American Journal of Mathematics and Statistics 2013, 3(5): 253-257
DOI: 10.5923/j.ajms.20130305.01

New Sequence Spaces of Fuzzy Numbers Defined by a Orlicz Function

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Abstract In the present paper we introduce new sequence spaces of fuzzy numbers defined by a Orlicz function. We also make an effort to study some topological properties and some inclusion relation beween these spaces.

Keywords Fuzzy Number, Orlicz Function, Sequence Space

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1. Introduction and Preliminaries

Fuzzy sets theory compared to other mathematical theories, is perhaps the most easily adaptable theory to practice. The main reason is that a fuzzy set has the property of relativity, variability and inexactness in the definition of its elements. Instead of defining an entity in calculus by assuming that its role is exactly known, we can use fuzzy sets to define the same entity by allowing possible deviations and inexactness in its role. This representation suits well the uncertainities encountered in practical life, which make fuzzy sets a valuable mathematical tool. The concepts of fuzzy sets and fuzzy set operations were first introduced by Zadeh [17] and subsequently several authors have discussed various aspects of the theory and applications of fuzzy sets such as fuzzy topological spaces, similarity relations and fuzzy orderings, fuzzy measures of fuzzy events, fuzzy mathematical programming etc. Matloka [5] introduced bounded and convergent sequences of fuzzy numbers and studied some of their properties. For details about sequence spaces (see [1], [2], [3], [9], [10], [11], [12], [13], [15], [16]) and references therein.

An Orlicz function M is a function, which is continuous, non-decreasing and convex with $M(0) = 0$, $M(x) > 0$ for $x > 0$ and $M(x) \to \infty$ as $x \to \infty$.

Lindenstrauss and Tzafriri [4] used the idea of Orlicz function to define the following sequence space. Let w be the space of all real or complex sequences $x = (x_k)$, then

$$l_M = \left\{x \in w : \sum_{k=1}^{\infty} M(\frac{|x_k|}{\rho}) < \infty\right\},$$

Which is called as an Orlicz sequence space. The space l_M is a Banach space with the norm

$$\| x \| = inf\left\{\rho > 0 : \sum_{k=1}^{\infty} M(\frac{|x_k|}{\rho}) \leq 1\right\}.$$

It is shown in [4] that every Orlicz sequence space l_M contains a subspace isomorphic to $l_p (p \geq 1)$. The Δ_2-condition is equivalent to $M(Lx) \leq k\, LM(x)$ for all values of $x \geq 0$, and for $L > 1$. An Orlicz function Mcan always be represented in the following integral form

$$M(x) = \int_0^x \eta(t)dt,$$

Where η is known as the kernel of M , is right differentiable for $t \geq 0, \eta(0) = 0, \eta(t) > 0, \eta$ is non-decreasing and $\eta(t) \to \infty$ and as $t \to \infty$.

A fuzzy number is a fuzzy set on the real axis, i.e., a mapping $X : \mathbb{R}^n \to [0,1]$ which satisfies the following four conditions:

(1) X is normal, i.e., there exist an $x_0 \in \mathbb{R}^n$ such that $X(x_0) = 1$;

(2) X is fuzzy convex, i.e., for $x, y \in \mathbb{R}^n$ and $0 \leq \boldsymbol{\lambda} \leq 1$, $X(\lambda x + (1-\lambda)y) \geq \min[X(x), X(y)]$;

(3) X is upper semi-continuous;

(4) the closure of $\{x \in \mathbb{R}^n : X(x) > 0\}$, denoted by $[X]^0$, is compact.

Let $C(\mathbb{R}^n) = \{A \subset \mathbb{R}^n : A \text{ is compact and convex }\}$. The space $C(\mathbb{R}^n)$ has a linear structure induced by the operations

$$A + B = \{a + b, a \in A, b \in B\}$$

and

$$\lambda A = \{\lambda a, \lambda \in A\}$$

for $A, B \in C(\mathbb{R}^n)$ and $\lambda \in \mathbb{R}$. The Hausdorff distance between A and B of $C(\mathbb{R}^n)$ is defined as

$$\delta_\infty(A, B) = max\left\{\sup_{a \in A} \inf_{b \in B} \|a - b\|, \sup_{b \in B} \inf_{a \in A} \|a - b\|\right\}$$

where $\|.\|$ denotes the usual Euclidean norm in $\mathbb{R}^n$. It is well known that $(C(\mathbb{R}^n), \delta_\infty)$ is a complete (non separable) metric space. For $0 < \alpha \leq 1$, the α-level set

$$X^\alpha = \{x \in \mathbb{R}^n : X(x) \geq \alpha\}$$

is a non-empty compact convex, subset of $\mathbb{R}^n$, as is the support $[X]^0$. Let $L(\mathbb{R}^n)$ denote the set of all fuzzy

