

Using Decision Theory Approach to Build a Model for Bayesian Sampling Plans

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Abstract The paper deals with constructing a model for Bayesian sampling plans for the system “Average out going quality level (AOQL)”, where the percentage of defectives is varied from lot to lot, so it considered to be a random variable, having a prior distribution $f(p)$, which must be fitted to represent the distribution of percentage of defectives efficiently. The parameters of this distribution must estimated, and then used in model construction. The aim of the model is to find the parameters of single Bayesian sampling plan (n, c) , the sample size, and the acceptance number (c) , from minimizing the total cost of the model, which comprises cost inspection and cost of repairing or replacement of defective units. In addition to cost of rejecting good items, which is a penalty cost. Also the construction depend on decision rule $[\delta(x)]$, for acceptance and decision rule for rejection $[1 - \delta(x)]$. Finally the build model can be applied to another distribution like Gamma – Poisson, Normal – Beta, to find the sampling plan (n, c) necessary to test the product of the lot and to have a production with accepted (AOQL) to satisfy consumer's and producer's risk. All the derivation required to build this cost function are explained and all the results of obtained samples and applications are illustrated in tables.

Keywords Bayesian Sampling Plan, (AOQL), Beta & Gamma Distribution, Total Cost Function, Producer's , Consumer's Risk

1. Introduction

The term sampling inspection plan, is used when the quality of product is evaluated by inspecting samples rather than by total inspects, which required cost and time. Here we introduce a model for total expected cost function by [7 & 9] whom gives simple information about the elements of this cost function, while [10] introduce the cost model when the percentage of defective is random variable follows Gamma distribution, and suggested to apply another distribution as we done in our research. The build model differs from them by including decision rule $[\delta(x)]$ for acceptance and $[1 - \delta(x)]$ for rejection, the posterior distribution $f(p|x)$ and finding the optimal value of acceptance number (c) is a closed form, also the sample size necessary to take a decision, in a closed form also. The programs required to obtain (n, c) , $[p_n(c)]$ and total cost, are executed.

The aim of this paper is to build the total cost of quality control model, used to find the Bayesian sampling plan (n, c) for (AOQL) system, the method considered the percentage

of defectives in lots is random variable having prior distribution $f(p)$. So the method derived used to find Bayesian sampling plan.

2. Methodology of Research

Since the aim of research insist on finding Bayesian sampling plans for system (AOQL), where the percentage of defective in production represents random variable varied from lot to lot and have some prior distribution $f(p)$, which determined from past data and experience, and it may be Beta distribution or Gamma, or Log-Normal or any other distributions. It is necessary to determine the parameters of Bayesian sampling plan (n, c) , taken from lot N by minimizing either (total inspection cost) or minimizing total expected risk, which is the risk due to taking the wrong decision.

To satisfy, the aim, first of all we define all notations necessary to build the model, these are;

N : Lot size of product.

X : Number of defective units in the lot N .

$P = \left(\frac{X}{N}\right)$: Percentage of defective in lot N .

$p = \left(\frac{x}{n}\right)$: Percentage of defective in the sample n .

x : Number of defective in sample.

X : Number of defective in Lot.

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C : Acceptance number

C_1 : Cost of sampling and testing unit in the sample.

C_2 : Cost of repairing or replacement of defective units in the sample.

C_3 : Cost of sampling and testing units in the remaining quantity $(N - n)$ after rejecting sample.

C_4 : Cost of repairing or replacement of defective unit in quantity $(N - n)$.

C_5 : Cost of accepting good unit and it is not a penalty cost.

C_6 : Cost of accepting defective unit.

$P(p)$: Probability of accepting the production with quality p and it is;

$$P(p) = pr(x \leq c)$$

$Q(p)$: Probability of rejecting the production with quality p .

AOQ : Average percentage of defectives in the lot after doing rectifying inspection on it.

$\delta(x)$: Decision rule for explaining the probability of acceptance the lot $(N - n)$, after knowing the sample n contains x defective.

$1 - \delta(x)$: Decision rule for rejection.

\bar{p} : Mean of the probability distribution of defective.

\bar{P} : Mean of prior distribution $f(p)$.

$p_n(x) = E(p|x)$: Mean of posterior distribution $f(p|x, n)$.

pr : Break even quality point;

$$pr = \frac{C_3 - C_5}{C_6 - C_4}$$

$f(p)$: Prior distribution of percentage of defectives in the lot of product.

