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Percentiles Based Construction of Acceptance Sampling Plans for the Truncated Type-I Generalized Logistic Distribution

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Abstract In this article, acceptance sampling plans are developed for the truncated type-I generalized logistic distribution percentiles when the life test is truncated at a pre-specified time. The minimum sample size necessary to ensure the specified life percentile is obtained under a given customer's risk. The operating characteristic values (and curves) of the sampling plans as well as the producer's risk are presented. Two examples with real data sets are also given as illustration.

Keywords Acceptance Sampling, Consumer's Risk, Operating Characteristic Function, Producer's Risk, Truncated Life Tests, Producer's Risk

1. Introduction

Acceptance sampling is concerned with inspection and decision making regarding products and is one of the oldest aspects of quality assurance. A typical application of acceptance sampling is as follows. A company receives a shipment of product from a vendor. This product is often a component or raw material used in the company's manufacturing process. A sample is taken from the lot and some quality characteristic of the units in the sample is inspected. On the basis of the information in this sample, a decision is made regarding lot disposition. Usually, this decision is either to accept or to reject the lot. Accepted lots are put into production; rejected lots may be returned to the vendor or may be subjected to some other lot disposition action. While it is customary to think of acceptance sampling as a receiving inspection activity, there are other uses of sampling methods. For example, frequently a manufacturer will sample and inspect its own product at various stages of production. Lots that are accepted are sent forward for further processing, while rejected lots may be reworked or scrapped. For the purpose of reducing the test time and cost, a truncated life test may be conducted to determine the smallest sample size to ensure a certain mean life of products when the life test is terminated at a pre-assigned time t_0, and the number of failures observed does not exceed a given acceptance number c.

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A sampling inspection plans in the case that the sample observations are lifetimes of products put to test aims at verifying that the actual population mean exceeds a required minimum. The population mean stands for the mean lifetime of the product, say μ. If μ_0 is a specified minimum value, then one would like to verify that $\mu \geq \mu_0$, this means that the true unknown population mean lifetime of the product exceeds the specified value. On the basis of a random sample of size n, the lot is accepted, if by means of a suitable decision criterion, the acceptance sampling plan decides in favor of $\mu \geq \mu_0$. Otherwise the lot is rejected. The decision criterion is naturally based on the number of observed failures in the sample of, n products during a specified time t_0 from which a lower bound for the unknown mean lifetime is derived. If the observed number of failures is large, say larger than a number c, the derived lower bound is smaller than μ_0 and the hypothesis $\mu \geq \mu_0$ is not verified. Hence, the lot cannot be accepted. Such a sampling plan is named Reliability test plan or Acceptance sampling plans on life tests.

A common practice in life testing is to terminate the life test by a pre-determined time t_0 and note the number of failures (assuming that a failure is well defined). One of the objectives of these experiments is to set a lower confidence limit on the mean life. It is then to establish a specified mean life with a given probability of at least p^* which provides protection to consumers. The decision to accept the specified mean life occurs if and only if the number of observed failures at the end of the fixed time t_0 does not exceed a given number 'c'- called the acceptance number. The test may get terminated before the time t_0 is reached when the number of failures exceeds 'c' in which case the decision is to reject the lot. For such a truncated life test and the associated decision rule; we are interested in obtaining the smallest sample size to arrive at a decision.

The acceptance sampling plans were developed based on the mean lifetime of items for assuring the quality and reliability of products. These type of acceptance sampling plans for truncated life tests can be found in Epstein[3], Sobel and Tischendrof[20], Goode and Kao[5], Gupta and Groll[7], Gupta[6], Fertig and Mann[4], Kantam and Rosaiah[8], Kantam et al.[9], Baklizi[1], Wu and Tsai[24], Rosaiah and Kantam[18], Tsai and Wu[22], Rao et.al.[15] and Rao et al.[16].

However, the sampling plans based on the population mean may not catch the specific percentile of product lifetime required for engineering design considerations. When the quality of interest is a low percentile, the sampling plans based on the population mean could pass the lot that has the low percentile below the pre-specified standard required by the customer. Therefore, engineers pay more attention to the percentile of lifetime than the mean life in life-testing applications. In view of this, recently more authors proposed the acceptance sampling plans based on percentile, see for example, Balakrishnan et al.[2], Lio et al.[11], Lio et al.[12], Rao and Kantam[14], Rao et al.[17] and Rao[13]. They argued that the sampling plans proposed at the mean life in a skewed distribution will pass out the product with lower percentiles. Gupta[6] also pointed that for a skewed distribution, median life as a quality parameter performs better than the mean lifetime. These reasons motivate to develop acceptance sampling plans based on the percentiles of the truncated type-I generalized logistic distribution under a truncated life test.

