American Journal of Intelligent Systems 2016, 6(1): 22-29
DOI: 10.5923/j.ajis.20160601.03

# A Hybrid High Order Fuzzy Time Series Forecasting Approach Based on PSO and ANNs Methods

**Erol Eğrioglu[1,*], Cagdas Hakan Aladag[2], Ufuk Yolcu[3], Ali Zafer Dalar[1]**

[1]Department of Statistics, Giresun University, Giresun, Turkey
[2]Department of Statistics, Hacettepe University, Ankara, Turkey
[3]Department of Statistics, Ankara University, Ankara, Turkey

**Abstract** Fuzzy time series approaches usually consist of three phases such as fuzzification, defining fuzzy relationships, and defuzzification. Depending on the forecasting model order, the method used to define fuzzy relation changes. Using methods such as genetic algorithms, fuzzy c-means or particle swarm optimization in the fuzzification step improves the forecasting performance. In recent studies, it has been observed that utilizing feed forward neural networks simplifies calculations and increases the forecasting accuracy. A new fuzzy time series approach is proposed to analyze high order fuzzy time series forecasting models in this paper. The proposed method is a hybrid method in which particle swarm optimization and feed forward neural networks are employed in the fuzzification and in the defining fuzzy relationships steps, respectively. To investigate the performance of the proposed hybrid fuzzy time series forecasting method, the method was applied to a real-world time series which is the data of index 100 in Istanbul Stock Exchange (ISE). In order to study the usefulness of the proposed method, its forecasting accuracy is compared with that of some other fuzzy time series approaches available in the literature. The application results accentuate the superiority of the proposed method over the other approaches for ISE data.

**Keywords** Feed forward neural networks, Forecasting, Fuzzy time series, High order model, Hybrid approach, Particle swarm optimization, Stock prices

## 1. Introduction

Time series forecasting is a classic prediction problem which is getting more and more attention from both academic and industry communities. In the literature, various techniques have been utilized to solve this problem. Forecasting approaches can be divided into four groups which are probability theory-based conventional methods [1], computational methods such as neural networks [2], fuzzy theory-based methods such as fuzzy time series [3] and hybrid forecasting techniques [4]. It is well-known fact that using a hybrid forecasting approach can produce better forecasts since a hybrid approach includes advantages of different methods used in this hybrid approach. Therefore, there have been many studies in which hybrid approaches are proposed for time series forecasting in order to reach high accuracy level [5-13].

Fuzzy time series approach based on fuzzy set theory was introduced as an alternative method for conventional forecasting time series models [3]. In recent years, fuzzy time series approaches for forecasting problems have drawn a great amount of attention [7]. Fuzzy time series forecasting models can be categorized into two subclasses which are the first order and the high order models [14]. Although it is easy to use the first order models, this type of models is inadequate to analyze real world time series since relations between high order fuzzy lagged variables are not included in a first order model. Despite of the fact that modeling capability of the high order models is better than this of the first order models, the main drawback of using the high order model is that defining fuzzy relations becomes very hard when this type of models are being used. To analyze the high order models Chen [15] proposed a method which utilizes fuzzy logic group relation tables to determine the fuzzy relations. The method proposed by Chen [15] is also the improved version of the method suggested by Chen [16]. Afterwards, Aladag et al. [17] proposed a high order fuzzy time series model in which artificial neural networks are employed to establish fuzzy relations.

Fuzzy time series method is composed of three main stages; these are fuzzification, determining fuzzy relations between observation, and defuzzification. Like other phases, fuzzification stage also plays an important role in forecasting performance of fuzzy time series. Decomposition of universe of discourse according to a specified length of interval is a favorite method for the fuzzification step. The length of interval which is required for partitioning the universe of discourse has an important impact on the forecasting results [18]. Huarng [18] suggested mean based and distribution

* Corresponding author:
erole1977@yahoo.com (Erol Eğrioglu)
Published online at http://journal.sapub.org/ajis

based methods to determine the length of interval. Egrioglu et al. [19, 20] calculated length of intervals in first order and high order models by using single variable constrained optimization. Huarng and Yu [21] proposed a method in which the length of the interval is not fixed and is exponentially increased with a ratio. That is, they partitioned the universe of discourse by using dynamic length of interval instead of fixed one. Yolcu et al. [22] improved the method proposed by Huarng and Yu [21]. They determined the ratio used in Huarng and Yu's [21] method by using optimization. Afterwards, some studies were inspired by the idea of using dynamic interval length. Davari et al. [23], Kuo et al. [24, 25], and Park et al. [26] utilized particle swarm optimization (PSO) to determine dynamic interval lengths. Bas et al. [27] utilized differential evolution algorithm to determine dynamic interval lengths. Uslu et al. [28] and Bas et al. [29] proposed two approaches based on weights using the number of recurrences of fuzzy relations. And also Uslu et al. [30] proposed an approach based on weights determined chronologically. It has been observed that using PSO method improves forecasting accuracy. Moreover, in fuzzification stage, there are some studies in the literature that have used fuzzy clustering techniques [31, 11].

In this study, a new hybrid high order fuzzy time series forecasting approach is proposed. In the proposed hybrid method, PSO method is used for fuzzification and artificial neural networks (ANNs) are utilized to define fuzzy relations. In the literature, the proposed method is the first fuzzy time series method in which PSO and ANNs are used together. Also, the proposed hybrid method provides some advantages given below.

