

Maximum Likelihood Estimation for Availability Measures of a Three Component Identical System in the Presence of Human Errors and CCS Failures

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Abstract This paper deals with the 3-component identical system under the influence of human errors and Common Cause Shock (CCS) failures. The M L estimates of system availability and frequency of failures were developed in the case of series and parallel systems. Also we developed empirical evidence to establish the validity of the proposed estimates by simulation study.

Keywords Availability, Frequency of Failures, CCS Failures, Human Errors, M L Estimation, Monte-Carlo Simulation

1. Introduction

In most of the studies relating to life testing and reliability, the lifetime is treated as continuous and accordingly continuous probability distributions are proposed as models. David & Epstein[4] have investigated exponential distribution in the life experiments. Interest in estimation is very important for statistical inference of random phenomena and time to failure is treated as random phenomena in reliability theory. There is extensive literature on reliability modeling, inference relating to reliability characteristics. However, there is very little discussion in assessing and estimating system performance measures like availability, frequency of failures etc. Vesely[10] discussed the Binomial failure rate model. Atwood[1], Atwood and Steverson[2] and Meachum and Atwood[7] have applied the Binomial failure rate model for Common cause shock failures to the data associated with nuclear power plants. Chari et al[3] discussed the CCS failures to arrive the expressions for reliability. Lin et al[6] derived the exact maximum likelihood estimates of the parameters included in the lives distribution of components with constant failure rates in a series system. Sarhan and El-Bassiouny[9] used M L estimation approach to derive the estimators for the parameters included in a parallel system under the assumption that the life times of the system's components

have complementary exponential distributions. Dhillon[5] discussed the role of CCS failures and human errors in estimating reliability of a system. Sagar et al[8] developed the reliability and availability measures in the presence of CCS failures and human errors. He considered three component identical system.

This paper presents, Maximum likelihood estimation approach to assess the performance of the redundant systems. In this connection, we considered three component identical system which is affected by human errors and Common cause shock failures. The M L estimates are proposed for reliability indices such as availability function, frequency of failure function in the case of series and parallel systems. Also the proposed estimates are assessed based on simulated samples.

2. Notations

λ_i, λ_c & λ_h : the failure rates of individual, CCS failures and human errors respectively.

p_1, p_2 & p_3 : the chance of individual, CCS failures and human errors respectively.

μ_0, μ_1 & μ_2 : service rates.

$Av_{chs}(t)$: time-dependent system availability for series configuration in the case of CCS failures as well as human errors.

$\hat{Av}_{chs}(t)$: M L estimate of time-dependent system availability for series configuration in the case of CCS failures as well as human errors.

$Av_{chp}(t)$: time-dependent system availability for parallel configuration in the case of CCS failures as well as human

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errors.

$\hat{A}v_{chp}(t)$: M L estimate of time-dependent system availability for parallel configuration in the case of CCS failures as well as human errors.

$Av_{chs}(\infty)$: steady-state system availability for series configuration in the case of CCS failures as well as human errors.

$\hat{A}v_{chs}(\infty)$: M L estimate of steady-state system availability for series configuration in the case of CCS failures as well as human errors.

$Av_{chp}(\infty)$: steady-state system availability for parallel configuration in the case of CCS failures as well as human errors.

$\hat{A}v_{chp}(\infty)$: M L estimate of steady-state system availability for parallel configuration in the case of CCS failures as well as human errors.

$f_{chs}^{ss}(T)$: steady-state frequency of failure for series configuration in the case of CCS failures as well as human errors.

$\hat{f}_{chs}^{ss}(T)$: M L estimate of steady-state frequency of failure for series configuration in the case of CCS failures as well as human errors.

$f_{chp}^{ss}(T)$: steady-state frequency of failure for parallel configuration in the case of CCS failures as well as human errors.

$\hat{f}_{chp}^{ss}(T)$: M L estimate of steady-state frequency of failure for parallel configuration in the case of CCS failures as well as human errors.

\bar{x}, \bar{y} & \bar{w} : sample means of the occurrence of individual, CCS failures and human errors respectively.

\bar{z} : sample mean of service time of the components.

\hat{x}, \hat{y} & \hat{w} : sample estimates of individual failure rate, CCS failure rate and human errors respectively.

\hat{z} : sample estimate of service time of the components.

n : sample size.

N : number of simulated samples.

M S E: mean square error.

3. Assumptions

(i) The system has three identical components, which are stochastically independent.

(ii) The system is affected by individual, CCS failures as well as human errors.

(iii) The components in the system will fail singly at the constant rate λ_i and failure probability is p_i

(iv) The components may fail due to CCS failures as well as human errors at the constant rates λ_c and λ_h with failure probabilities p_2 and p_3 s.t $p_1 + p_2 + p_3 = 1$.

(v) The occurrences of all failures follow an exponential distribution.

(vi) The failed components are serviced singly and service time follows an exponential distribution with rate of service ' μ '.

4. The Model

Under the assumptions given in section 3, we have formulated a Markov graph which represents a three component identical system given in Fig.4.1 to derive the availability function and the frequency of encountering the different states in the presence of individual failures, human errors and CCS failures. The numerals in Fig.4.1 denote the state numbers. The quantities appeared in Fig.4.1 are defined as

$$\begin{aligned} \lambda_0 &= 3\lambda_i p_1; \lambda_1 = 2\lambda_i p_1; \lambda_2 = \lambda_i p_1 \\ \lambda_{12} &= \lambda_c p_2; \lambda_{13} = \lambda_h p_3 \\ \mu_0 &= \mu; \mu_1 = 2\mu; \mu_2 = 3\mu \end{aligned} \quad (1)$$

