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Estimating Parameters and Reliability for a New Mixture Distribution ($\mathfrak{X}_p$)

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Abstract This paper deals with a new probability distribution obtained from mixing Pareto (two parameters γ, θ) with Wiebull (two Parameters λ, k), using mixing proportion (p).The researcher construct the probability density function, and cumulative distribution function, reliability, hazard functions, moments are obtained the parameters which estimated by moment method (MOM) and maximum likelihood method (MLE) using simulation procedure taking different sample size $(n = 25, 50, 100)$ and replicate $(R = 500)$ for each experiment.The comparison has been done through mean squares error (MSE).

Keywords Mixture distribution ($\mathfrak{X}_p$), Pareto and Weibull, Moment Estimators, Maximum Likelihood Estimators, Mean Squares Error, Reliability

1. Introduction [3][6][7]

A mixture distribution is a compounding of statistical distributions, which arise when sampling from inhomogene ous populations (or mixed populations) with a different probability density function in each component.For example the distribution of times to failure in a mixture of good and defective items, and the distribution of some diagnostic measure in a mixed population of patients, some of whom have a given disease and some of them do not have.

The application of finite mixture models arise in economics, medicine, psychology, agriculture, life testing and reliability. In many applications, the available data can be considered as data coming from a mixture population of two or more distributions.This idea enables us to mix statistical distribution to get a new distribution carrying the properties of its components. The distribution constructed here is denoted by ($\mathfrak{X}_p$), mixture of two parameters Pareto and two parameters Weibull.

2. $\mathfrak{X}$ Distribution

Here the ($\mathfrak{X}$) distribution is a mixture of some distributions with Weibull distribution, like exponential Weibull, Pareto Weibull, ErlangWeibull, Gamma Weibull. Now we explain ($\mathfrak{X}$) distribution which is a mixture of Pareto distribution with Weibull distribution and we shall denoted it by ($\mathfrak{X}_p$ $distribution$).

The statistical properties of ($\mathfrak{X}_p$ $distribution$) are explained, then we want to estimate its parameters;

a. The probability density function $(p.d.f)$ of ($\mathfrak{X}$) is;

$$f_{\mathfrak{X}_p}(x;\ \alpha) = p_1 \frac{\gamma\theta^\gamma}{x^{\gamma+1}} + p_2 \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} \exp\left[-\left(\frac{x}{\lambda}\right)^k\right] \quad (1)$$

$$x, \theta, \gamma, k, \lambda > 0$$

$(p_1 + p_2 = 1)$ parameters of mix $\underline{\alpha} = (p_1, p_2, \theta, \gamma, k, \lambda)$ vector of parameters.

b. The cumulative distribution function $(c.d.f)$ of ($\mathfrak{X}_p$) is;

$$F_{\mathfrak{X}_p}(x;\ \alpha) = p_1\left[1 - \left(\frac{\theta}{x}\right)^\gamma\right] + p_2\left[1 - \exp\ \{-\left(\frac{x}{\lambda}\right)^k\}\right] \quad (2)$$

Also this function can be drawn for various values of parameters.The reliability function of ($\mathfrak{X}_p$) distribution denoted by;

$$R_{\mathfrak{X}_p}(x;\ \alpha) = 1 - F_{\mathfrak{X}_p}(x;\ \alpha) = p_1\left[\left(\frac{\theta}{x}\right)^\gamma\right] + p_2\left[\exp\ \{-\left(\frac{x}{\lambda}\right)^k\}\right] \quad (3)$$

c. The hazard function of ($\mathfrak{X}_p$) is;

$$h_{\mathfrak{X}_p}(x;\ \alpha) = \frac{f_{\mathfrak{X}_p}(x;\ \alpha)}{R_{\mathfrak{X}_p}(x;\ \alpha)} = \frac{p_1\frac{\gamma\theta^\gamma}{x^{\gamma+1}} + p_2\frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1}\exp\left[-\left(\frac{x}{\lambda}\right)^k\right]}{p_1\left[\left(\frac{\theta}{x}\right)^\gamma\right] + p_2\left[\exp\ \{-\left(\frac{x}{\lambda}\right)^k\}\right]} \quad (4)$$

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d. The r^{th} moment about origin of ($\mathfrak{X}_p$) distribution which can be used in estimation method are defined as;

$$\mu'_{r\,\mathfrak{X}_p} = p_1 \frac{\gamma\theta^r}{\gamma-r} + p_2\lambda^r\,\Gamma\left[1+\frac{r}{k}\right] \quad (5)$$

Where;

$\mu'_r = \frac{\gamma\theta^r}{\gamma-r}$ existing only for $r < \gamma$.