The rest of the article is organized as follows. The proposed sampling plans are established for the truncated type-I generalized logistic percentiles under a truncated life test, along with the operating characteristic (OC) and some relevant tables, is given in Section 2. Two examples based on real fatigue life data sets are provided for the illustration in Section 3 and discussion and some conclusions are made in Section 4.

2. Acceptance Sampling Plans

Recently truncated type-I generalized logistic distribution (TTGLD) has been proposed by Rosaiah *et al.*[19] and they have studied estimation of scale parameter. The two-parameter truncated type-I generalized logistic distribution has the following density function

$$f(x) = \frac{2^{\alpha}(\alpha/\sigma)e^{-x/\sigma}}{[1+e^{-x/\sigma}]^{(\alpha+1)}(2^{\alpha}-1)};\ \text{for } x \ge 0 \qquad (1)$$

and the distribution function

$$F(x) = \frac{1}{(2^{\alpha}-1)}\left[\frac{2^{\alpha}}{(1+e^{-x/\sigma})^{\alpha}} - 1\right];\ \text{for } x \ge 0. \qquad (2)$$

Here $\alpha > 0$ and $\sigma > 0$ are shape and scale parameters respectively. The survival and hazard functions of truncated type-I generalized logistic distribution are respectively given by

$$r(x) = 1 - \frac{1}{(2^{\alpha}-1)}\left[\frac{2^{\alpha}}{(1+e^{-x/\sigma})^{\alpha}} - 1\right];\ \text{for } x \ge 0, \qquad (3)$$

and

$$h(x) = \frac{(\alpha/\sigma)e^{-x/\sigma}}{[1+e^{-x/\sigma}][(1+e^{-x/\sigma})^{\alpha}-1]}. \qquad (4)$$

They showed that TTGLD is an increasing failure rate (IFR) model. From now on truncated type-I generalized logistic distribution with the shape parameter α and scale parameter σ will be denoted by TTGLD (α, σ).

Given $0 < q < 1$ the $100q^{\text{th}}$ percentile (or the q^{th} quantile) is given by

$$t_q = -\sigma \ln\left[\left(\frac{2^{\alpha}}{q(2^{\alpha}-1)+1}\right)^{1/\alpha} - 1\right]. \qquad (5)$$

The t_q is increasing with respect to α and q. Therefore, the $100q^{\text{th}}$ percentile, t_q, is depend upon α. When q=0.5, then $t_q = -\sigma \ln\left[\left(\frac{2^{\alpha}}{(2^{\alpha}+1)}\right)^{1/\alpha} - 1\right]$ and $t_{0.5}$ is also the median of truncated type-I generalized logistic distribution. Let $\eta = -\ln\left[\left(\frac{2^{\alpha}}{q(2^{\alpha}-1)+1}\right)^{1/\alpha} - 1\right]$. Then, Eq. (5) implies that

$$\sigma = t_q / \eta. \qquad (6)$$

To develop acceptance sampling plans for the truncated type-I generalized logistic percentiles, the scale parameter σ in the truncated type-I generalized logistic cdf is replaced by Eq. (6) and the truncated type-I generalized logistic cdf is rewritten as

$$F(t) = \frac{1}{(2^{\alpha}-1)}\left[\left(\frac{2^{\alpha}}{(1+e^{-t/(t_q/\eta)})^{\alpha}}\right) - 1\right]$$

Letting $\delta = t/t_q$, *F(t)* can be rewritten emphasizing its dependence on δ as

$$F(t,\delta) = \frac{1}{(2^{\alpha}-1)}\left[\left(\frac{2^{\alpha}}{(1+e^{-\delta\eta})^{\alpha}}\right) - 1\right].$$

$$\frac{\partial F(t,\delta)}{\partial \delta} = \frac{1}{(2^{\alpha}-1)}\left(\frac{\alpha\eta 2^{\alpha} e^{-\delta\eta}}{(1+e^{-\delta\eta})^{\alpha+1}}\right);\delta \geq 0$$

A common practice in life testing is to terminate the life test by a pre-determined time t, the probability of rejecting a bad lot be at least p^*, and the maximum number of allowable bad items to accept the lot be c. The acceptance sampling plan for percentiles under a truncated life test is to set up the minimum sample size n for this given acceptance number c such that the consumer's risk, the probability of accepting a bad lot, does not exceed $1- p^*$. A bad lot means that the true $100q^{th}$ percentile, t_q, is below the specified percentile, t_q^0. Thus, the probability p^* is a confidence level in the sense that the chance of rejecting a bad lot with $t_q < t_q^0$ is at least equal to p^*. Therefore, for a given p^*, the proposed acceptance sampling plan can be characterized by the triplet $(n,c,t/t_q^0)$.

