

The New Fundamental Equation of Turbulent Phase Transition

Asya S. Skal

Post Box 1836, Ariel 44837, Israel

Abstract Coupling the Navier-Stokes (NS) and the diffusion of momentum equations we have obtained the new fundamental flow equation that is mathematically equivalent to conductivity in the weak and strong magnetic fields, wherein a magnetic field corresponds to a vorticity field and the electric current density corresponds to the mass flux (momentum per unit volume). We have used the Helmholtz technique to separate time and spatial variables. The new fundamental spatial equation of current (fluid) motions, $j(r) = \sigma(r) \nabla \phi(r) + \sigma(r) \mathbf{R}(r) \mathbf{H} \times \mathbf{j}(r) + \sigma(r) \mathbf{R}(r) (j(r) \cdot \mathbf{H}) \text{sign}(\nabla(\mathbf{H} \cdot \mathbf{e}_j)) \mathbf{e}_j$ includes two non-linear terms of NS equations, which can be solved by the Green function technique.

Keywords Spatial Navier Stokes Equation, Time-Independent Diffusion of Momentum Equations, Time Dependent Part of Diffusion Equation, Two Hall Coefficients

1. Introduction

Hydrodynamics equations are often closely analogous to the electrodynamics equations. The part of this analogy is also equivalent to the elasticity (Lame) equation and the Hall equation of conductivity in a weak magnetic field, which were carried out analytically and numerically using the Green function technique in our previous papers (Skal, 1995, 2002). We will extend this analogy to an NS equation. More precisely, the main purpose of this paper is to show the equivalence between the NS equation and conductivity in a strong magnetic field, which is responsible for turbulent phase transition in electro-, hydro- and aerodynamics.

The new physical laws of behavior in a strong magnetic field stagnated for almost 200 years in the Navier-Stokes (NS) equation. To solve the NS equations, Reynolds () constructed time-averaged equations. The basic tool required for derivation of these equations from instantaneous NS equation is separation of the flow variable (like velocity u) into the mean time averaged component and the fluctuating component. First of all we construct the new fundamental equation, not directly from NS equation as did Reynolds, but rather coupling it with the momentum diffusion equation, which satisfies Newton's third law. Secondly, we used macroscopic variables as one of averaging procedures of the instantaneous NS equation. Instead of the time averaged component and the fluctuating component, we have used Helmholtz's method of the separation of variables into

time and spatial-dependent parts of the equation, which in the end leads to similarity between hydrodynamic and electrodynamic equations of motion.

2. Averaging Instantaneous Conservation of Momentum Equation by the Macroscopic Variables Similar to Elec- Trodynamics Variables

The conservation of momentum equation for incompressible Newtonian viscous and chemically inactive fluids of constant density and temperature in the entire region of Reynolds numbers are presented by (Feynman 1966) in the form

$$\rho \partial u / \partial t + \nabla(\rho^2 u) / (2\rho) + \Psi \times (\rho u) = \nu \nabla^2(\rho u) - \nabla[p + \rho \phi] \quad (1a)$$

$$\Psi = \nabla \times u; \nabla \cdot \Psi = 0; \quad (1b)$$

where u [L/T] is the local velocity, Ψ [1/T] is the local vorticity, ρ is the momentum density, p [ML⁻¹T⁻²] is pressure and ϕ [L²T⁻²] is the gravity potential. ν [L²/T] and ρ [M/L³] are kinematical viscosity and mass density, respectively. In analogy to an electrical field, $E_{el} = -\nabla \phi$ one can create a hydraulic field as $E_{hyd} = -\nabla \phi$ where $-\phi$ is a hydraulic potential, that satisfies an equation

$$\phi = p / \gamma + \phi / g \quad (2)$$

where $\gamma = g\rho$ is specific weight and g is acceleration due to gravity [L/T²] and ϕ [L²T⁻²] is the gravity potential.

Introducing the macroscopic momentum density $q(r) = \rho u(r)$ similar to current density $j = enu(r)$ the conservation of momentum equations obtains the form:

* Corresponding author:

asyaskal@yahoo.com. (Asya S. Skal)

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$$\partial q / \partial t + \Theta \times q / \rho + \nabla q^2 / (2\rho) = -\gamma \nabla \varphi + \nu \nabla^2 q \quad (3a)$$

$$\Theta = \nabla \times q; \nabla \cdot \Theta = 0; \quad (3b)$$

Where $\Theta = \rho_*$ is the macroscopic vorticity.