The differential equations associated with the system states are

$$\begin{aligned} p'_0(t) &= -(\lambda_0 + \lambda_{12} + \lambda_{13}) p_0(t) + \mu_0 p_1(t) \\ p'_1(t) &= \lambda_0 p_0(t) - (\lambda_1 + \mu_0) p_1(t) + \mu_1 p_2(t) \\ p'_2(t) &= \lambda_1 p_1(t) - (\lambda_2 + \mu_1) p_2(t) + \mu_2 p_3(t) \\ p'_3(t) &= (\lambda_{12} + \lambda_{13}) p_0(t) - \lambda_2 p_2(t) - \mu_2 p_3(t) \end{aligned} \quad (2)$$

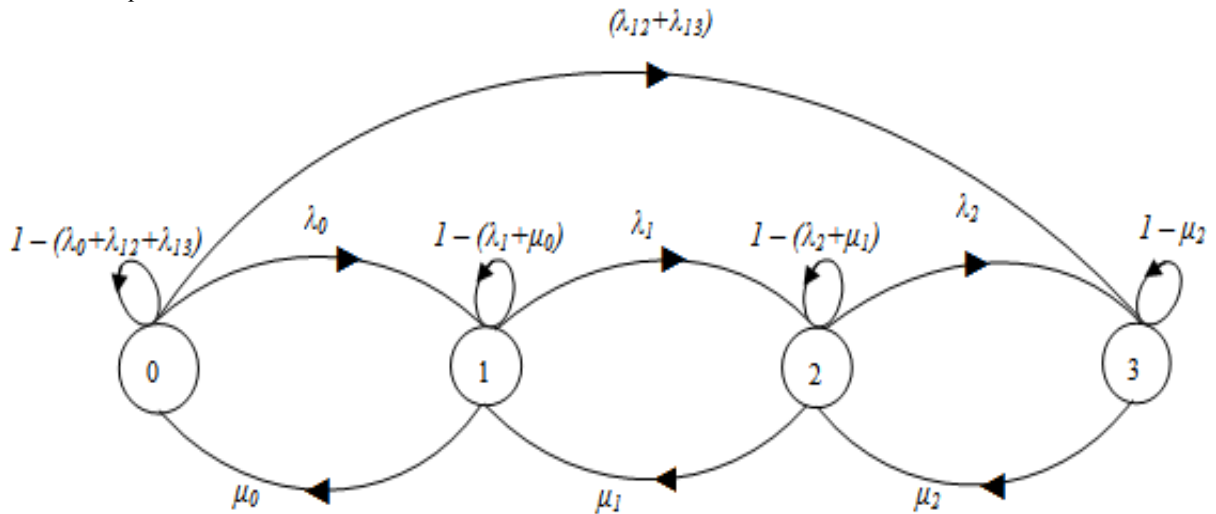


Figure 4.1. Markov graph for availability and frequency of failure functions of 3-component identical system with individual, CCS failures and human error

Using the Laplace transformation, the set of equations stated in (2) can be solved with given initial conditions, i.e at $t=0$, $P_0(t) = 1$ and $P_1(t) = P_2(t) = P_3(t) = 0$ and the solution is

$$p_0(t) = [(\gamma_1^3 + \gamma_1^2 \ell_1 + \gamma_1 \tau_1 + \psi_1) / \gamma_1 (\gamma_1 - \gamma_2)(\gamma_1 - \gamma_3)] \exp(\gamma_1 t) \\ - [(\gamma_2^3 + \gamma_2^2 \ell_2 + \gamma_2 \tau_2 + \psi_2) / \gamma_2 (\gamma_1 - \gamma_2)(\gamma_2 - \gamma_3)] \exp(\gamma_2 t) \\ + [(\gamma_3^3 + \gamma_3^2 \ell_3 + \gamma_3 \tau_3 + \psi_3) / \gamma_3 (\gamma_1 - \gamma_3)(\gamma_2 - \gamma_3)] \exp(\gamma_3 t) \\ - \psi_1 / (\gamma_1 \gamma_2 \gamma_3) \quad (3)$$

$$p_1(t) = [(\gamma_1^2 \ell_2 + \gamma_1 \tau_2 + \psi_2) / \gamma_1 (\gamma_1 - \gamma_2)(\gamma_1 - \gamma_3)] \exp(\gamma_1 t) \\ - [(\gamma_2^2 \ell_2 + \gamma_2 \tau_2 + \psi_2) / \gamma_2 (\gamma_1 - \gamma_2)(\gamma_2 - \gamma_3)] \exp(\gamma_2 t) \\ + [(\gamma_3^2 \ell_2 + \gamma_3 \tau_2 + \psi_2) / \gamma_3 (\gamma_1 - \gamma_3)(\gamma_2 - \gamma_3)] \exp(\gamma_3 t) \\ - \psi_2 / (\gamma_1 \gamma_2 \gamma_3) \quad (4)$$

$$p_2(t) = [(\gamma_1 \tau_3 + \psi_3) / \gamma_1 (\gamma_1 - \gamma_2)(\gamma_1 - \gamma_3)] \exp(\gamma_1 t) \\ - [(\gamma_2 \tau_3 + \psi_3) / \gamma_2 (\gamma_1 - \gamma_2)(\gamma_2 - \gamma_3)] \exp(\gamma_2 t) \\ + [(\gamma_3 \tau_3 + \psi_3) / \gamma_3 (\gamma_1 - \gamma_3)(\gamma_2 - \gamma_3)] \exp(\gamma_3 t) \\ - \psi_3 / (\gamma_1 \gamma_2 \gamma_3) \quad (5)$$

$$p_3(t) = 1 - [p_0(t) + p_1(t) + p_2(t)] \quad (6)$$

Where the quantities $\ell_1, \ell_2, \tau_1, \tau_2, \tau_3, \psi_1, \psi_2, \psi_3, \gamma_1, \gamma_2$ & γ_3 are defined as follows

$$\left. \begin{aligned} \ell_1 &= (\mu_0 + \mu_1 + \mu_2 + \lambda_1 + \lambda_2) \\ \ell_2 &= \lambda_0 \end{aligned} \right\} \quad (7)$$