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Published online at http://journal.sapub.org/ajms
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numbers. The linear structure of $L(\mathbb{R}^n)$ induces addition $X+Y$ and scalar multiplication $\lambda X, \boldsymbol{\lambda} \in \mathbb{R}$, in terms of α-level sets, by

$$[X+Y]^\alpha = X^\alpha + Y^\alpha$$

and

$$[\lambda X]^\alpha = \boldsymbol{\lambda}[X]^\alpha$$

for each $0 \leq \alpha \leq 1$. Define for each $1 \leq q < \infty$

$$d_q(X,Y) = \left\{\int_0^1 \delta_\infty(X^\alpha, Y^\alpha)^q \, d\alpha\right\}^{\frac{1}{q}}$$

and $d_\infty(X,Y) = \sup_{0 \leq \alpha \leq 1} \delta_\infty(X^\alpha, Y^\alpha)$. Clearly $d_\infty(X,Y) = \lim_{q\to\infty} d_q(X,Y)$ with $d_q \leq d_r$ if $q \leq r$. Moreover $(L(\mathbb{R}^n), d_\infty)$ is a complete, separable and locally compact metric space. We denote by $w(F)$ the set of all sequences $X = (X_k)$ of fuzzy numbers.

Let l^0 be the set of all complex sequences $x = (x_k)$ normed by $\|x\| = \sup_k |x_k|$, where $k \in \mathbb{N}$, the set of positive integers. Let C denote the space whose elements are the sets of distinct positive integers. Given any elements $\sigma \in C$, we denote by $c(\sigma)$ the sequence $\{c_n(\sigma)\}$ which is such that $c_n(\sigma) = 1$ if $n \in \sigma, c_n(\sigma) = 0$ otherwise. Further

$$C_s = \left\{\sigma \in C : \sum_{n=1}^{\infty} c_n(\sigma) \leq s\right\},$$

the set of those σ whose support has cardinality at most s, and

$$\Phi = \left\{x = \{x_k\} \in l^0 : \varphi_1 > 0, \Delta\varphi_k \geq 0 \; and \; \Delta\left(\frac{\varphi_k}{k}\right) \leq 0 \;\; (k = 1,2,\ldots)\right\},$$

Where $\Delta\varphi_k = \varphi_k - \varphi_{k-1}$, where $\{\varphi_k\}$ are real sequences see [5]. For $\varphi \in \Phi$, Sargent [14] define the following sequence space

$$m(\varphi) = \left\{x = \{x_k\} \in l^0 : \sup_{s\geq 1} \sup_{\sigma \in C_s} \left(\frac{1}{\varphi_s} \sum_{k\in\sigma} |x_k|\right) < \infty\right\}.$$

In [14], Sargent studied some of its properties and obtained its relationship with the space l_p. In [7], Nurray and Savas introduced the classes of sequences of fuzzy numbers,

$$l^F(p) = \left\{X = (X_k) \in w(F) : \sum_k d(X_k, \bar{0})^{p_k} < \infty\right\},$$

where $p = (p_k)$ is a bounded sequence of positive real numbers. They proved that $l^F(p)$ is a complete metric space with the metric defined by

$$h(X,Y) = \left(\sum_k d(X_k, \bar{0})^{p_k}\right)^{\frac{1}{M}},$$

where $M = \max(1, sup_k \, p_k)$.

Let σ be a one-to-one mapping of the set of positive integers into itself such that $\sigma^k(n) = \sigma(\sigma^{k-1}(n))$, $k = 1, 2, \ldots$, M be an Orlicz function and $p = (p_k)$ be a bounded sequence of positive real numbers. In this paper we define the following classes of sequences of fuzzy numbers:

$$l_\infty^F(M,\sigma,p) = \left\{X = (X_k) \in w(F) : \sup_{k,n}\left[M\left(\frac{d(X_{\sigma^k(n)},\bar{0})}{\rho}\right)\right]^{p_k} < \infty\right\},$$

$$l_1^F(M,\sigma,p) = \left\{X = (X_k) \in w(F) : \sup_n \sum_k \left[M\left(\frac{d(X_{\sigma^k(n)},\bar{0})}{\rho}\right)\right]^{p_k} < \infty\right\}$$

and

$$m^F(M,\varphi,\sigma,p) = \left\{X = (X_k) \in w(F) :\right.$$

: sup$n$sup$s \geq 1$, $\sigma \in Cs1\varphi s$ $k \in \sigma Md(X\sigma kn, \quad 0)\rho pk < \infty$.

