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Graphical Interpretation of the New Sequence of Functions Involving Mittage-Leffler Function Using Matlab

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Abstract The aim of the paper is an attempt to introduce a new sequence of functions $\left\{P_n^{(\beta,\gamma,\alpha)}\left(x;a,k,s\right)/n=0,1,2,...\right\}$, which involving the Mittage-Leffler function $E_{\alpha,\beta}(z)$ by using operational technique. Some interesting generating relations are obtained in sections 2. The remarkable thing of this paper is the crucial MATLAB coding of the new sequence of functions, Database and Graph established using the MATLAB (R2012a) in the section (4) and (5) for different values of parameters and n=1, 2, 3. The reader can establish Database and Graph using the same program for any value of n.

Keywords Mittag-Leffler Function, Generating Relation, Sequence of Functions, Operational Techniques Matlab

1. Introduction

MATLAB is a high-performance language for technical computing. It integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation. Typical uses include:

- Math and computation
- Algorithm development
- Modeling, simulation, and prototyping
- Data analysis, exploration, and visualization
- Scientific and engineering graphics
- Application development, including Graphical User Interface building

In pure mathematics, since Matlab is an integrated computer software which has three functions: symbolic computing, numerical computing and graphics drawing. Matlab is capable to carry out many functions including computing polynomials and rational polynomials, solving equations and computing many kind of mathematical expressions. One can also use Matlab to calculate the limit, derivative, integral and Taylor series of some mathematical expressions. With Matlab, The graphs of functions with one or two variables can be easily drawn in selected domain. Therefore, functions can be studied by visualization for their main Characteristics. Matlab is also a system which can be easily expanded. Matlab provides many powerful software packages which can be easily incorporated into the clients system. Recently, there are many papers in the literature which are devoted to the application of Matlab in mathematical analysis, see the work of Stephen (2006), Dunn (2003), Shampine and Robert (2005).

The distinct scientific communities that are working on various aspects of automatic analysis of data include Combinatorial Pattern Matching, Data Mining, Computational Statistics, Network Analysis, Text Mining, Image Processing, Syntactical Pattern Recognition, Machine Learning, Statistical Pattern Recognition, Computer Vision, and many others.

The special function

$$E_\alpha(z)=\sum_{n=0}^{\infty}\frac{(z)^n}{\Gamma(\alpha n+1)},\ z\in\mathbf{C} \tag{1}$$

And its general form

$$E_{\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{(z)^n}{\Gamma(\alpha n+\beta)}$$

$$\alpha,\beta\in\mathrm{C},\mathrm{R}(\alpha)>0,\mathrm{R}(\beta)>0,\ z\in\mathbf{C}, \tag{2}$$

with C being the set of complex numbers are called Mittag-Leffler functions (Erd´elyi, et al., 1955, Section 18.1). The former was introduced by Mittag-Leffler (Mittag-Leffler,1903) in connection with his method of summation of some divergent series. In his papers (Mittag-Leffler, 1903, 1905), he investigated certain properties of this function. The function defined by (2) first

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appeared in the work of Wiman (Wiman, 1905). The function (2) is studied, among others, by Wiman (1905), Agarwal (1953), Humbert (1953) and Humbert and Agarwal (1953) and others. The main properties of these functions are given in the book by Erd´elyi et al. (1955, Section 18.1) and a more comprehensive and a detailed account of Mittag-Leffler functions are presented in Dzherbashyan (1966, Chapter 2). In particular, the functions (1) and (2) are entire functions of order $\rho = 1/\alpha$ and type $\sigma = 1$; see, for example, (Erd´elyi, et al., 1955, p.118).

In recent years the interest in functions of Mittag-Leffler type among scientists, engineers and applications-oriented mathematicians has deepened. The Mittag-Leffler function arises naturally in the solution of fractional order integral equations or fractional order differential equations, and especially in the investigations of the fractional generalization of the kinetic equation, random walks, L´evy flights, super-diffusive transport and in the study of complex systems. The ordinary and generalized Mittag-Leffler functions interpolate between a purely exponential law and power-law like behavior of phenomena governed by ordinary kinetic equations and their fractional counterparts, see Lang (1999a, 1999b), Hilfer (2000), Saxena et al. (2002).