$$\left. \begin{aligned} \tau_1 &= (\mu_0 \mu_1 + \lambda_1 \mu_2 + \lambda_1 \lambda_2 + \mu_1 \mu_2 + \mu_0 \lambda_2 + \mu_0 \mu_2) \\ \tau_2 &= (\lambda_0 \mu_1 + \lambda_0 \mu_2 + \lambda_0 \lambda_2) \\ \tau_3 &= (\lambda_0 \lambda_1 + \lambda_{13} \mu_2 + \lambda_{12} \mu_2) \end{aligned} \right\} \quad (8)$$

$$\left. \begin{aligned} \gamma_1 &= -\gamma \sin(\alpha) - v_1/3 \\ \gamma_2 &= \gamma \sin(\pi/3 + \alpha) - v_1/3 \\ \gamma_3 &= \gamma \sin(-\pi/3 + \alpha) - v_1/3 \end{aligned} \right\} \quad (10)$$

Here

$$\begin{aligned} \gamma &= (2/3) (v_1^2 - 3v_2)^{1/2} \\ \alpha &= \sin^{-1}(-4q/\gamma^3)/3 \\ q &= v_3 - (v_1 v_2)/3 + 2v_1^3/27 \end{aligned}$$

Where v_1, v_2 & v_3 are defined as follows

$$\begin{aligned} v_1 &= (\mu_0 + \mu_1 + \mu_2 + \lambda_0 + \lambda_1 + \lambda_2 + \lambda_{12} + \lambda_{13}) \\ v_2 &= (\mu_0 \lambda_2 + \lambda_1 \lambda_2 + \lambda_1 \mu_2 + \lambda_0 \mu_1 + \mu_1 \lambda_{12} + \lambda_1 \lambda_{12} + \lambda_{12} \mu_2 + \mu_0 \mu_2 \\ &\quad + \lambda_{13} \mu_2 + \lambda_{12} \lambda_2 + \lambda_1 \lambda_{13} + \mu_0 \lambda_{12} + \mu_0 \mu_1 + \mu_1 \lambda_{13} + \lambda_0 \mu_2 \\ &\quad + \mu_1 \mu_2 + \lambda_0 \lambda_2 + \lambda_0 \lambda_1 + \mu_0 \lambda_{13} + \lambda_{13} \lambda_2) \\ v_3 &= (\lambda_{13} \mu_0 \mu_1 + \lambda_0 \lambda_1 \lambda_2 + \lambda_{13} \mu_0 \lambda_2 + \mu_2 \lambda_1 \lambda_{13} + \lambda_{13} \mu_0 \lambda_2 \\ &\quad + \mu_0 \mu_1 \mu_2 + \lambda_{12} \mu_0 \mu_1 + \mu_0 \mu_2 \lambda_{12} + \lambda_0 \lambda_1 \mu_2 + \lambda_{12} \lambda_1 \lambda_2 \\ &\quad + \mu_1 \mu_2 \lambda_{13} + \mu_2 \lambda_1 \lambda_{12} + \lambda_0 \mu_1 \mu_2 + \mu_1 \mu_2 \lambda_{12} + \lambda_{13} \lambda_1 \lambda_2 + \mu_2 \mu_0 \lambda_{13}) \end{aligned}$$

And the quantities $\lambda_0, \lambda_1, \lambda_2, \lambda_{12}, \lambda_{13}, \mu_0, \mu_1$ & μ_2 as given in (1) are to be substitute in equations (7) – (10)

5. Maximum Likelihood (M L) Estimation of the System Availability and Frequency of Failures

This section discusses the maximum likelihood estimation approach for estimating the reliability measures like system availability and frequency of failures of a 3-component identical system under the influence of individual, CCS failures as well as human errors. The M L estimates are proposed for series and parallel systems.

Let x_1, x_2, \dots, x_n be a sample of 'n' number of times between individual failures which will obey exponential law.

Let y_1, y_2, \dots, y_n be a sample of 'n' number of times between CCS failures which follow exponential as well.

Let w_1, w_2, \dots, w_n be a sample of 'n' number of times between human errors failures which follow exponential as well.

Let z_1, z_2, \dots, z_n be a sample of 'n' number of times repair of the components with exponential population law.

$\hat{x}, \hat{y}, \hat{w}$ & \hat{z} are the maximum likelihood estimates of individual failure rate (λ_i), CCS failure rate (λ_c), human errors rate (λ_h) and repair rate ' μ ' of the system respectively.

$$\text{Where, } \hat{x} = \frac{1}{\bar{x}}; \quad \hat{y} = \frac{1}{\bar{y}}; \quad \hat{w} = \frac{1}{\bar{w}}; \quad \hat{z} = \frac{1}{\bar{z}}$$

$$\text{and } \bar{x} = \frac{\sum x_i}{n}; \quad \bar{y} = \frac{\sum y_i}{n}; \quad \bar{w} = \frac{\sum w_i}{n}; \quad \bar{z} = \frac{\sum z_i}{n}$$

are the sample estimates of the rate of individual failure times, rate of CCS failure times, rate of human error times and rate of repair times of the components respectively.

5.1. Estimation of Time-dependent System Availability

The M L estimates of time-dependent availability function for series and parallel systems are obtained in this section.