The mean and variance of ($\mathfrak{X}_p$) distribution can be obtained from (5), where;

$$\mu'_{\mathfrak{X}_p} = p_1 \frac{\gamma\theta}{\gamma-1} + p_2\,\lambda\Gamma\left[1+\frac{1}{k}\right] \quad (6)$$

While the variance of ($\mathfrak{X}_p$) is;

$$\sigma^2_{\mathfrak{X}_p} = p_1\left[\frac{\gamma\theta^2}{\gamma-2} - p_1\frac{\gamma^2\theta^2}{(\gamma-1)^2}\right] + p_2\left[\Gamma(2) - p_2\Gamma^2(1)\right] - 2p_1p_2\frac{\gamma\,\theta}{\gamma-1}\Gamma(1) \quad (7)$$

To estimate parameters by moments we must find, $\mu'_1(when\ r=1)$, and another four moments.

$$\left.\begin{aligned}
\mu'_1 &= p_1\frac{\gamma\theta}{\gamma-1} + p_2\,\lambda\Gamma\left[1+\frac{1}{k}\right]\\
\mu'_2 &= p_1\frac{\gamma\theta^2}{\gamma-2} + p_2\lambda^2\Gamma\left[1+\frac{2}{k}\right]\\
\mu'_3 &= p_1\frac{\gamma\theta^3}{\gamma-3} + p_2\lambda^3\,\Gamma\left[1+\frac{3}{k}\right]\\
\mu'_4 &= p_1\frac{\gamma\theta^4}{\gamma-4} + p_2\lambda^4\Gamma\left[1+\frac{4}{k}\right]\\
\mu'_5 &= p_1\frac{\gamma\theta^5}{\gamma-5} + p_2\lambda^5\Gamma\left[1+\frac{5}{k}\right]
\end{aligned}\right\} \quad (8)$$

When (k) is integer $(k=1)$ for example then solving these equations yield moment estimator.

$$\mu'_1 = \frac{\sum x_i}{n}$$
$$\bar{x} = p_1\frac{\gamma\theta}{\gamma-1} + (1-p_1)\,\lambda\Gamma\,(2) \quad (9)$$

$$\mu'_2 = \frac{\sum x_i^2}{n}$$
$$\frac{\sum x_i^2}{n} = p_1\frac{\gamma\theta^2}{\gamma-2} + (1-p_1)\lambda^2\,\Gamma(3) \quad (10)$$

$$\mu'_3 = \frac{\sum x_i^3}{n}$$
$$\frac{\sum x_i^3}{n} = p_1\frac{\gamma\theta^3}{\gamma-3} + (1-p_1)\,\lambda^3\Gamma\,(4) \quad (11)$$

$$\mu'_4 = \frac{\sum x_i^4}{n}$$
$$\frac{\sum x_i^4}{n} = p_1\frac{\gamma\theta^4}{\gamma-4} + (1-p_1)\,\lambda^4\Gamma\,(5) \quad (12)$$

$$\mu'_5 = \frac{\sum x_i^5}{n}$$
$$\frac{\sum x_i^5}{n} = p_1\frac{\gamma\theta^5}{\gamma-5} + (1-p_1)\lambda^5\Gamma(6) \quad (13)$$

$$\Gamma(2) = 1! = 1$$
$$\Gamma(3) = 2! = 2$$
$$\Gamma(4) = 3! = 6$$
$$\Gamma(5) = 4! = 24$$
$$\Gamma(6) = 5! = 120$$

Here we assumed $(k=1)$, if we assume $(\gamma = 6)$, the parameters reduced to (p_1, θ, λ), since $(p_2 = 1-p_1)$, so we construct three equations for moment methods;

$$\frac{\sum x_i}{n} = 6\frac{p_1\theta}{5} + (1-p_1)\,\lambda$$
$$\frac{\sum x_i^2}{n} = 6\frac{p_1\theta^2}{4} + 2(1-p_1)\,\lambda^2$$
$$\frac{\sum x_i^3}{n} = 6\frac{p_1\theta^3}{3} + 6(1-p_1)\,\lambda^3$$

