American Journal of Mathematics and Statistics 2013, 3(3): 130-134
DOI: 10.5923/j.ajms.20130303.05

On the Efficiency of Ratio Estimator Based on Linear Combination of Median, Coefficients of Skewness and Kurtosis

Kazeem A. Adepoju*, **Olanrewaju I. Shittu**

Department of Statistics University of Ibadan Ibadan, Nigeria

Abstract In this paper, an estimator which is robust in the present of outlier and Skewness is proposed. This is achieved by incorporating median, a good measure of location in this regard to the modified ratio estimator developed as in[22]. Using well analyzed data to illustrate the procedure for the Ratio estimator, its Mean square error was observed the minimum of the existing estimators considered. The proposed modified estimator is uniformly better than all other estimators and thus most preferred over the existing modified ratio estimators for the use in practical applications for certain population with peculiar characteristics

Keywords Ratio, Product, Estimator, Parameter, Mean Square Error

1. Introduction

Ratio estimation involves the use of known population totals for auxiliary variables to improve the weighting from sample values to population estimates of interest. It operates by comparing the sample estimate for an auxiliary variable with the known population total for the same variable on the frame. The ratio of the sample estimate of the auxiliary variable to its population total on the frame is used to adjust the sample estimate for the variable of interest. The ratio weights are given by X| x (where X is the known population total for the auxiliary variable and x is the corresponding estimate of the total based on all responding units in the sample). These weights assume that the population for the variable of interest will be estimated by the sample equally as well (or poorly) as the population total for the auxiliary variable is estimated by the sample.

Consider a finite population U = $\{U_1, U_2, .U_N\}$ of N distinct and identifiable units. Let Y is a study variable with value Y_i measured on Ui, i = 1, 2, 3… N giving a vector Y= $\{Y_1, Y_2 \ldots, Y_N\}$. The problem is to estimate the population mean $\bar{y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$ with some desirable properties on the basis of a random sample selected from the population U. The simplest estimator of population mean is the sample mean obtained by using simple random sampling without replacement, when there is no additional information on the auxiliary variable available. Sometimes in sample surveys, along with the study variable Y, information on auxiliary variable x correlated with Y is also collected. This information on auxiliary variable x, may be utilized to obtain a more efficient estimator of the population mean.

The two broad categories of estimators using auxiliary information are the Ratio method and product method of estimation.

Among those who have worked on these estimators see[15],[18],[19] and[22]. This study is motivated by the success recorded as in[18] on their respective works on “Modified ratio estimators using known median and coefficient of Kurtosis” and “Efficiency of some modified ratio and product Estimators using known value of some population parameters”.

The essence of conducting research is to provide dependable solution to practical problems. This paper will uncover some estimators in the literature for better modeling potentials, therefore, the study is of high significance as it tends to improve on the existing ratio estimators thereby facilitate increase in precision of estimator.

2. Motivation

The present work is motivated by recent works by statisticians on modified ratio estimators. Therefore further modification can be made to bring about gain in precision.

3. Literature Review

The historical development of the ratio method of estimation began as far back as 1662. Auxiliary information

* Corresponding author:
ka.adepoju@ui.edu.ng (Kazeem A. Adepoju)
Published online at http://journal.sapub.org/ajms

is any information closely related to the study variable. The use of auxiliary information usually leads to the sampling strategy with higher efficiency compared to those in which no auxiliary information is used. Higher precision can also be achieved by using the auxiliary information for the dual purposes of selection and the estimation procedure.

It is important to note that proper use of the knowledge of auxiliary information may result in appreciable gain in precision of the estimates. But indiscriminate use of auxiliary information might not provide the desired precision and in some extreme cases might even lead to loss in precision.

Many authors since the discovery of ratio method of estimation have come up with various degree of modification of the conventional ratio estimations for better performance. See also[10],[11],[12] and[22] for more.