2.1. Minimum Sample Size

For a fixed p^* our sampling plan is characterized by $(n,c,t/t_q^0)$. Here we consider sufficiently large sized lots so that the binomial distribution can be applied. The problem is to determine for given values of p^* ($0 < p^* <1$), t_q^0 and c, the smallest positive integer, n required to assert that $t_q > t_q^0$ must satisfy

$$\sum_{i=0}^{c}\binom{n}{i} p_0^i \left(1-p_0\right)^{n-i} \leq 1-p^*, \quad (7)$$

where $p = F(t;\delta_0)$ is the probability of a failure during the time t given a specified 100 q^{th} percentile of lifetime t_q^0 and depends only on $\delta_0 = t/t_q^0$, since $\partial F(t;\delta)/\partial\delta > 0$, $F(t;\delta)$ is a non-decreasing function of δ. Accordingly, we have

$F(t,\delta) < F(t,\delta_0) \Leftrightarrow \delta \leq \delta_0$, Or equivalently, $F(t,\delta) \leq F(t,\delta_0) \Leftrightarrow t_q \geq t_q^0$.

The smallest sample size n satisfying the inequality (7) can be obtained for any given q, t/t_q^0, p^* and α. To save space, only the results of small sample sizes for q=0.1, t/t_q^0 =0.7, 0.9, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5; p^*=0.75, 0.90, 0.95, 0.99; c = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and α =2 are reported in Tables 1.

If $p = F(t;\delta_0)$ is small and n is large the binomial probability may be approximated by Poisson probability with parameter $\lambda = n\,p$ so that the left side of (7) can be written as

$$\sum_{i=0}^{c}\frac{e^{-\lambda}\lambda^i}{i\,!} \leq 1-p^*, \quad (8)$$

where $\lambda = n\,F(t;\delta_0)$. The minimum values of n satisfying (8) are obtained for the same combination of q, t/t_q^0, p^* and α values as those used for (7). The results are reported in Table 2.

2.2. Operating Characteristic of the Sampling Plan $(n,c,t/t_q^0)$

The operating characteristic (OC) function of the sampling plan $(n,c,t/t_q^0)$ is the probability of accepting a lot. It is given as

$$L(p) = \sum_{i=0}^{c}\binom{n}{i} p^i \left(1-p\right)^{n-i}, \quad (9)$$

where $p = F(t;\delta)$. It should be noticed that $F(t;\delta)$ can be represented as a function of $\delta = t/t_q$. Therefore, $p = F(\frac{t}{t_q^0}\frac{1}{d_q})$ where $d_q = t_q/t_q^0$. Using Eq. (9), the OC values and OC curves can be obtained for any sampling plan, $(n,c,t/t_q^0)$, and any α. To save space, we present Tables 3 to show the OC values for the sampling plan $(n,c=5,t/t_{0.1}^0)$ with α =2. Figure 1 shows the OC curves for the sampling plan $(n,c,t/t_{0.1}^0)$ with p^* =0.90 for $\delta_0 = 1, \alpha = 2$, where c = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

2.3. Producer's Risk

The producer's risk is defined as the probability of rejecting the lot when $t_q > t_q^0$. For a given value of the producer's risk, say γ, we are interested in knowing the value of d_q to ensure the producer's risk is less than or equal to γ if a sampling plan $(n,c,t/t_q^0)$ is developed at a specified confidence level p^*. Thus, one needs to find the smallest value d_q according to Eq. (9) as

$$\sum_{i=0}^{c}\binom{n}{i} p^i \left(1-p\right)^{n-i} \geq 1-\gamma, \quad (10)$$

where $p = F(\frac{t}{t_q^0}\frac{1}{d_q})$, $d_q = t_q/t_q^0$. To save space, based on sampling plans $(n,c,t/t_q^0)$ established in Tables 1 the minimum ratios of $d_{0.1}$ for the acceptability of a lot under α =2, at the producer's risk of γ =0.05 are presented in Table 4.

Table 1. Minimum sample sizes necessary to assert the 10th percentile to exceed a given values, $t_{0.1}^{0}$, with probability p^* and the corresponding acceptance number, *c*, for the truncated type-I generalized logistic distribution with $\alpha = 2$ using the binomial approximation