When $\sigma(n) = n+1$, we obtain the classes of sequences of fuzzy numbers as follows:

$$l_\infty^F(M,p) = \left\{X = (X_k) \in w(F) : \sup_{k,n}\left[M\left(\frac{d(X_{k+n},\bar{0})}{\rho}\right)\right]^{p_k} < \infty\right\},$$

$$l_1^F(M,p) = \left\{X = (X_k) \in w(F) : \sup_n \sum_k \left[M\left(\frac{d(X_{k+n},\bar{0})}{\rho}\right)\right]^{p_k} < \infty\right\}$$

and

$$m^F(M,\varphi,p) = \left\{X = (X_k) \in w(F) :\right.$$

: sup$n$sup$s \geq 1$, $\sigma \in Cs1\varphi s$ $k \in \sigma Md(Xk + n, \quad 0)\rho pk < \infty$.

If we take $p = (p_k) = 1$, we obtain the classes of sequences of fuzzy numbers as follows:

$$l_\infty^F(M,\sigma) = \left\{X = (X_k) \in w(F) : \sup_{k,n} M\left(\frac{d(X_{\sigma^k(n)},\bar{0})}{\rho}\right) < \infty\right\},$$

$$l_1^F(M,\sigma) = \left\{X = (X_k) \in w(F) : \sup_n \sum_k M\left(\frac{d(X_{\sigma^k(n)},\bar{0})}{\rho}\right) < \infty\right\}$$

and

$$m^F(M,\varphi,\sigma) = \left\{X = (X_k) \in w(F) :\right.$$

: sup$n$sup$s \geq 1$, $\sigma \in Cs1\varphi s$ $k \in \sigma Md(X\sigma kn, \quad 0)\rho < \infty$.

The following inequality will be used throughout the paper. Let $p = (p_k)$ be a bounded sequence of positive real numbers with $0 < p_k \leq \sup_k = H$ and let $D = \max\{1, 2^{H-1}\}$.

Then for the factorable sequences $\{a_k\}$ and $\{b_k\}$ in the complex plane, we have

$$|a_k + b_k|^{p_k} \leq D(|a_k|^{p_k} + |b_k|^{p_k}). \quad (1)$$

The main aim of this paper is to study some topological properties and some inclusion relations between above defined sequence spaces.

2. Main Results

Theorem 2.1. Let M be an Orlicz function and $p = (p_k)$ be a bounded sequence of positive real numbers, then the spaces $l_\infty^F(M,\sigma,p)$, $l_1^F(M,\sigma,p)$ *and* $m^F(M,\varphi,\sigma,p)$ are linear spaces over the field of complex numbers $\mathbb{C}$.

Proof. Let $X = (X_k), Y = (Y_k) \in m^F(M,\varphi,\sigma,p)$ and $\alpha, \beta \in \mathbb{C}$, then there exist positive numbers ρ_1, ρ_2 such that

$$\sup_n \sup_{s\geq 1, \sigma \in C_s} \frac{1}{\varphi_s} \sum_{k\in\sigma}\left[M\left(\frac{d(X_{\sigma^k(n)}, \bar{0})}{\rho_1}\right)\right]^{p_k} < \infty$$

and

$$\sup_{n}\ \sup_{s\ge 1,\sigma\in C_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{d(Y_{\sigma^k(n)},\,\bar{0})}{\rho_1}\right)\right]^{p_k} < \infty.$$

