

Fuzzy Time Series Method Based on Multiplicative Neuron Model and Membership Values

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Abstract Many fuzzy time series forecasting methods have been suggested to forecast time series in the literature. Most of these methods use various artificial intelligence methods. The determining of the fuzzy relation is one of the most important stage in fuzzy time series methods. Some methods determine fuzzy relations by using index number of the fuzzy sets. Recently, several methods that use membership values to determine fuzzy relation have been proposed in the literature. In this paper, a new fuzzy time series forecasting algorithm is proposed. The algorithm use fuzzy c-means method in fuzzification stage. The determining of fuzzy relations stage is performed by multiplicative neuron model in proposed algorithm. In the proposed method, membership values are used instead of index number of the fuzzy sets. Three data sets are used to compare proposed method with some other methods in the literature. It is concluded that proposed method outperforms the some other methods in the literature.

Keywords Fuzzy Time Series, Forecasting, Multiplicative Neuron Model, Fuzzy C-means

1. Introduction

Fuzzy time series methods are based on fuzzy set theory that was proposed by Zadeh[1]. For some time series, the observations of time series can be represented by fuzzy sets. If the observations of the time series are fuzzy sets, traditional time series methods cannot be used. Time series that have got fuzzy observations are called fuzzy time series. The fuzzy time series firstly were defined by Song and Chissom[2]. Moreover, the first fuzzy time series forecasting method was proposed by Song and Chissom[3]. There are two kind of fuzzy time series as time variant and time invariant. The inner relations of time invariant fuzzy time series do not change in time. In generally, fuzzy time series methods have got three stages. These stages are called fuzzification, determining fuzzy relation and defuzzification, respectively. Various artificial intelligence methods can be used in these stages. Genetic algorithm, particle swarm optimization and fuzzy c-means methods for fuzzification of time series; genetic algorithm, particle swarm optimization and artificial neural networks for determining fuzzy relation and artificial neural networks for defuzzification have been used in the literature. it is known that each stages are very effective on forecasting performance of fuzzy time series.

Artificial neural networks were firstly used for determining fuzzy relation by Huang and Yu[4]. They determined fuzzy relation by using artificial neural networks in first order fuzzy time series forecasting model. Then, Yu and Huang[5] applied neural networks to fuzzy time series forecasting in bivariate first order fuzzy time series forecasting model. Artificial neural networks were applied to determine fuzzy relation by Egrioglu et al.[6] in bivariate high order fuzzy time series forecasting model. Egrioglu et al. ([7-8]), Aladag et al. ([9-10]), Egrioglu et al.[11], Aladag[12], Yu and Huang[13] and Yolcu et al.[14]. Egrioglu et al. ([6-7-8]), Aladag et al. ([9-10]), Egrioglu et al.[15], Aladag[16] used index number of fuzzy sets for training artificial neural networks in determining fuzzy relations stage. In these studies, membership values of fuzzy sets are not taken into consideration. Yu and Huang[13] and Yolcu et al.[14] used membership values of fuzzy sets for training artificial neural networks in determining fuzzy relations stage. Aladag[16] different from others since it used multiplicative neuron model neural network instead of multilayer perceptron neural network. The multiplicative neuron model neural network was use to determine fuzzy relation of seasonal fuzzy time series in Aladag S. et al.[12].

In this study, novel fuzzy time series method is proposed. Proposed method use fuzzy c-means clustering in fuzzification stage and multiplicative neuron model neural network in determining fuzzy relations stage. Particle swarm optimization method is applied for training multiplicative neuron model neural network. Fuzzy c-means, multiplicative neuron model neural network and particle swarm

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Published online at <http://journal.sapub.org/ajis>

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optimization are summarized respectively in section 2,3 and 4. In section 5, the proposed method is given. Applications of proposed method and conclusions are given in section 6 and in section 7.