p^*	c	$t / t_{0.1}^{0}$							
		0.7	0.9	1.0	1.5	2.0	2.5	3.0	3.5
0.75	0	20	15	14	9	6	5	4	3
0.75	1	39	30	27	17	13	10	8	7
0.75	2	56	43	39	25	18	14	12	10
0.75	3	74	56	51	33	24	19	15	13
0.75	4	90	69	62	40	29	23	19	16
0.75	5	107	82	73	48	35	27	22	19
0.75	6	123	95	85	55	40	32	26	22
0.75	7	140	107	96	62	46	36	29	25
0.75	8	156	120	107	69	51	40	33	28
0.75	9	172	132	118	77	56	44	36	31
0.75	10	188	144	129	84	61	48	40	34
0.90	0	33	25	22	14	10	8	6	5
0.90	1	56	42	38	24	18	14	11	9
0.90	2	76	58	52	33	24	19	15	13
0.90	3	96	73	65	42	31	24	19	16
0.90	4	115	88	78	51	37	29	23	20
0.90	5	133	102	91	59	43	33	27	23
0.90	6	151	116	104	67	49	38	31	26
0.90	7	169	130	116	75	54	43	35	29
0.90	8	187	143	128	83	60	47	38	32
0.90	9	204	157	140	90	66	52	42	36
0.90	10	221	170	152	98	72	56	46	39
0.95	0	42	32	29	18	13	10	8	7
0.95	1	67	52	46	29	21	16	13	11
0.95	2	90	69	61	39	28	22	18	15
0.95	3	111	85	76	49	35	27	22	19
0.95	4	131	100	89	57	42	32	26	22
0.95	5	150	115	103	66	48	37	30	26
0.95	6	169	130	116	75	54	42	34	29
0.95	7	188	144	129	83	60	47	38	32
0.95	8	207	158	142	91	66	52	42	36
0.95	9	225	173	154	99	72	56	46	39
0.95	10	243	186	167	107	78	61	50	42
0.99	0	65	50	44	28	20	16	12	10
0.99	1	94	72	64	41	29	23	18	15
0.99	2	119	91	81	52	37	29	23	19
0.99	3	143	109	97	62	45	35	28	23
0.99	4	165	126	113	72	52	40	33	27
0.99	5	187	143	127	82	59	46	37	31
0.99	6	208	159	142	91	66	51	41	35
0.99	7	228	175	156	100	73	56	46	38
0.99	8	248	190	170	109	79	61	50	42
0.99	9	268	205	183	118	85	66	54	45
0.99	10	288	220	197	126	92	71	58	49

Table 2. Minimum sample sizes necessary to assert the 10th percentile to exceed a given values, $t_{0.1}^{0}$, with probability p^* and the corresponding acceptance number, *c*, for the truncated type-I generalized logistic distribution with $\alpha = 2$ using the Poisson approximation

p^*	c	$t / t_{0.1}^{0}$							
		0.7	0.9	1.0	1.5	2.0	2.5	3.0	3.5
0.75	0	21	16	14	10	7	6	5	4
0.75	1	33	26	23	15	11	9	8	6
0.75	2	55	43	38	25	19	15	12	11
0.75	3	74	57	51	33	25	20	16	14
0.75	4	91	70	63	41	31	24	20	17
0.75	5	108	83	75	49	36	29	24	20
0.75	6	125	96	86	56	42	33	27	23
0.75	7	141	109	97	64	47	37	31	26
0.75	8	157	121	109	71	52	41	34	29
0.75	9	173	134	120	78	58	46	38	32
0.75	10	190	146	131	85	63	50	41	35
0.90	0	34	26	24	15	12	9	8	7
0.90	1	53	41	36	24	18	14	12	10
0.90	2	77	59	53	35	26	20	17	15
0.90	3	97	75	67	44	32	26	21	18
0.90	4	116	90	80	53	39	31	25	22
0.90	5	135	104	93	61	45	36	30	25
0.90	6	153	118	106	69	51	40	33	29
0.90	7	171	132	118	77	57	45	37	32
0.90	8	189	146	130	85	63	50	41	35
0.90	9	207	159	143	93	69	54	45	38
0.90	10	224	173	155	101	74	59	49	42
0.95	0	44	34	30	20	15	12	10	8
0.95	1	66	51	46	30	22	18	15	13
0.95	2	91	70	63	41	30	24	20	17
0.95	3	113	87	78	51	38	30	25	21
0.95	4	133	103	92	60	44	35	29	25
0.95	5	153	118	106	69	51	40	33	29
0.95	6	172	133	119	78	57	45	38	32
0.95	7	191	147	132	86	64	50	42	36
0.95	8	210	162	145	95	70	55	46	39
0.95	9	229	176	158	103	76	60	50	42
0.95	10	247	190	170	111	82	65	54	46
0.99	0	67	52	47	30	23	18	15	13
0.99	1	94	73	65	43	32	25	21	18
0.99	2	122	94	84	55	41	32	27	23
0.99	3	146	113	101	66	49	39	32	27
0.99	4	169	130	117	76	56	44	37	31
0.99	5	191	147	132	86	63	50	41	35
0.99	6	212	163	146	95	70	56	46	39
0.99	7	233	179	160	105	77	61	51	43
0.99	8	253	195	175	114	84	66	55	47
0.99	9	273	210	188	123	91	72	59	51
0.99	10	293	226	202	132	97	77	64	54

