American Journal of Fluid Dynamics 2015, 5(1): 17-22
DOI: 10.5923/j.ajfd.20150501.02

Numerical Simulations of Co- and Counter-Taylor-Couette Flows: Influence of the Cavity Radius Ratio on the Appearance of Taylor Vortices

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Abstract Taylor-Couette flows in the annular region between rotating concentric cylinders are studied numerically to determine the combined effects of the co - and counter-rotation of the outer cylinder and the radius ratio on the system response. The computational procedure is based on a finite volume method using staggered grids. The axisymmetric conservative governing equations are solved using the SIMPLER algorithm. One considers the flow confined in a finite cavity with radius ratios η = 0.25, 0.5, 0.8 and 0.97. One has determined the critical points and properties for the bifurcation from the basic circular Couette flow (CCF) to the Taylor Vortex Flow (TVF) state. Indeed, the results are presented in terms of the critical Reynolds number Re_i of the inner cylinder that depends on the rotational Reynolds number of the outer cylinder Re_o and η. To show the capability of the present code, excellent quantitative agreement has been obtained between the calculations and previous experimental measurements for a wide range of radius ratios and rotation rates.

Keywords Taylor-Couette flow, Rotating cylinders, Taylor vortex, Finite volume method

1. Introduction

In recent years, the Taylor vortex flow pattern has been applied intensively to enhance thermal exchange in food processing industry or mixing in bio-industry and medical field such as catalytic chemical reactors, dynamic filtration devices and cell culture bioreactors. This flow is induced by the force balance between the centrifugal force and the pressure gradient in the radial direction within the gap of two concentric rotating cylinders. If the outer cylinder is held stationary and the inner one rotates at low angular velocities, the flow is steady and purely azimuthal (circular Couette flow CCF). Taylor [1] showed that when the angular velocity of the inner cylinder is increased above a certain threshold, CCF becomes unstable and is replaced by a series of axisymmetric counter-rotating toroidal vortices known as Taylor Vortex Flow (TVF). A further increase in the rotation rate of the inner cylinder gives rise to series of fluid transitions with following flow modes, Wavy Vortex Flow (WVF), Modulated Wavy Vortex Flow (MWVF) and ending with featureless turbulence.

After a brief review of the previous works focused on the stability of Taylor-Couette flows in Section 2, the numerical method is presented in Section 3. The influence of the radius ratio for the co- and counter-rotating cases on the appearance of the TVF regime is discussed in detail in Section 4, before some concluding remarks in Section 5.

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Published online at http://journal.sapub.org/ajfd

2. State-of-Art

In most of the cases, rotation is not limited to the inner cylinder. In fact, many investigations have been carried out where both cylinders rotate. Andereck et *al.* [2] have well examined this problem in the small gap size and shown experimentally that the simplest flow CCF can bifurcate out to the three flow modes, Taylor vortex flow (TVF), spiral vortices (SPI) and interpenetrating spirals (IPS) in the case of counter-rotating cylinders. On the other side, when the cylinders rotate in the same direction, more complex flow patterns appear only for high values of angular velocities. The wide gap case was addressed indetail experimentally by Schulz et *al.* [3, 4] and numerically by Hoffmann et *al.* [5, 6]. For a radius ratio η = 0.5, they have determined the spatio-temporal properties and the bifurcation behaviour of TVF and SPI states that bifurcate out of CCF. Recently, Khali et *al.* [7] have studied this problem in the case of non-Newtonian fluids using the Lattice Boltzmann Method (LBM) and Viazzo and Poncet [8] investigated the influence of a radial temperature gradient on the stability of enclosed

Taylor-Couette flows using high-order methods. A large number of flow parameters control the stability of Taylor-Couette flows, such that the reader should refer to the review of Fénot et *al*. [9] for a more exhaustive state-of-art.

The present paper deals with the numerical examination of the structure and the dynamic properties of the Taylor vortex flow (TVF) that bifurcates out of the base state of circular Couette flow (CCF). To our knowledge, little attention has been paid to the influences of the gap width and rotational Reynolds number of the outer cylinder on the transition to the TVF regime. A particular emphasis has been then placed on the gap width effects on this transition for radius ratios covering the wide-gap, mixed-gap and narrow-gap cavities in the co- and counter-rotating cases.