Assume $(p_1 = \frac{1}{2})$;

$$\frac{\sum x_i}{n} = 3\frac{\theta}{5} + \frac{1}{2}\lambda \;\Rightarrow\; 10\bar{x} = 6\,\theta + 5\lambda \quad (14)$$
$$\frac{\sum x_i^2}{n} = 3\frac{\theta^2}{4} + \lambda^2 \;\Rightarrow\; 4\frac{\sum x_i^2}{n} = 3\,\theta^2 + 4\lambda^2 \quad (15)$$
$$\frac{\sum x_i^3}{n} = \theta^3 + 3\lambda^3 \;\Rightarrow\; \frac{\sum x_i^3}{n} = \theta^3 + 3\lambda^3 \quad (16)$$

From equation (14), find θ

If $\theta = \frac{1}{6}(10\bar{x} - 5\lambda)$, substitute in (15);

$$4\frac{\sum x_i^2}{n} = 3\left[\frac{1}{6}(10\bar{x} - 5\lambda)\right]^2 + 4\lambda^2$$
$$4\frac{\sum x_i^2}{n} = \frac{3}{36}\left[100\bar{x}^2 - 100\bar{x}\lambda + 25\lambda^2\right] + 4\lambda^2$$
$$4\frac{\sum x_i^2}{n} = \frac{100}{12}\bar{x}^2 - \frac{100}{12}\bar{x}\lambda + \frac{25}{12}\lambda^2 + 4\lambda^2$$
$$4\frac{\sum x_i^2}{n} = \frac{100}{12}\bar{x}^2 - \frac{100}{12}\bar{x}\lambda + \frac{73}{12}\lambda^2$$
$$\frac{73}{12}\lambda^2 + \frac{100}{12}\bar{x}\lambda + \frac{100}{12}\bar{x}^2 - 4\frac{\sum x_i^2}{n} = 0$$

By using fixed point method where $\lambda_{i+1} = g(\lambda_i)$ get;

$$\lambda = \bar{x} + \frac{73}{100}\frac{\lambda^2}{\bar{x}} - \frac{12}{25\bar{x}}\frac{\sum x_i^2}{n} \quad (17)$$

Using simple program, the moment estimator of (λ) is obtained, therefore;

$$\hat{\theta}_{mom} = \frac{1}{6}(10\bar{x} - 5\hat{\lambda}_{mom}) \quad (18)$$

From $\hat{\lambda}_{mom}, \hat{\theta}_{mom}$ we can estimate (p_1) by moment estimator from solving;

$$\bar{x} = \frac{6}{5}\hat{\theta}_{mom}\;\; p_1 + (1-p_1)\hat{\lambda}_{mom}$$

And from equation (8) since;

$$\bar{x} = \frac{p_1\theta\gamma}{\gamma-1} + \lambda(1-p_1)$$

Use $\hat{p}_{1(mom)}, \hat{\theta}_{mom}, \hat{\lambda}_{mom}$ to obtain $\hat{\gamma}_{mom}$

3. Maximum Likelihood Method[5]

To apply the ML for the mixture distribution ($\mathfrak{X}_p$) from two subpopulation (S_{p_1}, S_{p_2}), where (S_{p_1}) is Pareto population and (S_{p_2}) is Weibull population and we assume (p) is mixing proportion parameters, where $(p_1 + p_2 = 1)$.Let (t_{ij}) is the time failure for random sample (n) taken from mixed distribution ($\mathfrak{X}_p$), if we can determine the units of each sub population ($S_{p_i}, i = 1,2$), then choosing the random variable (T), which is the time through it the failure time of (r_i) units can be determined before (T), (i.e $t_{i1}, t_{i2}, \dots, t_{ir_i}$) such that;

$$x_{ij} = \frac{t_{ij}}{T} \leq 1$$

Therefore the sample units are of type (time censored sampling Type 1) and the conditional function for time failure from (S_{p_i}) for units that failed before time (T), is;

$$f_i(t|t \leq T) = \frac{f_i(t)}{F_i(T)} \qquad 0 \leq t \leq T$$

The probability that (r_1) unit from (S_{p_1}) failed before (T), (r_2) unit from (S_{p_2}) failed before (T) also, and ($n-r$) is remained until time (T), is;

$$\frac{n!}{r_1! r_2! (n-r)!}[p_1 F_1(t)]^{r_1}[p_2 F_2(T)]^{r_2}[R(T)]^{n-r} \tag{9}$$