$f(p|x)$: Posterior distribution of percentage of defectives.

3. Construction of the Model

Suppose that x be $r.v \sim \text{Binomial}(n, p)$

$$p(x; n, p) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, 2, \dots, n \quad (1)$$

And p the percentage of defectives in process is $r.v \sim \text{Beta}(\alpha, \beta)$

$$f(p; \alpha, \beta) = \frac{1}{\text{Beta}(\alpha, \beta)} p^{\alpha-1} (1-p)^{\beta-1}, \quad 0 \leq p \leq 1 \quad (2)$$

Therefore

$$\begin{aligned} g_n(x) &= \int_0^1 p(x; n, p) f(p) dp \\ &= C_x^n \frac{1}{\beta(\alpha, \beta)} \int_0^1 p^{x+\alpha-1} (1-p)^{n+\beta-x-1} dp \\ &= C_x^n \frac{1}{\beta(\alpha, \beta)} \text{Beta}(x+\alpha, n+\beta-x) \end{aligned} \quad (3)$$

$g_n(x)$ Is used to find the posterior distribution which is:

$$\begin{aligned} f(p|x, n) &= \frac{f(p) p(x; n, p)}{g_n(x)} \\ &= \frac{1}{\beta(x+\alpha, n+\beta-x)} p^{x+\alpha-1} (1-p)^{n+\beta-x-1} \end{aligned} \quad (4)$$

Therefore, the posterior distribution of percentage of defectives is also Beta, but with parameters $(x+\alpha, n+\beta-x)$

and with average;

$$p_n(x) = E(p|x) = \frac{x + \alpha}{\alpha + \beta + n}$$

Which can be written in terms of the mean of prior (\bar{P}) i.e.

$$\bar{P} = \frac{\alpha}{\alpha + \beta}$$

as:

$$\begin{aligned} p_n(x) &= \frac{x + \alpha}{\alpha + \beta + n} \\ &= \frac{\alpha(1 + \frac{x}{\alpha})}{(\alpha + \beta)(1 + \frac{n}{\alpha + \beta})} \\ &= \frac{\bar{P}(x + \alpha)}{\alpha + n\bar{P}} \end{aligned}$$

Now; let

$$\gamma(n, c) = pr \sum_{x=0}^c g_n(x) - \sum_{x=0}^c g_n(x) E(p|x) \quad (5)$$

$$= pr \sum_{x=0}^c g_n(x) - \sum_{x=0}^c p_n(x) g_n(x) \quad (6)$$

And let pr be break even quality level, which is the point of percentage of defective, at which we cannot distinguish between acceptance decision and rejection, and is defined by $pr = \frac{x-x}{N-n}$, and it is also defined in terms of quality control cost parameters.

Therefore;

$$\begin{aligned} \gamma(n, c) &= pr \sum_{x=0}^c C_x^n \frac{1}{\beta(\alpha, \beta)} \beta(x + \alpha, n + \beta - x) \\ &\quad - \sum_{x=0}^c \frac{x + \alpha}{\alpha + \beta + n} C_x^n \frac{1}{\beta(\alpha, \beta)} \beta(x + \alpha, n + \beta - x) \\ &= \frac{1}{\beta(\alpha, \beta)} \sum_{x=0}^c C_x^n [pr B(x + \alpha, n + \beta - x) \\ &\quad - B(x + \alpha + 1, n + \beta - 1)] \end{aligned} \quad (7)$$

We know that the loss equal zero, when the decision is correct, and the loss is $|k_a(p) - k_r(p)|$ for wrong decision, i.e.

$$\begin{aligned} |k_a(p) - k_r(p)| &= |(C_5 + C_6 p) - (C_3 + C_4 p)| \\ &= |(C_5 - C_3) + (C_6 - C_4)p| \\ &= |C_6 - C_4| |p - pr| \end{aligned} \quad (8)$$

According to that, the equation of expected risk $R[f(p), n, c]$ is defined;

$$\begin{aligned} R[f(p), n, c] &= nk_s + k \int_0^{pr} (pr - p) Q(p) f(p) dp \\ &\quad + k \int_{pr}^1 (p - pr) P(p) f(p) dp \end{aligned} \quad (9)$$

where

$$k = (N - n)(C_6 - C_4), k_s = C_1 + C_2 \bar{P}$$

$$\begin{aligned} \therefore R[f(p), n, c] &= nk_s + k \left[\int_0^{pr} (p - pr) P(p) f(p) dp \right. \\ &\quad \left. + \int_0^{pr} (pr - p) f(p) dp \right] \end{aligned} \quad (10)$$

Using the $R[f(p), n, c]$, can found the value of expected risk under Binomial process and continues prior distribution. See[9].