Table 3. Operating characteristic values of the sampling plan $(n, c=5, t/t_{0.1}^0)$ for a given p^* under truncated type-I generalized logistic distribution with $\alpha = 2$

p^*	n	$t/t_{0.1}^0$	$t_{0.1}/t_{0.1}^0$							
			1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75
0.75	107	0.7	0.2465	0.4667	0.6439	0.7664	0.8465	0.8979	0.9311	0.9526
0.75	82	0.9	0.2466	0.4709	0.6499	0.7725	0.8517	0.9022	0.9343	0.9551
0.75	73	1.0	0.2499	0.4766	0.6561	0.7779	0.8561	0.9055	0.9368	0.9570
0.75	48	1.5	0.2336	0.4667	0.6531	0.7787	0.8584	0.9081	0.9392	0.9590
0.75	35	2.0	0.2331	0.4744	0.6648	0.7905	0.8684	0.9160	0.9453	0.9636
0.75	27	2.5	0.2440	0.4944	0.6862	0.8087	0.8825	0.9265	0.9529	0.9692
0.75	22	3.0	0.2466	0.5037	0.6977	0.8192	0.8909	0.9328	0.9576	0.9725
0.75	19	3.5	0.2248	0.4833	0.6844	0.8117	0.8869	0.9307	0.9565	0.9720
0.90	133	0.7	0.0981	0.2604	0.4384	0.5909	0.7073	0.7914	0.8508	0.8925
0.90	102	0.9	0.0971	0.2621	0.4431	0.5972	0.7138	0.7974	0.8559	0.8967
0.90	91	1.0	0.0976	0.2647	0.4474	0.6020	0.7184	0.8014	0.8593	0.8994
0.90	59	1.5	0.0931	0.2649	0.4541	0.6126	0.7299	0.8122	0.8687	0.9072
0.90	43	2.0	0.0910	0.2684	0.4640	0.6256	0.7431	0.8240	0.8786	0.9153
0.90	33	2.5	0.0975	0.2870	0.4899	0.6522	0.7667	0.8434	0.8939	0.9271
0.90	27	3.0	0.0942	0.2867	0.4944	0.6594	0.7744	0.8504	0.8998	0.9319
0.90	23	3.5	0.0866	0.2772	0.4883	0.6571	0.7745	0.8516	0.9013	0.9334
0.95	150	0.7	0.0497	0.1663	0.3225	0.4762	0.6059	0.7071	0.7830	0.8389
0.95	115	0.9	0.0490	0.1675	0.3267	0.4826	0.6132	0.7143	0.7895	0.8445
0.95	103	1.0	0.0479	0.1666	0.3267	0.4835	0.6148	0.7161	0.7913	0.8461
0.95	66	1.5	0.0482	0.1738	0.3426	0.5048	0.6372	0.7372	0.8099	0.8618
0.95	48	2.0	0.0470	0.1769	0.3525	0.5192	0.6531	0.7523	0.8232	0.8731
0.95	37	2.5	0.0489	0.1871	0.3712	0.5420	0.6757	0.7725	0.8402	0.8869
0.95	30	3.0	0.0491	0.1923	0.3828	0.5570	0.6911	0.7864	0.8520	0.8965
0.95	26	3.5	0.0386	0.1692	0.3564	0.5343	0.6740	0.7743	0.8437	0.8909
0.99	187	0.7	0.0096	0.0544	0.1460	0.2674	0.3948	0.5122	0.6124	0.6943
0.99	143	0.9	0.0096	0.0555	0.1499	0.2745	0.4042	0.5226	0.6229	0.7042
0.99	127	1.0	0.0100	0.0577	0.1552	0.2824	0.4137	0.5325	0.6324	0.7128
0.99	82	1.5	0.0092	0.0577	0.1595	0.2925	0.4283	0.5496	0.6499	0.7296
0.99	59	2.0	0.0095	0.0620	0.1721	0.3133	0.4541	0.5768	0.6762	0.7534
0.99	46	2.5	0.0087	0.0615	0.1751	0.3211	0.4656	0.5901	0.6897	0.7661
0.99	37	3.0	0.0091	0.0658	0.1871	0.3403	0.4887	0.6138	0.7118	0.7856
0.99	31	3.5	0.0088	0.0663	0.1912	0.3489	0.5004	0.6265	0.7242	0.7968