2. The fuzzy c-means Clustering Method

Song and Chissom[2] suggested partition of universe of discourse method for using the fuzzification stage. However there are several problems such as determination of the number of intervals, arbitrarily choice of interval length and membership degrees in this method. To solve these problems, Cheng et al.[17] and Li et al.[18] used fuzzy c-means clustering method for fuzzification. This method that is commonly used for clustering was firstly introduced by Bezdek[19]. It is based on objective function and advantage of this method is objects can have membership degrees in clusters. In this method, fuzzy clustering is conducted by minimizing the least squared errors within groups. Fuzzy c-means clustering algorithm has two main steps. In the first step, the algorithm finds centroids of each cluster and in the next step every object is assigned to a cluster depending on its distance to centroids. Let u_{ij} be the membership values, v_i be the center of cluster, n be the number of variables, and c be the number of clusters. Then the objective function, which is tried to be minimized in fuzzy clustering, is

$$J_\beta(X, V, U) = \sum_{i=1}^c \sum_{j=1}^n u_{ij}^\beta d^2(x_j, v_i) \quad (1)$$

where β is a constant ($\beta > 1$) and called the fuzzy index. $d(x_j, v_i)$ is a similarity measure between an observation and the center of corresponding fuzzy cluster. The objective function J_β is minimized subject to constraints given below.

$$0 \leq u_{ij} \leq 1, \quad \forall i, j$$

$$0 < \sum_{j=1}^n u_{ij} \leq n, \quad \forall i$$

$$\sum_{i=1}^c u_{ij} = 1, \quad \forall j$$

In fuzzy c-means clustering method, to solve the minimization problem given above, an iterative algorithm is used. In each iteration, the values of v_i and u_{ij} are updated by using the formulas given in (3) and (4), respectively.

$$v_i = \frac{\sum_{j=1}^n u_{ij}^\beta x_j}{\sum_{j=1}^n u_{ij}^\beta} \quad (2)$$

$$u_{ij} = \frac{1}{\sum_{k=1}^c \left(\frac{d(x_j, v_i)}{d(x_j, v_k)} \right)^{\frac{2}{\beta-1}}} \quad (3)$$

3. Particle Swarm Optimization

Particle swarm optimization is one of the heuristic technique that first developed by Kennedy and Eberhart[20]. Although a general problem with optimization algorithms is that of becoming trapped in a local optimum, particle swarm optimization avoid from it.

In the literature, particle swarm optimization method has been applied for fuzzification stage in fuzzy time series. Davari et al.[21] was applied PSO algorithm for fuzzification stage in the first order fuzzy time series forecasting model. Then Kuo et al. ([22-23]) used particle swarm optimization method in high order models and in both first and high order models, respectively. Particle swarm optimization method was used for two factor high order model in Park et al.[24]

It was shown that using some time varying parameters can increase the convergence speed of the algorithm. Shi and Eberhart[25] used time varying inertia weight. Time varying acceleration coefficient in standard particle swarm optimization method was applied by Ma et al.[26]. In modified particle swarm optimization, these time varying constituents are used together unlike standard particle swarm optimization.

Algorithm 1: The modified particle swarm optimization.

Step 1. Positions of each k^{th} ($k = 1, 2, \dots, pn$) particles' positions are randomly generated in the initial satge and kept in a vector X_k given as follows:

$$X_k = \{x_{k,1}, x_{k,2}, \dots, x_{k,d}\}, k = 1, 2, \dots, pn \quad (4)$$

Where $x_{k,i}$ ($i=1, 2, \dots, d$) represents i th position of k th particle. pn and d represents the number of particles in a swarm and positions of any particle, respectively.