But here the proposed model depend on definition $r(p, \delta)$ in terms of p and decision rule $[\delta(x)]$, and it is defined as follows;

$$r(p, \delta) = \delta(x) [C_1 n + C_2 np + C_5(N - n) + C_6 p(N - n)] p(x|p) + (1 - \delta(x)) [C_1 n + C_2 np + C_3(N - n) + C_4 p(N - n)] p(x|p) \quad (11)$$

And since P is random variable has $f(p)$ then the expected value of risk in terms of $f(p)$ is denoted by $R[f(p), \delta]$ and it is defined as:

$$\begin{aligned} R[f(p), \delta] &= E\{\delta(x) [C_1 n + C_2 np + C_5(N - n) + C_6 p(N - n)] p(x|p)\} \\ &\quad + E\{(1 - \delta(x)) [C_1 n + C_2 np + C_3(N - n) + C_4 p(N - n)] p(x|p)\} \\ &= E\{\delta(x) [(C_5 - C_3)(N - n) + (C_6 - C_4)(N - n)p] p(x|p)\} \\ &\quad + E\{[C_3(N - n) + C_4(N - n)p] p(x|p)\} + E\{[C_1 n + C_2 np] p(x|p)\} \\ &\because E\{\delta(x) (C_6 - C_4)(N - n)p\} p(x|p) = (C_6 - C_4)(N - n) E\{\delta(x)p\} p(x|p) \\ &= (C_6 - C_4)(N - n) \int_0^1 p \sum_{x \in \{x, \delta(x)=d_1\}} b(x, n, p) f(p) dp \\ &= (C_6 - C_4)(N - n) \sum_x b_w(x, n) \int_0^1 p f(p|x) dp \\ &= (C_6 - C_4)(N - n) \sum_x b_w(x, n) E(p|x) \\ &= (C_6 - C_4)(N - n) E\{\delta(x) E(p|x)\} \end{aligned} \quad (12)$$

After some steps we have;

$$\begin{aligned} &= E\{\delta(x) [(C_5 - C_3)(N - n) + (C_6 - C_4)(N - n)p] p(x|p)\} \\ &\quad + E\{[C_3(N - n) + C_4(N - n)p] p(x|p)\} + E\{[C_1 n + C_2 np] p(x|p)\} \\ R[f(p), \delta] &= (N - n) E\{\delta(x) [(C_5 - C_3) + (C_6 - C_4) E(p|x)]\} \\ &\quad + [C_2 n + C_4(N - n)] E(p) + C_1 n + C_3(N - n) \\ R[f(p), \delta] &= (N - n) E\{\delta(x) (E(p|x) - pr)(C_6 - C_4)\} + [C_2 n + C_4(N - n)] E(p) + C_1 n + C_3(N - n) \end{aligned} \quad (13)$$

The minimum value for formula (13) can be verified by;

$$\delta(x) = \begin{cases} 1 & \text{when } E(p|x) \leq pr \\ 0 & \text{otherwise} \end{cases}$$

where;

$$pr = \frac{C_3 - C_5}{C_6 - C_4}, \quad C_3 < C_5, \quad C_4 > C_6 \text{ or } C_3 > C_5 \text{ \& } C_4 < C_6$$

Since equation (13), define the formula of expected risk function in terms of expectation, we want to find it in terms of sampling distribution, this required to take the distribution of defectives in the lot $f_N(x)$, and distribution of (x) in sample (n) , $p(x|X)$ in consideration, and also taking $f(p)$ (the prior distribution of quality of a process, and also define the loss $L(x, \delta(x))$, this tend us to find the expected value of the loss due to the decision of accepting and rejecting, which is now defined by the following equation;