- In the fuzzification step, determination on length of interval is not subjective since PSO is employed to calculate interval lengths. It prevents arbitrarily determination of the interval length so that the proposed method works in a systematic way.
- Instead of using a fixed length of interval, dynamic interval lengths are employed. Therefore, the data can be modeled better.
- The proposed method does not require constructing complex fuzzy logic group relation tables to define fuzzy relations since ANNs method is utilized to determine these relations. This substantially reduces the computational complexity of the method since it is easy to use ANNs approach.
- The proposed hybrid method can benefit from capabilities of methods PSO and ANNs to analyze high order fuzzy time series forecasting models. For example, the proposed method has the ability of flexible modeling which is also included by ANNs.
- By virtue of the advantages mentioned above, it is expected that the proposed hybrid high order fuzzy time series forecasting method has high forecasting accuracy.

The basic definitions on fuzzy time series are given in the next section. The method proposed in this paper is introduced step by step in Section 3. Section 4 covers the comparative applications of all methods mentioned in this paper. The final section is about a conclusion driven from the whole study.

# 2. Basic Fuzzy Time Series Definitions

Fuzzy time series was firstly put forward by Song and Chissom [32-34]. Fuzzy time series can be shortly defined as time series whose observations are fuzzy sets. General definitions of fuzzy time series are given as follows:

Let $U$ be the universe of discourse, where $U = \{u_1, u_2, \cdots, u_b\}$. A fuzzy set $A_i$ of $U$ is defined as $A_i = f_{A_i}(u_1)/(u_1) + f_{A_i}(u_2)/(u_2) + \cdots + f_{A_i}(u_b)/(u_b)$, where $f_{A_i}$ is the membership function of the fuzzy set $A_i; f_{A_i} : U \to [0,1]$. $u_a$ is a generic element of fuzzy set $A_i; f_{A_i}(u_a)$ is the degree of belongingness of $u_a$ to $A_i; f_{A_i}(u_a) \in [0,1]$ and $1 \le a \le b$.

**Definition 1.** Fuzzy time series Let $Y(t)$ ($t$=0, 1, 2, …) a subset of real numbers, be the universe of discourse by which fuzzy sets $f_j(t)$ are defined. If $F(t)$ is a collection of $f_1(t)$, $f_2(t)$, … then $F(t)$ is called a fuzzy time series defined on $Y(t)$.

**Definition 2.** Let $F(t)$ be a fuzzy time series. If $F(t)$ is a caused by $F(t-1)$, then this fuzzy logical relationship is represented by

$$F(t-1) \to F(t) \tag{1}$$

and it is called the first order fuzzy time series forecasting model.

**Definition 3.** Let $F(t)$ be a fuzzy time series. If $F(t)$ is a caused by $F(t-1)$, $F(t-2)$, …, $F(t-m)$, then this fuzzy logical relationship is represented by

$$F(t-1), F(t-2), \ldots, F(t-m) \to F(t) \tag{2}$$

and it is called the $m^{\text{th}}$ order fuzzy time series forecasting model.

# 3. The Proposed Hybrid Forecasting Method

In the literature, it has been seen that utilizing artificial intelligent techniques in various steps of fuzzy time series approach can provide three important advantages. First of all, the approach can work more systematically since arbitrary decisions are avoided. Secondly, using artificial intelligent methods makes the usage of fuzzy time series approach easier since computational complexity of the method is reduced. Finally, in many applications, it was observed that utilizing artificial intelligent techniques considerably increase the forecasting performance of fuzzy time series. Therefore, using hybrid methods combine fuzzy time series and artificial intelligent techniques have been preferred in recent studies. For example, Chen and Chung [35] and Lee et al. [36] utilized genetic algorithms in fuzzification step.

Davari et al. [23], Kuo et al. [24, 25], and Park et al. [26] employed PSO method to fuzzify observations. Cheng et al. [37] and Li et al. [38] used fuzzy c-means method in fuzzification step.

Particle swarm optimization, which is a population based heuristic algorithm, was firstly proposed by Kennedy & Eberhart [39]. Distinguishing feature of this heuristic algorithm is that it simultaneously examines different points in different regions of the solution space to obtain the global optimum solution. Local optimum traps can be avoided because of this feature of the method [40]. In the fuzzy time series literature, there are some studies in which PSO method was used for fuzzification. Davari et al. [23] employed PSO method for fuzzification step of first order fuzzy time series. Kuo et al. [24] utilized this method for both first and high order fuzzy time series. PSO method was used by Kuo et al. [25] to fuzzify observations in high order models. Park et al. [26] exploited PSO method for fuzzification in two factor high order fuzzy time series. In the proposed hybrid approach, the modified particle swarm optimization (MPSO) method is used for fuzzification. The detailed information about MPSO method can be found in Aladag et al. [6]. The MPSO algorithm has time varying inertia weight like in Shi and Eberhart [41]. In a similar way, this algorithm also has time varying acceleration coefficient like in Ma et al. [42]. In the proposed hybrid forecasting technique, MPSO optimization method is utilized to fuzzify the observations.