Table 4. Minimum ratio of true $d_{0.1}$ for the acceptability of a lot for the truncated type-I generalized logistic distribution with $\alpha = 2$ and producer's risk of 0.05

p^*	c	$t / t_{0.1}^0$							
		0.7	0.9	1.0	1.5	2.0	2.5	3.0	3.5
0.75	0	24.9190	25.8398	24.9875	22.2916	23.2612	22.4014	19.6850	22.5124
0.75	1	7.1685	7.1685	6.7431	6.8446	6.5963	6.3211	6.4103	7.3421
0.75	2	4.4603	4.5045	4.3122	4.1356	4.0080	4.1169	3.9904	4.5725
0.75	3	3.4855	3.5261	3.4200	3.3201	3.2841	3.1046	3.1260	3.5817
0.75	4	2.9824	2.9824	2.8877	2.7988	2.7732	2.7480	2.6911	3.0731
0.75	5	2.6831	2.6518	2.6288	2.5549	2.4716	2.4125	2.4254	2.7732
0.75	6	2.4783	2.4649	2.3998	2.3321	2.3381	2.2795	2.2457	2.5694
0.75	7	2.3026	2.3026	2.2401	2.2237	2.1810	2.1101	2.1200	2.4190
0.75	8	2.1968	2.1758	2.1200	2.0956	2.0623	2.0437	2.0210	2.3084
0.75	9	2.0907	2.0812	2.0483	1.9988	1.9685	1.9350	1.9433	2.2237
0.75	10	2.0121	2.0032	1.9728	1.9186	1.8947	1.8986	1.8829	2.1501
0.90	0	41.5455	40.6174	38.7147	36.9822	36.9822	33.4448	32.6052	37.2856
0.90	1	10.0100	10.1215	9.5877	9.5877	9.2937	8.6730	8.2781	9.4877
0.90	2	6.0277	5.9880	5.7241	5.5157	5.4825	5.1706	5.2301	5.9488
0.90	3	4.5496	4.5045	4.3535	4.2918	4.1545	3.9386	3.8715	4.4170
0.90	4	3.8066	3.7439	3.6832	3.5676	3.4990	3.3322	3.3818	3.8551
0.90	5	3.3322	3.3080	3.2258	3.1368	3.0120	2.9630	2.9438	3.3693
0.90	6	3.0221	3.0120	2.9155	2.8514	2.7732	2.7152	2.6596	3.0423
0.90	7	2.7988	2.7732	2.7071	2.6062	2.5988	2.5478	2.4582	2.8161
0.90	8	2.6137	2.5988	2.5407	2.4649	2.4190	2.3502	2.3143	2.6441
0.90	9	2.4851	2.4649	2.3935	2.3502	2.3202	2.2568	2.2568	2.5840
0.90	10	2.3685	2.3563	2.2967	2.2568	2.2075	2.1810	2.1603	2.4649
0.95	0	53.2198	53.5332	49.7265	48.1464	46.1894	44.3853	45.4959	52.0021
0.95	1	12.4844	12.3153	11.5340	11.1111	10.5932	10.3520	10.1215	11.6822
0.95	2	7.1685	7.0572	6.7431	6.4558	6.3211	6.2344	6.0277	6.8966
0.95	3	5.2910	5.2604	5.0839	4.8403	4.6664	4.5725	4.5956	5.2604
0.95	4	4.3328	4.2717	4.1169	4.0437	3.8551	3.7594	3.7133	4.2517
0.95	5	3.7594	3.7439	3.6101	3.4990	3.3818	3.2960	3.3322	3.8066
0.95	6	3.3818	3.3568	3.2605	3.1476	3.0628	2.9824	2.9727	3.3944
0.95	7	3.0941	3.0836	2.9922	2.8877	2.8425	2.7647	2.7152	3.1046
0.95	8	2.8877	2.8785	2.7816	2.7071	2.6752	2.5988	2.5988	2.9727
0.95	9	2.7397	2.7071	2.6288	2.5621	2.4988	2.4716	2.4450	2.7988
0.95	10	2.5913	2.5840	2.4988	2.4450	2.3998	2.3685	2.3261	2.6596
0.99	0	82.7815	81.3008	77.8210	74.0192	74.0192	66.4452	65.0195	74.0192
0.99	1	1.8116	1.7867	1.7159	1.6181	1.6038	1.5031	1.4596	1.6686
0.99	2	0.9948	0.9841	0.9471	0.8985	0.8802	0.8372	0.8061	0.9212
0.99	3	0.7142	0.7065	0.6775	0.6560	0.6380	0.6127	0.5870	0.6710
0.99	4	0.5733	0.5715	0.5470	0.5277	0.5079	0.5031	0.4805	0.5490
0.99	5	0.4914	0.4852	0.4711	0.4530	0.4422	0.4274	0.4182	0.4778
0.99	6	0.4352	0.4321	0.4169	0.4043	0.3914	0.3783	0.3772	0.4311
0.99	7	0.3958	0.3924	0.3788	0.3700	0.3558	0.3516	0.3395	0.3880
0.99	8	0.3648	0.3631	0.3508	0.3404	0.3296	0.3252	0.3193	0.3650
0.99	9	0.3410	0.3386	0.3292	0.3177	0.3095	0.3049	0.2971	0.3396
0.99	10	0.3223	0.3210	0.3098	0.3030	0.2937	0.2888	0.2854	0.3262