$$\begin{aligned} R[f(x), n, c] &= \sum_{x=0}^N \sum_{x=0}^c L(x, \delta(x)) p(x|X) f_N(x) \\ &= \sum_{x=0}^N \sum_{x=0}^c [C_1 n + C_2 x + C_5(N - n) + C_6(X - x)] p(x|X) f_N(x) \\ &\quad + \sum_{x=0}^N \sum_{x=c+1}^c [C_1 n + C_2 x + C_3(N - n) + C_4(X - x)] p(x|X) f_N(x) \end{aligned} \quad (14)$$

$$\begin{aligned} &= \sum_{x=0}^N \sum_{x=0}^c [(N - n)(C_5 - C_3) + (C_6 - C_4)(X - x)] p(x|X) f_N(x) \\ &\quad + C_3(N - n) + C_4 \sum_{x=0}^N \sum_{x=0}^c (X - x) p(x|X) f_N(x) + C_1 n + C_4 \sum_{x=0}^N \sum_{x=0}^c x p(x|X) f_N(x) \end{aligned} \quad (15)$$

Putting the following simplification in equation (15), we obtain equation (16);

$$\begin{aligned} E(x) &= n\bar{p} \\ E(X) &= N\bar{p} \\ g_n(x) &= \sum_{x=0}^N p(x|X) f_N(x) \end{aligned}$$

Also substituting;

$$\sum_{x=0}^N \sum_{x=0}^c (X - x) p(x|X) f_N(x) = \sum_{x=0}^c g_n(x) E(X - x|x)$$

Where;

$$\begin{aligned} E(X - x|x) &= E\{E(X - x|x, p|x)\} \\ &= E[E(X - x)|p|x] \\ &= E[(N - n)p|x] \end{aligned}$$

$$= (N - n)E(p|x)$$

Where, $E(p|x)$ is the posterior mean.

By applying these expectations in equation (15), we get;

$$R[N, n, c] = C_1 n + C_2 n \bar{p} + (N - n)(C_3 + C_4 \bar{p}) + (N - n) [\sum_{x=0}^c g_n(x)(C_6 - C_4)E(p|x) + C_5 - C_3] \quad (16)$$

Which can be simplified to;

$$R[N, n, c] = n(C_1 + C_2 \bar{p}) + (N - n)(C_3 + C_4 \bar{p}) + (N - n)(C_6 - C_4) \gamma(n, c) \quad (17)$$

Where;

$$\begin{aligned} \gamma(n, c) &= pr \sum_{x=0}^c g_n(x) - \sum_{x=0}^c g_n(x)E(p|x) \\ &= pr \sum_{x=0}^c g_n(x) - \sum_{x=0}^c p_n(x) g_n(x) \\ &\quad \text{since } p_n(x) = E(p|x) \end{aligned}$$

The function in equation (17) considered to be a function of Bayesian single plan to test the product (n, c) and searching for the two values (n, c) which gives minimum value for function (17), can be done by applying the first partial derivatives for the function equal to zero, then solving the two equations, since the distribution of (X, x) is a discrete distribution, so we cannot apply differentiation method, we can apply forward differences method for (n, c) which required writing $\gamma(n, c)$ in a fitness way;

$$\begin{aligned} \gamma(n, c) &= pr \sum_{x=0}^c g_n(x) - \sum_{x=0}^c p_n(x) g_n(x) \\ &= pr G_n(c) - \bar{p}_n(c) G_n(c) \end{aligned}$$

Where;

$$\bar{p}_n(c) = \sum_{x=0}^c p_n(x) g_n(x) / \sum_{x=0}^c g_n(x) \quad (18)$$

Which represents the weighted mean for $p_n(x)$ when $(x \leq c)$ according to the rule of [10], we can summarize the following rule;

$$\Delta_n^{(r)} = G_n^{(r)}(x) = (-1)^r g_n(x) p_n(x) p_{n+1}(x+1) \dots p_{n+r-1}(x+r-1) \quad (19)$$