The Mittag-Leffler function is not given in the tables of Laplace transforms, where it naturally occurs in the derivation of the inverse Laplace transform of the functions of the type $\mathrm{p}^{\alpha}(\mathrm{a}+\mathrm{bp}^{\beta})$, where p is the Laplace transform parameter and a and b are constants. This function also occurs in the solution of certain boundary value problems involving fractional integro-differential equations of Volterra type (Samko et al., 1993). During the various developments of fractional calculus in the last four decades this function has gained importance and popularity on account of its vast applications in the fields of science and engineering. Hille and Tamarkin (1930) have presented a solution of the Abel-Volterra type equation in terms of Mittag-Leffler function. During the last 15 years the interest in Mittag-Leffler function and Mittag-Leffler type functions is considerably increased among engineers and scientists due to their vast potential of applications in several applied problems, such as fluid flow, rheology, diffusive transport akin to diffusion, electric networks, probability, statistical distribution theory etc. For a detailed account of various properties, generalizations, and application of this function, the reader may refer to earlier important works of Blair (1974), Bagley and Torvik (1984), Caputo and Mainardi (1971), Dzherbashyan (1966), Gorenflo and Vessella (1991), Gorenflo and Rutman (1994), Kilbas and Saigo (1995), Gorenflo et al. (1997), Gorenflo and Mainardi (1994, 1996, 1997), Gorenflo, Luchko and Rogosin (1997), Gorenflo, Kilbas and Rogosin (1998), Luchko (1999), Luchko and Srivastava (1995), Kilbas, Saigo and Saxena (2002, 2004), Saxena and Saigo (2005), Kiryakova (2008a, 2008b), Saxena, Kalla and Kiryakova (2003), Saxena, Mathai and Haubold (2002, 2004, 2004a, 2004b, 2006), Saxena and Kalla (2008), Mathai, Saxena and Haubold (2006), Haubold andMathai (2000), Haubold, Mathai and Saxena (2007), Srivastava and Saxena (2001), and others.

Operational techniques have drawn the attention of several researchers in the study of sequences of functions and polynomials. In this paper, we introduce a new sequence of functions $\left\{P_n^{(\beta,\gamma,\alpha)}(x;a,k,s)/n=0,1,2,\ldots\right\}$, which involving the Mittage-Leffler function $E_{\alpha,\beta}(z)$ in equation (17) by using operational technique. Some interesting generating relations are obtained in sections 2. The remarkable thing of this paper is the crucial MATLAB coding of the new sequence of functions, Database and Graph established by using the MATLAB (R2012a) in the section (4) and (5) for different values of parameters and n=1, 2, 3. The reader can establish Database and Graph using the same program for any value of n.

In 1956, Chak defined a class of polynomials as,

$$G_{n,k}^{(\alpha)}(x)=x^{-\alpha-kn+n}e^{x}\left(x^{k}D\right)^{n}\left[x^{\alpha}e^{-x}\right] \tag{3}$$

Where k is constant and $n=0,1,2,\ldots,D\equiv\dfrac{d}{dx}$.

Gould and Hopper (1962) introduced generalized Hermite polynomials as,

$$H_n^{r}(x,a,p)=(-1)^{n}x^{-a}\exp\left(px^{r}\right)D^{n}\left[x^{a}\exp\left(-px^{r}\right)\right] \tag{4}$$

Chatterjea (1964) studied as a class of polynomials for generalized Laguerre polynomials,

$$T_{rn}^{(\alpha)}(x,p)=\frac{1}{n!}x^{-\alpha-n-1}\exp\left(px^{r}\right)\left(x^{2}D\right)^{n}\left[x^{\alpha+1}\exp\left(-px^{r}\right)\right] \tag{5}$$

In 1968, Singh studied the generalized Truesdell polynomials defined as,

$$T_n^{(\alpha)}(x,r,p)=x^{-\alpha}\exp\left(px^{r}\right)(xD)^{n}\left[x^{\alpha}\exp\left(-px^{r}\right)\right] \tag{6}$$

Srivastava and Singh (1971) introduced a general class of polynomials as,

$$G_n^{(\alpha)}(x,r,p,k)=\frac{1}{n!}x^{-\alpha-kn}\exp\left(px^{r}\right)\left(x^{k+1}D\right)^{n}\left[x^{\alpha}\exp\left(-px^{r}\right)\right] \tag{7}$$

In 1971, the Rodrigues formulae for generalized Lagurre polynomials is given by Mittal (1971) as,

$$T_{kn}^{(\alpha)}(x)=\frac{1}{n!}x^{-\alpha}\exp\left(p_k(x)\right)D^n\left[x^{\alpha+n}\exp\left(-p_k(x)\right)\right] \tag{8}$$

Where $p_k(x)$ is a polynomial in x of degree k.

Mittal (1971a) also proved following relation for (8) as,

$$T_{kn}^{(\alpha+s-1)}(x)=\frac{1}{n!}x^{-\alpha-n}\exp\left(p_k(x)\right)T_s^n\left[x^{\alpha}\exp\left(-p_k(x)\right)\right] \tag{9}$$

Where s is constant and $T_s\equiv x(s+xD)$.