ANNs method is an effective inference method which can learn complex relationships between the observations. An important kind of ANNs is feed forward neural networks. In fuzzy time series literature, feed forward neural networks were firstly used by Huarng and Yu [43] for defining fuzzy relations stage in fuzzy time series. Aladag et al. [17] employed feed forward neural networks for this stage in high order fuzzy time series. In Egrioglu et al. [8], feed forward neural networks were utilized to define fuzzy relations in bivariate fuzzy time series. Egrioglu et al. [44] also used this type of neural networks to determine fuzzy relations in multi variable fuzzy time series. Yu and Huarng [45] and Yolcu et al. [46] utilized feed forward neural networks which include more than one neuron in the output layer for determining fuzzy relations. In the proposed approach, feed forward neural networks are used to define fuzzy relations. And also the fuzzy lagged variable selection in fuzzy time series with genetic algorithm was proposed by [47].

As mentioned above, in the proposed hybrid fuzzy time series forecasting method, MPSO and feed forward neural networks methods are used for fuzzification and establishing fuzzy relations, respectively. The proposed hybrid fuzzy time series method is the first fuzzy time series method uses PSO and ANNs approaches together. The algorithm of the proposed approach is given below.

**Algorithm 2:** The proposed method

**Step 1.** Boundaries of universe of discourse *min* and *max*, parameters of PSO method such as $w_1$, $w_2$, $c_{1i}$, $c_{2i}$, $c_{1f}$, and $c_{2f}$. The number of particles *pn*, and maximum number of iterations *tmax* are specified.

**Step 2.** For variables, which will be optimized with PSO, starting positions and velocities are determined. Positions and velocities of particles are given in formula (3) and (4).

Universe of discourse and its partition according to variables $(x_1, x_2, \cdots, x_{N-1})$ which will be optimized with PSO are as follows

$$U = [\min, \max], u_1 = [\min, x_1], u_2 = [x_1, x_2],$$
$$u_3 = [x_2, x_3], \cdots, u_{N-1} = [x_{N-1}, \max]$$

where $U$ is the universe of discourse, *min* and *max* are maximum and minimum values of time series, respectively, $x_1, x_2, \cdots, x_{N-1}$ are positions of particles, and $N$ represents the number of sub intervals. For instance, let $N = 4$, and the number of particles $m$ equals to 4. In this case, particles and velocities are like in Table 1.

**Table 1.** Presentations of Positions and Velocities

| Positions | | | |
|---|---|---|---|
| Particle 1 | $x_{11}$ | $x_{21}$ | $x_{31}$ |
| Particle 2 | $x_{12}$ | $x_{22}$ | $x_{32}$ |
| Particle 3 | $x_{13}$ | $x_{23}$ | $x_{33}$ |
| Particle 4 | $x_{14}$ | $x_{24}$ | $x_{34}$ |
| Velocities | | | |
| Particle 1 | $v_{11}$ | $v_{21}$ | $v_{31}$ |
| Particle 2 | $v_{12}$ | $v_{22}$ | $v_{32}$ |
| Particle 3 | $v_{13}$ | $v_{23}$ | $v_{33}$ |
| Particle 4 | $v_{14}$ | $v_{24}$ | $v_{34}$ |

$$X_k = \left\{x_1^k, x_2^k, \cdots, x_d^k\right\},\ k = 1, 2, \cdots, pn \qquad (3)$$

$$V_k = \left\{v_1^k, v_2^k, \cdots, v_d^k\right\},\ k = 1, 2, \cdots, pn \qquad (4)$$

**Step 3**. For each particle, Steps 3.1 to 3.3 are performed iteratively. That is, Step 3 is repeated for the number of particles *m*.

***Step 3.1*** Observations of time series are fuzzified according to the current particle.

For instance, partition of universe of discourse for Particle 1 given in Table 1 is as follows:

$$u_1 = [\min, x_{11}], u_2 = [x_{11}, x_{21}], u_3 = [x_{21}, x_{31}],\ u_4 = [x_{31}, \max]$$

According to this, fuzzy sets can be defined as follows:

$$A_1 = 1/u_1 + 0.5/u_2 + 0/u_3 + 0/u_4$$
$$A_2 = 0.5/u_1 + 1/u_2 + 0.5/u_3 + 0/u_4$$
$$A_3 = 0/u_1 + 0.5/u_2 + 1/u_3 + 0.5/u_4$$
$$A_4 = 0/u_1 + 0/u_2 + 0.5/u_3 + 1/u_4$$

Fuzzy time series is obtained by mapping each observation of time series into a fuzzy set which has the maximum membership degree for the interval including this observation.

***Step 3.2*** Fuzzy relationships are established with feed forward neural networks.

An example will be given to explain this step more clearly for the second order fuzzy time series. Because of dealing with second order fuzzy time series, two inputs are employed in neural network model, so that lagged variables $F_{t-2}$ and $F_{t-1}$ are obtained from fuzzy time series $F_t$. These series are given in Table 2. The index numbers ($i$) of $A_i$ of $F_{t-2}$ and $F_{t-1}$ series are taken as input values whose titles are Input-1 and Input-2 in Table 2 for the neural network model. Also, the index numbers of $A_i$ of $F_t$ series are taken as target values whose title is Target in Table 2 for the neural network model. When the third observation is taken as an example, inputs values for the learning sample [$A_6$, $A_2$] are 6 and 2. Then, target value for this learning sample is 3.