3. Helmholtz's Method of Separation of Variables into Time and Spatial-Dependent Parts

Helmholtz (Moon and Spencer, 1988) separates variables for the wave, momentum diffusion and

Schrodinger equations using orthogonal coordinate systems in Euclidean 3-space, (skew coordinate systems do not allow separation of variables; however, for engineering problems this situation is handled without difficulty).

3.1. Separation of the Time and Spatial Variables for Diffusion of Momentum

$\Phi(r, t)$. Separation of the variables begins by assuming that function $\Phi(r, t)$ is in fact separable

$$\Phi(r, t) = q(r)T(t) \quad (4)$$

where $q(r)$ is spatial mass flux and $T(t)$ is a time depended solution of equation

$$\nabla^2 \Phi(r, t) = \frac{\partial \Phi(r, t)}{\alpha^2 \partial t} \quad (5)$$

Substituting the equation (4) into Equation (5), and simplifying we obtain

$$T(t) \nabla^2 (q(r)) = q(r) \frac{\partial T(t)}{\alpha^2 \partial t} \quad (6)$$

Or

$$\frac{dT(t)}{\alpha^2 T(t) dt} = \frac{\nabla^2 q(r)}{q(r)} = \beta^2 \quad (7)$$

Since the right side of the equation depends on r , the lefthand expression depends only on t , this equation is valid

in general if and only if both sides of the equation are equal to a constant value. As a result we obtain two equations: one for $q(r)$, the other for $T(t)$, where β^2 is the separation constant. Therefore the spacial part or timeindependent diffusion equation can be presented as:

$$\nabla^2 q(r) - q(r) / k(r) = 0 \quad (8)$$

where $1/k(r) = \alpha^2 \beta^2$ has dimension $1/L^2$.

3.2. Separation of the Variables for the Conservation of Momentum Equation

Unfortunately we cannot employ Helmholtz separation of variable to conservation of momentum equation because of it non-linearity. Therefore we make the second assumption that the spatial part of conservation of the momentum equation can be obtained by omitting only the time-depended term $\partial q / \partial t$.

How valid Helmholtz's and our assumptions of the separation of variable for creation the new fundamental equation are can be ascertained by comparing our solution with experimental data in our next paper. We admit that except for these two assumptions, all other following results will be

absolutely mathematically strong.

4. New Spatial Fundamental Equation for Current Density and Fluid Flows

Substitution of Equation (8) into space-depended part of NS Equation (3a), yields

$$q(r) = \frac{k(r)\gamma}{\nu} \nabla \varphi(r) + \frac{k(r)}{\nu \rho} \Theta(r) \times q(r) + \frac{k(r)}{2\nu \rho} \nabla q^2(r) \quad (9)$$

By introducing two kinetic coefficients of: hydraulic conductivity $Q(r) = \frac{k(r)\gamma}{\nu}$ and the hydraulic Hall coefficient $R_{hyd}(r) = 1 / \gamma \rho(r)$ the equation can be rewritten as:

$$q(r) = Q(r) \nabla \varphi(r) + Q(r) R(r) \Theta(r) \times q(r) + Q(r) R(r) \nabla q^2(r) / 2 \quad (10)$$

The new fundamental spatial hydrodynamic equation describes entire range of Reynolds numbers with the

same two non-linear terms as the NS equation. Using our analogy wherein a magnetic field H corresponds to a vorticity field Θ and the electric current density $j(r)$ corresponds to the mass flux $q(r)$ (momentum per unit volume), hydraulic conductivity $Q(r)$ corresponds to electric conductivity $\sigma(r)$, the equation can be rewritten for electrodynamics as:

$$j(r) = \sigma(r) \nabla \varphi(r) + \sigma(r) R(r) H \times q(r) + \sigma(r) R(r) \nabla j^2(r) / 2 \quad (11)$$

To understand the different roles of each of the non-linear terms let us consider a region of a small velocity where the second non-linear term in right-hand part of Equation (10) may be dropped; the equation becomes:

$$q(r) = Q(r) \nabla \varphi(r) + Q(r) R(r) \Theta(r) \times q(r) \quad (12)$$

This equation with the first non-linear term is identical to the well known electrodynamic Hall equation or current density in a weak magnetic field:

$$j(r) = \sigma(r) \nabla \varphi(r) + \sigma(r) R(r) H \times j(r) \quad (13)$$

Therefore, only the second nonlinear (Bernoulli) term is responsible for the region of a strong magnetic field. Understanding the mystery of turbulence lies in understanding the physical meaning of this term. So, we need to interpret the nonlinear Bernoulli term and make a new presentation.