Recently, Shukla and Prajapati (2007) obtained several properties of (9).

Chandel (1973) also studied a class of polynomials defined as:

$$T_n^{\alpha,k}(x,r,p)=x^{-\alpha}\exp\left(px^r\right)\left(x^kD\right)^n\left[x^{\alpha}\exp\left(-px^r\right)\right] \tag{10}$$

In 1974, Chandel established a generalization of polynomial system as:

$$G_n\left(h,g(x),k\right)=e^{-hg(x)}\left(x^kD\right)^n\left[e^{-hg(x)}\right] \tag{11}$$

Where $g(x)$ is a function of x, h and k are constants.

In the same year, Srivastava (1974) discussed some operational formulas generalized function $F_n^{(r,m)}(x,a,k,p)$ in the form,

$$F_n^{(r,m)}(x,a,k,p)=x^{-a}\exp\left(px^r\right)\left(x^kD\right)^n\left[x^{a+km}\exp\left(-px^r\right)\right] \tag{12}$$

In 1975, Joshi and Parjapat introduced the a class of polynomial,

$$M_{vn}^{(\alpha)}(x,a,k)=\frac{x^{-\alpha-nk}}{n!}\exp\left(p_v(x)\right)\left[x^k(a+xD)\right]^n\left[x^{\alpha}\exp\left(-p_v(x)\right)\right] \tag{13}$$

Subsequently in 1975, Patil and Thakare have obtained several formulae and generating relation for

$$P_n^{(\alpha)}(x,r,s,a,k,\lambda)=x^{-\alpha}\exp\left(px^r\right)\left[x^k(\lambda+xD)\right]^n\left[x^{\alpha+sn}\exp\left(-px^r\right)\right] \tag{14}$$

In 1979, Srivastava and Singh studied a sequence of functions $V_n^{(\alpha)}(x;a,k,s)$ defined as:

$$V_n^{(\alpha)}(x;a,k,s)=\frac{x^{-\alpha}}{n!}\exp\left\{p_k(x)\right\}\theta^n\left[x^{\alpha}\exp\left\{-p_k(x)\right\}\right] \tag{15}$$

By employing the operator $\theta\equiv x^a(s+xD)$, where s is constant and $p_k(x)$ is a polynomial in x of degree k.

J.C. Parjapati and N.K. Ahudia (Accepted on: 27.08.2012) introduced the sequence of function defined as,

$$V_n^{(\alpha,\beta,\delta)}(x;a,k,s)=\frac{1}{n!}x^{-\beta}W\left(\alpha,\delta;p_k(x)\right)\left(T_x^{a,s}\right)^n\left[x^{\beta}W\left(\alpha,\delta;-p_k(x)\right)\right] \tag{16}$$

A new sequence of function $\left\{P_n^{(\beta,\gamma,\alpha)}(x;a,k,s)/n=0,1,2,...\right\}$ is introduced in this paper as:

$$P_n^{(\beta,\gamma,\alpha)}(x;a,k,s)=\frac{1}{n!}x^{-\alpha}E_{\beta,\gamma}\left(p_k(x)\right)\left(T_x^{a,s}\right)^n\left[x^{\alpha}E_{\beta,\gamma}\left(-p_k(x)\right)\right] \tag{17}$$

Where $T_x^{a,s}\equiv x^a(s+xD), D\equiv\frac{d}{dx}$, a and s are constants, k is finite and non-negative integer, $p_k(x)$ is a polynomial in x of degree k and $E_{\alpha,\beta}(z)$ is a Mittage-Muffler function defined in equation (2).

Some generating relations of class of polynomials or sequence of function have been obtained by using the properties of the differential operators. $T_x^{a,s}\equiv x^a(s+xD), T_x^{a,1}\equiv x^a(1+xD)$, where $D\equiv\frac{d}{dx}$, is based on the work of Mittal (1977), Patil and Thakare (1975), Srivastava and Singh (1979).

Some useful Operational Techniques are given below:

$$\exp\left(tT_x^{a,s}\right)\left(x^{\beta} f\left(x\right)\right) = x^{\beta}\left(1-ax^{a}t\right)^{-\left(\frac{\beta+s}{a}\right)} f\left(x\left(1-ax^{a}t\right)^{-1/a}\right) \tag{18}$$

$$\exp\left(tT_x^{a,s}\right)\left(x^{\alpha-an} f\left(x\right)\right) = x^{\alpha}\left(1+at\right)^{-1+\left(\frac{\alpha+s}{a}\right)} f\left(x\left(1+at\right)^{1/a}\right) \tag{19}$$