**Table 2.** Notations for Second Order Fuzzy Time Series

| Observation No | $F_{t-2}$ | $F_{t-1}$ | $F_t$ | Input-1 | Input-2 | Target |
|---|---|---|---|---|---|---|
| 1 | --- | --- | $A_6$ | --- | --- | --- |
| 2 | --- | $A_6$ | $A_2$ | --- | --- | --- |
| 3 | $A_6$ | $A_2$ | $A_3$ | 6 | 2 | 3 |
| 4 | $A_2$ | $A_3$ | $A_7$ | 2 | 3 | 7 |
| 5 | $A_3$ | $A_7$ | $A_4$ | 3 | 7 | 4 |
| 6 | $A_7$ | $A_4$ | $A_2$ | 7 | 4 | 2 |

***Step 3.3*** According to outputs obtained from neural networks, fuzzy and crisp forecasts are obtained. Then, over the test set, mean square error (MSE) is calculated by the formula given below.

$$MSE = \frac{1}{n}\sum_{t=1}^{n}\left(Actual_t - Forecast_t\right)^2 \quad (5)$$

where $Actual_t$ and $Forecast_t$ represent actual and forecasted values, respectively, for $t^{\text{th}}$ observation in the test set. $n$ is the number of observations in the test set.

If the output of a neural network is $i$, fuzzy forecast is $A_i$ and the corresponding crisp forecast is the midpoint of interval $u_i$ with the maximum membership degree in $A_i$.

**Step 4.** According to MSE values calculated in Step 3, *pbest* and *gbest* are computed.

$$pbest_i = \left(p_{i1}, p_{i2}, \cdots, p_{id}\right), i = 1,2,\cdots,d \quad (6)$$

$$pbest_g = gbest = \left(p_{g1}, p_{g2}, \cdots, p_{gd}\right), i = 1,2,\cdots,d \quad (7)$$

Positions and velocities of particles are calculated by using the formulas in (8) and (9), respectively.

$$v_{ij}^{k+1} = \left[w \times v_{ij}^{k} + c_1 \times rand_1 \times \left(pbest_{ij} - x_{ij}\right) + \cdots + c_2 \times rand_2 \times \left(gbest_j - x_{ij}\right)\right] \quad (8)$$

$$x_{ij}^{k+1} = x_{ij} + v_{ij}^{k+1} \quad (9)$$

**Step 5.** If the maximum number of iterations is not reached, the algorithm goes to Step 3. Otherwise, the algorithm goes to the next step.

**Step 6.** *gbest* is obtained as the optimal solution.

# 4. The Application

In order to evaluate the performance of the proposed hybrid fuzzy time series forecasting method, the method was applied to the data of index 100 in Istanbul Stock Exchange (ISE). ISE data includes daily observations was divided into three data sets. Each data set belongs to a different year. The periods, 03/10/2008 - 31/12/2008, 01/10/2009 - 31/12/2009 and 01/10/2010 - 23/12/2010 of ISE were referred as Series 1, 2, and 3, respectively. The proposed method and the other fuzzy time series approaches available in the literature were applied to all three data sets. All the other fuzzy time series approaches are codded in MATLAB package program.

Then, all obtained forecasting results were compared to each other. The graphs of these time series are given in Fig. 1, 2, and 3, respectively.