3. Numerical Modeling

3.1. Geometrical Configuration

One considers the flow confined between two concentric cylinders of radii R_i and R_o respectively and height h, as shown in Figure 1. The working fluid is assumed to be incompressible and Newtonian of density ρ and kinematic viscosity ν. Both cylinders can rotate independently around their common axis z at the rotation rates Ω_i and Ω_o respectively, while the top and bottom end-walls are kept stationary.

The system is characterized by two geometric parameters: the radius ratio $\eta = R_i/R_o$ and aspect ratio $\Gamma = h/d$, where $d = R_o - R_i$ is the gap width. Four values of η have been here investigated: $\eta = 0.25$ (large-gap), 0.5 (middle-gap) and $\eta = 0.8$ and 0.97 (small-gap). In order to minimize the influence of the bottom and top disks, a sufficiently large aspect ratio of $\Gamma = 20$ has been chosen.

In enclosed systems, the stability of the isothermal flow depends mainly on two other parameters: the inner and outer Reynolds numbers, $Re_i = \Omega_i R_i d/\nu$ and $Re_o = \Omega_o R_o d/\nu$ respectively, also referred as Taylor numbers in the literature.

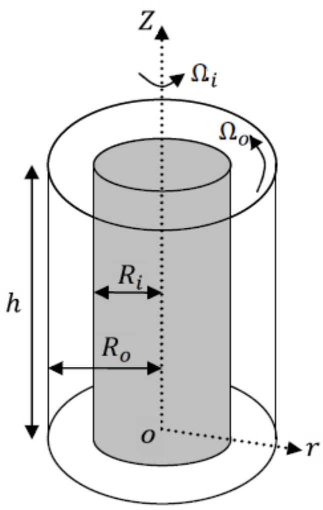

Figure 1. Sketch of the Taylor-Couette system with relevant notations

3.2. Numerical Method

The flow pattern is described by the balance equations for mass and momentum equations, written in cylindrical coordinate system (r,φ,z). To solve this set of coupled equations numerically, one uses an in-house axisymmetric code based on the finite volume method using staggered grids in a (r,z) plane fully described by Elena [10]. The numerical procedure is based on the SIMPLER algorithm to solve the velocity-pressure coupling. A (40×200) mesh in the (r,z) frame has proved to be sufficient to get grid independent solutions for both configurations ($Re_o = 0$ and $Re_o \neq 0$). For this grid, the size of the thinner mesh is $\Delta_1 r/h = 6.2\times10^{-4}$ and $\Delta_1 z/h = 1.85\times10^{-3}$ in the radial and axial directions respectively. It will be used for all cases considered in the following. About 3.10^4 iterations are necessary to obtain the numerical convergence of the calculations.

3.3. Basic Flow State and Validation

The steady base flow solution for differentially- rotating cylinders is described by:

$$u = w = 0, \quad v(r) = Ar + \frac{B}{r} \tag{1}$$

where u,v,w are the velocity components (m/s). The constants A and B are chosen to satisfy the no-slip conditions, which gives:

$$A = (\kappa - \eta^2)\Omega_i/(1-\eta^2), \quad B = R_i^2\Omega_i(1-\kappa)/(1-\eta^2) \tag{2}$$

where $\kappa = \Omega_o/\Omega_i$ is the rotation rate ratio. The associated pressure field P is given by:

$$P(r) = \rho\left[A^2r^2/2 + 2AB\ln(r) - B^2/(2r^2)\right] \tag{3}$$

with a constant axial pressure gradient.