2. Generating Relations

$$\sum_{n=0}^{\infty} P_n^{(\beta,\gamma,\alpha)}(x;a,k,s) x^{-an} t^n = (1-at)^{-\left(\frac{\alpha+s}{a}\right)} E_{\beta,\gamma}\left(p_k\left(x\right)\right) E_{\beta,\gamma}\left(-p_k\left(x\left(1-at\right)^{-1/a}\right)\right) \tag{20}$$

$$\sum_{n=0}^{\infty} \binom{m+n}{m} x^{-an} P_n^{(\beta,\gamma,\alpha)}(x;a,k,s) t^n$$
$$= (1-at)^{-\left(\frac{\alpha+s}{a}\right)} \frac{E_{\beta,\gamma}\left(p_k\left(x\right)\right)}{E_{\beta,\gamma}\left(p_k\left(x\left(1-at\right)^{-1/a}\right)\right)} P_n^{(\beta,\gamma,\alpha)}\left(x\left(1-at\right)^{-1/a};a,k,s\right) \tag{21}$$

$$\sum_{n=0}^{\infty} P_n^{(\beta,\gamma,\alpha-an)}(x;a,k,s) x^{-an} t^n = (1+at)^{-1+\left(\frac{\alpha+s}{a}\right)} E_{\beta,\gamma}\left(p_k\left(x\right)\right) E_{\beta,\gamma}\left(-p_k\left(x\left(1+at\right)^{1/a}\right)\right) \tag{22}$$

Proof of (20)

From (17), we consider:

$$\sum_{n=0}^{\infty} P_n^{(\beta,\gamma,\alpha)}(x;a,k,s) t^n = x^{-\alpha} E_{\beta,\gamma}\left(p_k(x)\right) \exp(tT_x^{a,s})\left[x^{\alpha} E_{\beta,\gamma}\left(-p_k(x)\right)\right] \tag{23}$$

Using operational technique (18), above equation (23) reduces to:

$$\sum_{n=0}^{\infty} P_n^{(\beta,\gamma,\alpha)}(x;a,k,s) t^n = x^{-\alpha} E_{\beta,\gamma}\left(p_k(x)\right) x^{\alpha}(1-ax^{a}t)^{-\left(\frac{\alpha+s}{a}\right)} E_{\beta,\gamma}\left(-p_k\left(x(1-ax^{a}t)^{-1/a}\right)\right)$$
$$= (1-ax^{a}t)^{-\left(\frac{\alpha+s}{a}\right)} E_{\beta,\gamma}\left(p_k(x)\right) E_{\beta,\gamma}\left(-p_k\left(x(1-ax^{a}t)^{-1/a}\right)\right) \tag{24}$$

And replacing t by tx^{-a}, this gives (20).

Proof of (21)

We can write (17) as:

$$\left(T_x^{a,s}\right)^n \left[x^{\alpha} E_{\beta,\gamma}(-p_k(x))\right] = n! x^{\alpha} \frac{1}{E_{\beta,\gamma}(p_k(x))} P_n^{(\beta,\gamma,\alpha)}(x;a,k,s) \tag{25}$$

Or

$$\exp\left(t\left(T_x^{a,s}\right)\right)\left\{\left(T_x^{a,s}\right)^n \left[x^{\alpha} E_{\beta,\gamma}(-p_k(x))\right]\right\} = n! \exp\left(tT_x^{a,\alpha}\right)\left[x^{\alpha} \frac{1}{E_{\beta,\gamma}(p_k(x))} P_n^{(\beta,\gamma,\alpha)}(x;a,k,s)\right]$$

$$\sum_{m=0}^{\infty} \frac{t^m}{m!}\left(T_x^{a,s}\right)^{m+n}\left[x^{\alpha} E_{\beta,\gamma}(-p_k(x))\right] = n! \exp\left(tT_x^{a,s}\right)\left[x^{\alpha} \frac{1}{E_{\beta,\gamma}(p_k(x))} P_n^{(\beta,\gamma,\alpha)}(x;a,k,s)\right] \tag{26}$$

Using the operational technique (18), above equation can be written as:

$$\sum_{m=0}^{\infty} \frac{t^m}{m!}\left(T_x^{a,s}\right)^{m+n}\left[x^{\alpha} E_{\beta,\gamma}(-p_k(x))\right]$$
$$= n! x^{\alpha}\left(1-ax^{a}t\right)^{-\left(\frac{\alpha+s}{a}\right)} \frac{1}{E_{\beta,\gamma}\left(p_k\left(x\left(1-ax^{a}t\right)^{-1/a}\right)\right)} P_n^{(\beta,\gamma,\alpha)}\left(x\left(1-ax^{a}t\right)^{-1/a};a,k,s\right) \tag{27}$$

