

On the Efficiency of Some Modified Ratio and Product Estimators – The Optimal C_x Approach

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Abstract In this paper, an optimal estimator for estimating the population mean is proposed. This is achieved by minimizing the coefficient of variation (C_x) of the auxiliary variable in the Mean Square Error (MSE) from the existing estimators. Using well analyzed data to illustrate the procedure for both the Ratio and Product estimators, a minimum of 10 percent reduction in the MSE were observed from each of the existing estimators considered. The proposed optimal estimators is uniformly better than all other estimators and thus most preferred over the existing modified ratio and product estimators for the use in practical applications for certain population with peculiar characteristics

Keywords Ratio, Product, Estimator, Parameter, Mean Square Error

1. Introduction

Ratio estimation involves the use of known population totals for auxiliary variables to improve the weighting from sample values to population estimates of interest. It operates by comparing the sample estimate for an auxiliary variable with the known population total for the same variable on the frame. The ratio of the sample estimate of the auxiliary variable to its population total on the frame is used to adjust the sample estimate for the variable of interest. The ratio weights are given by X/x (where X is the known population total for the auxiliary variable and x is the corresponding estimate of the total based on all responding units in the sample). These weights assume that the population for the variable of interest will be estimated by the sample equally as well (or poorly) as the population total for the auxiliary variable is estimated by the sample.

Consider a finite population $U = \{U_1, U_2, \dots, U_N\}$ of N distinct and identifiable units. Let Y be a study variable with value Y_i measured on U_i , $i = 1, 2, 3, \dots, N$ giving a vector $Y = \{Y_1, Y_2, \dots, Y_N\}$. The problem is to estimate the

population mean $\bar{y} = \frac{1}{N} \sum_{i=1}^N Y_i$ with some desirable

properties on the basis of a random sample selected from the population U . The simplest estimator of population mean is the sample mean obtained by using simple random sampling without replacement, when there is no additional information on the auxiliary variable available. Sometimes in sample surveys, along with the study variable Y , information on

auxiliary variable x correlated with Y is also collected. This information on auxiliary variable x , may be utilized to obtain a more efficient estimator of the population mean.

The two broad categories of estimators using auxiliary information are the ratio method and product method of estimation.

Among those who have worked on these estimators are Sisodia and Dwivedi[12], Pandey and Dubey[7], Singh, Taylor[10], Adewara and Singh[2]. The study is motivated by the success recorded by J. Subramani and G. Kumarapandian[8] on their respective works on “Modified ratio estimators using known median and coefficient of Kurtosis” and “Efficiency of some modified ratio and product Estimators using known value of some population parameters”.

This work is focused on improving the efficiency of some Ratio and Product estimators in the literature by obtaining the optimal values of coefficient of variation of the auxiliary variable C_x in the MSE and then substitute back with some re-arrangement to obtain an improved estimate of the estimator and their respective MSE.

2. Literature Review

Sen., A.R.[9] presented an historical development of the ratio method of estimation starting from the year 1662. Auxiliary information is any information closely related to the study variable. The use of auxiliary information usually leads to the sampling strategy with higher efficiency compared to those in which no auxiliary information is used. Higher precision can also be achieved by using the auxiliary information for the dual purposes of selection and the estimation procedure.

It is important to note that proper use of the knowledge of

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auxiliary information may result in appreciable gain in precision of the estimates. But indiscriminate use of auxiliary information might not provide the desired precision and in some extreme cases might even lead to loss in precision.

Many authors since the discovery of ratio method of estimation have come up with various degree of modification of the conventional ratio estimations for better performance. These among others include Sisodia and Dwivedi[12], Singh and Tailor[10], Pandey and Dubey[7], Adewara and Singh[10] among others.

3. Methodology

3.1. Existing Modified Ratio and Product Estimators

Suppose a pairs (x, y) (i=1, 2, ..., n) observations are taken on n units sampled from N population units using simple random sampling without replacement scheme, \bar{X} and \bar{Y} are the population means for the auxiliary variable (x) and variable of interest (y) and \bar{x} and \bar{y} are the sample means based on the sample drawn.