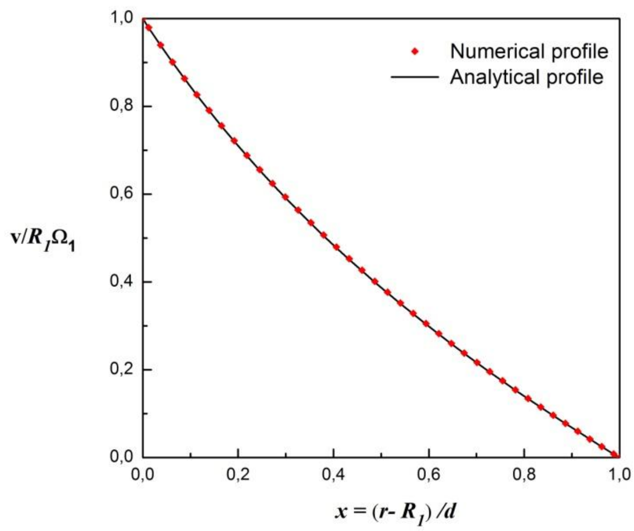

Figure 2. Radial distribution of the mean azimuthal velocity component for z/h=0.1, Re_i=50, Re_o=0, Γ=20 and η=0.5

To validate the present flow solver, the first state to be computed was that of circular Couette flow (CCF). In this sense, Figure 2 shows the tangential velocity profile along the radius for Re_i=50, Re_o=0 and η=0.5. It can be observed an excellent agreement between the calculated solution and the analytical one for circular Couette flow. The radial distributions of the radial and axial velocity components (not shown here) are zero, as expected, except from some flow regions very close to the end-walls.

4. Results and Discussion

4.1. Case "Re_o=0" – Stationary Outer Cylinder

One first focuses on the middle-gap case with the outer cylinder keeping at rest (Re_o=0). For η = 0.5, the critical inner Reynolds number Re_{ic} for the onset of Taylor vortices is 68.4. This value perfectly agrees with that obtained by Di Prima and Swinney [11].

Figure 3 shows some typical maps of the azimuthal velocity, pressure and stream function for two values of the inner Reynolds number, representing the base flow (Re_i=50<Re_{ic}) and the first bifurcation to Taylor vortices (Re_i=72>Re_{ic}). From the azimuthal velocity contours, one can locate inflow and outflow regions with respect to the walls of the inner cylinderand see the direction of rotation of the cell. From the streamlines, one can clearly see the classical axial periodicity of the flow, characterized by a number of vortices fixed by the value of the aspect ratio Γ = 20.

In the case of a stationary outer cylinder and for the four values of the radius ratio considered here, the critical inner Reynolds number Re_{ic} for the onset of Taylor vortices is compared with values given by the linear stability of Gebhardt and Grossmann [12] on Figure 4. One can note that the numerical values agree particularly well with the linear stability analysis for all types of cavity (wide-, mixed- or narrow gap cavities as 0.25≤η≤0.97), which continues to valid the present numerical flow solver.