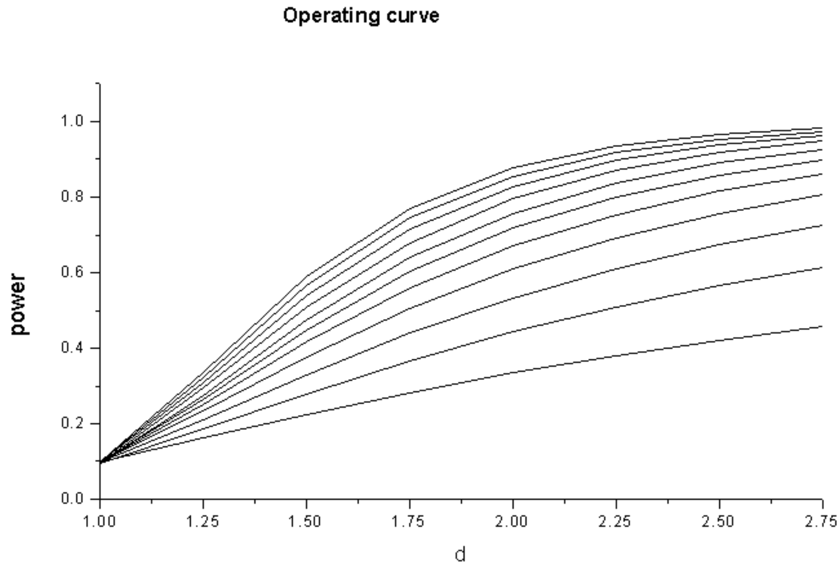

Figure 1. OC curves for c=0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, respectively under p^*=0.90, $\delta_0 = 1$ and $\alpha = 2$, based on the 10th percentile, $d = d_{0.1}$, of truncated type-I generalized logistic distribution

3. Illustrative Examples

In this section, we consider two examples with real data sets are given to illustrate the proposed acceptance sampling plans. The first data set is of the data given arisen in tests on endurance of deep groove ball bearings ([10], p.28). The data are the number of million revolutions before failure for each of the 23 ball bearings in life test and they are: 51.84, 51.96, 54.12, 55.56, 67.80, 68.44, 68.64, 68.88, 84.12, 93.12, 98.64, 105.12, 105.84, 127.92, 128.04 and 173.40. The second data set regarding the software reliability was presented by Wood[23], analyzed via the acceptance sampling viewpoint by Rosaiah and Kantam[18], Balakrishnan *et al.*[2], Lio *et al.*[11] and Rao and Kantam[14]. The software reliability data set was reported in hours as 519, 968, 1430, 1893, 2490, 3058, 3625, 4422, and 5218. As the confidence level is assured by this acceptance sampling plan only if the lifetimes are from the truncated type-I generalized logistic distribution. Then, we should check if it is reasonable to admit that the given sample comes from the truncated type-I generalized logistic distribution by the goodness of fit test and model selection criteria. The first data set was used by Sultan[21] to demonstrate the goodness of fit for generalized exponential distribution. Balakrishnan *et al.*[2] compared the goodness of fits among the Rayleigh, generalized BS, and BS distributions for the software reliability data set presented here using probability plots and showed that the generalized BS model (R-square (RS) = 0.97) was slightly better than the BS model (RS = 0.96) and both models were much better than the Rayleigh model (RS=0.87). However, the acceptance sampling plans under the truncated life test based on the truncated type-I generalized logistic distribution for percentiles have not yet been developed. We have applied QQ plot and RS method to test the goodness of fit for both data sets for truncated type-I generalized logistic distribution and we got RS= 0.9811 for first data set and RS= 0.9896 for second data set. Hence, the truncated type-I generalized logistic distribution could also provide reasonable goodness of fits for both data sets.

3.1. Example 1

Assume that the lifetime distribution is truncated type-I generalized logistic distribution with α =2 and that the experimenter is interested to establish the true unknown 10th percentile lifetime for the ball bearings to be at least 20 million revolutions with confidence p^*=0.75 and the life test would be ended at 50 million revolutions, which should have led to the ratio $t/t_{0.1}^0$ = 2.5. Thus, for an acceptance number c = 4 and the confidence level p^*=0.75, the required sample size n found from Table 1 should be at least 23. Therefore, in this case, the acceptance sampling plan from truncated life tests for the truncated type-I generalized logistic distribution 10th percentile should be $(n, c, t/t_q^0)$= (23, 4, 2.5). Based on the ball bearings data, the experimenter must have decided whether to accept or reject the lot. The lot should be accepted only if the number of items of which lifetimes were less than or equal to the scheduled test lifetime, 50 million revolutions, was at most 4 among the first 23 observations. Since there is no failure time less than or equal to 50 million revolutions in the given sample of n =23 observations, the experimenter would accept the lot, assuming the 10th percentile lifetime $t_{0.1}$ of at least 20 million revolutions with a confidence level of p^*=0.75. The OC values for the acceptance sampling plan $(n, c, t/t_q^0)$ = (23,4,2.5) and confidence level p^*=0.75 under truncated type-I generalized logistic distribution with α =2 from Table 3 is as follows:

$t_{0.1}/t_{0.1}^0$	1.00	1.25	1.50	1.75	2.00	2.25	2.50	2.75
OC	0.2338	0.4600	0.6402	0.7631	0.8427	0.8938	0.9269	0.9486

This shows that if the true 10th percentile is equal to the required 10th percentile ($t_{0.1}/t_{0.1}^0$ = 1.00) the producer's risk is approximately 0.7662 (=1- 0.2338). The producer's risk is almost equal to 0.0731 when the true 10th percentile is greater than or equal to 2.5 times the specified 10th percentile.

From Table 4, the experimenter could get the values of $d_{0.1}$ for different choices of *c* and $t/t_{0.1}^0$ in order to assert that the producer's risk was less than 0.05. In this example, the value of $d_{0.1}$ should be 2.8877 for *c* = 4, $t/t_{0.1}^0$ =1.0 and p^* =0.75. This means the product can have a 10th percentile life of 2.8877 times the required 10th percentile lifetime in order that under the above acceptance sampling plan the product is accepted with probability of at least 0.95.

Alternatively, assume that products have a truncated type-I generalized logistic distribution with α =2, and consumers wish to reject a bad lot with probability of p^* =0.75. What should the true 10th percentile life of products be so that the producer's risk is 0.05 if the acceptance sampling plan is based on an acceptance number *c*=3 and $t/t_{0.1}^0$ =0.7? From Table 4, we can find that the entry for α =2, p^*=0.75, *c*=3, and $t/t_{0.1}^0$ =0.7 is $d_{0.1}$ =3.4855. Thus, the manufacturer's product should have a 10th percentile life at least 3.4855 times the specified 10th percentile life in order for the products to be accepted with probability 0.75 under the above acceptance sampling plan. Table 1 indicates that the number of products required to be tested is *n*=74 so that the sampling plan is $(n, c, t/t_{0.1}^0)$ = (74, 3, 0.7).

3.2. Example 2

Suppose an experimenter would like to establish the true unknown 10th percentile lifetime for the software mentioned above to be at least 100h and the life test would be ended at 350 h, which should have led to the ratio $t/t_{0.1}^0$ =3.5. The goodness of fit test for these nine observations were verified and showed that truncated type-I generalized logistic model as a reasonable goodness of fit for these nine observations. Thus, with *c*=1 and p^*=0.90, the experimenter should find from Table 1 the sample size *n* must be at least 9 and the sampling plan to be $(n, c, t/t_{0.1}^0)$ = (9, 1, 3.5). Since there were no items with a failure time less than or equal to 350h in the given sample of *n* =9 observations, the experimenter would accept the lot, assuming the 10th percentile lifetime $t_{0.1}$ of at least 100h with a confidence level of p^*=0.90.

4. Discussion and Conclusions

The acceptance sampling plans based on the truncated type-I generalized logistic population mean could have less chance to report a failure than the acceptance sampling plans based on 10th percentile. The acceptance sampling plans based on population mean could accept the lot of bad quality of the 10th percentiles. The minimum sample sizes are reported in Table 1 of this article for the 10th percentiles are compared with the minimum sample sizes are reported in Table 1 of Lio *et al.*[11] and Rao and Kantam[14]. It shows that the minimum sample sizes using truncated type-I generalized logistic population are smaller than those reported in Tables 1 of Lio *et al.*[11] whereas, the minimum sample sizes using truncated type-I generalized logistic population are smaller than those reported in Tables 1 of Rao and Kantam[14] for the 10th percentile when $\delta_0 \le 1.0$ and larger than those reported in Tables 1 of Rao and Kantam[14] for log-logistic population for the 10th percentile when $\delta_0 > 1.0$.

This article has derived the acceptance sampling plans based on the truncated type-I generalized logistic percentiles when the life test is truncated at a pre-fixed time. The procedure is provided to construct the proposed sampling plans for the percentiles of the truncated type-I generalized logistic distribution with known parameter α =2. To ensure that the life quality of products exceeds a specified one in terms of the life percentile, the acceptance sampling plans based on percentiles should be used. Some useful tables are provided and applied to establish acceptance sampling plans for two examples.

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