Step 2. Velocities are randomly determined and stored in a vector V_k given below.

$$V_k = \{v_{k,1}, v_{k,2}, \dots, v_{k,d}\}, k = 1, 2, \dots, pn \quad (5)$$

Step 3. According to the evaluation function, $Pbest$ and $Gbest$ particles given in (6) and (7), respectively, are determined.

$$Pbest_k = (p_{k,1}, p_{k,2}, \dots, p_{k,d}), k = 1, 2, \dots, pn \quad (6)$$

$$Gbest = (p_{g,1}, p_{g,2}, \dots, p_{g,d}) \quad (7)$$

where $Pbest_k$ is a vector stores the positions corresponding to the k^{th} particle's best individual performance, and $Gbest$ represents the best particle, which has the best evaluation function value, found so far.

Step 4. Let c_1 and c_2 represents cognitive and social coefficients, respectively, and w is the inertia parameter. Let $(c_{1i}, c_{1f}), (c_{2i}, c_{2f})$, and (w_1, w_2) be the intervals which includes possible values for c_1 , c_2 and w , respectively. At each iteration, these parameters are calculated by using the formulas given in (8), (9) and (10).

$$c_1 = (c_{1f} - c_{1i}) \frac{t}{\max t} + c_{1i} \quad (8)$$

$$c_2 = (c_{2f} - c_{2i}) \frac{\max t - t}{\max t} + c_{2i} \quad (9)$$

$$w_1 = (w_1 - w_2) \frac{t}{\max t} + w_1 \quad (10)$$

where $maxt$ and t represent maximum iteration number and current iteration number, respectively.

Step 5. Values of velocities and positions at t^{th} iteration are updated by using the formulas given in (11) and (12), respectively.

$$v_{i,j}^{t+1} = \left[w \times v_{i,j}^t + c_1 \times r \times a \times n_1 \times (p_{i,j} - x_{i,j}) + c_2 \times r \times a \times n_2 \times (p_{g,j} - x_{i,j}) \right] \quad (11)$$

$$x_{i,j}^{t+1} = x_{i,j}^t + v_{i,j}^{t+1} \quad (12)$$

where $rand_1$ and $rand_2$ are random values from the interval [0 1].

Step 6. Steps 3 to 5 are repeated until a predetermined maximum iteration number ($maxt$) is reached.

4. Multiplicative Neuron Model

The single multiplicative artificial neuron model which is a special case of neuron model was first presented by Yadav et al.[27]. This model has only one neuron and the general structure of the model, which includes only five inputs is illustrated in Figure 1.

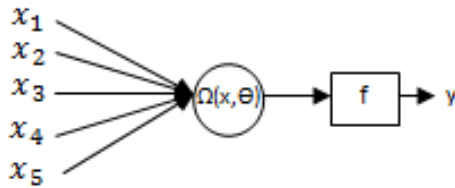


Figure 1. Single multiplicative artificial neuron model

In this model, x_i ($i=1, \dots, 5$) is input pattern and when output values are computed by using multiplication function instead of sum function.

The operator $\Omega(x, \theta)$ is a multiplicative operation which is composed of multiplication of weighted inputs. Activation function and target of the model are represented by f and y , respectively. In the literature, for training the multiplicative neuron model, there are several learning algorithms. Back propagation and cooperative particle swarm optimization learning algorithms were used in the process of multiplicative neuron model, Yadav et al[27] and Zhao and Yang [28], respectively.

5. Proposed Method

In the literature, various techniques have been applied to determine fuzzy relations in fuzzy time series forecasting methods. These techniques are fuzzy logic group relation tables, genetic algorithm, particle swarm optimization and neural networks. Some of these methods passed over membership values of fuzzy sets. The memberships of fuzzy sets should be taken into consideration in determining fuzzy relation stage. In this study, a novel fuzzy time series forecasting method is proposed. Fuzzy c-means clustering in

fuzzification stage and multiplicative neuron model neural network in determining fuzzification stage are used in this method. To train multiplicative neuron model neural network, particle swarm optimization is applied. The some advantages of proposed method are given as below:

- Because of using fuzzy c-means in fuzzification stage, there is no necessity subjective decisions like determining interval length.
- The proposed method take into consideration membership values.
- The proposed method has modeling ability of multiplicative neuron neural network.
- The proposed method can forecast nonlinear time series because multiplicative neuron neural network are employed. The proposed method is given in algorithm 2.