Khoshnevison et al[5] defined their family of estimators as:

$$t = \bar{y} \left[\frac{a\bar{x} + b}{\alpha(a\bar{x} + b) + (1-\alpha)(a\bar{x} + b)} \right]^g$$

where $\alpha \neq 0$, b are either real numbers or a functions of the known parameters of the auxiliary variable x such as standard deviation σ_x , coefficient of variation, C_x , Skewness $\beta_{1(x)}$, Kurtosis $\beta_{2(x)}$ and correlation coefficient ρ .

i. When $\alpha=1$ a=1, b= 0, g=0, we have the usual ratio estimator, $\theta_0 = \bar{y}$ with

$$MSE(\theta_0) = \frac{N-n}{Nn} \bar{Y}^2 C_y^2 \quad (3.1)$$

ii. When $\alpha=0$ a=1, b= 0, g=1, we have the usual ratio estimator, $\theta_1 = \bar{y} \frac{\bar{X}}{\bar{x}}$ with

$$MSE(\theta_1) = \frac{N-n}{Nn} \bar{Y}^2 (C_y^2 + C_x^2 - 2\rho C_x C_y) \quad (3.2)$$

iii. When $\alpha=1$ a=1, b= 0, g=-1, we have the usual product estimator, $\theta_2 = \bar{y} \frac{\bar{x}}{\bar{X}}$ with

$$MSE(\theta_2) = \frac{N-n}{Nn} \bar{Y}^2 (C_x^2 + C_y^2 + 2\rho C_x C_y) \quad (3.3)$$

iv. When $\alpha=1$ a=1, b= C_x , g=1, Sisodia and Dwivedi[12]

ratio estimator $\theta_3 = \bar{y} \left(\frac{\bar{x} + C_x}{\bar{X} + C_x} \right)$ with

$$MSE(\theta_3) = \frac{N-n}{Nn} \bar{Y}^2 \left[C_y^2 + \left(\frac{\bar{x}}{\bar{X} + C_x} \right)^2 C_x^2 - 2 \frac{\bar{x}}{\bar{X} + C_x} \rho C_x C_y \right] \quad (3.4)$$

v. When $\alpha=1$ a=1, b= C_x , g=1, we have Pandey and Dubey[7] product estimator with $\theta_4 = \bar{y} \left(\frac{\bar{X} + C_x}{\bar{x} + C_x} \right)$ with

$$MSE(\theta_4) = \frac{N-n}{Nn} \bar{Y}^2 \left[C_y^2 + \left(\frac{\bar{x}}{\bar{x} + C_x} \right)^2 C_x^2 + 2 \frac{\bar{x}}{\bar{x} + C_x} \rho C_x C_y \right] \quad (3.5)$$

vi. When $\alpha=1$ a=1, b= ρ , g=1, we have Singh, Taylor[10]

ratio estimator as $\theta_5 = \bar{y} \left(\frac{\bar{X} + \rho}{\bar{x} + \rho} \right)$ with

$$MSE(\theta_5) = \frac{N-n}{Nn} \bar{Y}^2 \left[C_y^2 + \left(\frac{\bar{x}}{\bar{x} + \rho} \right)^2 C_x^2 - 2 \frac{\bar{x}}{\bar{x} + \rho} \rho C_x C_y \right] \quad (3.6)$$

vii. When $\alpha=1$ a=-1, b= ρ , g=1, we have Singh, Taylor[10]

product estimator $\theta_6 = \bar{y} \left(\frac{\bar{x} + \rho}{\bar{X} + \rho} \right)$ with

$$MSE(\theta_6) = \frac{N-n}{Nn} \bar{Y}^2 \left[C_y^2 + \left(\frac{\bar{X}}{\bar{X} + \rho} \right)^2 C_x^2 + 2 \frac{\bar{X}}{\bar{X} + \rho} \rho C_x C_y \right] \quad (3.7)$$

It should be noted that there are other ratio and product estimators from the family mentioned above, but here attention is focused on those estimators that uses the coefficient of variation C_x and correlation coefficient ρ . Conventionally, for ratio estimators to hold, $\rho > 0$ and also for product estimators to hold, $\rho < 0$.