5.1.1. Series System

For a series system, state 1 itself is a absorbing state and hence no transition is allowed from state 1 to state 2 and state 2 to state 3. Hence $\lambda_1 = \lambda_2 = 0$. Therefore, the time-dependent system availability function for a series system is given by

$$Av_{chs}(t) = \phi_1 \exp(\gamma_1 t) - \phi_2 \exp(\gamma_2 t) + \phi_3 \exp(\gamma_3 t) \\ - 6\mu^3 / (\gamma_1 \gamma_2 \gamma_3) \quad (11)$$

Where

$$\begin{aligned} \phi_1 &= [(\gamma_1^3 + 6\mu \gamma_1^2 + 11\mu^2 \gamma_1 + 6\mu^3) / \gamma_1 (\gamma_1 - \gamma_2)(\gamma_1 - \gamma_3)] \\ \phi_2 &= [(\gamma_2^3 + 6\mu \gamma_2^2 + 11\mu^2 \gamma_2 + 6\mu^3) / \gamma_2 (\gamma_1 - \gamma_2)(\gamma_2 - \gamma_3)] \\ \phi_3 &= [(\gamma_3^3 + 6\mu \gamma_3^2 + 11\mu^2 \gamma_3 + 6\mu^3) / \gamma_3 (\gamma_1 - \gamma_3)(\gamma_2 - \gamma_3)] \end{aligned}$$

and

$$\left. \begin{aligned} \gamma_1 &= -\gamma \sin(\alpha) - m_1/3 \\ \gamma_2 &= \gamma \sin(\pi/3 + \alpha) - m_1/3 \\ \gamma_3 &= \gamma \sin(-\pi/3 + \alpha) - m_1/3 \end{aligned} \right\} \quad (12)$$

Here

$$\begin{aligned} \gamma &= (2/3) (m_1^2 - 3m_2)^{1/2} \\ \alpha &= \sin^{-1}(-4q/\gamma^3)/3 \\ q &= m_3 - (m_1 m_2)/3 + 2m_1^3/27 \end{aligned}$$

Where m_1, m_2 & m_3 are defined as follows

$$\begin{aligned} m_1 &= (6\mu + 3\lambda_i p_1 + \lambda_c p_2 + \lambda_h p_3) \\ m_2 &= (15\lambda_i p_1 \mu + 6\lambda_c p_2 \mu + 6\lambda_h p_3 \mu + 11\mu^2) \\ m_3 &= (11\lambda_h p_3 \mu^2 + 6\mu^3 + 11\lambda_c p_2 \mu^2 + 18\lambda_i p_1 \mu^2) \end{aligned}$$

The expression given in (11) agrees with the result already developed by Sagar et al[8]. Therefore, the maximum likelihood estimate of the time-dependent system availability function of the series system in the presence of individual, CCS failures as well as human errors can be obtained by

$$\hat{Av}_{chs}(t) = \phi'_1 \exp(D_1 t) - \phi'_2 \exp(D_2 t) + \phi'_3 \exp(D_3 t) - \frac{6(\hat{z})^3}{D_1 D_2 D_3} \quad (13)$$

Where

$$\begin{aligned} \phi'_1 &= \frac{[D_1^3 + 6(\hat{z})D_1^2 + 11(\hat{z})^2 D_1 + 6(\hat{z})^3]}{D_1(D_1 - D_2)(D_1 - D_3)} \\ \phi'_2 &= \frac{[D_2^3 + 6(\hat{z})D_2^2 + 11(\hat{z})^2 D_2 + 6(\hat{z})^3]}{D_2(D_1 - D_2)(D_2 - D_3)} \end{aligned}$$

$$\phi_3' = \frac{[D_3^3 + 6(\hat{z})D_3^2 + 11(\hat{z})^2D_3 + 6(\hat{z})^3]}{D_3(D_1 - D_3)(D_2 - D_3)}$$

And

$$\left. \begin{aligned} D_1 &= -D \sin(\alpha') - m_1'/3 \\ D_2 &= D \sin(\pi/3 + \alpha') - m_1'/3 \\ D_3 &= D \sin(-\pi/3 + \alpha') - m_1'/3 \\ D &= (2/3)((m_1')^2 - 3m_2')^{1/2} \\ \alpha' &= (\sin^{-1}(-4q'/D^3))/3 \\ q' &= m_3' - (m_1'm_2')/3 + 2(m_1')^3/27 \\ m_1' &= (3\hat{x}p_1 + \hat{y}p_2 + \hat{w}p_3 + 6\hat{z}) \\ m_2' &= \hat{z}(15\hat{x}p_1 + 6\hat{y}p_2 + 6\hat{w}p_3 + 11\hat{z}) \\ m_3' &= (\hat{z})^2(18\hat{x}p_1 + 11\hat{y}p_2 + 11\hat{w}p_3 + 6\hat{z}) \end{aligned} \right\} \quad (14)$$

and $\hat{x}, \hat{y}, \hat{w}$ and \hat{z} are sample estimates given in "section 5".

5.1.2. Parallel System

The time-dependent system availability function of the parallel system can be obtained by

$$\begin{aligned} Av_{chp}(t) &= p_0(t) + p_1(t) + p_2(t) \\ &= (K_1 + Y_1 + \phi_1)\exp(\gamma_1 t) - (K_2 + Y_2 + \phi_2)\exp(\gamma_2 t) \\ &\quad + (K_3 + Y_3 + \phi_3)\exp(\gamma_3 t) + (K_4 + Y_4 + \phi_4) \end{aligned} \quad (15)$$