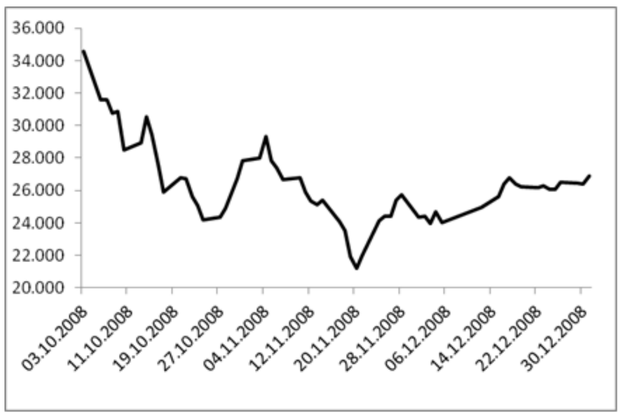


**Figure 1.** The graph of Series 1, the period between 03/10/2008 and 31/12/2008

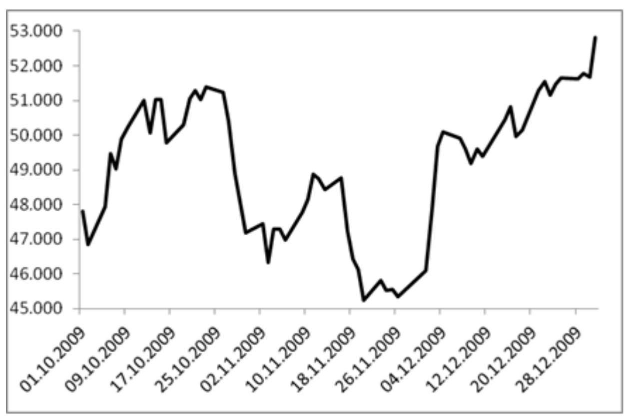


**Figure 2.** The graph of Series 2, the period between 01/10/2009 and 31/12/2009

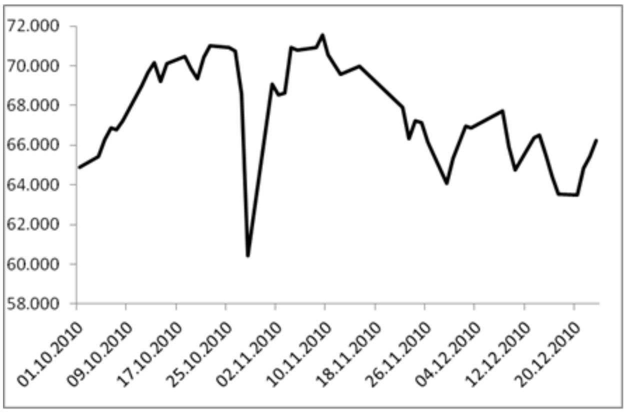


**Figure 3.** The graph of Series 3, the period between 01/10/2010 and 23/12/2009

In the implementation, for all series, the last 7 observations were used for test sets and the rest of the data was used for training. When time series were analyzed, the best models for all methods were tried to be determined by examining different cases. For doing that, the length of interval was changed from 100 to 1500, increment by 100. Also, the number of fuzzy sets was changed between 5 and 15. Thus, for all methods, the best cases which give the best forecasts for the test set were determined. After the best case of each fuzzy time series was determined, forecasts obtained from these models were evaluated. In order to evaluate the forecasting performance of all forecasting fuzzy time series methods, two performance measures, root mean square error (RMSE) and mean absolute percentage error (MAPE), were used. These criteria are calculated over the sets and the related formulas are given below.

$$RMSE = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\left(Actual_t - Forecast_t\right)^2} \quad (10)$$

$$MAPE = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{Actual_t - Forecast_t}{Actual_t}\right| \quad (11)$$

where $Actual_t$ and $Forecast_t$ represent actual and forecasted values, respectively, for $t^{th}$ observation in the test set. And, $n$ is the number of observations in the test set.

- When the number of fuzzy sets is 12, for the method of Song and Chissom [33].
- When the length of interval is 1200 for the method of Chen [16].
- When the length of interval is 800 for the method of Huarng Distribution Based [18].
- When the length of interval is 200 for the method of Huarng Average Based [18].
- When the ratio sample percentile is 0.5 for the method of Huarng and Yu Ratio Based [21].
- When the number of fuzzy sets is 5 for the method of Cheng et al. [37].
- When the number of fuzzy sets is 11 and the number of neurons in hidden layer of neural network, which was used to define fuzzy relations, is 5 for the method of Yolcu et al. [46].
- When the first order model is used and the number of intervals is 15 for the proposed hybrid method.

For Series 1, forecasts produced by all fuzzy time series methods and the proposed hybrid method are presented in Table 3. Also, corresponding RMSE and MAPE values are presented in this table. The cases which we got the superlative results given in Table 3:

**Table 3.** All obtained results for Series 1

| Date | Test Set | Song-Chissom [33] | Chen [16] | Huarng [18]¹ | Huarng [18]² | Huarng and Yu [21] | Cheng et al. [37] | Yolcu et al. [46] | The Proposed Method |
|---|---|---|---|---|---|---|---|---|---|
| **23.12.2008** | 26294 | 26410 | 26400 | 26200 | 26100 | 26091 | 26390 | 26274 | 26130 |
| **24.12.2008** | 26055 | 26410 | 26400 | 26200 | 26367 | 26091 | 26390 | 26273 | 26130 |
| **25.12.2008** | 26059 | 26410 | 26400 | 26200 | 26100 | 26091 | 26390 | 26339 | 26130 |
| **26.12.2008** | 26499 | 26410 | 26400 | 26200 | 26100 | 26091 | 26390 | 26337 | 26130 |
| **29.12.2008** | 26424 | 26410 | 26400 | 26200 | 26500 | 26608 | 26390 | 26565 | 26545 |
| **30.12.2008** | 26411 | 26410 | 26400 | 26200 | 26500 | 26608 | 26390 | 26429 | 26545 |
| **31.12.2008** | 26864 | 26410 | 26400 | 26200 | 26500 | 26091 | 26390 | 26460 | 26545 |
| | **RMSE** | 261.02 | 259.76 | 310.47 | 251.30 | 354.60 | 258.89 | 219.09 | 209.79 |
| | **MAPE** | 0.0075 | 0.0075 | 0.0096 | 0.0080 | 0.0098 | 0.0076 | 0.0067 | 0.0068 |

[1]Distribution Based Method, [2]Average Based Method

According to Table 3, in terms of RMSE, the most accurate forecasts are obtained when the proposed forecasting technique is used for Series 1. In terms of MAPE, the best methods which give the most accurate results are the method suggested by Yolcu et al. [46] and the method proposed in this study.

**Table 4.** All obtained results for Series 2

| Date | Test Set | Song-Chissom [33] | Chen [16] | Huarng [18][1] | Huarng [18][2] | Huarng and Yu [21] | Cheng et al. [37] | Yolcu et al. [46] | The Proposed Method |
|---|---|---|---|---|---|---|---|---|---|
| 23.12.2009 | 51162 | 51137 | 52150 | 51900 | 51573 | 51033 | 50872 | 51317 | 51781 |
| 24.12.2009 | 51461 | 51137 | 50850 | 50700 | 50373 | 51033 | 50763 | 51317 | 51781 |
| 25.12.2009 | 51661 | 51137 | 50850 | 50700 | 51240 | 51033 | 50763 | 51317 | 51781 |
| 28.12.2009 | 51619 | 51137 | 52150 | 51900 | 51573 | 51033 | 50763 | 51317 | 51157 |
| 29.12.2009 | 51786 | 51137 | 52150 | 51900 | 51573 | 51033 | 50763 | 51317 | 51781 |
| 30.12.2009 | 51668 | 51137 | 52150 | 51900 | 51773 | 52004 | 50763 | 51317 | 51781 |
| 31.12.2009 | 52825 | 51137 | 52150 | 51900 | 51573 | 51033 | 50763 | 51317 | 51781 |
| | RMSE | 771.36 | 666.50 | 659.95 | 671.45 | 830.56 | 1084.16 | 640.48 | 509.36 |
| | MAPE | 0.0116 | 0.0123 | 0.0111 | 0.0097 | 0.0128 | 0.0185 | 0.0090 | 0.0074 |