3.2. Proposed Ratio and Product Estimators

The proposed optimal estimators were obtained by minimizing the $MSE(.)$ from each of the existing estimators with respect to C_x and equate to zero. The solution for C_x is then substituted back to initial $MSE(.)$ to get the modified optimal estimators and corresponding $MSE(.*.)$ as follows in equations 3.8 to 3.12 below:

$$i. \theta_1^* = \bar{y} \frac{\bar{X}}{\bar{x}}$$

$$MSE(\theta_1^*) = \frac{N-n}{Nn} \bar{Y}^2 C_y^2 (1 - \rho^2) \quad (3.8)$$

$$ii. \theta_2^* = \bar{y} \frac{\bar{x}}{\bar{X}}$$

$$MSE(\theta_2^*) = \frac{N-n}{Nn} \bar{y}^2 C_y^2 (1-\rho^2) \quad (3.9)$$

$$\text{iii. } \theta_3 = \bar{y} \left(\frac{\bar{X} + C_x}{\bar{X} + C_x} \right) \quad C_y = \frac{\bar{X} \rho C_x}{\bar{X} + C_x}$$

$$MSE(\hat{\theta}_3) = \frac{N-n}{Nn} \bar{Y}^2 C_y^2 \left\{ \frac{\bar{X}}{\bar{X} + C_x} \right\}^2 (1-\rho^2) \quad (3.10)$$

$$\text{iv. } \theta_4 = \bar{y} \left[\frac{\bar{x} + C_x}{\bar{X} + C_x} \right] \quad C_y = \left(\frac{\bar{X}}{\bar{X} + C_x} \right) \rho C_x$$

$$MSE(\hat{\theta}_4) = \frac{N-n}{Nn} \bar{Y}^2 C_x^2 \left(\frac{\bar{X}}{\bar{X} + C_x} \right)^2 (1-\rho^2)$$

$$\text{v. } \hat{\theta}_5 = \left\{ \frac{\bar{X} + \rho}{x + \rho} \right\}$$

$$MSE(\hat{\theta}_5) = \frac{N-n}{Nn} \bar{Y}^2 C_x^2 \left(\frac{\bar{X}}{\bar{X} + C_x} \right)^2 (1-\rho^2) \quad (3.11)$$

$$\text{vi. } \hat{\theta}_6 = \bar{y} \left[\frac{\bar{x} + \rho}{\bar{X} + \rho} \right]$$

$$MSE(\hat{\theta}_6) = \frac{N-n}{Nn} \bar{Y}^2 C_x^2 \left(\frac{\bar{X}}{\bar{X} + C_x} \right)^2 (1-\rho^2) \quad (3.12)$$

3.3. Data Used

Following Adewara et. al.[2], data sets from two populations are used:

Population I: Kadilar and Cingi[3] with the parameters:

$$N = 106, n = 20, \rho = 0.86, C_x = 2.1,$$

$$\bar{Y} = 2212.59 \text{ and } \bar{X} = 27421.70$$

Population II: Maddala (1977) with the parameters:

$$N = 16, n = 4, \rho = 0.6823, C_y = 0.2278,$$

$$C_x = 0.0986 \quad \bar{Y} = 7.6375 \text{ and } \bar{X} = 75.4313$$

Since conventionally, for ratio estimators to hold, $\rho > 0$ and also for product estimators to hold, $\rho < 0$.

4. Results and Discussions

The results obtained from the application of the two data sets to the Mean square error of the conventional and proposed ratio and product estimators in equations 3.8 to 3.12 are shown tables 1 and 2 below:

It can be observed from table 1 that the MSE for the proposed estimators are less than each of the existing ratio estimators.

Also it can be observed from table 2 that the MSE for the proposed estimators are less than each of the existing product estimators.