Define $\rho_3=\max(2|\alpha|\rho_1,\ 2|\beta|\rho_2)$. Since M is non-decreasing, convex and so by using inequality (1), we have

$$\begin{aligned}
&\sup_{n}\ \sup_{s\ge 1,\sigma\in C_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{d\big(\alpha X_{\sigma^k(n)}+\beta Y_{\sigma^k(n)},\,\bar{0}\big)}{\rho_3}\right)\right]^{p_k}\\
&\le \sup_{n}\ \sup_{s\ge 1,\sigma\in C_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{\alpha d\big(X_{\sigma^k(n)},\,\bar{0}\big)}{\rho_3}\right.\right.\\
&\qquad\left.\left.+\frac{\beta d\big(Y_{\sigma^k(n)},\,\bar{0}\big)}{\rho_3}\right)\right]^{p_k}\\
&\le \frac{1}{2}\sup_{n}\ \sup_{s\ge 1,\sigma\in C_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{d\big(X_{\sigma^k(n)},\,\bar{0}\big)}{\rho_1}\right)\right]^{p_k}\\
&\quad+\sup_{n}\ \sup_{s\ge 1,\sigma\in C_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{d\big(Y_{\sigma^k(n)},\,\bar{0}\big)}{\rho_2}\right)\right]^{p_k}\\
&<\infty.
\end{aligned}$$

This proves that $m^F(M,\varphi,\sigma,p)$ is a linear space. Similarly, we can prove that $l_\infty^F(M,\sigma,p)$ and $l_1^F(M,\sigma,p)$ are linear spaces.

Theorem 2.2. Let M be an Orlicz function and $p=(p_k)$ be a bounded sequence of positive real numbers, then the space $m^F(M,\varphi,\sigma,p)$ is a complete metric space, with the metric defined by

$$\begin{aligned}
&g(X,Y)\\
&=\sup_{n}\ \sup_{s\ge 1,\sigma\in C_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{d\big(X_{\sigma^k(n)},\,Y_{\sigma^k(n)}\big)}{\rho}\right)\right]^{p_k}.
\end{aligned}$$

Proof. Let (X^i) be a Cauchy sequence in $m^F(M,\varphi,\sigma,p)$. Then,

$$\begin{aligned}
&g(X^i,X^j)\\
&=\sup_{n}\ \sup_{s\ge 1,\sigma\in C_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{d\left(X^i_{\sigma^k(n)},\,X^j_{\sigma^k(n)}\right)}{\rho}\right)\right]^{p_k}\\
&\to 0\ as\ i,j\to\infty.
\end{aligned}$$

Hence

$$\left[M\left(\frac{d\left(X^i_{\sigma^k(n)},X^j_{\sigma^k(n)}\right)}{\rho}\right)\right]^{p_k}\to 0\ as\ i,j\to\infty,\text{for all } n.$$

Therefore (X^i) is a Cauchy sequence in $L(\mathbb{R}^n)$. Since $L(\mathbb{R}^n)$ is complete, it is convergent so that $\lim_{i\to\infty}X_k^i=X_k$, for each $k\in\mathbb{N}$. Since (X^i) is a Cauchy sequence for each $\epsilon>0$, there exists $n_0=n_0(\epsilon)$ such that

$g(X^i,X^j)<\epsilon$, for all $i,j\ge n_0$.

So, we have

$$\begin{aligned}
&\limsup_{j}\ \sup_{n}\ \sup_{s\ge 1,\sigma\in C_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{d\left(X^i_{\sigma^k(n)},\,X^j_{\sigma^k(n)}\right)}{\rho}\right)\right]^{p_k}\\
&=\sup_{n}\ \sup_{s\ge 1,\sigma\in C_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{d\left(X^i_{\sigma^k(n)},\,X_{\sigma^k(n)}\right)}{\rho}\right)\right]^{p_k}\\
&\qquad<\epsilon,\ \text{for all } i\ge n_0.
\end{aligned}$$

This implies that $g(X^i,X^j)<\epsilon$, for all $i,j\ge n_0$, i.e. $X^i\to X$ as $i\to\infty$, where $X=(X_k)$. Since

$$\begin{aligned}
&\sup_{n}\ \sup_{s\ge 1,\sigma\in C_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{d\big(X_{\sigma^k(n)},\,X_0\big)}{\rho}\right)\right]^{p_k}\\
&\le\sup_{n}\ \sup_{s\ge 1,\sigma\in C_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{d\left(X^{n_0}_{\sigma^k(n)},\,X_{\sigma^k(n)}\right)}{\rho}\right)\right]^{p_k}\\
&\quad+\sup_{n}\ \sup_{s\ge 1,\sigma\in C_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{d\big(X_{\sigma^k(n)},\,X_0\big)}{\rho}\right)\right]^{p_k}
\end{aligned}$$

then we obtain $X=(X_k)\in m^F(M,\varphi,\sigma,p)$. Therefore $m^F(M,\varphi,\sigma,p)$ is a complete metric space. This completes the proof of the theorem.