where

$$\begin{aligned} K_1 &= [(\gamma_1^3 + \gamma_1^2 \ell_1 + \tau_1 \gamma_1 + \psi_1) / \gamma_1 (\gamma_1 - \gamma_2) (\gamma_1 - \gamma_3)] \\ K_2 &= [(\gamma_2^3 + \gamma_2^2 \ell_1 + \tau_1 \gamma_2 + \psi_1) / \gamma_2 (\gamma_1 - \gamma_2) (\gamma_2 - \gamma_3)] \\ K_3 &= [(\gamma_3^3 + \gamma_3^2 \ell_1 + \tau_1 \gamma_3 + \psi_1) / \gamma_3 (\gamma_1 - \gamma_3) (\gamma_2 - \gamma_3)] \\ Y_1 &= [(\gamma_1^2 \ell_2 + \gamma_1 \tau_2 + \psi_2) / \gamma_1 (\gamma_1 - \gamma_2) (\gamma_1 - \gamma_3)] \\ Y_2 &= [(\gamma_2^2 \ell_2 + \gamma_2 \tau_2 + \psi_2) / \gamma_2 (\gamma_1 - \gamma_2) (\gamma_2 - \gamma_3)] \\ Y_3 &= [(\gamma_3^2 \ell_2 + \gamma_3 \tau_2 + \psi_2) / \gamma_3 (\gamma_1 - \gamma_3) (\gamma_2 - \gamma_3)] \\ \phi_1 &= [(\gamma_1 \tau_3 + \psi_3) / \gamma_1 (\gamma_1 - \gamma_2) (\gamma_1 - \gamma_3)] \\ \phi_2 &= [(\gamma_2 \tau_3 + \psi_3) / \gamma_2 (\gamma_1 - \gamma_2) (\gamma_2 - \gamma_3)] \\ \phi_3 &= [(\gamma_3 \tau_3 + \psi_3) / \gamma_3 (\gamma_1 - \gamma_3) (\gamma_2 - \gamma_3)] \\ K_4 &= -\psi_1 / (\gamma_1 \gamma_2 \gamma_3) \\ Y_4 &= -\psi_2 / (\gamma_1 \gamma_2 \gamma_3) \\ \phi_4 &= -\psi_3 / (\gamma_1 \gamma_2 \gamma_3) \end{aligned}$$

The quantities $\ell_1, \ell_2, \tau_1, \tau_2, \tau_3, \psi_1, \psi_2, \psi_3$, and $\gamma_1, \gamma_2, \gamma_3$ associated with $K_1, K_2, K_3, K_4, Y_1, Y_2, Y_3, Y_4, \phi_1, \phi_2, \phi_3, \phi_4$ are defined in (7), (8), (9) & (10) respectively.

The time-dependent system availability function given in (15) agrees with the result already developed by Sagar et al[8]. Therefore, the maximum likelihood estimate of the time-dependent system availability function of the parallel system in the presence of individual, CCS failures as well as human errors can be obtained by

$$\begin{aligned} Av_{chp}(t) &= (K_1' + Y_1' + \phi_1') \exp(D_1 t) \\ &\quad - (K_2' + Y_2' + \phi_2') \exp(D_2 t) \\ &\quad + (K_3' + Y_3' + \phi_3') \exp(D_3 t) \\ &\quad + (K_4' + Y_4' + \phi_4') \end{aligned} \quad (16)$$

Where

$$\begin{aligned} K_1' &= [(D_1^3 + D_1^2 \ell_1' + \tau_1' D_1 + \psi_1') / D_1 (D_1 - D_2) (D_1 - D_3)] \\ K_2' &= [(D_2^3 + D_2^2 \ell_1' + \tau_1' D_2 + \psi_1') / D_2 (D_1 - D_2) (D_2 - D_3)] \\ K_3' &= [(D_3^3 + D_3^2 \ell_1' + \tau_1' D_3 + \psi_1') / D_3 (D_1 - D_3) (D_2 - D_3)] \\ Y_1' &= [(D_1^2 \ell_2' + \tau_2' D_1 + \psi_2') / D_1 (D_1 - D_2) (D_1 - D_3)] \\ Y_2' &= [(D_2^2 \ell_2' + \tau_2' D_2 + \psi_2') / D_2 (D_1 - D_2) (D_2 - D_3)] \\ Y_3' &= [(D_3^2 \ell_2' + \tau_2' D_3 + \psi_2') / D_3 (D_1 - D_3) (D_2 - D_3)] \\ \phi_1' &= [(D_1 \tau_3' + \psi_3') / D_1 (D_1 - D_2) (D_1 - D_3)] \\ \phi_2' &= [(D_2 \tau_3' + \psi_3') / D_2 (D_1 - D_2) (D_2 - D_3)] \end{aligned}$$

$$\begin{aligned} \phi_3' &= [(D_3 \tau_3' + \psi_3') / D_3 (D_1 - D_3) (D_2 - D_3)] \\ K_4' &= -\psi_1' / (D_1 D_2 D_3) \\ Y_4' &= -\psi_2' / (D_1 D_2 D_3) \\ \phi_4' &= -\psi_3' / (D_1 D_2 D_3) \end{aligned}$$

And the quantities $D_1, D_2, D_3, \ell_1', \ell_2', \tau_1', \tau_2', \tau_3', \psi_1', \psi_2', \psi_3'$ are defined as follows

$$\left. \begin{aligned} D_1 &= -D \sin(\alpha') - v_1'/3 \\ D_2 &= D \sin(\pi/3 + \alpha') - v_1'/3 \\ D_3 &= D \sin(-\pi/3 + \alpha') - (v_1')v_1'/3 \end{aligned} \right\} \quad (17)$$

Here

$$\begin{aligned} D &= (2/3)((v_1')^2 - 3v_2')^{1/2} \\ \alpha' &= (\sin^{-1}(-4q'/D^3))/3 \\ q' &= v_3' - (v_1'v_2')/3 + 2(v_1')^3/27 \end{aligned}$$