Algorithm 2: Proposed Method

Step 1. Apply FCM.

Firstly, FCM presented in Section 2 with the number of clusters c ($2 \leq c \leq n$) is applied to time series includes crisp values. Values of cluster centers are calculated then, values of memberships are obtained by using these values of cluster centers. Sorted fuzzy clusters L_r , $r=1, 2, \dots, c$ are obtained by sorting the cluster centers in ascending order. The membership values are sorted by same order.

Step 2. Rearrange membership values.

Firstly, threshold value ($tv1$) are chosen. Then, the membership values that are lower than threshold value are taken as zero. For the each observations, membership values are standardized. For example, for any t , membership values are given in Table 1 when five fuzzy sets are used.

Table 1. Membership values of t^{th} observation

Set 1	Set 2	Set3	Set 4	Set 5
0.1	0.4	0.3	0.2	0

Let the threshold value $th1=0.25$. The membership values are rearranged in Table 2.

Table 2. Rearranged membership values of t^{th} observation

Set 1	Set 2	Set3	Set 4	Set 5
0	0.4/(0.4+0.3)	0.3/(0.4+0.3)	0	0

Step 3. Determining of fuzzy relations.

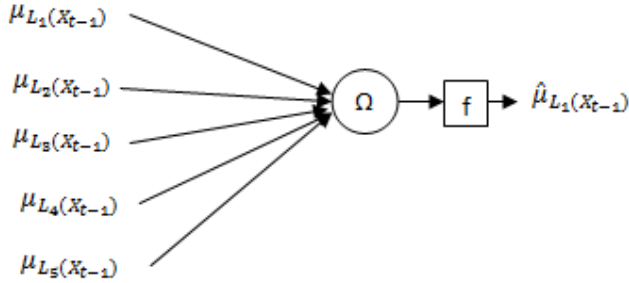
The different multiplicative neuron model for each fuzzy sets is used. When fuzzy sets (L_r , $r=1, 2, \dots, c$) are used, the number of multiplicative neuron models that are used to determine fuzzy relations is c . For example, the membership values of three consecutive observations are given in Table 3, input and target of the multiplicative neuron model for first fuzzy set are given in Table 4 when 5 fuzzy sets are used in the method. Moreover, the architecture of multiplicative neuron model are shown in Fig. 2.

Table 3. Membership values of three consecutive observations

t	Set 1	Set 2	Set3	Set 4	Set 5
1	0.1	0.4	0.3	0.2	0
2	0.2	0.3	0.5	0	0
3	0.1	0.1	0.7	0.1	0

Table 4. Input and target of the multiplicative neuron model for first fuzzy set

Training sample	Input 1	Input 2	Input 3	Input 4	Input 5	Target
1	0.1	0.4	0.3	0.2	0	0.2
2	0.2	0.3	0.5	0	0	0.1

**Figure 2.** The architecture of multiplicative neuron model for first fuzzy set**Step 4.**Obtaining fuzzy forecasts.

The forecasts of membership values are obtained from multiplicative neuron models for each corresponding fuzzy sets.

Step 5. Rearrange membership values.

Firstly, threshold value (tv2) is chosen. Then, the membership values that are lower than threshold value are taken as zero. For the each observation, membership values are standardized. This step is applied like Step 2.