Theorem 2.3. Let M be an Orlicz function and $p=(p_k)$ be a bounded sequence of positive real numbers, then we have the following

(i) the space $l_1^F(M,\sigma,p)$ is a complete metric space, with the metric defined by

$$g(X,Y)=\sup_{n}\sum_{k}\left[M\left(\frac{d\big(X_{\sigma^k(n)},\,Y_{\sigma^k(n)}\big)}{\rho}\right)\right]^{p_k},$$

(ii) the space $l_\infty^F(M,\sigma,p)$ is a complete metric space, with the metric defined by

$$g(X,Y)=\sup_{k,n}\left[M\left(\frac{d\big(X_{\sigma^k(n)},\,Y_{\sigma^k(n)}\big)}{\rho}\right)\right]^{p_k}.$$

Proof. It is easy to prove in view of Theorem 2.2, so we omit the details.

Theorem 2.4. Let M be an Orlicz function and $p=(p_k)$ be a bounded sequence of positive real numbers, then $m^F(M,\varphi,\sigma,p)\subset m^F(M,\psi,\sigma,p)$ if and only if $\sup_{s\ge 1}\frac{\varphi_s}{\psi_s}<\infty$.

Proof. Let $\sup_{s\ge 1}\frac{\varphi_s}{\psi_s}<\infty$ and $(X_k)\in m^F(M,\varphi,\sigma,p)$. Then

$$\begin{aligned}
&\sup_{n}\ \sup_{s\ge 1,\sigma\in C_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{d(X_{\sigma^k(n)},\,\bar{0})}{\rho}\right)\right]^{p_k}<\infty,\\
&\Rightarrow \sup_{n}\ \sup_{s\ge 1,\sigma\in C_s}\frac{1}{\psi_s}\sum_{k\in\sigma}\left[M\left(\frac{d(X_{\sigma^k(n)},\,\bar{0})}{\rho}\right)\right]^{p_k}\\
&\le\left\{\sup_{s\ge 1}\frac{\varphi_s}{\psi_s}\right\}\sup_{n}\ \sup_{s\ge 1,\sigma\in C_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{d(X_{\sigma^k(n)},\,\bar{0})}{\rho}\right)\right]^{p_k}\\
&<\infty.
\end{aligned}$$

Therefore $(X_k)\in m^F(M,\psi,\sigma,p)$. Hence $m^F(M,\varphi,\sigma,p)\subset m^F(M,\psi,\sigma,p)$.

Conversely, let $m^F(M,\varphi,\sigma,p)\subset m^F(M,\psi,\sigma,p)$. Suppose that $\sup_{s\ge 1}\frac{\varphi_s}{\psi_s}=\infty$, then there exists a sequence of natural numbers (s_i) such that $\lim_{i\to\infty}\frac{\varphi_{s_i}}{\psi_{s_i}}=\infty$. Let $(X_k)\in m^F(M,\varphi,\sigma,p)$. Then,

$$\sup_{n}\ \sup_{s\ge 1,\sigma\in C_s}\frac{1}{\varphi_s}\sum_{k\in\sigma}\left[M\left(\frac{d(X_{\sigma^k(n)},\,\bar{0})}{\rho}\right)\right]^{p_k}<\infty.$$

Now, we have

$$\Rightarrow \sup_{n} \sup_{s \geq 1, \sigma \in C_s} \frac{1}{\psi_s} \sum_{k \in \sigma} \left[M\left(\frac{d\left(X_{\sigma^k(n)}, \bar{0}\right)}{\rho} \right) \right]^{p_k}$$

$$\geq \left\{ \sup_{i \geq 1} \frac{\varphi_{s_i}}{\psi_{s_i}} \right\} \sup_{n} \sup_{i \geq 1, \sigma \in C_s} \frac{1}{\varphi_{s_i}} \sum_{k \in \sigma} \left[M\left(\frac{d(X_{\sigma^k(n)}, \bar{0})}{\rho} \right) \right]^{p_k}$$

$$= \infty.$$

Therefore $(X_k) \notin m^F(M, \psi, \sigma, p)$, which is a contradiction. Therefore $\sup_{s \geq 1} \frac{\varphi_s}{\psi_s} < \infty$. This completes the proof of the theorem.