Then the likelihood function for sample is;

$$L(t_{11}, t_{12}, \dots, t_{1r_1}; t_{21}, t_{22}, \dots, t_{2r_2} | parameters\ \underline{\alpha})$$

$$= \frac{n!}{r_1! r_2! (n-r)!}[p_1 F_1(t)]^{r_1}[p_2 F_2(T)]^{r_2}[R(T)]^{n-r} \frac{\prod_{j=1}^{r_1} f_1(t_{1j})}{[F_1(T)]^{r_1}} \times \frac{\prod_{j=1}^{r_2} f_2(t_{2j})}{[F_2(t)]^{r_2}}$$

$$L = \frac{n!}{r_1! r_2! (n-r)!} p_1^{r_1} p_2^{r_2} \left[p_1 \left(\frac{\theta}{x}\right)^{\gamma} + p_2 \exp\left\{-\left(\frac{x}{\lambda}\right)^{k}\right\}\right]^{n-r}$$

$$\left[\prod_{j=1}^{r_1} \frac{p_1 \gamma \theta^{\gamma}}{(x_j)^{\gamma+1}}\right]\left[\prod_{j=1}^{r_2} \frac{p_2 k}{\lambda}\left(\frac{x_j}{\lambda}\right)^{k-1} exp\left\{-\left(\frac{x_j}{\lambda}\right)^{k}\right\}\right]$$

$$\ln L = \ln C + r_1 \ln(p_1) + r_2 \ln(1-p_1) + (n-r)\ln\left[p_1 \left(\frac{\theta}{x}\right)^{\gamma} + p_2 \exp\left\{-\left(\frac{x}{\lambda}\right)^{k}\right\}\right]$$

$$+ r_1 \ln(p_1) + r_1 \ln(\gamma) + r_1 \gamma \ln(\theta) - (\gamma+1)\sum_{j=1}^{n} \ln x_j$$

$$+ r_2 \ln(p_2) + r_2 \ln(k) - r_2 \ln(\lambda) + (k-1)\sum_{j=1}^{n} \ln\left(\frac{x_j}{\lambda}\right) - \sum_{j=1}^{n}\left(\frac{x_j}{\lambda}\right)^{k}$$

$$\ln L = \ln C + r_1[2\ln(p_1) + \ln(\gamma) + \gamma \ln(\theta)] + r_2[2\ln(p_2) + \ln(k) - \ln(\lambda)]$$

$$+(n-r)\ln\left[p_1 \left(\frac{\theta}{x}\right)^{\gamma} + p_2 \exp\left\{-\left(\frac{x}{\lambda}\right)^{k}\right\}\right] - (\gamma+1)\sum_{j=1}^{n} \ln x_j + (k-1)\left[\sum_{j=1}^{n} \ln x_j - n\ln(\lambda)\right] - \sum_{j=1}^{n}\left(\frac{x_j}{\lambda}\right)^{k}$$

The parameters here are ($p_1, \gamma, \theta, k, \lambda$).

$$\frac{\partial \ln L}{\partial p_1} = 2\frac{r_1}{p_1} - 2\frac{r_2}{p_2} + (n-r)\frac{\left[\left(\frac{\theta}{x}\right)^{\gamma} - \exp\left\{-\left(\frac{x}{\theta}\right)^{k}\right\}\right]}{\left[p_1 \left(\frac{\theta}{x}\right)^{\gamma} + p_2 \exp\left\{-\left(\frac{x}{\lambda}\right)^{k}\right\}\right]}$$

$$\Rightarrow 2\frac{r_1}{p_1} - 2\frac{r_2}{p_2} + (n-r)\frac{\left[\left(\frac{\theta}{x}\right)^{\gamma} - \exp\left\{-\left(\frac{x}{\theta}\right)^{k}\right\}\right]}{\left[p_1 \left(\frac{\theta}{x}\right)^{\gamma} + p_2 \exp\left\{-\left(\frac{x}{\lambda}\right)^{k}\right\}\right]} = 0$$

Therefore;

$$\left(2\frac{r_1}{p_1} - 2\frac{r_2}{p_2}\right)\left[p_1 \left(\frac{\theta}{x}\right)^{\gamma} + p_2 \exp\left\{-\left(\frac{x}{\theta}\right)^{k}\right\}\right] + (n-r)\left[\left(\frac{\theta}{x}\right)^{\gamma} - \exp\left\{-\left(\frac{x}{\theta}\right)^{k}\right\}\right] = 0$$

$$\frac{\partial \ln L}{\partial \gamma} = (n-r)\frac{\left(\frac{\theta}{x}\right)^{\gamma}\ln\left(\frac{\theta}{x}\right)}{\left[p_1 \left(\frac{\theta}{x}\right)^{\gamma} + p_2 \exp\left\{-\left(\frac{x}{\lambda}\right)^{k}\right\}\right]} + \frac{r_1}{\gamma} + r_1 \ln\theta - \sum_{j=1}^{m} \ln x_j$$