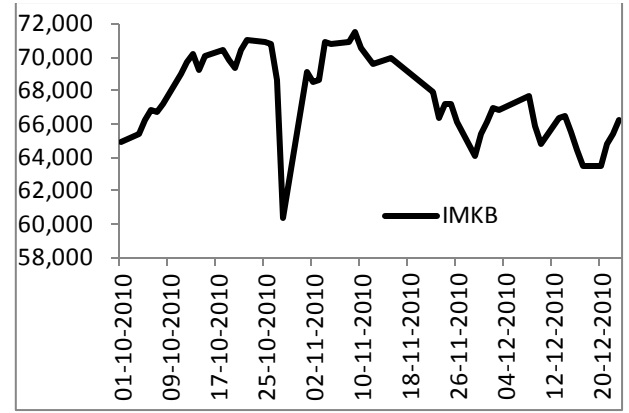
Step 6.Defuzzification of fuzzy forecasts.

The defuzzification is applied like Song and Chissom[3] method. There are three different situation while the defuzzification is applied. The forecasts are obtained in these situations as follow:

- If the membership values of the fuzzy forecast have only one maximum, then take the center value of this set as the defuzzified forecasted value.
- If membership values of fuzzy forecast have two or more consecutivemaximums, then select arithmetic mean of centers of corresponding clusters as the defuzzified forecasted value.
- Otherwise, standardize the fuzzy output and use the center of the fuzzy sets as the forecasted value.

6. Application

Firstly, the data of Index 100 in stocks and bonds exchange market of İstanbul (IMKB) between 01/10/2010 and 23/12/2010 are used in the application. The time series graph of the data set is given in Fig.3. The last 7 observations are used for testing, the other observations are used as training set.

**Figure 3.** The time series graph of IMKB, the period between 01.10.2010 and 23.12.2010

The proposed method is compared with the first order fuzzy time series forecasting models and the classical forecasting techniques. Firstly, for comparison, some methods available in the literature such as Song and Chissom[2], Chen[29], Hunarng and Yu[4] are applied to these data sets. Secondly, simple exponential smoothing and autoregressive integrated moving average (ARIMA) models are also applied. The all results are obtained from fuzzy time series and classical methods are shown in Table 5. The root of mean square error (RMSE) and mean absolute percentage error (MAPE) values for all methods are given in the Table 5. RMSE and MAPE values can be calculated from (13) and (14). In the application of the proposed method, th1=0.1 and th2=0.1, the best number of fuzzy sets is obtained as 12.

$$RMSE = \left(\frac{\sum_{i=1}^T (y_i - \hat{y}_i)^2}{T} \right)^{1/2} \quad (13)$$

$$MAPE = \frac{1}{T} \sum_{i=1}^T \left| \frac{y_i - \hat{y}_i}{y_i} \right| \quad (14)$$

where y_i is the actual value; $\hat{y}_i$ is the predicted value; T is the number of data.

The proposed method and benchmark methods are applied to TAIFEX data whose observations are between 03.08.1998 and 30.09.1998. The time series, which is also used in Lee at al.[30] has 47 observations. The first 31 and the last 16 observations are used for the training and the test sets, respectively. TAIFEX data is also forecasted by using proposed method. The results of other methods are taken from Lee et al. ([30-31]) and Aladag et al.[9]. The all results for TAIFEX data are shown in Table 6. In the application of the proposed method, th1= 0.05 and th2=0.05, the best number of fuzzy sets is obtained as 8.

Table 5. The forecasting results of all methods for test data of IMKB 2010

IMKB	Song and Chissom Method (1993)	Chen Method (1996)	Huarn g Distribution Based Method (2001)	Huarn g Average Based Method (2001)	Huarn g Ratio Based Method (2006)	ARIMA	Simple Exponential smoothing	Proposed Method
65499	65355	65500	66167	66500	67254	67064.5	66299.8	66655
64429	65355	65500	65500	66300	66035	66441	65831.4	66655
63524	65974	65500	66000	64500	65435	65781	65011.3	66655
63502	64736	64950	63500	63500	63668	65222.4	64141.3	66655
64820	64736	64950	63500	63500	63668	65208.5	63767.1	65008
65440	65974	65500	66000	65500	66038	66021.7	64382.8	65008
66219	65355	65500	65500	66300	66035	66404.6	65001.4	66655
RMSE	1161.21	1047.84	1212.81	1014.73	1256.90	1467.50	1129.84	730.77
MAPE	0.014	0.012	0.015	0.012	0.016	0.020	0.016	0.004