4. Methodology

In this section, review will be made on the existing modified ratio estimators and the new class of modified ratio estimators. Consider a finite population U ($U_1, U_2 \ldots U_N$) of N distinct and identifiable units. Let Y is a study variable with value Yi measured on U_i, i = 1, 2, 3… N giving a vector $Y = (Y_1, Y_2, ..., Y_N)$. The problem is to estimate the population mean $\overline{Y} = \frac{1}{N}\sum Y_i$ with some desirable properties on the basis of a random sample selected from population U. The simplest estimator of a population mean is the sample mean obtained by using random sampling without replacement, when there is no additional information on the auxiliary variable available. Sometimes in sample survey, along with study variable Y, information on auxiliary variable X correlated with Y is also collected. This information on auxiliary variable X may be utilized to obtain a more efficient estimator of the population mean. Ratio method of estimation, using auxiliary information is an attempt made in this direction. Before discussing further about the modified ratio estimators and the proposed modified ratio estimators of notations to be used are described.

N - Population Size
n - Sample Size
f - n/N, Sampling fraction
Y - Study Variable
X - Auxiliary Variable
$\overline{X}, \overline{Y}$ - Population means
$\overline{x}, \overline{y}$ - Sample means
S_X, S_Y - Population Standard deviations
C_X, C_Y - Coefficient of Variations
ρ - Coefficient of Correlation

$\beta_1 = \frac{N\sum(X_i - \overline{X})^3}{(N-1)(N-2)S^3}$, coefficient of skewness of the auxiliary variable

$$\beta_2 = \frac{N(N+1)\sum(X_i - \overline{X})^4}{(N-1)(N-2)(N-3)S^4} - \frac{3(N-1)^2}{(N-2)(N-3)},$$

coefficient of the auxiliary variable
Md – Median of the auxiliary variable
B(.) – Bias of the estimator
MSE (.) – Mean squared error of the estimator
$\hat{Y}_i$ – Existing modified ratio estimator

4.1. Existing Modified Radio Estimators with their Biases, Mean Squared Errors and their Constants

Table 1. Existing Modified Radio Estimators with their Biases, Mean Squared Errors and their Constants

Estimator	Bias	Mean Square Error	Constant
$\hat{Y}_1 = \overline{y}\left(\frac{\overline{X}C_x + \beta_2}{\overline{x}C_x + \beta_2}\right)$ See[20]	$\frac{(1-f)}{n}\overline{Y}(\theta_1^2 C_1^2 - \theta_1 C_x C_y \rho)$	$\frac{(1-f)}{n}\overline{Y}^2(C_y^2 + \theta_1^2 C_x^2 - 2\theta_1 C_x C_y \rho)$	$\theta_1 = \frac{\overline{X}C_x}{\overline{X}C_x + \beta_2}$
$\hat{Y}_2 = \overline{y}\left(\frac{\overline{X}\beta_2 + C_x}{\overline{x}\beta_2 + C_x}\right)$ See[20]	$\frac{1-f}{n}\overline{Y}(\theta_2^2 C_x^2 - \theta_2 C_x C_y \rho)$	$(-f)\overline{Y}^2(C_y^2 + \theta_2^2 C_x^2 - 2\theta_2 C_x C_y \rho)$	$\theta_2 = \frac{\overline{X}\beta_2}{\overline{X}\beta_2 + C_x}$
$\hat{Y}_3 = \overline{y}\left(\frac{\overline{X}\beta_2 + \beta_1}{\overline{x}\beta_2 + \beta_1}\right)$ See[22]	$\frac{(1-f)}{n}\overline{Y}(\theta_3^2 C_x^2 - \theta_3 C_x C_y \rho)$	$(1-f)\overline{Y}^2(C_y^2 + \theta_3^2 C_x^2 - 2\theta_3 C_x C_y \rho)$	$\theta_3 = \frac{\overline{X}\beta_2}{\overline{X}\beta_2 + \beta_1}$
$\hat{Y}_3 = \overline{y}\left(\frac{\overline{X}\beta_1 + \beta_2}{\overline{x}\beta_1 + \beta_2}\right)$ See[19]	$\frac{(1-f)}{n}\overline{Y}(\theta_y^2 C_x^2 - \theta_y C_x C_y \rho)$	$\frac{(1-f)}{n}\overline{Y}^2(C_y^2 + \theta_Y^2 C_x^2 - 2\theta_Y C_x C_y \rho)$	$\theta_Y = \frac{\overline{X}\beta_1}{\overline{X}\beta_1 + \beta_2}$