Where

$$\begin{aligned} v_1' &= (6\hat{x}p_1 + \hat{y}p_2 + \hat{w}p_3 + 6\hat{z}) \\ v_2' &= (\hat{x}p_1)(\hat{x}p_1 + \hat{y}p_2 + \hat{w}p_3) \\ &\quad + 2\hat{x}p_1(5\hat{x}p_1 + \hat{y}p_2 + \hat{w}p_3) \\ &\quad + \hat{z}(\hat{x}p_1 + \hat{y}p_2 + \hat{w}p_3) \\ &\quad + 2\hat{z}(3\hat{x}p_1 + \hat{y}p_2 + \hat{w}p_3) \\ &\quad + 3\hat{z}(6\hat{x}p_1 + \hat{w}p_3) + 11\hat{z} \\ v_3' &= \hat{z}(2\hat{x}p_1\hat{w}p_3) \\ &\quad + 2\hat{z}[2\hat{x}p_1\hat{y}p_2 + \hat{x}p_1\hat{w}p_3 + 6(\hat{x}p_1)^2] \\ &\quad + 2(\hat{z})^2(\hat{y}p_2 + \hat{w}p_3 + 3\hat{z}) + 3(\hat{z})^2(\hat{y}p_2 + \hat{w}p_3) \\ &\quad + 6(\hat{z})^2(3\hat{x}p_1 + \hat{y}p_2 + \hat{w}p_3) \\ &\quad + 2(\hat{x}p_1)^2(\hat{x}p_1 + \hat{y}p_2 + \hat{w}p_3) \end{aligned}$$

And

$$\left. \begin{aligned} \ell_1' &= (3\hat{x}p_1 + 6\hat{z}) \\ \ell_2' &= (3\hat{x}p_1) \end{aligned} \right\} \quad (18)$$

$$\left. \begin{aligned} \tau_1' &= 2(\hat{x}p_1)^2 + \hat{z}(4\hat{x}p_1) + 11(\hat{z})^2 \\ \tau_2' &= 3(\hat{x}p_1)^2 + 15\hat{x}p_1\hat{z} \\ \tau_3' &= 6(\hat{x}p_1)^2 + 3\hat{z}(\hat{y}p_2 + \hat{w}p_3) \\ \psi_1' &= 6(\hat{z})^3 \end{aligned} \right\} \quad (19)$$

$$\left. \begin{aligned} \psi_2' &= 6(\hat{z})^2(3\hat{x}p_1 + \hat{y}p_2 + \hat{w}p_3) \\ \psi_3' &= 18(\hat{x}p_1)^2(\hat{z}) + 6\hat{x}p_1\hat{z}(\hat{y}p_2 + \hat{w}p_3) \\ &\quad + 3(\hat{z})^2(\hat{y}p_2 + \hat{w}p_3) \end{aligned} \right\} \quad (20)$$

where $\hat{x}, \hat{y}, \hat{w}$ and \hat{z} are sample estimates given in "section 5".

5.2. Estimation of Steady-State System Availability

The maximum likelihood estimates of steady-state system availability functions for series and parallel systems are obtained in this section.

5.2.1. Series System

The steady-state availability of series system can be obtained by using the final value theorem of Laplace transformation.

$$\begin{aligned} Av_{chs}(\infty) &= \lim_{t \rightarrow \infty} Av_{chs}(t) \\ Av_{chs}(\infty) &= -\psi_1 / (\gamma_1 \gamma_2 \gamma_3) \end{aligned} \quad (21)$$

Where $\psi_1, \gamma_1, \gamma_2$ & γ_3 are defined in (9) and (12).

The above expression given in (21) agrees with the result already developed by Sagar et al[8]. Therefore, the maximum likelihood estimate of steady-state system availability function of series system in the presence of individual, CCS failures as well as human errors can be obtained by

$$\hat{A}v_{chs}(\infty) = -\frac{6(\hat{z})^3}{D_1 D_2 D_3} \quad (22)$$

Where D_1, D_2, D_3 are defined in (14)

5.2.2. Parallel System

The steady-state availability function of parallel system can be obtained by using the final value theorem of Laplace transformation.

$$Av_{chp}(\infty) = \lim_{t \rightarrow \infty} Av_{chp}(t) \\ Av_{chp}(\infty) = -(\psi_1 + \psi_2 + \psi_3) / (\gamma_1 \gamma_2 \gamma_3) \quad (23)$$

Where ψ_1, ψ_2, ψ_3 and $\gamma_1, \gamma_2, \gamma_3$ are defined in (9) and (10)

The above expression given in (23) agrees with the result already developed by Sagar et al[8]. Therefore, the maximum likelihood estimate of steady-state system availability function of parallel system in the presence of individual, CCS failures as well as human errors can be obtained by

$$\hat{A}v_{chp}(\infty) = -\frac{(\psi'_1 + \psi'_2 + \psi'_3)}{(D_1 D_2 D_3)} \quad (24)$$

Where D_1, D_2, D_3 and $\psi'_1, \psi'_2, \psi'_3$ are defined in (17) and (20)

5.3. Estimation of Frequency of Failures

The maximum likelihood estimates of the frequency of failure functions for series and parallel systems in the presence of individual, CCS failures and human errors are obtained in this section.

5.3.1. Series System

The frequency of failure function for series system is given by

$$f_{chs}^{ss}(T) = -\psi_1 (3\lambda_i p_1 + \lambda_c p_2 + \lambda_h p_3) / (\gamma_1 \gamma_2 \gamma_3) \quad (25)$$

Where ψ_1 and $\gamma_1, \gamma_2, \gamma_3$ are given in (9) and (12)

The above expression given in (25) agrees with the result already developed by Sagar et al[8]. Therefore, the M L estimate of the frequency of failure function for series system is given by

$$\hat{f}_{chs}^{ss}(T) = -\frac{6(\hat{z})^3(3\hat{x}p_1 + \hat{y}p_2 + \hat{w}p_3)}{(D_1 D_2 D_3)} \quad (26)$$

Where D_1, D_2 & D_3 are defined in (14)

5.3.2. Parallel System

The frequency of failure function for parallel system is given by

$$f_{chp}^{ss}(T) = -3\mu[(\lambda_c p_2 + \lambda_h p_3)(2\mu^2 + \lambda_i p_1 \mu + 2(\lambda_i p_1)^2 + 6(\lambda_i p_1)^3)] / (\gamma_1 \gamma_2 \gamma_3) \quad (27)$$

Where γ_1, γ_2 & γ_3 are given in (10)

The above expression given in (27) agrees with the result already developed by Sagar et al[8]. Therefore, the maximum likelihood estimate of the frequency of failure function of parallel system is given by

$$\hat{f}_{chp}^{ss}(T) = -\frac{3\hat{z}[(\hat{y}p_2 + \hat{w}p_3)(2(\hat{z})^2 + \hat{x}p_1\hat{z} + 2(\hat{x}p_1)^2 + 6(\hat{x}p_1)^3)]}{(D_1 D_2 D_3)} \quad (28)$$

Where D_1, D_2 & D_3 are defined in (17)

6. Simulation and Results

The M L estimates of system availability (time-dependent & steady-state) and frequency of failure functions were not identified with exact or analytical form of probability density function since they are complex functions of the sample information. As such, the probability density function of the estimates is not identified analytically. Hence an attempt is made to establish the validity and to find precision of the estimates of the above measures; Monte-Carlo simulation is used.