[1] Distribution based method, [2] Average based method

**Table 5.** All obtained results for Series 3

| Date | Test Set | Song-Chissom [33] | Chen [16] | Huarng [18][1] | Huarng [18][2] | Huarng and Yu [21] | Cheng et al. [37] | Yolcu et al. [46] | The Proposed Method |
|---|---|---|---|---|---|---|---|---|---|
| 15.12.2010 | 65499 | 65355 | 65500 | 66167 | 66500 | 67254 | 65992 | 65999 | 66094 |
| 16.12.2010 | 64429 | 65355 | 65500 | 65500 | 66300 | 66035 | 65992 | 64813 | 65394 |
| 17.12.2010 | 63524 | 65974 | 65500 | 66000 | 64500 | 65435 | 65992 | 64864 | 64513 |
| 20.12.2010 | 63502 | 64736 | 64950 | 63500 | 63500 | 63668 | 65992 | 64871 | 64513 |
| 21.12.2010 | 64820 | 64736 | 64950 | 63500 | 63500 | 63668 | 65992 | 64858 | 64513 |
| 22.12.2010 | 65440 | 65974 | 65500 | 66000 | 65500 | 66038 | 65992 | 65522 | 65394 |
| 23.12.2010 | 66219 | 65355 | 65500 | 65500 | 66300 | 66035 | 65992 | 64813 | 65394 |
| | RMSE | 1161.41 | 1047.84 | 1212.84 | 1014.73 | 1256.83 | 1544.90 | 929.84 | 761.81 |
| | MAPE | 0.0139 | 0.0120 | 0.0151 | 0.0117 | 0.0163 | 0.0200 | 0.0113 | 0.0105 |

[1] Distribution based method, [2] Average based method

For Series 2, forecasting results obtained from all fuzzy time series methods are summarized in Table 4. For this series, the best cases are;

- When the number of fuzzy sets is 9, for the method of Song and Chissom [33].
- When the length of interval is 1300 for the method of Chen [16].
- When the length of interval is 800 for the method of Huarng Distribution Based [18].
- When the length of interval is 200 for the method of Huarng Average Based [18].
- When the ratio sample percentile is 0.5 for the method of Huarng and Yu Ratio Based [21].
- When the number of fuzzy sets is 15 for the method of Cheng et al. [37].
- When the number of fuzzy sets is 13 and the number of neurons in the hidden layer is 7 for the method of Yolcu et al. [46].
- When the third order model is used and the number of intervals is 15 for the proposed hybrid method.

When Table 4 is examined, it is obvious that the proposed method produces the most accurate forecasts for Series 2 in terms of both performance measures RMSE and MAPE. For Series 2, the proposed hybrid forecasting method is superior to the other fuzzy time series approaches.

For Series 3, forecasting results obtained from all fuzzy time series methods are presented in Table 5. For this series, the best cases are;

- When the number of fuzzy sets is 9, for the method of Song and Chissom [33].
- When the length of interval is 1100 for the method of Chen [16].
- When the length of interval is 1000 for the method of Huarng Distribution Based [18].
- When the length of interval is 200 for the method of Huarng Average Based [18].
- When the ratio sample percentile is 0.5 for the method of Huarng and Yu Ratio Based [21].
- When the number of fuzzy sets is 9 for the method of Cheng et al. [37].
- When the number of fuzzy sets is 7 and the number of neurons in the hidden layer is 6 for the method of Yolcu et al. [46].
- When the fourth order model is employed and the number of intervals is 15 for the proposed hybrid method.

According to Table 5, it is clearly seen that the most accurate forecasts are obtained when the proposed approach is utilized in terms of bot criteria RMSE and MAPE. For Series 3, the proposed hybrid forecasting method is superior to the other fuzzy time series approaches.

## 5. Conclusions and Discussion

The most proper approach to analyze time series whose observations include uncertainty is fuzzy time series. During the last ten years, there has been a substantial increase in the interest on fuzzy time series. Specifically, they are good for tasks involving data sets have uncertainty. However, there have been some problems with using this approach. The main drawbacks of this approach are some arbitrary decisions that directly affect the performance of the approach, and complex computations. In this study, to overcome these issues, a novel hybrid high order fuzzy time series approach utilizes artificial intelligent techniques is firstly introduced. In the fuzzification step of the proposed method, PSO is employed to determine the length of interval instead of determining it by an arbitrary decision. Also, in the suggested method, feed forward neural networks are utilized to define fuzzy relations between observations of time series so the method does not require complex fuzzy group relation tables operations. In the literature, the proposed hybrid approach is the first fuzzy time series method in which PSO and ANNs methods are exploited together. In order to show the applicability of the proposed method, it was applied to a real-world time series which is index 100 in stocks and bonds exchange market of İstanbul. Some other fuzzy time series approaches were also applied to the data for the aim of comparison. As a result of the implementation, it was observed that the proposed hybrid forecasting approach produces very promising results. Therefore, the suggested method can be used as a good alternative method to forecast fuzzy time series.