use of (25) gives:

$$\sum_{m=0}^{\infty}\frac{t^m (m+n)!}{m!n!}x^{\alpha}\frac{1}{E_{\beta,\gamma}(p_k(x))}P_{m+n}^{(\beta,\gamma,\alpha)}(x;a,k,s)$$
$$=x^{\alpha}\left(1-ax^a t\right)^{-\left(\frac{\alpha+s}{a}\right)}\frac{1}{E_{\beta,\gamma}\left[p_k\left(x\left(1-ax^a t\right)^{-1/a}\right)\right]}P_n^{(\beta,\gamma,\alpha)}\left(x\left(1-ax^a t\right)^{-1/a};a,k,s\right) \quad (28)$$

Therefore

$$\sum_{m=0}^{\infty}\binom{m+n}{n}P_{m+n}^{(\beta,\gamma,\alpha)}(x;a,k,s)t^m$$
$$=\left(1-ax^a t\right)^{-\left(\frac{\alpha+s}{a}\right)}\frac{E_{\beta,\gamma}(p_k(x))}{E_{\beta,\gamma}\left(p_k\left(x\left(1-ax^a t\right)^{-1/a}\right)\right)}P_n^{(\beta,\gamma,\alpha)}\left(x\left(1-ax^a t\right)^{-1/a};a,k,s\right) \quad (29)$$

And replacing t by tx^{-a}, this gives the result (21).

Proof of (22)

Again from (17), we have:

$$\sum_{n=0}^{\infty}x^{-an}P_n^{(\beta,\gamma,\alpha-an)}(x;a,k,s)t^n = x^{-\alpha}E_{\beta,\gamma}\left(p_k(x)\right)\exp(tT_x^{a,s})\left[x^{\alpha-an}E_{\beta,\gamma}\left(-p_k(x)\right)\right] \quad (30)$$

applying the operational technique (19), we get:

$$\sum_{n=0}^{\infty}x^{-an}P_n^{(\beta,\gamma,\alpha-an)}(x;a,k,s)t^n = x^{-\alpha}E_{\beta,\gamma}\left(p_k(x)\right)x^{\alpha}\left(1+at\right)^{-1+\frac{\alpha+s}{a}}E_{\beta,\gamma}\left[-p_k\left(x\left(1+at\right)^{1/a}\right)\right]$$
$$=\left(1+at\right)^{-1+\frac{\alpha+s}{a}}E_{\beta,\gamma}\left(p_k(x)\right)E_{\beta,\gamma}\left[-p_k\left(x\left(1+at\right)^{1/a}\right)\right] \quad (31)$$

This proves (22).

3. Special Cases

The interesting special and particular cases between (17) and class of polynomials (3)-(17) can also be obtained for appropriate values of β, γ, α, a, k and *s*.

The MATLAB is one of the important aspects mainly in the field of sciences and engineering, Therefore, the imperative MATLAB coding established for each parameter of equation (17) and some interesting Database and Graphs also established in the section 5. Using this coding reader may obtain large number of graphs of equation (17), which gives the eccentric characteristics in the area of sequence of functions or class of polynomials.

4. Programming of the New Sequence of Function in MATLAB

Code is divided in parts as a new sequence of function is composition of two functions.

Code of Generalized Mittage-Muffler function:

```
function [E1] = GMLF(b,c,x)
% GMLF returns sum(k=0:inf,(x^k/(gamma(kb+c))
% Format of call: GMLF(b,c,x)
syms x
E1 = 1/gamma(c);
for k=1:100
E1 = E1 + (x.^k./gamma(b.*k+c));
end
end
```

Code of New sequence of functions:

```
function [Pn] = gnsGMLF1(beta,gamma,alpha,a,k,s,x)
%Graph of Pn(beta,gamma,alpha,a,k,s,x)
%MLF(a,b,x)=sum(k=0:inf,(gamma(a+k))x^k/(gamma(a))(k!*gamma(kb+c))
%P=Pn(beta,gamma,alpha,a,k,s,x)=(1/n!)*x^-alpha*GMLF(beta,gamma,x^k)*Tn^(a,s)
%(x^a*(s+x*D)(x^alpha)*GMLF(beta,gamma,-x^k)), where n=1,2,3,…
syms x
%n=input('please enter n:');
%n=1;
E1= GMLF(beta,gamma,-x.^k);
y=(x.^alpha).*E1;
for i=1:n
y=(x.^a).*(s.*y+x.*diff(y));
end
E2=GMLF(beta,gamma,x.^k);
v=(1./factorial(n)).*(1./(x.^alpha)).*E2.*y;
```

```
%Pn=subs(v,x,-2:.5:2);
Pn=subs(v,x);
end
Plot Graph:
ezplot(gnsGMLFn(beta,gamma,alpha,a,k,s,x),[-2:.5:2])
```

5. Different Databases and Graphs Using MATLAB

The new sequence $P_n^{(\beta,\gamma,\alpha)}(x;a,k,s)$ introduced in equation (17), takes place in the form of Pn(β,γ,α,a, k,s,x) to establish Database and Graph for different values of parameters β, γ, α, a, k, s and (n = 0, 1, 2, 3,…). We establish here four different Database for different values of parameters for n=1,2,3 in the interval $-2 \le x \le 2$ with difference .5, as shown in Database (a), (b), (c) & (d) and their corresponding Graphs are plotted.