4. Simulation Procedure

We have;

$$F = \left[p_1\left(1 - \left(\frac{\theta}{x}\right)^{\gamma}\right) + p_2\left(1 - \exp\left\{-\left(\frac{x}{\lambda}\right)^{k}\right\}\right)\right]$$

$$F = p_1 F_1 + p_2 F_2$$

$$F_1 = \left(1 - \left(\frac{\theta}{x}\right)^{\gamma}\right)$$
$$R_1 = \left(1 - \left(\frac{\theta}{x}\right)^{\gamma}\right)$$
$$\left(\frac{\theta}{x}\right)^{\gamma} = 1 - R_1$$
$$\frac{\theta}{x} = (1 - R_1)^{\frac{1}{\gamma}}$$
$$x_i = \frac{\theta}{(1 - R_1)^{\frac{1}{\gamma}}} 0 \leq R_1 \leq 1$$
$$x_i = \lambda[\ln(1 - R_2)]^k 0 \leq R_2 \leq 1$$

To find the estimators of (MOM & MLE), we perform simulation experiments using Monte Carlo assuming that;

p_1	0.3			0.5			0.6		
n	150	200	300	150	200	300	150	200	300
n_1	45	60	90	75	100	150	90	120	180

Chosen values of parameters are;

$Model$	γ	θ	k	λ
I	1	1	2	1
II	2	1.5	1	0.9
III	0.7	1.2	4	3

Also chosen values for predetermined censoring time (T).

$Model$	T	
I	2	4
II	4	5
III	3	4

Also the total number of time intervals (m) are;

$Model$	m	
I	4	8
II	8	10
III	6	8

The results for estimators of parameters are explained in the following tables;

Table (1). Values of (MLE) estimatorsfor parameters of ($\mathfrak{X}_p$) distribution for model I, (L=1000)

p_1	n	T	M	γ	k	θ	λ	p_1
0.3	150	2	4	0.94817	0.96025	2.26568	2.81401	0.31732
		4	8	0.96879	0.97201	2.11288	2.86317	0.31059
	200	2	4	0.95429	0.96251	2.27825	2.82030	0.31670
		4	8	0.97418	0.97513	2.10554	2.88890	0.30765
	300	2	4	0.95881	0.96698	2.26012	2.82243	0.31459
		4	8	0.97532	0.97768	2.08876	2.90333	0.30642
0.5	150	2	4	0.95450	0.95737	2.13124	2.84009	0.41021
		4	8	0.97226	0.97079	2.06327	2.86003	0.40850
	200	2	4	0.96073	0.95871	2.14588	2.83887	0.40882
		4	8	0.97724	0.97407	2.08122	2.87188	0.40759
	300	2	4	0.96322	0.96618	2.11040	2.86380	0.40531
		4	8	0.97733	0.97663	2.03648	2.91000	0.40451
0.6	150	2	4	0.95797	0.95533	2.03063	2.90377	0.50053
		4	8	0.97389	0.96861	2.02212	2.86665	0.50640
	200	2	4	0.96528	0.95461	2.06363	2.87442	0.50234
		4	8	0.97854	0.97150	2.01580	2.91009	0.50397
	300	2	4	0.96585	0.96389	2.03774	2.90311	0.50771
		4	8	0.97809	0.97645	2.00214	2.91356	0.50178

Table (2). Values of (MOM) estimatorsfor parameters of ($\mathfrak{X}_p$) distribution for model I, (L=1000)