From equation (19), we can get;

$$(1) \Delta_c \gamma(n, c) = pr g_n(c+1) + (-1) g_n(c+1) p_n(c+1) = g_n(c+1) [pr - p_n(c+1)]$$

$$(2) \Delta_c \gamma(n, c-1) = pr g_n(c) + \Delta_n G_n(c) = g_n(c) [pr - p_n(c)]$$

$$(3) \Delta_n \gamma(n, c) = pr \Delta_n G_n(c) + \Delta_n^2 G_n^2(c) = pr (-1) g_n(c) p_n(c) + (-1)^2 g_n(c) p_n(c) p_{n+1}(c+1)$$

$$\Delta_n \gamma(n, c) = g_n(c) p_n(c) [p_{n+1}(c+1) - pr]$$

$$(4) \Delta_n \gamma(n-1, c) = pr \Delta_n G_{n-1}(c) + \Delta_n^2 G_{n-1}^2(c) = pr (-1) g_{n-1}(c) p_{n-1}(c) + (-1)^2 g_{n-1}(c) p_{n-1}(c) p_n(c+1)$$

$$\Delta_n \gamma(n-1, c) = g_{n-1}(c) p_{n-1}(c) [p_n(c+1) - pr]$$

According to Beta – Binomial distribution when $x \sim \text{Binomial}(n, p)$;

$$p(x) = C_x^n p^x (1-p)^{n-x}, \quad x = 0, 1, \dots, n$$

And, $p \sim \text{Beta}(\alpha, \beta)$;

$$f(p) = \frac{1}{B(\alpha, \beta)} p^{\alpha-1} (1-p)^{\beta-1}, \quad 0 \leq p \leq 1$$

We find $f(p|x) \sim \text{Beta}(x + \alpha, n + \beta - x)$, so the posterior mean is;

$$\begin{aligned} p_n(x) &= E(p|x) = \frac{x + \alpha}{\alpha + \beta + n} \\ p_n(x) &= \bar{p} \left(\frac{x + \alpha}{\alpha + n \bar{p}} \right) \end{aligned}$$

Hence,

$$\gamma(n, c) = \frac{1}{B(\alpha, \beta)} \sum_{x=0}^c C_x^n [pr B(x + \alpha, n + \beta - x) - B(x + \alpha + 1, n + \beta - x)]$$

From applying $\Delta_n \gamma(n, c)$, $\Delta_n \gamma(n-1, c)$, we find the optimal value of $C = (C^*)$ acceptance number, which is the value that satisfy;

$$(\alpha + \beta + n)pr - (\alpha + 1) < C^* \leq (\alpha + \beta + n)pr - \alpha \quad (20)$$

And it depend on the estimated values of parameters (α, β) and also on the value of sample size which is obtained from minimizing;

$$R[f(p), n, c] = An + BN + C(N - n) \frac{1}{n} \quad (21)$$

With respect to (n) , where;

$$A = [(C_2 - C_4)\bar{p} + (C_1 - C_3) + (C_4 - C_6) \int_0^{pr} (p - pr)f(p)dp]$$

$$B = C_4\bar{p} + C_3 + (C_6 - C_4) \int_0^{pr} (p - pr)f(p)dp$$

$$C = \frac{1}{2} (C_3 - C_5)(1 - pr)f(p = pr)$$

The result of n^* now is;

$$n^* = \begin{cases} N & A \leq 0 \\ \left\lceil \frac{CN}{A} \right\rceil^{1/2} & A > 0 \end{cases} \quad (22)$$

Where A depend on parameters of quality control cost and on $f(p)$, so for Beta – Binomial distribution, the values of n^* is simplified to:

$$n^* = \left[\frac{(C_3 - C_5)p_r^{\alpha-1}(1-pr)^{\beta} N}{2B(\alpha, \beta)[(C_2 - C_4)\bar{p} + (C_1 - C_3) + (C_4 - C_6)(\bar{p}IB_{pr}(\alpha+1, \beta) - prIB_{pr}(\alpha, \beta))]} \right]^{1/2} \quad (23)$$

Where;

$$IB_{pr}(\alpha, \beta) = \sum_{x=\alpha}^{\alpha+\beta-1} C_x^{\alpha+\beta-1} pr^x (1-pr)^{\alpha+\beta-x-1}$$

$$IB_{pr}(\alpha, \beta) = E(\alpha, \alpha + \beta - 1, pr)$$

$$IB_{pr}(\alpha + 1, \beta) = E(\alpha + 1, \alpha + \beta, pr)$$

$$\therefore n^* = \left[\frac{(C_3 - C_5)p_r^{\alpha-1}(1-pr)^{\beta} N}{2B(\alpha, \beta)[(C_2 - C_4)\bar{p} + (C_1 - C_3) + (C_4 - C_6)(\bar{p}E(\alpha + 1, \alpha + \beta, pr) - prE(\alpha, \alpha + \beta - 1, pr))]} \right]^{1/2} \quad (24)$$