For a range of specified values of the rates of individual (λ_i), CCS failures (λ_c), human errors (λ_h) and service rate (μ) for the samples of sizes $n=5(5)30$ were simulated in each case with $N=10,000(20,000)90,000$ in order to evolve mean square error (MSE) in each case and given in numerical illustration. It is interesting to note that estimation gives a very close estimate in the case of very small samples of size $n=5$. This shows that M L approach and estimators are quite useful in estimating reliability and availability measures.

TABLE 6.1

Time-dependent system availability function for three component identical Series System with $\lambda_i = 0.05$; $\lambda_c = 0.04$; $\lambda_h = 0.03$; $\mu = 5$; $p_1 = 0.5$; $p_2 = 0.25$; $p_3 = 0.25$; $t = 2$

Sample size n = 5			
N	$Av_{chs}(t)$	$\hat{A}v_{chs}(t)$	M S E
10000	0.979279	0.974438	0.000276
30000	0.979279	0.974150	0.000307
50000	0.979279	0.974424	0.000298
70000	0.979279	0.974385	0.000297
90000	0.979279	0.974464	0.000294

Sample size n = 10			
N	$Av_{chs}(t)$	$\hat{A}v_{chs}(t)$	M S E
10000	0.979279	0.976992	0.000100
30000	0.979279	0.977122	0.000095
50000	0.979279	0.977091	0.000097
70000	0.979279	0.977022	0.000098
90000	0.979279	0.977078	0.000098

Sample size n = 15			
N	$Av_{chs}(t)$	$\hat{A}v_{chs}(t)$	M S E
10000	0.979279	0.977804	0.000057
30000	0.979279	0.977867	0.000059
50000	0.979279	0.977793	0.000058
70000	0.979279	0.977835	0.000058
90000	0.979279	0.977829	0.000059

Sample size n = 20			
N	$Av_{chs}(t)$	$\hat{A}v_{chs}(t)$	M S E
10000	0.979279	0.978066	0.000042
30000	0.979279	0.978238	0.000041
50000	0.979279	0.978214	0.000041
70000	0.979279	0.978216	0.000041
90000	0.979279	0.978163	0.000042

Sample size n = 25			
N	$Av_{chs}(t)$	$\hat{Av}_{chs}(t)$	M S E
10000	0.979279	0.978402	0.000032
30000	0.979279	0.978450	0.000031
50000	0.979279	0.978428	0.000032
70000	0.979279	0.978416	0.000032
90000	0.979279	0.978386	0.000032

Sample size n = 30			
N	$Av_{chs}(t)$	$\hat{Av}_{chs}(t)$	M S E
10000	0.979279	0.978621	0.000026
30000	0.979279	0.978532	0.000026
50000	0.979279	0.978521	0.000026
70000	0.979279	0.978512	0.000026
90000	0.979279	0.978549	0.000026

TABLE 6.2

Time-dependent system availability function for three component identical Parallel System with $\lambda_i = 0.05$; $\lambda_c = 0.04$; $\lambda_h = 0.03$; $\mu = 5$; $p_1 = 0.5$; $p_2 = 0.25$; $p_3 = 0.25$; $t = 2$

Sample size n = 5			
N	$Av_{chp}(t)$	$\hat{Av}_{chp}(t)$	M S E
10000	0.998260	0.998728	0.000010
30000	0.998260	0.998667	0.000010
50000	0.998260	0.998773	0.000009
70000	0.998260	0.998905	0.000009
90000	0.998260	0.999144	0.000009

Sample size n = 10			
N	$Av_{chp}(t)$	$\hat{Av}_{chp}(t)$	M S E
10000	0.998260	0.998391	0.000002
30000	0.998260	0.998334	0.000002
50000	0.998260	0.998504	0.000002
70000	0.998260	0.998656	0.000002
90000	0.998260	0.998941	0.000002

Sample size n = 15			
N	$Av_{chp}(t)$	$\hat{Av}_{chp}(t)$	M S E
10000	0.998260	0.998294	0.000001
30000	0.998260	0.998237	0.000001
50000	0.998260	0.998418	0.000001
70000	0.998260	0.998594	0.000001
90000	0.998260	0.998905	0.000001

Sample size n = 20			
N	$Av_{chp}(t)$	$\hat{Av}_{chp}(t)$	M S E
10000	0.998260	0.998263	0.000000
30000	0.998260	0.998198	0.000000
50000	0.998260	0.998398	0.000000
70000	0.998260	0.998577	0.000000
90000	0.998260	0.998892	0.000000

Sample size n = 25			
N	$Av_{chp}(t)$	$\hat{Av}_{chp}(t)$	M S E
10000	0.998260	0.998243	0.000000
30000	0.998260	0.998171	0.000000
50000	0.998260	0.998377	0.000000
70000	0.998260	0.998579	0.000000
90000	0.998260	0.998898	0.000000

Sample size n = 30			
N	$Av_{chp}(t)$	$\hat{Av}_{chp}(t)$	M S E
10000	0.998260	0.998238	0.000000
30000	0.998260	0.998153	0.000000
50000	0.998260	0.998383	0.000000
70000	0.998260	0.998580	0.000000
90000	0.998260	0.998894	0.000000