4.2. Proposed Modified Ratio Estimators

In this paper, the following modified ratio estimator is proposed $\hat{Y}_p = \bar{y}\left[\dfrac{\bar{X}\beta_2 + Md\beta_2 + \beta_1}{\bar{x}\beta_2 + Md\beta_2 + \beta_1}\right]$, the corresponding Bias, the Mean Square Error and constant are derived respectively as

$$Bias\left(\hat{Y}_p\right) = \frac{(1-f)}{n}\bar{Y}\left[\theta_p^2 C_x^2 - \theta_p C_x C_y \rho\right]$$

$$MSE\left(\hat{Y}_p\right) = \frac{(1-f)}{n}\bar{Y}^2\left[C_y^2 + \theta_p^2 C_x^2 - 2\theta_p C_x C_y \rho\right]$$

The constant $\hat{\theta}_p = \dfrac{\bar{X}C_x + \rho}{\bar{X}C_x + \rho + \beta_2}$

5. Computation of Constants, Bias and Mean Square Error

5.1. Overview

For the sake of convenience to the readers and for the case of comparison, the computation will be made of the Bias, MSE and Constant of existing modified ratio estimators and the proposed ratio estimator. This is to enable the assessment to be made of the various estimators. The parameters from the natural populations stated earlier will be in use.This

Table 2. Parameters and Constant of the Populations

	Population			
Parameters	1	2	3	4
N	34	34	22	49
N	20	20	5	20
$\bar{Y}$	856.4117	856.4117	22.6209	116.1633
$\bar{X}$	208.8823	199.4412	1467.5455	98.6735
P	0.4491	0.4453	0.9022	0.6904
S_y	733.1407	733.1407	33.6469	98.8286
C_y	0.8561	0.8561	1.4609	0.8508
S_x	150.5059	150.2150	2562.1449	102.9709
C_x	0.7205	0.7532	1.7459	1.0436
β_2	0.0978	1.0445	13.3694	5.9878
β_1	0.9782	1.1823	3.3914	2.4224
M_d	150.0000	142.500	534.5000	64.0000

5.2. The Constants of the Existing and Proposed Modified Ratio Estimators

Table 3. Constants of the Existing and Proposed Modified Ratio Estimators

	Constants			
Estimator	Population 1	Population 2	Population 3	Population 4
As in[20]	0.9994	0.9931	0.9948	0.9450
As in[20]	0.9658	0.9964	0.9999	0.9982
As in[22]	0.9542	0.9944	0.9998	0.9959
As in[22]	0.9995	0.9956	0.9973	0.9756
As in[19]	0.1195	0.5938	0.9735	0.9023
Proposed estimator	0.566	0.5813	0.7329	0.6051

5.3. The Biases of the Existing and Proposed Modified Ratio Estimators

Table 4. Biases of the Existing and Proposed Modified Ratio Estimators

	Bias (.)			
Estimator	Population 1	Population 2	Population 3	Population 4
As in[20]	4.2607	4.8369	2.5432	1.3519
As in[20]	3.8212	4.8860	2.6106	1.6268
As in[22]	3.6732	4.8556	2.6095	1.6144
As in[22]	4.2629	4.8739	2.5763	1.5070
As in[19]	0.4529	0.5207	2.2674	1.1462
Proposed estimator	0.1677	0.4371	-0.1720	0.0957

5.4. The Mean Square Errors of the Existing and Proposed Modified Ratio Estimators

Table 5. Mean Square Errors of the Existing and Proposed Modified Ratio Estimators

	Mean Square Error MSE (.)			
Estimator	Population 1	Population 2	Population 3	Population 4
As in[20]	10534.5417	10902.7384	45.2894	214.7486
As in[20]	10298.4432	10930.3879	45.8857	233.6573
As in[22]	10220.4736	10913.2804	45.8758	232.7813
As in[22]	10535.7860	10923.6103	45.5814	225.2956
As in[19]	10178.2990	8937.4062	42.9321	201.3263
Proposed estimator	8842.7098	8920.9689	31.5157	152.0470

6. Conclusions

In this paper, a new ratio estimator has been proposed using linear combination of the Coefficient of Skewness, Coefficient of Kurtosis and Median of the auxiliary variable. We have reviewed the existing ratio estimators along with their properties such as Constants, Biases and Mean Square errors. The biases and mean squared errors of the proposed estimators are obtained and compared with that of existing modified ratio estimators.

The performances of the proposed modified ratio estimators are assessed with that of existing modified ratio estimators for certain natural populations. In this direction, we have considered four natural populations for the assessment of the performances of the proposed modified ratio estimator with that of existing modified ratio estimators in statistics literature. The population 1, 2 and 3 are taken from[10], while population from[1].