The estimated values of (α, β) using moment's method are;

$$\hat{\alpha} = \bar{x}_p (\bar{x}_p \bar{x}_q - S_p^2) / S_p^2$$

$$\hat{\beta} = (1 - \bar{x}_p) (\bar{x}_p \bar{x}_q - S_p^2) / S_p^2 \quad (25)$$

Where,

$$\bar{p} = \frac{\alpha}{\alpha + \beta}$$

$$\sigma^2 = \frac{\bar{p}(1 - \bar{p})}{\alpha + \beta + 1}$$

4. Application

The following data represents the distribution of percentage of defectives for (100) lots taken from Iraqi general company for producing liquid oils, after tests it found to be Beta distribution (α, β)

The last three classes were grouped since its observed frequencies is less than 5, also we compute the mean of percentage of defectives $\bar{x}_p = 0.0042625$ and average lot $N = 247328$ units, the variance of percentage of defectives $S_p^2 = 0.0000079613$.

From equation (25), the estimated values of parameters α, β by method of moments are:

$$\hat{\alpha} = 2.268248, \quad \hat{\beta} = 529.872768$$

$$\chi_{cal}^2 = \sum_{i=1}^m \frac{(O_i - E_i)^2}{E_i}$$

The value of calculated $(\chi_{cal}^2 = 0.762854)$ is compared at $(\alpha = 0.05)$ and degree of freedom $(k = m - p - 1 = 9 - 3 = 6)$, $(\chi_{6,0.05}^2 = 12.6)$ we find $\chi_{cal}^2 < \chi_{tab}^2$ for this reason the hypothesis:

$$H_0: p_i \sim \text{Beta}(\alpha, \beta)$$

$$H_1: p_i \not\sim \text{Beta}(\alpha, \beta)$$

is accepted

Therefore, we depend on prior Beta distribution at estimated parameters;

$$\hat{\alpha} = 2.268248, \quad \hat{\beta} = 529.872768$$

In finding the parameter of single Bayesian sampling plan for the system *AOQL* where;

$$\bar{p} = E(p) = \frac{\hat{\alpha}}{\hat{\alpha} + \hat{\beta}} = 0.0042625$$

Also, we have obtained the estimated parameter of total cost function, for the lots of product (from state company vegetable oil), these are;

$$C_1 = 0.009 \text{ ID per unit}$$

$$C_2 = 0.260 \text{ ID per unit}$$

$$C_3 = 0.021 \text{ ID per unit}$$

$$C_4 = 0.382 \text{ ID per unit}$$

$$C_5 = 0.00 \text{ ID per unit}$$

$$C_6 = 0.742 \text{ ID per unit}$$

$$\text{product risk } \alpha_0 = 0.05, \quad \text{consumer's risk } \beta_0 = 0.10$$

$$p_0 = \bar{p} = 0.0042625, \quad p_1 = 0.05$$

Also, we find;

$$\int_0^{pr} (pr - p)f(p)dp = prIB_{pr}(\alpha, \beta) - \bar{p}IB_{pr}(\alpha + 1, \beta)$$

$$= (0.0187)(0.9374) - (0.00426)(0.9532) = 0.013468$$

$$\bar{x}_p = 0.0042625, \text{ is the estimated value of percentage of defective in normal condition which is also denoted by } (p_0 = 0.0042625), \text{ while } (p_0) \text{ is the percentage of defective when the conditions of production is not normal.}$$

$$p_1 = 0.05$$

$$\alpha_0 = 0.05 \text{ producer risk}$$

$$\beta_1 = 0.10 \text{ consumer risk}$$

From equation (20) and (24) by using the estimated values of $(\hat{\alpha}, \hat{\beta})$ and $(C_1, C_2, C_3, C_4, C_5, C_6)$, also for different values of process average (\bar{p}) , the results for different size of lot N are tabulated in table (2).

After testing the distribution of quality in table (1), and estimating parameters, and also computing different parameters of total cost model, we find different values of

Bayesian single sampling plan (n, c) and the results are explained in table (2).

