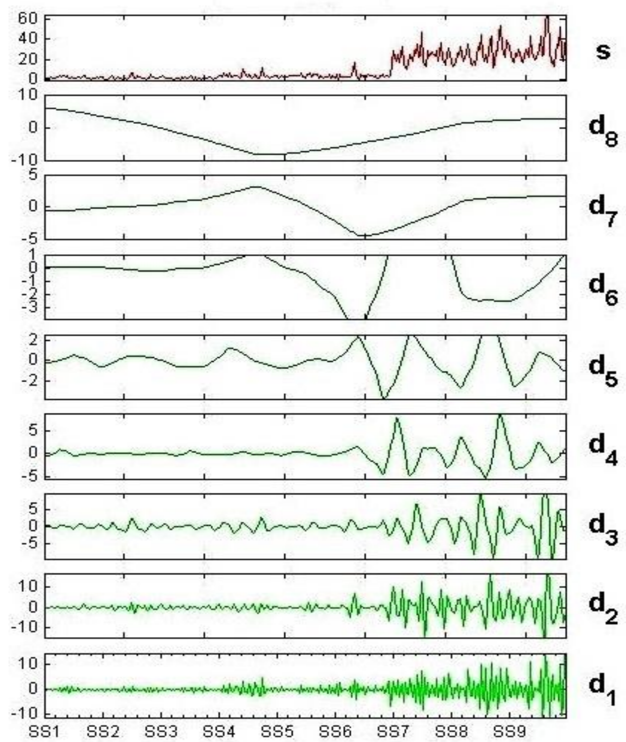

Figure 4(b). Wavelet decomposition of BOD time series. Original time series **s** and its details

Original signal S is taken as monthly interval. Approximation a_1 represents the bi-monthly behavior of the signal S, a_2 represents quarterly behavior of the signal, a_3 is bi-quarterly or 8-months mode behavior, a_4 is the behavior of signal in 16-months mode and so on. Likewise details d_1 exhibits monthly variation, d_2 shows bi-monthly variation and d_3 is quarterly variation in the value of BOD also d_4, d_5, d_6, d_7 and d_8 are variation in the value of BOD in 8, 16, 32, 64, and 128 months mode, respectively. A peak in a detail shows rapid fall or rise in the value of BOD in that time mode. The measurement of BOD depends on anthropogenic driving forces. Fluctuation in downstream over time may be as a result of effluent load (Fig. 4(b)).

In signal analysis, the low-frequency content of the signal is an important part. Because it gives the identity of the signal. Trend is the slowest part of the signal means lowest frequency part of the signal. In wavelet analysis terms, this is correspond to the greatest scale value. In figure 4(a) it is corresponds to a_8. As the scale increases, the resolution decreases, producing a better estimate of the unknown trend. A trend of the BOD signal is exhibits in figure 5. It is clear that the value of BOD level is lie between 0-30 ppm during the time period from 2005 to 2008.

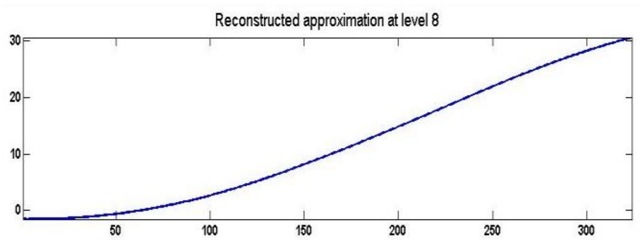

Figure 5. Trend of BOD in downstream of river Rāmgangā

Figure 6 depicts the histograms for Approximation a_5. A cumulative histogram is a mapping that counts the cumulative number of observations in all of the bins up to the specified bin. The histogram provides important information about the shape of a distribution. A variance in the value of BOD during three years in individual sampling spot is clearly depicts in approximation a_5 of the decomposition. It is clear that the BOD level is minimum in Sampling spots Kalagargh, Bhutpuri, Seohara, Mishripur, Awganpur, varied between 3-10 ppm, while in sampling spots Jamamasjid and Kathgargh shows maximum BOD level.

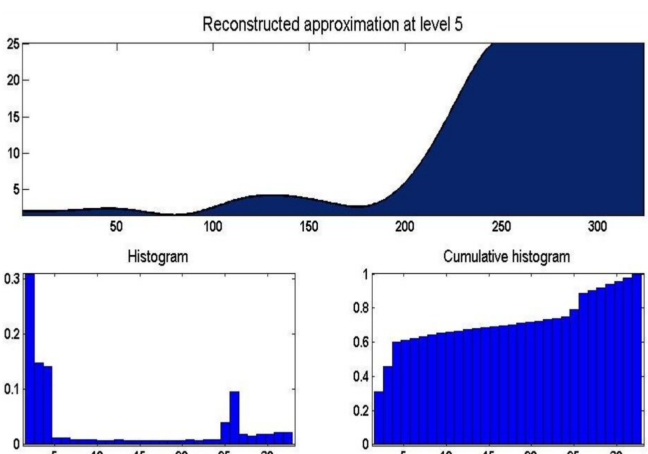

Figure 6. Approximation a5 (upper) and its histogram and cumulative histogram (lower)

Histogram of approximation a5 shows that 60 percent water of river is below the prescribe limit of BOD by WHO and Indian Standards. In other words, 40 percent of river segment is highly polluted.