## ACKNOWLEDGEMENTS

This work was in supported by “The Scientific and Technological Research Council of Turkey (TUBITAK)”, Turkey, under project number 210T150.

## REFERENCES

[1] Zhaoa W., Wanga J., Lu, H., 2014, Forecasting monthly electricity consumption in China using a time-varying-weight combining method based on the high-order Markov chain model, Omega, 45, 80-91.

[2] Curry, B., 2004, ‘Simple’ neural networks for forecasting, Omega, 32(2), 97–100.

[3] Aladag, C. H., Egrioglu, E., Yolcu, U., Uslu, V. R., 2014, A high order seasonal fuzzy time series model and application to international tourism demand of Turkey, Journal of intelligent and fuzzy systems, 26(1), 295–302.

[4] Zhua, B., Wei, Y., 2013, Carbon price forecasting with a novel hybrid ARIMA and least squares support vector machines methodology, Omega, 41(3), 517–524.

[5] Aladag, C. H., 2011, A new architecture selection method based on tabu search for artificial neural networks, Expert Systems with Applications, 38(4), 3287–3293.

[6] Aladag, C. H., Egrioglu, E., Yolcu, U., Dalar, A. Z., 2012, A new time invariant fuzzy time series forecasting method based on particle swarm optimization, Applied Soft Computing, 12(10), 3291-3299.

[7] Aladag, C. H., 2013, Using multiplicative neuron model to establish fuzzy logic relationships, Expert Systems with Applications, 40(3), 850-853.

[8] Egrioglu, E., Aladag, C. H., Yolcu, U., Basaran, M. A., Uslu, V. R., 2009, A New Hybrid Approach Based on SARIMA and Partial High Order Bivariate Fuzzy Time Series Forecasting Model, Expert Systems with Applications, 36(4), 7424-7434.

[9] Pai, P-F., Lin, C-S., 2005, A hybrid ARIMA and support vector machines model in stock price forecasting, Omega, 33(6), 497–505.

[10] Wanga, J-J., Wanga, J-Z., Zhang, Z-G., Guo, S-P., 2012, Stock index forecasting based on a hybrid model, Omega, 40(6), 758–766.

[11] Cagcag Yolcu, O., 2013, A Hybrid Fuzzy Time Series Approach Based on Fuzzy Clustering and Artificial Neural Network with Single Multiplicative Neuron Model, Mathematical Problems in Engineering, Volume 2013, Article ID 560472, 9 pages.

[12] Cagcag Yolcu, O., 2014, The Forecasting of Istanbul Stock Exchange by Using a Hybrid Fuzzy Time Series Approach, Turkish Journal of Fuzzy Systems, 5(1), 10-26.

[13] Bas, E., Uslu, V. R., Yolcu, U., Egrioglu, E., 2014, A modified genetic algorithm for fuzzy time series to find the optimal interval lengths, Applied Intelligence, 41(2), 453-463.

[14] Aladag, C. H., Egrioglu, E., 2012, Advanced time series forecasting methods, Advances in time series forecasting, (Editors: Aladag, C. H., and Egrioglu, E.), Bentham Science Publishers Ltd., eISBN: 978-1-60805-373-5, 3-10.

[15] Chen, S. M., 2002, Forecasting enrollments based on high order fuzzy time series, Cybernetics and Systems, 33, 1-16.

[16] Chen, S. M., 1996, Forecasting enrollments based on fuzzy time-series, Fuzzy Sets and Systems, 81, 311-319.

[17] Aladag, C. H., Basaran, M. A., Egrioglu, E., Yolcu, U., Uslu, V. R., 2009, Forecasting in high order fuzzy time series by using neural networks to define fuzzy relations, Expert Systems with Applications, 36(3), 4228-4231.

[18] Huarng, K., 2001, Effective length of intervals to improve forecasting in fuzzy time-series, Fuzzy Sets and Systems, 123(3), 387-394.

[19] Egrioglu, E., Aladag, C. H., Yolcu, U., Uslu, V. R., Basaran, M. A., 2010, Finding an optimal interval length in high order fuzzy time series, Expert Systems with Applications, 37(7), 5052–5055.

[20] Egrioglu, E., Aladag, C. H., Basaran, M. A., Uslu, V. R., Yolcu, U., 2011, A New Approach Based on the Optimization of the Length of Intervals in Fuzzy Time Series, Journal of Intelligent and Fuzzy Systems, 22(1), 15–19.

[21] Huarng, K., Yu, T. H-K., 2006, Ratio-based lengths of intervals to improve fuzzy time series forecasting, IEEE Transactions on Systems, Man, and Cybernetics-Part B: Cybernetics, 36(2), 328-340.

[22] Yolcu, U., Egrioglu, E., Uslu, V. R., Basaran, M. A., Aladag, C. H., 2009, A New Approach for Determining the Length of Intervals for Fuzzy Time Series, Applied Soft Computing, 9(2), 647-651.

[23] Davari, S., Zarandi, M. H. F., Turksen, I. B., 2009, An Improved fuzzy time series forecasting model based on particle swarm intervalization, The 28th North American Fuzzy Information Processing Society Annual Conferences (NAFIPS2009), Cincinnati, Ohio, USA, June 14-17.