First Database and Graph

Database (a)

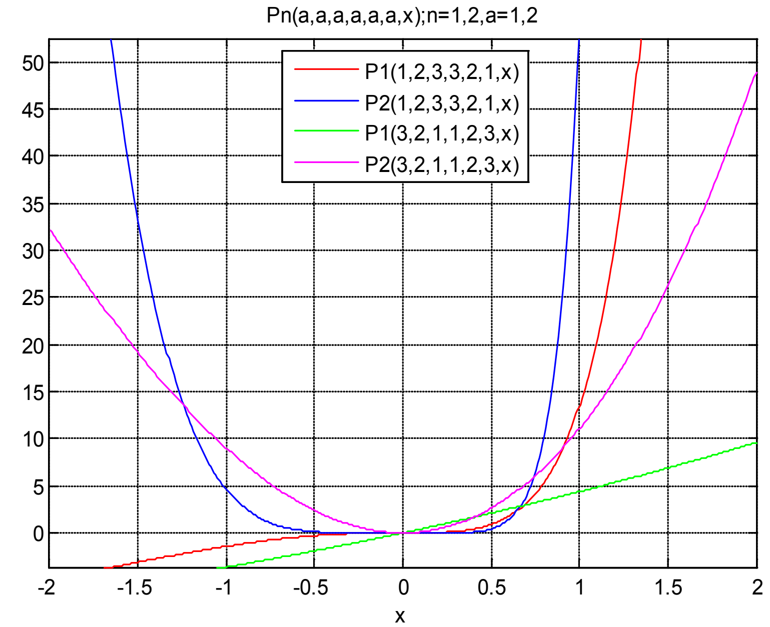
Pn(β,γ,α,a,k,s,x);n=1,2				
X	P1(1,2,3,3,2,1,x)	P2(1,2,3,3,2,1,x)	P1(3,2,1,1,2,3,x)	P2(3,2,1,1,2,3,x)
-2.0	-4.9539	$0.0123\text{x}10^4$	-6.5838	32.1720
-1.5	-3.1059	$0.0033\text{x}10^4$	-5.1918	19.1450
-1.0	-1.4313	$0.0005\text{x}10^4$	-3.6356	8.9902
-.50	-0.2919	$0.0000\ \text{x}10^4$	-1.9076	2.3718
0	0	0	0	0
.50	0.8986	$0.0000\text{x}10^4$	2.0951	2.6322
1.0	13.5283	$0.0052\text{x}10^4$	4.3858	11.0742
1.5	89.6584	$0.1234\text{x}10^4$	6.8802	26.1810
2.0	433.7566	$1.4942\text{x}10^4$	9.5868	48.8584

Command Window code For Plot Graph of database (a)

```
hold on
h1= ezplot(gnsGMLF1(1,2,3,3,2,1,x),[-2:.5:2]);
h2= ezplot(gnsGMLF2(1,2,3,3,2,1,x),[-2:.5:2]);
h3= ezplot(gnsGMLF1(3,2,1,1,2,3,x),[-2:.5:2]);
h4= ezplot(gnsGMLF2(3,2,1,1,2,3,x),[-2:.5:2]);
title('Pn(a,a,a,a,a,a,x);n=1,2,a=1,2')
hold off
set(h1,'color','r')
set(h2,'color','b')
set(h3,'color','g')
set(h4,'color','m')
legend('P1(1,2,3,3,2,1,x)','P2(1,2,3,3,2,1,x)','P1(3,2,1,1,2,3,x)','P2(3,2,1,1,2,3,x)')
```

Graph of the New Sequence based on Database (a)

To establish database (a) first save the files of programming given as above then apply the code >>gnsGMLF1(1,2,3,3,2,1,x) in command window of MATLAB (R2012a), we have the first column of the database, in the same way we can obtain the other values of database for different parameters. For plot the graph of database use command window code (a) for plot graph in command window, we have the graph (a) for the database (a).