Table 6. The forecasting results of all methods for test data of TAIFEX

Date	TAIFEX	Lee et al. method (2007)	Lee et al. Method (2008)	Aladağ et al. (2009)	Proposed Method
10.09.1998	6709.75	6621.43	6917.40	6850	6822.46
11.09.1998	6726.50	6677.48	6852.23	6850	6713.21
14.09.1998	6774.55	6709.63	6805.71	6850	6713.21
15.09.1998	6762.00	6732.02	6762.37	6850	6822.46
16.09.1998	6952.75	6753.38	6793.06	6850	6822.46
17.09.1998	6906.00	6756.02	6784.40	6850	6950.53
18.09.1998	6842.00	6804.26	6970.74	6850	6822.46
19.09.1998	7039.00	6842.04	6977.22	6850	6822.46
21.09.1998	6861.00	6839.01	6874.46	6850	6822.46
22.09.1998	6926.00	6897.33	7126.05	6850	6822.46
23.09.1998	6852.00	6896.83	6862.49	6850	6822.46
24.09.1998	6890.00	6919.27	6944.36	6850	6822.46
25.09.1998	6871.00	6903.36	6831.88	6850	6822.46
28.09.1998	6840.00	6895.95	6843.24	6850	6822.46
29.09.1998	6806.00	6879.31	6858.45	6850	6822.46
30.09.1998	6787.00	6878.34	6825.64	6850	6822.46
	RMSE	93.5	102.96	83.58	82.29
	MAPE	1.09	1.14	0.96	0.92

Table 7. The forecasting results of all methods for test data of IMKB 2008

IMKB	Song and Chissom Method (1993)	Chen Method (1996)	Huarn g Dis. Ba. Method (2001)	Huarn g Average Based Method (2001)	Huarn g Ratio Based Method (2006)	ARIMA	Simple Exponential smoothing	Proposed Method
26294	26410	26400	26200	26100	26091	26205	26205	26205
26055	26410	26400	26200	26367	26091	26294	26294	26205
26059	26410	26400	26200	26100	26091	26055	26055	26205
26499	26410	26400	26200	26100	26091	26059	26059	26205
26424	26410	26400	26200	26500	26608	26499	26499	26757
26411	26410	26400	26200	26500	26608	26424	26424	26757
26864	26410	26400	26200	26500	26091	26411	26411	26757
RMSE	261.0109	259.7625	310.4688	251.239	354.7194	259.33	259.33	233.1
MAPE	0.0075	0.0075	0.0096	0.008	0.0098	0.0072	0.0072	0.006

Finally, proposed method is applied to the data of Index 100 in stocks and bonds exchange market of İstanbul (IMKB) between 3 October 2008 and 31 December 2008. The last 7 observations are used for testing; the other observations are used as training set. The application results are given in Table 7. In the application of the proposed method, $th1=0.2$ and $th2=0.2$, the best number of fuzzy sets is obtained as 12. It can be seen that proposed method is better than others for third data set, too.

7. Discussion and Conclusions

Some fuzzy time series methods passed over membership values. Recent years, fuzzy time series methods based on membership values have been proposed in the literature. In this study, a novel fuzzy time series forecasting method that take into consideration membership values are proposed. The performance of the proposed method is experimentally compared with the other methods by using three real time series data. It is obtained from experimental study that the forecasting of the proposed method is better than benchmark methods for two time series. Besides good forecasting performance of the proposed method, it has got ability to forecast nonlinear time series because of employing multiplicative neuron model neural network. In the future studies, threshold parameter can be optimized in the PSO algorithm. This optimization may provide better forecasting results.

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