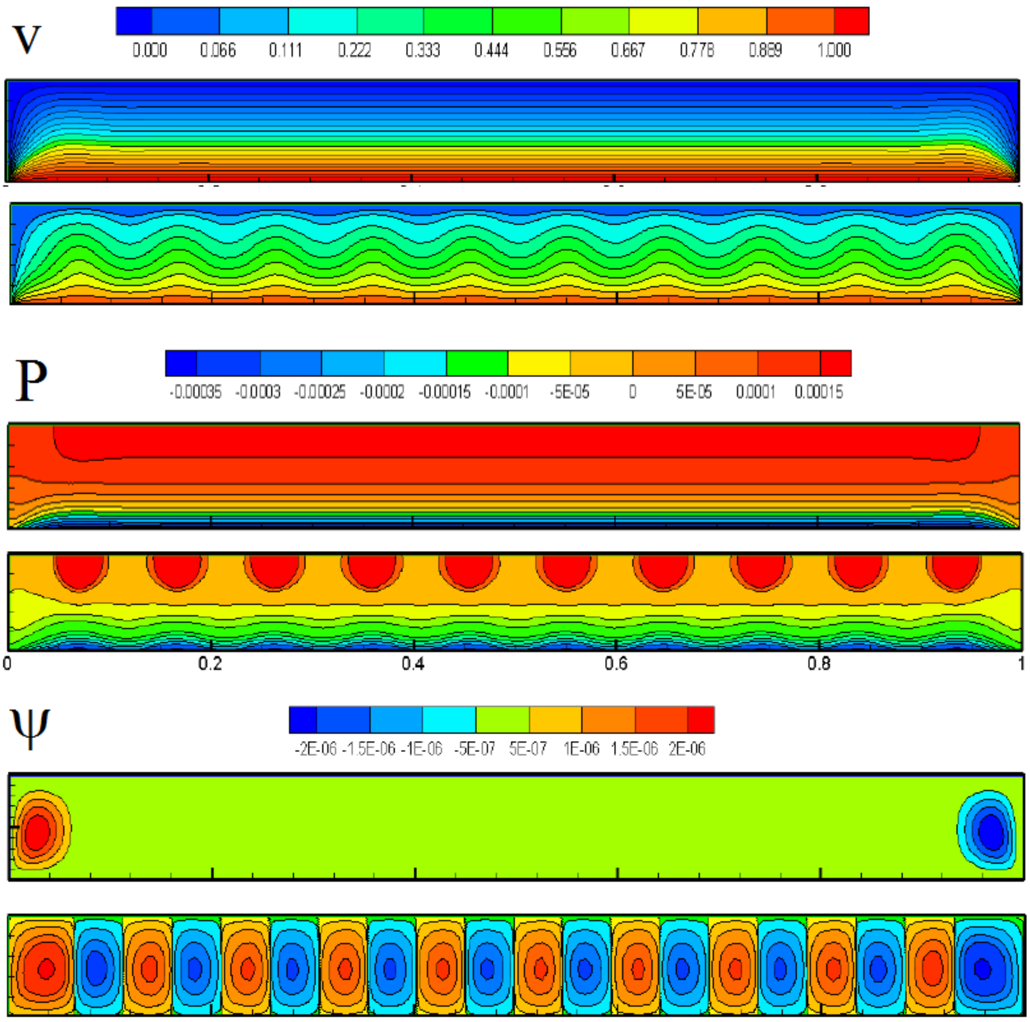

Figure 3. Contours of the azimuthal velocity *v*, pressure *P* and stream function *Ψ* in the meridional planes for η = 0.5, Γ = 20 and Re_o = 0: Re_i = 50 for the upper plot and Re_i = 72 for the lower one. The whole fluid is shown with the inner cylinder being at the bottom of each subfigure

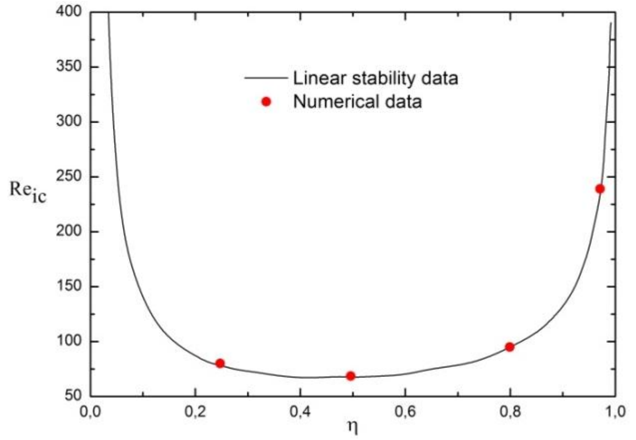

Figure 4. Transition between the CCF and TVF regimes for Γ=20 and Re_o=0. The solid line represents the values given by the linear stability analysis of Gebhardt and Grossmann [12] and the symbols represent the present calculations