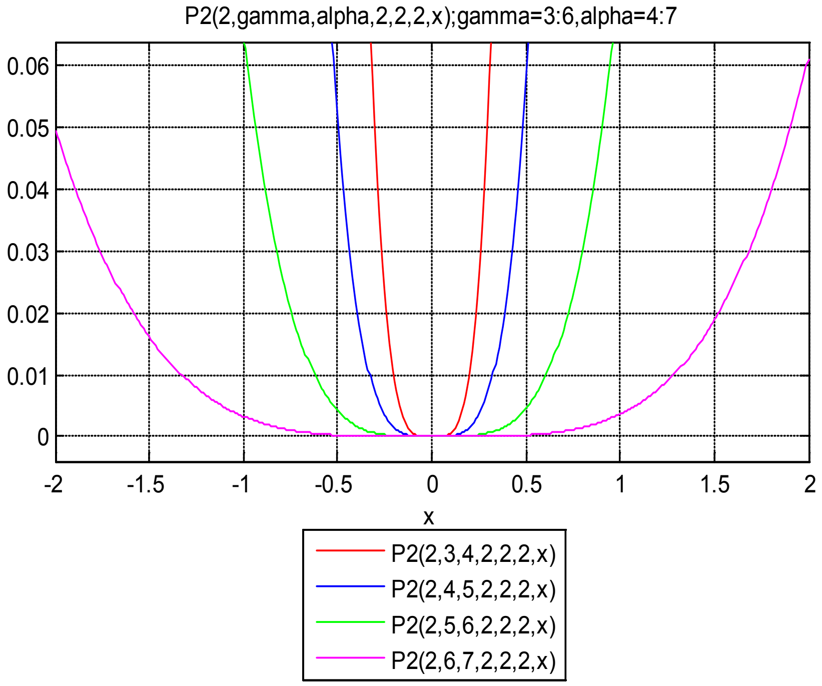


Graph (a)

Second Database and Graph

Database (b)

P2(β,γ,α,a,k,s,x)				
X	P2(2,3,4,2,2,2,x)	P2(2,4,5,2,2,2,x)	P2(2,5,6,2,2,2,x)	P2(2,6,7,2,2,2,x)
-2.0	64.7402	11.1483	0.9575	0.0495
-1.5	22.6419	3.7346	0.3144	0.0161
-1.0	4.9380	0.7810	0.0645	0.0033
-.50	0.3404	0.0517	0.0042	0.0002
0	0	0	0	0
.50	0.4127	0.0579	0.0045	0.0002
1.0	7.2606	0.9800	0.0748	0.0036
1.5	40.3753	5.2493	0.3932	0.0188
2.0	140.0371	17.5525	1.2902	0.0611

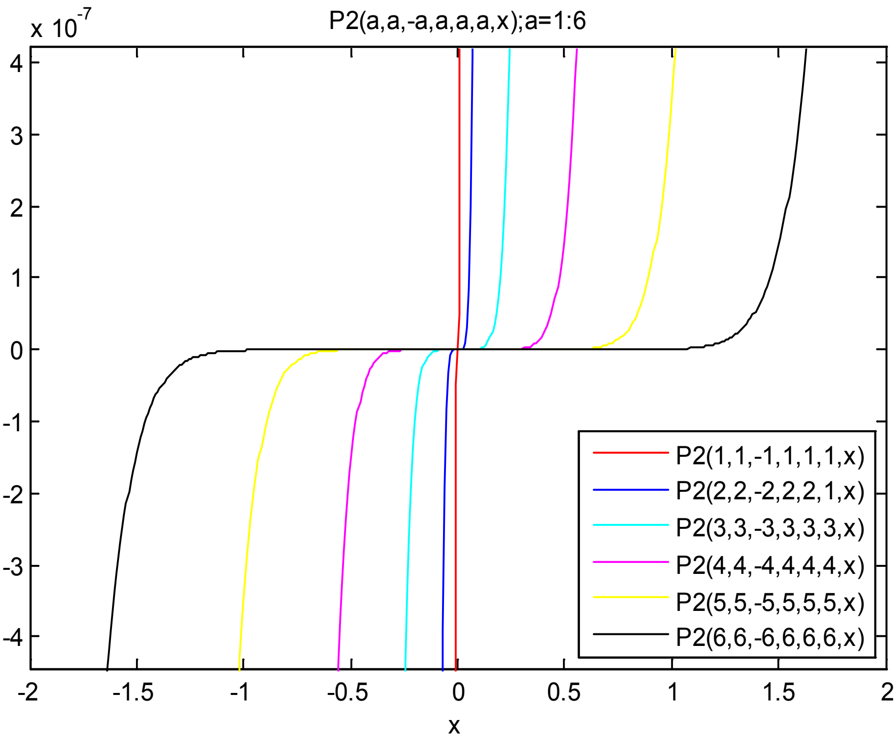


Graph (b)