While the first nonlinear term corresponds to the Hall effect, the Bernoulli term corresponds to the driving force, which is the gradient of a kinetic energy. Let us introduce a more common acceleration field containing both derivations of velocity field: gradient of a velocity and rotor of velocity $\Theta_1(r) = \nabla \times q(r) + \nabla q(r)$, where the usual vorticity field, which correspond to the rotor of velocity is part of it. Using identity $\Theta_1(r) \times q(r) = \Theta(r) \times q(r) + \nabla q(r) \times q(r) = \Theta(r) \times q(r)$ we can use the same symbol $\Theta(r)$ as a vorticity field. Now using identity $\nabla q^2(r) = q(r) \nabla q(r) = (q(r) \cdot \Theta(r)) \sin g(\nabla(\Theta(r) e_q)) e_q$ we can present the Bernoulli term as $Q(r) R(r) (q(r) \cdot \Theta(r)) \sin g(\nabla(\Theta(r) e_q)) e_q$ where e_q is a unite vector in a flow direction. To explain appearance of the function $\sin g(\nabla(\Theta(r) e_q)) e_q$, which may equal +1, 0 or -1, let us consider the formula of a velocity $v = v_0 + at$, where a is an

acceleration. From the latter formula follows that the former formula if $a > 0$ gives +1, if $a = 0$ gives 0 and if $a < 0$ gives -1. As a result one obtains

$$q(r) = Q(r) \nabla \phi(r) + Q(r) \mathbf{R}(r) \Theta \times q(r) + Q(r) \mathbf{R}(r) (q(r) \Theta) \text{sign}(\nabla(\Theta \cdot e_q)) e_q \quad (14)$$

where e_q is a unity vector in the current flow direction. Using the analogy of an acceleration field and magnetic field, one can obtain

$$j(r) = \sigma(r) \nabla \phi(r) + \sigma(r) \mathbf{R}(r) \mathbf{H} \times \mathbf{j}(r) + \sigma(r) \mathbf{R}(r) (\mathbf{j}(r) \cdot \mathbf{H}) \text{sign}(\nabla(\mathbf{H} \cdot e_j)) e_j \quad (15)$$

where e_j is a unity vector in the current flow direction. This equation shows that turbulent phase transition is the Hall effect with the two Hall coefficients but exceedingly much greater than usual. Batchelor (2001) used opposite analogy: an electric current $\mathbf{j} \rightarrow$ a vorticity Θ , a magnetic field $\mathbf{H} \rightarrow$ a velocity field \mathbf{u} , that leads to the presentation of velocity field as an irrotational solenoidal vector field. He explained that just as the electric current produces a distribution of a magnetic field, so vorticity produces the velocity distribution in the surrounding fluid. We need to admit that neither our analogy nor Batchelor's has no relevance with a real magnetic field that is expelled from a region of a vorticity field because it just a mathematical analogy.

5. Conclusions

Using the Navier Stokes equation we have constructed two identical fundamental equations (Equations 14 and 15) of

turbulent phase transition in nature.

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REFERENCES

- [1] Batchelor GK (2001). 'An introduction to fluid dynamics, Cambridge University press. 5
- [2] Feynman RP, Leighton R.B, Sands M (1966). 'Lectures on Physics', New York
- [3] Moon P., Spencer DE (1988). 'Field theory', Handbook, 2nd ed. New York: Springer-Verland
- [4] Skal AS (2002). 'Equivalence between fundamental equations of elasticity and conductivity in a magnetic field', Mathematical, Physical and Engineering Science 458: 2099-2117
- [5] Skal AS (1995). 'Conductivity in a magnetic field and elasticity of disordered systems', Solid St. Communication. 96: 411-415