p_1	n	T	M	γ	k	θ	λ	p_1
0.3	150	2	4	1.07047	1.10602	2.70719	2.63734	0.37072
		4	8	1.00445	1.04807	2.37566	2.86217	0.32875
	200	2	4	1.05734	1.08148	2.71002	2.62503	0.36699
		4	8	1.00407	1.03259	2.35115	2.85014	0.32492
	300	2	4	1.04093	1.07482	2.70679	2.61052	0.36602
		4	8	1.00621	1.02536	2.30721	2.86619	0.31875
0.5	150	2	4	1.06441	1.11208	2.35627	2.73382	0.44579
		4	8	1.01034	1.05092	2.20410	2.90634	0.42048
	200	2	4	1.05257	1.08803	2.38766	2.70362	0.44475
		4	8	1.00114	1.03235	2.20711	2.87189	0.41625
	300	2	4	1.03108	1.08597	2.41001	2.65076	0.45076
		4	8	1.00810	1.02654	2.16223	2.90017	0.41140
0.6	150	2	4	1.06044	1.12125	2.16817	2.84709	0.52421
		4	8	1.00722	1.05342	2.10760	2.96130	0.51035
	200	2	4	1.04695	1.10627	2.20484	2.76686	0.52761
		4	8	1.00557	1.03380	2.11034	2.91883	0.50832
	300	2	4	1.03119	1.10022	2.20445	2.76728	0.52576
		4	8	1.00712	1.03136	2.10258	2.92706	0.50539

Table (3). Values of MSE for MLE estimators of ($\mathfrak{X}_p$) distribution for model I, (L=1000)

p_1	n	T	M	γ	k	θ	λ	p_1
0.3	150	2	4	0.00741	0.00430	1.20449	0.44203	0.00873
		4	8	0.00207	0.00150	0.38894	0.16799	0.00055
	200	2	4	0.00560	0.00325	0.94482	0.36081	0.00605
		4	8	0.00125	0.00179	0.24089	0.11403	0.00093
	300	2	4	0.00417	0.00233	0.78017	0.28677	0.00454
		4	8	0.00176	0.00141	0.23403	0.10093	0.00072
0.5	150	2	4	0.00552	0.00499	0.55513	0.53331	0.00674
		4	8	0.00140	0.00188	0.22415	0.22203	0.00151
	200	2	4	0.00412	0.00402	0.50877	0.44563	0.00542
		4	8	0.00172	0.00109	0.17719	0.16098	0.00106
	300	2	4	0.00332	0.00261	0.37515	0.33206	0.00378
		4	8	0.00144	0.00159	0.14206	0.12792	0.00080
0.6	150	2	4	0.00443	0.00569	0.32196	0.65743	0.00581
		4	8	0.00107	0.00228	0.17298	0.30052	0.00157
	200	2	4	0.00292	0.00470	0.30246	0.56811	0.00497
		4	8	0.00143	0.00141	0.12474	0.20015	0.00108
	300	2	4	0.00260	0.00326	0.22784	0.43362	0.00353
		4	8	0.00126	0.00170	0.10844	0.16203	0.00075

Table (4). Values of MSE for MOMestimators of ($\mathfrak{X}_p$) distribution for model I, (L=1000)

p_1	n	T	M	γ	k	θ	λ	p_1
0.3	150	2	4	0.11823	0.07218	2.91774	1.15049	0.02202
		4	8	0.04052	0.01895	0.83502	0.38275	0.00540
	200	2	4	0.10537	0.04744	2.82819	1.00396	0.02028
		4	8	0.03056	0.01048	0.65555	0.28381	0.00448
	250	2	4	0.07447	0.04062	2.63863	0.91678	0.01758
		4	8	0.02526	0.00599	0.54854	0.24075	0.00349
0.4	150	2	4	0.08661	0.08484	1.45658	1.40219	0.01714
		4	8	0.02720	0.02263	0.40187	0.48058	0.00436
	200	2	4	0.06792	0.06221	1.44320	1.21336	0.01518
		4	8	0.02048	0.01362	0.34111	0.32836	0.00356
	250	2	4	0.05071	0.05067	1.30184	1.07319	0.01376
		4	8	0.01636	0.01799	0.27066	0.28342	0.00272
0.5	150	2	4	0.06843	0.10824	0.86702	1.85042	0.01364
		4	8	0.02101	0.03045	0.25179	0.62560	0.00364
	200	2	4	0.05217	0.07690	0.83012	1.51801	0.01197
		4	8	0.01507	0.01864	0.20662	0.41136	0.00287
	250	2	4	0.03791	0.06061	0.73103	1.40799	0.01011
		4	8	0.01028	0.01211	0.16062	0.35462	0.00218