Table 1. Mean Square Errors (MSE) of the estimates of Existing and Proposed Estimators (Ratio Estimators) with Population I

Estimator	MSE for existing Ratio Estimator	MSE for proposed optimal Ratio Estimator
θ_1 (usual ratio estimator)	2542740	1409115.093
θ_3 (Sisodia,Dwivedi (as in[12]))	2542893	228039.874
θ (Singh,Taylor(as in[10]))	2542803	228043.0337
θ_1^8 (Adewara (as in[2]))	137519.8	76209.6230
θ_3^8 (Adewara (as in[2]))	137528	12333.1462
θ^*_{5} (Adewara (as in[2]))	137523.1	12332.5673

Table 2. Mean Square Errors (MSE) of the estimates of Existing and Proposed Estimators (Product Estimators) with Population II

Estimator	MSE for existing Product Estimator	MSE for proposed optimal Product Estimator
θ_2 θ_1 (usual ratio estimator)	0.3387	0.3033
θ_4 (Sisodia,Dwivedi (as in[12]))	0.3388	0.0568
θ_6 θ (Singh,Taylor (as in[10]))	0.3376	0.3039
$\theta_2 \theta_1^8$ (Adewara (as in[2]))	0.03763	0.0337
$\theta_4 \theta_3^8$ (Adewara (as in[2]))	0.03765	0.0063
$\theta_6 \theta^*_{5}$ (Adewara (as in[2]))	0.03751	0.0338

5. Conclusions

In this study, we have proposed an optimal estimators for the ratio estimators and product estimators by minimizing the coefficient of variation of the auxiliary variable x. Using the Kadilar and Cingi[3] and Maddala[6] data for the proposed estimators, a gain of a least 10% precision was achieved over the existing estimators for both the ratio and product estimators in the literature.

Therefore, the proposed optimal estimators is uniformly better than all other estimators and thus most preferred over the existing modified ratio and product estimators for the use

in practical applications for certain population with peculiar characteristics.

REFERENCES

- [1] Adewara, A.A. (2006): Effects of improving both the auxiliary and variable of interest in ratio and product estimators. *Proc. Pakistan Acad. Sci.* 43(4): 275-278.
- [2] Adewara, A.A, Rajesh S and Mukesh Kumar (2012): "Efficiency of some Modified Ratio and Product Estimators Using Known Values of Some Population Parameters" *Intl. Jour. Of Applied sciences and Technology*, Vol. 2 (2) Pgs. 76-79.
- [3] Kadilar, C. and Cingi, H. (2004). Ratio Estimators in Simple Random Sampling. *Appl. Mathe. and Comput.* 151, 893-902.
- [4] Kadilar, C. and Cingi, H. (2006): Ratio estimators for the population variance in sample and stratified random sampling. *Appl. Mathe. and Comput.* 173, 1047-1059.
- [5] Khoshnevisan, M., Singh, R., Chauhan, P., Sawan, N. and Smarandache, F. (2007): A general family of estimators for estimating population mean using known value of population parameter(s). *For East Jour. of theor. Statist.* 22(2), 181-191.
- [6] Maddala, G.S. (1977): *Econometrics*. "McGraw Hills Pub. Co." New York.
- [7] Pandey, B.N. and Dubey, Vyas (1988): Modified product estimator using coefficient of variation of auxiliary variate, *Assam Statistical Rev.*, 2(2), 64 – 66.
- [8] Subramani, J. and Kumarapandiyan, G. (2012). Modified Ratio Estimator for Population Mean Using Median of the Auxiliary Variable, *Proceedings of National Conference on Recent developments in the Applications of Reliability Theory and Survival Analysis held on 2nd and 3rd February 2012 at the Department of Statistics, Pondicherry University*
- [9] Sen, A.R. (1993): Some early developments in ratio estimation, *Biometric Journal* 35(1), 3-13
- [10] Singh, H.P. and Tailor, R. (2003): Use of known correlation coefficient in estimating the finite population mean. *Statist. in Trans* 6(4). 555-560.
- [11] Srivenkataramana, T. and Srinath, K.P. (1976): Ratio and Product methods of estimation in sample surveys when the two variables are moderately correlated. *Vignana Bharathi* 2:54-58.
- [12] Sisodia, B.V.S and Dwivedi, V.K. (1981): A modified ratio estimator using coefficient of variation of auxiliary variable. *Journal Ind. Soc. Agril. Statist.*, 33(2), 13-18 .
- [13] Yan, Z. and Tian, B. (2010). Ratio Method to the Mean Estimation Using Co-efficient of Skewness of Auxiliary Variable, *ICICA 2010, Part II, CCIS 106*, pp. 103–110