4.2. Case "$Re_o \neq 0$" – Rotating Outer Cylinder

The transition from circular Couette flow (CCF) to Taylor vortex flow (TVF) is considered when the outer cylinder may also rotate. It is located by fixing Re_o and slowly increasing Re_i. In Figure 5, one examines the effect of the outer cylinder rotation in a Taylor-Couette apparatus of small (η=0.25) intermediate (η=0.5) and large (η=0.8 and 0.97) radius ratio. The base flow is unstable to axisymmetric Taylor vortex flow. Indeed, for a given radius ratio, one can see that the rotation of the outer cylinder in opposite direction is at first weakly destabilizing before getting stabilizing for co-rotation of the cylinders. As an example, for a radius ratio η = 0.5, the minimal value of the transition point is for an outer Reynolds number Re_o=−15 and Re_i=66.3, which is close to the point found by Schulz et *al.* [3] (Re_{omin}=-15.26, Re_i=66.05). Beyond this value, the inner Reynolds number Re_i increases monotonically as Re_o increases. However, the co-rotation stabilizes the symmetric flow and the vortices appear for relatively high values of angular velocities of the inner cylinder and the delay of the transition is more marked. It can be seen that the rotation of the outer cylinder has the same qualitative effect as the imposed axial through flow. In fact, an axial flow stabilizes the circular Couette flow and delays the transition to TVF, which appears in a very narrow range of the parameters [13].

One can also see that the transition behaviour is found to depend strongly on the gap size. Indeed, for wide- and middle-gap cavities $\eta<0.5$, increasing the radius ratio has a destabilizing effect on the circular Couette flow. Beyond this value, as the annular gap decreases, the critical inner Reynolds number increases and the flow gets stable.

Figure 6 presents some comparisons with published results, the measurements of Schulz et *al.* [3] and the analytical data of Hoffmann et *al.* [5] for a radius ratio of $\eta=0.5$. One can first observe a very good agreement concerning the onset of the first instability for outer Reynolds numbers $-77.5 \leq Re_o \leq 0$. In this graph, the Rayleigh criterion is also plotted and corresponds to the straight-line ($Re_i=2Re_o$) asymptote of the stability threshold of Couette flow at large and positive Re_o values. All the results being above this line, the flow is always unstable regarding the centrifugal instability in an unbounded domain.

One has examined also the effects of the outer cylinder rotation rate on the development of the vortices within the gap. Figure 7 shows the numerical streamline contours. In the case where the cylinders rotate in the same direction (Figure 7a), 18 vortices appear instead of 20 and the size of the Ekman vortices increases. This phenomenon can be explained by some end-wall influence. However, when the cylinders rotate in opposite directions (Figure 7b), the number of vortices increases from 20 to 22 and the Ekman vortices size decreases. Similar observation has also been made by Khali et *al.* [7]. One can note that increasing the co-rotation has a more stabilizing effect compared to the counter-rotation case.

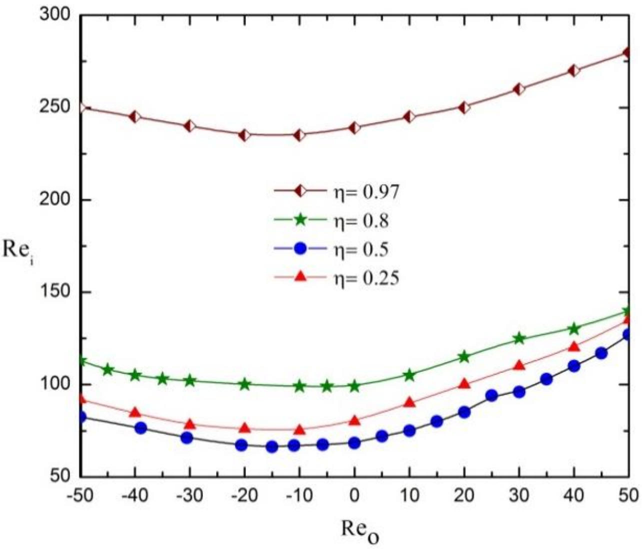

Figure 5. Stability diagram for the primary bifurcation to the TVF regime: Re_i versus Re_o for four values of η and Γ=20