Theorem 2.5. Let M be an Orlicz function and $p = (p_k)$ be a bounded sequence of positive real numbers, then $m^F(M, \varphi, \sigma, p) = m^F(M, \psi, \sigma, p)$ if and only if $\sup_{s \geq 1} \frac{\varphi_s}{\psi_s} < \infty$ and $\sup_{s \geq 1} \frac{\psi_s}{\varphi_s} < \infty$.

Proof. The proof directly follows from Theorem 2.4.

Theorem 2.6. Let M be an Orlicz function and $p = (p_k)$ be a bounded sequence of positive real numbers, then $l_1^F(M, \sigma, p) \subset m^F(M, \varphi, \sigma, p) \subset l_\infty^F(M, \sigma, p)$.

Proof. Let $(X_k) \in l_1^F(M, \sigma, p)$, then we have

$$\sup_{n} \sum_{k} \left[M\left(\frac{d(X_{\sigma^k(n)}, \bar{0})}{\rho} \right) \right]^{p_k} < \infty.$$

Since (φ_n) is monotone increasing, so we have

$$\frac{1}{\varphi_s} \sum\nolimits_{k \in \sigma} \left[M\left(\frac{d\left(X_{\sigma^k(n)}, \bar{0}\right)}{\rho} \right) \right]^{p_k} \leq$$

$$\frac{1}{\varphi_1} \sum\nolimits_{k \in \sigma} \left[M\left(\frac{d\left(X_{\sigma^k(n)}, \bar{0}\right)}{\rho} \right) \right]^{p_k}$$

$$\leq$$

$$\frac{1}{\varphi_1} \sum\nolimits_{k} \left[M\left(\frac{d\,(X_{\sigma^k(n)}, \bar{0})}{\rho} \right) \right]^{p_k} < \infty.$$

Hence

$$\sup_{n} \sup_{s \geq 1, \sigma \in C_s} \frac{1}{\varphi_s} \sum_{k \in \sigma} \left[M\left(\frac{d(X_{\sigma^k(n)}, \bar{0})}{\rho} \right) \right]^{p_k} < \infty.$$

Thus $(X_k) \in m^F(M, \varphi, \sigma, p)$. Therefore $l_1^F(M, \sigma, p) \subset m^F(M, \varphi, \sigma, p)$. Next, let $(X_k) \in m^F(M, \varphi, \sigma, p)$. Then, we have

$$sup_n \, sup_{s \geq 1, \sigma \in C_s} \frac{1}{\varphi_s} \sum_{k \epsilon \sigma} [M(\frac{d(X_{\sigma^k(n)}, \bar{0})}{\rho})]^{p_k} < \infty.$$

Thus

$$sup_{k,n} \frac{1}{\varphi_1} \, [M(\frac{d(X_{\sigma^k(n)}, \bar{0})}{\rho})]^{p_k} < \infty,$$

(on taking cardinality of σ to be 1). Thus $(X_k) \in l_\infty^F(M, \sigma, p)$. Hence

$$m^F(M, \varphi, \sigma, p) \subset l_\infty^F(M, \sigma, p).$$

This completes the proof of the theorem.

Theorem 2.7. Let M be an orlicz function and $p = (p_k)$ be a bounded sequence of positive real numbers, then $m^F(M, \varphi, \sigma, p) = l_1^F(M, \sigma, p)$ if and only if $sup_{s \geq 1} \varphi_s < \infty$.

Proof. It is clear that $m^F(M, \varphi, \sigma, p) = l_1^F(M, \sigma, p)$ when $\psi_s = 1$ for all $s \epsilon \mathbb{N}$. By Theorem 2.4, $m^F(M, \varphi, \sigma, p) \subset m^F(M, \psi, \sigma, p)$ if and only if $sup_{s \geq 1} \frac{\varphi_s}{\psi_s} < \infty$ i.e. $sup_{s \geq 1} \varphi_s < \infty$. Therefore by Theorem 2.6, $m^F(M, \varphi, \sigma, p) = l_1^F(M, \sigma, p)$ if and only if $sup_{s \geq 1} \varphi_s < \infty$.

3. Conclusions

In this paper we have introduce some new sequence spaces defined by a Orlicz function. We have also studied some topological properties like linearity, completeness and interested inclusion relations between the spaces. According to our opinion these results are new and interesting and beneficial for young researchers those working in this area.

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