The above table represents the results of sampling plan (n, c) according to quality level $\bar{x}_p = 0.001(0.001)0.005$ and lot size $N = 1000(1000)25000$, for equation (24) and (20), while for given data in research, we find that the proposed sampling plan when $N = 16818$ units, and $\bar{x}_p = 0.0042625$ and $pr = 0.0187$ is $(n, c) = (1295, 25)$ and also we compute $R[f(p)n, c]$ from its results found to be equal to (71.720 \$)

Table 1. Distribution of quality process for 100 lots

Percent of defective	Number of lots (O_i)	Expected frequency	$(O_i - E_i)^2/E_i$
0.00 - 0.103	7	8.3	0.203614
0.103 - 0.206	13	12.05	0.074896
0.206 - 0.309	21	20.71	0.004061
0.309 - 0.412	19	17.02	0.230430
0.412 - 0.505	9	9.30	0.009677
0.505 - 0.608	9	9.70	0.050515
0.608 - 0.711	5	5.01	0.000020
0.711 - 0.804	7	8.22	0.181070
0.804 - 0.907	3	10.71	0.008661
0.907 - 1.010	3		
1.010 - 1.113	2		
1.113 - 1.216	2	10.71	
Total	100	100.02	0.762854

Table 2. Bayesian sampling plans for $AOQL$ under Beta prior and due to decision making proposed model

Lot size	Quality level process									
	0.001		0.002		0.003		0.004		0.005	
	n	c	n	c	n	c	n	c	n	c
1000	89	2	143	3	175	4	189	5	225	4
2000	155	3	289	5	297	4	297	6	364	7
3000	247	5	327	6	327	5	395	7	415	9
4000	317	6	425	7	455	6	502	8	528	12
5000	389	7	513	8	575	8	593	10	610	13
6000	425	8	587	9	618	9	599	11	726	14
7000	526	10	669	10	695	10	778	14	809	15
8000	563	11	710	11	730	12	847	15	886	16
9000	612	12	763	13	850	13	886	15	973	17
10000	689	13	801	15	920	15	969	16	1013	18
11000	710	13	860	16	996	16	1040	17	1080	20
12000	763	14	895	18	1010	18	1131	18	1143	22
13000	802	15	926	18	1078	18	1189	19	1165	23
14000	846	16	957	19	1119	19	1212	20	1298	24
15000	885	17	992	20	1172	21	1265	21	1313	26
16000	912	17	946	20	1214	21	1283	22	1364	28
17000	943	18	1098	21	1206	22	1298	24	1385	29
18000	978	18	1109	22	1296	23	1310	26	1403	30
19000	1016	19	1155	23	1320	24	1364	27	1461	32
20000	1047	20	1179	23	1378	25	1393	27	1489	33
21000	1096	20	1219	24	1395	26	1402	28	1505	34
22000	1120	21	1269	25	1410	27	1458	29	1561	34
23000	1165	22	1302	26	1456	28	1506	30	1599	35
24000	1194	22	1338	26	1512	29	1585	32	1630	36
25000	1225	23	1396	27	1566	30	1613	34	1689	36

5. Conclusions

(1) The distribution of quality of 100 lots, found Beta prior with ($\hat{\alpha} = 2.268248, \hat{\beta} = 529.872768$).

(2) The calculated value of $\chi^2_{cal} = 0.762854$ compared with $\chi^2_{tab,6,0.05} = 12.6$ and for $\alpha_0 = 0.05$, degree of freedom ($k = 9 - 3 = 6$), since the last class of distribution of quality where grouped, it is found that $\chi^2_{cal} < \chi^2_{tab}$ for this reason, the hypothesis

$$H_0: p_i \sim \text{Beta}(\alpha, \beta)$$

$H_1: p_i \neq \text{Beta}(\alpha, \beta)$ is accepted

(3) The value ($\alpha_0 = 0.05$) is producer's risk and ($\beta_1 = 0.10$) is consumer risk, where considered in designing the various sampling plans, and we can use any set of (α_0, β_1).

(4) From table (2) we find the sample size (n) is increasing when the percentage of defective is increased, which is logical result.

(5) The proposed model can be applied to another kind of Bayesian sampling which have Gamma prior, and log – Normal, since it is a general model.

(6) From the data, we found that $\bar{x}_p = 0.0042625$, which is the estimated value of percentage of defectives in the normal condition of production process, but its true value found ($\bar{x}_p = 0.08$) which is greater than proposed ($AOQL = 5\%$) for the factory, this results is logical also because of complex and bad conditions of process of production, we look at the percentage (0.08) is very good now.

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