The constants biases and the mean square errors of the existing and the proposed modified ratio estimators for the above populations have been displayed in the table in the earlier section. It is observed that the biases and the mean square errors of the proposed estimator are less than the biases and mean square errors of the existing modified ratio estimators for most of the populations.

REFERENCES

[1] Cochran, W. G. (1977). Sampling Techniques, Third Edition, Wiley Eastern Limited

[2] Kadilar, C. and Cingi, H. (2004). Ratio estimators in simple random sampling, Applied Mathematics and Computation 151, 893-902

[3] Kadilar, C. and Cingi, H. (2006). An Improvement in Estimating the Population mean by using the Correlation Co-efficient, Hacettepe Journal of Mathematics and Statistics Volume 35 (1), 103-109

[4] Koyuncu, N. and Kadilar, C. (2009). Efficient Estimators for the Population mean, Hacettepe Journal of Mathematics and Statistics, Volume 38(2), 217-225

[5] Murthy, M.N. (1967). Sampling theory and methods, Statistical Publishing Society, Calcutta, India

[6] Prasad, B. (1989). Some improved ratio type estimators of population mean and ratio in finite population sample surveys, Communications in Statistics: Theory and Methods 18, 379–392

[7] Rao, T.J. (1991). On certain methods of improving ratio and regression estimators, Communications in Statistics: Theory and Methods 20 (10), 3325–3340

[8] Sen, A.R. (1993): Some early developments in ratio estimation, Biometric Journal 35(1), 3-13

[9] Sodipo, A. A. (2011) .The unpublished lecture note of Design and Analysis of Sample Survey

[10] Singh, D. and Chaudhary, F.S. (1986). Theory and Analysis of Sample Survey Designs, New Age International Publisher

[11] Singh, G.N. (2003). On the improvement of product method of estimation in sample surveys, Journal of the Indian Society of Agricultural Statistics 56 (3), 267–265

[12] Singh, H.P. and Tailor, R. (2003). Use of known correlation co-efficient in estimating the finite population means,

Statistics in Transition 6 (4), 555-560

[13] Singh, H.P. and Tailor, R. (2005). Estimation of finite population mean with known co-efficient of variation of an auxiliary, STATISTICA, anno LXV, n.3, pp 301-313

[14] Singh, H.P., Tailor, R., Tailor, R. and Kakran, M.S. (2004). An Improved Estimator of population means using Power transformation, Journal of the Indian Society of Agricultural Statistics 58(2), 223-230

[15] Sisodia, B.V.S. and Dwivedi, V.K. (1981). A modified ratio estimator using co-efficient of variation of auxiliary variable, Journal of the Indian Society of Agricultural Statistics 33(1), 13-18

[16] Srivastava, S.K. (1967). An estimator using auxiliary information in sample surveys, Bulletin/Calcutta Statistical Association, vol. 16, pp.121-132

[17] Srivastava, S.K. (1980). A Class of estimators using auxiliary information in sample surveys, The Canadian Journal of Statistics, vol. 8, no.2, pp.253-254

[18] Subramani, J. and Kumarapandiyan, G. (2012). A Class of almost unbiased modified ratio estimators for population mean with known population parameters, Elixir Statistics 44, 7411-7415

[19] Subramani, J. and Kumarapandiyan, G. (2012). Modified Ratio Estimator for Population Mean Using Median of the Auxiliary Variable, Proceedings of National Conference on Recent developments in the Applications of Reliability Theory and Survival Analysis held on 2nd and 3rd February 2012 at the Department of Statistics, Pondicherry University

[20] Subramani J, Kumarapandiyan ,G Estimation of Population Mean Using Known Median and Coefficient of Skewness, American Journal of Mathematics and Statistics 2012,

[21] Upadhyaya, L.N. and Singh, H.P. (1999). Use of transformed auxiliary variable in estimating the finite populations mean, Biometrical Journal 41 (5), 627-636

[22] Walsh, J.E. (1970). Generalization of ratio estimator for population total, Sankhya, A, 32, 99-106

[23] Yan, Z. and Tian, B. (2010). Ratio Method to the Mean Estimation Using Co-efficient of Skewness of Auxiliary Variable, ICICA 2010, Part II, CCIS 106, pp. 103–110