A comparative statistics of BOD time series in different time modes is shown in Table 2 and Table 3.

Table 2. Statistical analysis of BOD time series at different time mode

S. N.	Time mode	level	Mean	Median	Mode	Standard deviation
1	Monthly	S	10.66	3.84	3.40	12.48
2	Bi-monthly	a_1	10.67	3.78	2.76	12.04
3	Quarterly	a_2	10.67	3.79	2.66	11.47
4	Bi-quarterly	a_3	10.64	4.04	1.82	11.09
5	16-months mode	a_4	10.66	3.93	2.05	11.04
6	32-months mode	a_5	10.69	3.93	2.04	10.93
7	64-months mode	a_6	10.81	3.04	2.29	10.95
8	128-months mode	a_7	10.79	3.92	2.03	10.93
9	256-months mode	a_8	11.47	9.74	1.01	10.59

Table 3. Statistical analysis of variation in BOD time series at different time mode

S.N.	Time mode	level	Max	Min	Mode
1	Monthly variation	d_1	**16.34**	**-14.88**	0.191
2	Bi-monthly variation	d_2	**14.46**	**-18.75**	-0.483
3	Quarterly variation	d_3	8.36	-8.75	0.098
4	Bi-quarterly variation	d_4	5.96	-5.88	0.240
5	16-months mode variation	d_5	6.42	-5.36	0.341
6	32-months mode variation	d_6	4.89	-3.74	0.147
7	64-months mode variation	d_7	1.32	-0.91	0.019
8	128-months mode variation	d_8	4.32	-5.94	-5.769

Table 2 explains the statistic of original time series and its approximations in different time mode. A high value of standard deviation indicates that the data points are spread out in a wide range. Because of high standard deviation, data is spread widely in all time mode. Table 3 tells about the amplitude of the change. Large value of amplitude indicates that there is a rapid change in the value of BOD in that time interval. According to Table 3, details d_1 and d_2 have a large Maximum of 16.34, 14.46) and Minimum of -14.88, -18.75. It reveals that monthly and bi-monthly variation is relatively high.

A frequency distribution of the variation in the value of BOD is shown in fig.7. It is clear from the frequency distribution that the sharp variations have a small contribution as compared with the low variations at all scale. From the analysis presented above we note that the monthly and bi- monthly variation in the value of BOD of river Ramganga is relatively high. Which is supported by the decomposition of time series mode as presented in figure 4b and Table 3. However, we note that the anthropogenic activity has play an important role in increasing BOD level in the water system in this parameter.

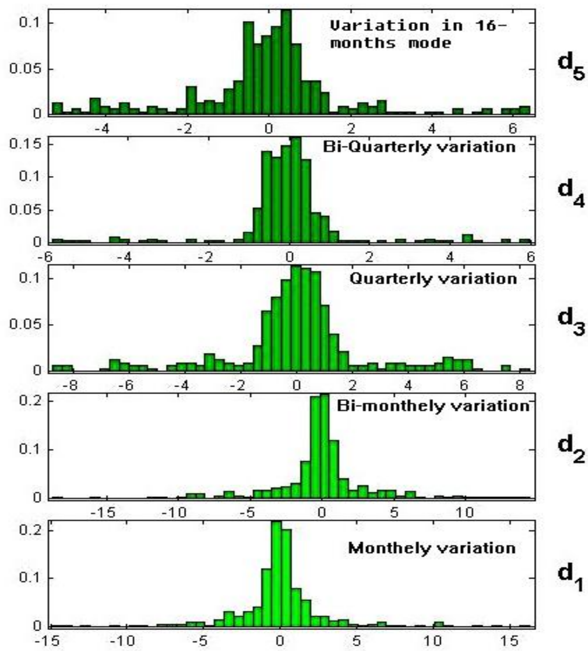

Figure 7. Frequency distribution of variation in BOD time series at different scale

5. Conclusions

Signal analysis of BOD a stressor indicator by Daubechies wavelet (db 8 level 8) make it possible to quantify the variations on a particular time frame and variations and interrelationships existing between natural and anthropogenic interferences of water quality indicators. The wavelet method allows the decomposition of signal according to different frequency levels which characterize the intensity of natural and man-made disturbances.

A histogram is a graphical representation showing a visual impression of the distribution of data. According to the behaviour studied, it is possible to conjecture that the difference between all-time series can be directly associated with the large number of anthropogenic activity. Taking into account these results we have shown, the wavelet analytical approach provides a simple and accurate framework for modelling the statistical behaviour of BOD turbulence.

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