Third Database and Graph

Database (c)

P2(β,γ,α,a,k,s,x)						
X	P2(1,1,-1,1,1,1,x)	P2(2,2,-2,2,2,1,x)	P2(3,3,-3,3,3,3,x)	P2(4,4,-4,4,4,41,x)	P2(5,5,-5,5,5,5,x)	P2(6,6,-6,6,6,6,x)
-2.0	0	-4.2249	-1.0160	-0.0422	-0.7054x10^{-3}	-0.5986x10^{-5}
-1.5	-0.0420	-1.1854	-0.1373	-0.0032	-0.0298 x10^{-3}	-0.0142 x10^{-5}
-1.0	-0.0677	-0.1835	-0.0081	-0.0001	-0.0003 x10^{-3}	-0.0001x10^{-5}
-.50	-0.0345	-0.0067	-0.0001	-0.0000	-0.0000 x10^{-3}	-0.0000x10^{-5}
0	0	0	0	0	0	0
.50	0.4247	0.0091	0.0001	0.0000	0.0000 x10^{-3}	0.0000x10^{-5}
1.0	11.0836	0.3348	0.0085	0.0001	0.0003 x10^{-3}	0.0001x10^{-5}
1.5	118.6302	2.9253	0.1476	0.0032	0.0298 x10^{-3}	0.0142x10^{-5}
2.0	873.5704	14.1380	1.1189	0.0425	0.7056 x10^{-3}	0.5986x10^{-5}

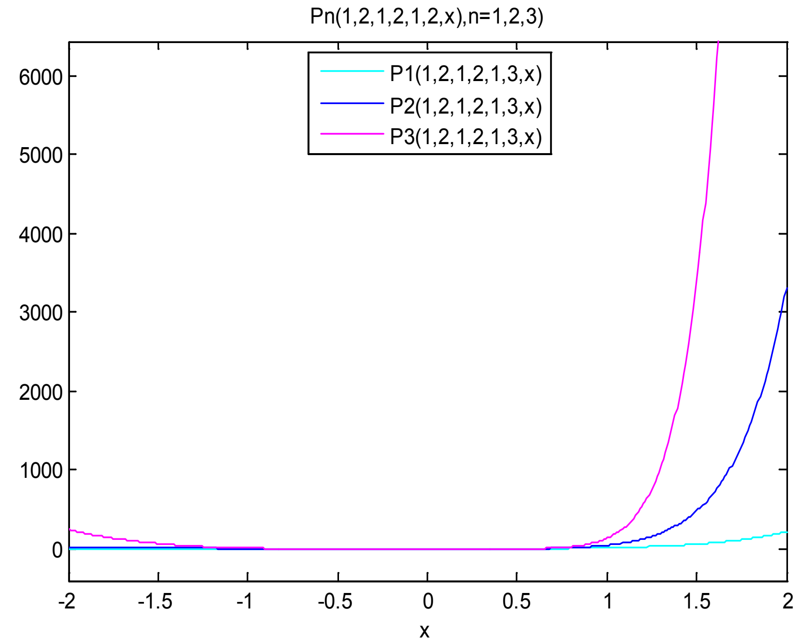


Graph (c)

Fourth Database and Graph

Database (d)

Pn(β,γ,α,a,k,s,x);n=1,2,3			
X	P1(1,2,1,2,1,3,x)	P2(1,2,1,2,1,3,x)	P3(1,2,1,2,1,3,x)
-2.0	2.4770	0.0257x10^{3}	0.0244x10^{4}
-1.5	2.0706	0.0124 x10^{3}	0.0067 x10^{4}
-1.0	1.4313	0.0039 x10^{3}	0.0010 x10^{4}
-.50	0.5838	0.0004 x10^{3}	0.0000 x10^{4}
0	0	0	0
.50	1.7973	0.0014 x10^{3}	0.0001 x10^{4}
1.0	13.5283	0.0455 x10^{3}	0.0134 x10^{4}
1.5	59.7722	0.4810 x10^{3}	0.3370 x10^{4}
2.0	216.8783	3.3018 x10^{3}	4.3406 x10^{4}



Graph (d)

Database and graph (b), (c) and (d) can be established parallel as established for database and graph of (a).

6. Conclusions

In the section (5), for the different values of parameters and value of n in the sequence of function $\left\{P_n^{(\beta,\gamma,\alpha)}(x;a,k,s)/n=0,1,2,...\right\}$ can easily interpreted and can be compared with the help of database and graph. The present paper has enabled us to find the trends of different functions in various ranges and have paved the way for comparison of trends.

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