TABLE 6.3

Frequency of failure function for three component identical Series System with $\lambda_i = 0.05$; $\lambda_c = 0.04$; $\lambda_h = 0.03$; $\mu = 5$; $p_1 = 0.5$; $p_2 = 0.25$; $p_3 = 0.25$

Sample size n = 5			
N	$f_{chs}^{ss}(T)$	$\hat{f}_{chs}^{ss}(T)$	M S E
10000	0.090575	0.124330	0.004737
30000	0.090575	0.125515	0.005201
50000	0.090575	0.124715	0.005116
70000	0.090575	0.124492	0.005092
90000	0.090575	0.124494	0.005046

Sample size n = 10			
N	$f_{chs}^{ss}(T)$	$\hat{f}_{chs}^{ss}(T)$	M S E
10000	0.090575	0.109678	0.001549
30000	0.090575	0.109739	0.001489
50000	0.090575	0.109426	0.001479
70000	0.090575	0.109670	0.001496
90000	0.090575	0.109648	0.001503

Sample size n = 15			
N	$f_{chs}^{ss}(T)$	$\hat{f}_{chs}^{ss}(T)$	M S E
10000	0.090575	0.105261	0.000849
30000	0.090575	0.105099	0.000857
50000	0.090575	0.105447	0.000853
70000	0.090575	0.105338	0.000861
90000	0.090575	0.105359	0.000860

Sample size n = 20			
N	$f_{chs}^{ss}(T)$	$\hat{f}_{chs}^{ss}(T)$	M S E
10000	0.090575	0.103467	0.000606
30000	0.090575	0.102931	0.000593
50000	0.090575	0.103171	0.000596
70000	0.090575	0.103219	0.000597
90000	0.090575	0.103351	0.000607

Sample size n = 25			
N	$f_{chs}^{ss}(T)$	$\hat{f}_{chs}^{ss}(T)$	M S E
10000	0.090575	0.101821	0.000459
30000	0.090575	0.101842	0.000460
50000	0.090575	0.102022	0.000464
70000	0.090575	0.102090	0.000466
90000	0.090575	0.102105	0.000469

Sample size n = 30			
N	$f_{chs}^{ss}(T)$	$\hat{f}_{chs}^{ss}(T)$	M S E
10000	0.090575	0.100984	0.000375
30000	0.090575	0.101249	0.000384
50000	0.090575	0.101309	0.000385
70000	0.090575	0.101358	0.000386
90000	0.090575	0.101266	0.000382

Sample size n = 25			
N	$f_{chp}^{ss}(T)$	$\hat{f}_{chp}^{ss}(T)$	M S E
10000	0.008884	0.010000	0.000005
30000	0.008884	0.010002	0.000005
50000	0.008884	0.010020	0.000005
70000	0.008884	0.010027	0.000005
90000	0.008884	0.010029	0.000005

Sample size n = 30			
N	$f_{chp}^{ss}(T)$	$\hat{f}_{chp}^{ss}(T)$	M S E
10000	0.008884	0.009914	0.000004
30000	0.008884	0.009942	0.000004
50000	0.008884	0.009949	0.000004
70000	0.008884	0.009953	0.000004
90000	0.008884	0.009943	0.000004

TABLE 6.4

Frequency of failure function for three component identical Parallel System with $\lambda_i = 0.05$; $\lambda_c = 0.04$; $\lambda_h = 0.03$; $\mu = 5$; $p_1 = 0.5$; $p_2 = 0.25$; $p_3 = 0.25$

Sample size n = 5			
N	$f_{chp}^{ss}(T)$	$\hat{f}_{chp}^{ss}(T)$	M S E
10000	0.008884	0.012316	0.000052
30000	0.008884	0.012453	0.000059
50000	0.008884	0.012366	0.000059
70000	0.008884	0.012345	0.000058
90000	0.008884	0.012341	0.000057

Sample size n = 10			
N	$f_{chp}^{ss}(T)$	$\hat{f}_{chp}^{ss}(T)$	M S E
10000	0.008884	0.010805	0.000017
30000	0.008884	0.010807	0.000016
50000	0.008884	0.010776	0.000016
70000	0.008884	0.010802	0.000016
90000	0.008884	0.010799	0.000016

Sample size n = 15			
N	$f_{chp}^{ss}(T)$	$\hat{f}_{chp}^{ss}(T)$	M S E
10000	0.008884	0.010350	0.000009
30000	0.008884	0.010334	0.000009
50000	0.008884	0.010369	0.000009
70000	0.008884	0.010358	0.000009
90000	0.008884	0.010361	0.000009

Sample size n = 20			
N	$f_{chp}^{ss}(T)$	$\hat{f}_{chp}^{ss}(T)$	M S E
10000	0.008884	0.010169	0.000006
30000	0.008884	0.010113	0.000006
50000	0.008884	0.010137	0.000006
70000	0.008884	0.010142	0.000006
90000	0.008884	0.010156	0.000006

7. Conclusions

The maximum likelihood estimates of system availability (time-dependent and steady-state) and frequency of failure functions [$Av(t)$, $Av(\infty)$, $f(T)$] were developed for three-component identical system with CCS failures as well as human errors. The estimates are proposed for all the measures said above both for series and parallel systems. We presented numerical simulation studies in order to establish empirically, the validity of the proposed M L estimates of the above measures in the absence of analytical approach. From the simulation results, we observed that

(i) The point estimates become more accurate when the sample size is large.

(ii) Each of MSE decreases with increasing the sample size.

Therefore, this paper suggest that the use of Maximum likelihood estimation approach is found satisfactory for estimation process of some important reliability measures of the system performance, since estimation gives a very close estimate of the above measures even in the case of very small samples of size n=5.

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