Table (5). Values of reliability function estimator by MLE , MOM & MSE ($\hat{R}$) for model I (T=2, $p_1 = 0.3$)

n	t_i	$Real$	$\hat{R}_{MLE}$	$\hat{R}_{MOM}$	$MSE(\hat{R})$ $\hat{R}_{MLE}$	$MSE(\hat{R})$ $\hat{R}_{MOM}$
150	0.5	0.82617	0.80837	0.81114	0.00032	0.00076
	1.0	0.68353	0.65507	0.65915	0.00066	0.00108
	1.5	0.56628	0.54032	0.53973	0.00111	0.00116
	2.0	0.46975	0.44786	0.44889	0.00137	0.00146
	2.5	0.39017	0.37272	0.37885	0.00170	0.00208
	3.0	0.32445	0.31125	0.32333	0.00106	0.00209
	3.5	0.27011	0.26069	0.27825	0.00123	0.00310
	4.0	0.22511	0.21890	0.24101	0.00132	0.00402
	4.5	0.18781	0.18421	0.20985	0.00133	0.00477
	5.0	0.15683	0.15531	0.18355	0.00129	0.00530
	5.5	0.13109	0.13113	0.16118	0.00119	0.00561
200	0.5	0.82617	0.80131	0.81099	0.00032	0.00058
	1.0	0.68353	0.65696	0.66003	0.00061	0.00086
	1.5	0.56628	0.54173	0.54104	0.00092	0.00104
	2.0	0.46975	0.44866	0.44980	0.00124	0.00134
	2.5	0.39017	0.37294	0.37916	0.00152	0.00192
	3.0	0.32445	0.31095	0.32320	0.00173	0.00276
	3.5	0.27011	0.25997	0.27788	0.00187	0.00369
	4.0	0.22511	0.21784	0.24054	0.00194	0.00456
	4.5	0.18781	0.18289	0.20937	0.00194	0.00526
	5.0	0.15683	0.15379	0.18307	0.00189	0.00576
	5.5	0.13109	0.12948	0.16072	0.00180	0.00606
300	0.5	0.82617	0.80262	0.81080	0.00026	0.00056
	1.0	0.68353	0.65757	0.66008	0.00052	0.00075
	1.5	0.56628	0.54130	0.54075	0.00078	0.00091
	2.0	0.46975	0.44720	0.44849	0.00103	0.00110
	2.5	0.39017	0.37055	0.37669	0.00123	0.00147
	3.0	0.32445	0.30780	0.31974	0.00138	0.00206
	3.5	0.27011	0.25622	0.27367	0.00146	0.00276
	4.0	0.22511	0.21365	0.23578	0.00148	0.00343
	4.5	0.18781	0.17840	0.20421	0.00145	0.00399
	5.0	0.15683	0.14911	0.17765	0.00139	0.00440
	5.5	0.13109	0.12471	0.15511	0.00130	0.00464

Table (6). Values of reliability function estimator by MLE, MOM & MSE ($\hat{R}$) for model I (T=2, $p_1 = 0.5$)

n	t_i	$Real$	$\hat{R}_{MLE}$	$\hat{R}_{MOM}$	$MSE(\hat{R})$ $\hat{R}_{MLE}$	$MSE(\hat{R})$ $\hat{R}_{MOM}$
150	0.5	0.81940	0.80141	0.80319	0.00035	0.00078
	1.0	0.67253	0.65187	0.64693	0.00069	0.00110
	1.5	0.55286	0.52579	0.52526	0.00103	0.00128
	2.0	0.45520	0.43229	0.43343	0.00146	0.00155
	2.5	0.37536	0.35697	0.36317	0.00175	0.00215
	3.0	0.30997	0.30588	0.30789	0.00198	0.00304
	3.5	0.25635	0.24606	0.26332	0.00213	0.00400
	4.0	0.21229	0.20520	0.22673	0.00220	0.00485
	4.5	0.17603	0.17154	0.20631	0.00221	0.00551
	5.0	0.14615	0.14369	0.17076	0.00215	0.00594
	5.5	0.12149	0.12055	0.15014	0.00206	0.00616
200	0.5	0.81940	0.80346	0.80298	0.00034	0.00070
	1.0	0.67253	0.64480	0.64778	0.00063	0.00087
	1.5	0.55286	0.52714	0.52640	0.00092	0.00105
	2.0	0.45520	0.43296	0.43406	0.00121	0.00132
	2.5	0.37536	0.35701	0.36319	0.00146	0.00187
	3.0	0.30997	0.30538	0.30754	0.00165	0.00267
	3.5	0.25635	0.24512	0.26281	0.00178	0.00355
	4.0	0.21229	0.20393	0.22618	0.00183	0.00435
	4.5	0.17603	0.17003	0.20578	0.00183	0.00498
	5.0	0.14615	0.14202	0.17126	0.00178	0.00540
	5.5	0.12149	0.11878	0.14866	0.00170	0.00563
300	0.5	0.81940	0.80470	0.80279	0.00028	0.00059
	1.0	0.67253	0.64556	0.64805	0.00053	0.00076
	1.5	0.55286	0.52709	0.52670	0.00078	0.00090
	2.0	0.45520	0.43207	0.43367	0.00102	0.00109
	2.5	0.37536	0.35535	0.36180	0.00121	0.00147
	3.0	0.30997	0.30308	0.30517	0.00135	0.00205
	3.5	0.25635	0.24231	0.26064	0.00143	0.00272
	4.0	0.21229	0.20174	0.22240	0.00146	0.00334
	4.5	0.17603	0.16657	0.20152	0.00144	0.00384
	5.0	0.14615	0.13838	0.16566	0.00138	0.00416
	5.5	0.12149	0.11505	0.14380	0.00130	0.00433