[24] Kuo, I-H., Horng, S-J., Kao, T-W., Lin, T-L., Lee, C-L., Pan, Y., 2009, An improved method for forecasting enrollments based on fuzzy time series and particle swarm optimization, Expert Systems with Application, 36(3), Part 2, 6108-6117.

[25] Kuo, I-H., Horng. S-J., Chen, Y-H., Run, R-S., Kao, T-W., Chen, R-J., Lai, J-L., Lin, T-L., 2010, Forecasting TAIFEX based on fuzzy time series and particle swarm optimization, Expert Systems with Application, 37(2), 1494-1502.

[26] Park, J-I., Lee, D-J., Song, C-K., Chun, M-G., 2010, TAIFEX and KOSPI 200 forecasting based on two factors high order fuzzy time series and particle swarm optimization, Expert Systems with Application, 37(2), 959-967.

[27] Bas, E., Uslu, V. R., Yolcu, U., Egrioglu, E., 2014, A fuzzy time series approach using de/best/1 mutation strategy of differential evolution algorithm, Aloy Journal of Soft Computing and Applications, 2(2), 60-69.

[28] Uslu, V. R., Bas, E., Yolcu, U., Egrioglu, E., 2014, A fuzzy time series approach based on weights determined by the number of recurrences of fuzzy relations, Swarm and Evolutionary Computation, 15, 19–26.

[29] Bas, E., Uslu, V. R., Yolcu, U., Egrioglu, E., 2013, A fuzzy time series analysis approach by using differential evolution algorithm based on the number of recurrences of fuzzy relations, American Journal of Intelligent systems, 3(2), 75-82.

[30] Uslu, V. R., Bas, E., Yolcu, U., Egrioglu, E., 2013, A new fuzzy time series analysis approach by using differential evolution algorithm and chronologically-determined weights, Journal of Social and Economic Statistics, 2(1), 18-30.

[31] Alpaslan, F., Cagcag, O., 2012, A Seasonal Fuzzy Time Series Forecasting Method Based on Gustafson-Kessel Fuzzy Clustering, Journal of Social and Economic Statistics, 1(2), 1-13.

[32] Song, Q., Chissom, B. S., 1993, Fuzzy time series and its models, Fuzzy Sets and Systems, 54, 269-277.

[33] Song, Q., Chissom, B. S., 1993, Forecasting enrollments with fuzzy time series- Part I, Fuzzy Sets and Systems, 54(1), 1-9.

[34] Song, Q., Chissom, B. S., 1994, Forecasting enrollments with fuzzy time series - Part II, Fuzzy Sets and Systems, 62(l), 1-8.

[35] Chen, S. M., Chung, N. Y., 2006, Forecasting enrolments using high order fuzzy time series and genetic algorithms, International Journal of Intelligent Systems, 21, 485-501.

[36] Lee, L. W., Wang, L. H., Chen, S. M., 2007, Temperature prediction and TAIFEX forecasting based on fuzzy logical relationships and genetic algorithms, Expert Systems with Applications, 33(3), 539-550.

[37] Cheng, C-H., Cheng, G-W., Wang, J-W., 2008, Multi-attribute fuzzy time series method based on fuzzy clustering, Expert Systems with Applications, 34(2), 1235-1242.

[38] Li, S-T., Cheng, Y-C., Lin, S-Y., 2008, A FCM-based deterministic forecasting model for fuzzy time series, Computers and Mathematics with Applications, 56(12), 3052-3063.

[39] Kennedy, J., Eberhart, R. C., 1995, Particle Swarm Optimization, IEEE International Conference on Neural Network, 4, 1942-1948.

[40] Aladag, C. H., Egrioglu, E., Yolcu, U., 2014, Robust multilayer neural network based on median neuron model, Neural Computing & Applications, 24(3), 945-956.

[41] Shi, Y., Eberhart, R.C., 1999, Empirical study of particle swarm optimization, Proceedings of the 1999 Congress on Evolutionary Computation, 3, 1945-1950.

[42] Ma, Y., Jiang, C., Hou, Z., Wang, C., 2006, The formulation of the optimal strategies for the electricity producers based on the particle swarm optimization algorithm, IEEE Transactions on Power Systems, 21(4), 1663–1671.

[43] Huarng, K., Yu, T. H-K., 2006, The application of neural networks to forecast fuzzy time series, Physica A: Statistical Mechanics and its Applications, 363(2), 481-491.

[44] Egrioglu, E., Aladag, C. H., Yolcu, U., Uslu, V. R., Basaran, M. A., 2009, A New Approach Based on Artificial Neural Networks for High Order Multivariate Fuzzy Time Series. Expert Systems with Applications, 36(7), 10589-10594.

[45] Yu, T. H-K., Huarng, K-H., 2010, A neural network- based fuzzy time series model to improve forecasting, Expert Systems with application, 37(4), 3366-3372.

[46] Yolcu, U., Aladag, C. H., Egrioglu, E., Uslu, V. R., 2013, Time-series forecasting with a novel fuzzy time series approach: an example for Istanbul stock market, Journal of Statistical Computation and Simulation, 83(4), 599-612.

[47] Aladag, C. H., Yolcu, U., Egrioglu, E., Bas, E., 2014, Fuzzy lagged variable selection in fuzzy time series with genetic algorithms, Applied Soft Computing, 22, 465-473.