Table (7). Values of reliability function estimator by MLE , MOM & MSE ($\hat{R}$) for model I (T=2, $p_1 = 0.6$)

n	t_i	$Real$	$\hat{R}_{MLE}$	$\hat{R}_{MOM}$	$MSE(\hat{R})$ $\hat{R}_{MLE}$	$MSE(\hat{R})$ $\hat{R}_{MOM}$
150	0.5	0.81264	0.82141	0.82319	0.00045	0.00088
	1.0	0.66153	0.68187	0.66693	0.00079	0.00120
	1.5	0.53944	0.55579	0.54526	0.00203	0.00138
	2.0	0.44064	0.45229	0.45343	0.00246	0.00255
	2.5	0.36055	0.37697	0.38317	0.00275	0.00315
	3.0	0.29550	0.32588	0.33789	0.00298	0.00314
	3.5	0.24258	0.26606	0.28332	0.00313	0.00500
	4.0	0.19946	0.22520	0.224673	0.00320	0.00585
	4.5	0.16426	0.19154	0.23631	0.00321	0.00651
	5.0	0.13548	0.16369	0.19076	0.00315	0.00694
	5.5	0.11190	0.14055	0.17014	0.00306	0.00626
200	0.5	0.81264	0.82346	0.82298	0.00054	0.00072
	1.0	0.66153	0.66480	0.66778	0.00065	0.00089
	1.5	0.53944	0.54714	0.54640	0.00094	0.00107
	2.0	0.44064	0.45296	0.45406	0.00124	0.00133
	2.5	0.36055	0.37701	0.38319	0.00148	0.00188
	3.0	0.29550	0.32538	0.32754	0.00167	0.00268
	3.5	0.24258	0.26512	0.28281	0.00179	0.00357
	4.0	0.19946	0.22393	0.24618	0.00184	0.00436
	4.5	0.16426	0.18003	0.22578	0.00184	0.00499
	5.0	0.13548	0.16202	0.18126	0.00179	0.00550
	5.5	0.11190	0.13878	0.16866	0.00172	0.00564
300	0.5	0.81264	0.82470	0.82279	0.00029	0.00069
	1.0	0.66153	0.65556	0.65805	0.00055	0.00075
	1.5	0.53944	0.53709	0.53670	0.00079	0.00099
	2.0	0.44064	0.45207	0.45367	0.00103	0.00119
	2.5	0.36055	0.37535	0.38180	0.00122	0.00148
	3.0	0.29550	0.32308	0.32517	0.00136	0.00206
	3.5	0.24258	0.26231	0.28064	0.00144	0.00274
	4.0	0.19946	0.22174	0.24240	0.00147	0.00335
	4.5	0.16426	0.18657	0.22152	0.00145	0.00385
	5.0	0.13548	0.15838	0.17566	0.00139	0.00417
	5.5	0.11190	0.12505	0.16380	0.00131	0.00434

5. Conclusions

From simulation experiments, the results indicate that:

1-The maximum likelihood estimator for reliability function is better than the moment estimator of reliability function.

2- When ($T = 2, p_1 = 0.5$), the values of estimator ($\hat{R}$) by MLE and MOM, and also of MSE ($\hat{R}$), are convergent inspite of increasing sample size.

3- The estimator of ($\hat{R}$) by MLE, and MOM approximated from real values of R by increasing the size of sample (n_1) from (S_{p_1}) .

4- The values of real function (R) are increasing with (T).

5- The values of mean square errors for estimated parameters are decreased when (T) increasing, this is due to decreasing to failure for units drawn from each sub – population.

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