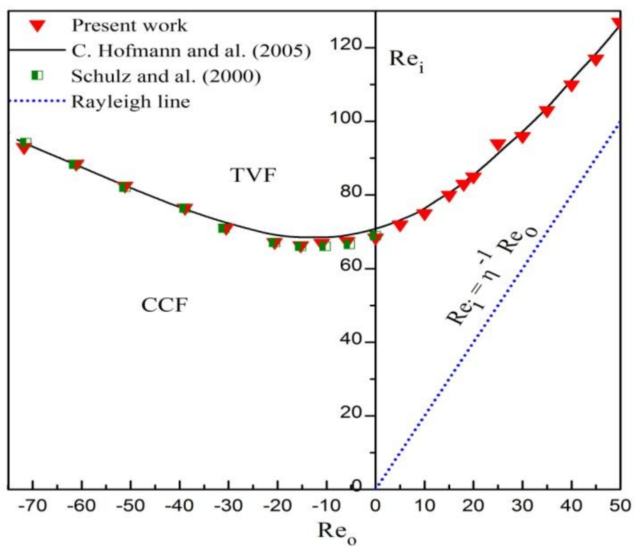

Figure 6. Comparison of the critical inner Reynolds number for Γ=20 and η=0.5 with different works

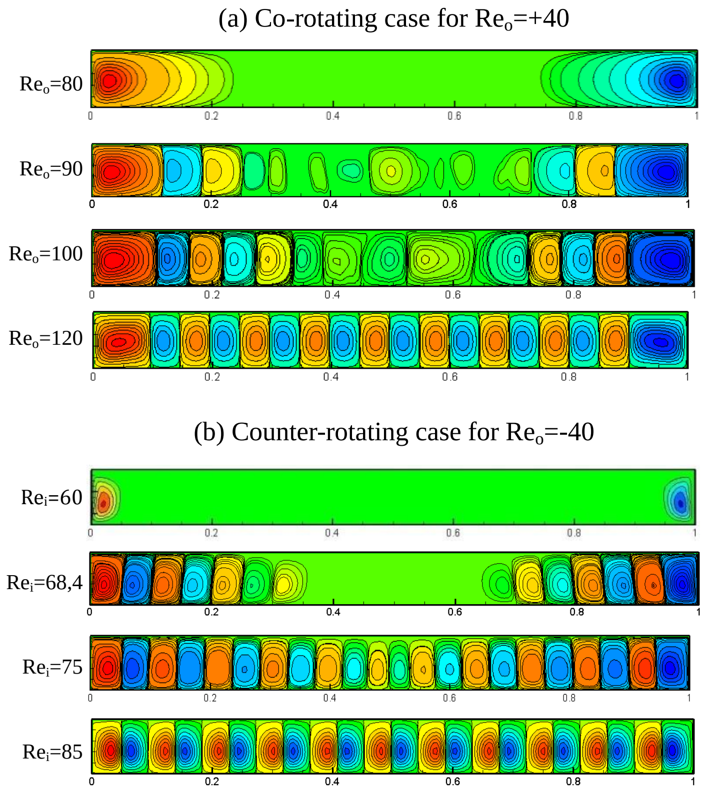

Figure 7. Streamline patterns in a meridional plane for Γ=20, η=0,5 and (Re_i, Re_o) as indicated. The whole fluid is shown with the inner cylinder being at the bottom of each subfigure

5. Conclusions

The effects of the radius ratio and the inner Reynolds number on the transition between the circular Couette flow regime (CCF) and the axisymmetric Taylor vortex flow (TVF) has been investigated numerically. An excellent quantitative agreement has been obtained between finite-volume calculations with previous experimental and analytical studies for a wide range of radius ratios and rotation rates.

It has been found that the size of the annular gap plays an important role on the stability threshold of circular Couette flow. In the wide- and middle-gap cases, as the radius ratio $\eta \leq 0.5$ is decreased, the critical inner Reynolds number increases contrary to the small gap case ($\eta > 0.5$), for which the critical inner Reynolds number is increased as the radius ratio is increased.

In addition, one can note that for a given radius ratio, the rotation of the outer cylinder in co- or counter direction delays the transition from the CCF to the TVF regimes. The flow in the co-rotating case is more stable than in the counter-rotating case. In the co-rotating case, the number of vortices is decreased from N to N-2. On the other hand, the number of vortices increases from N to N+2 in the case where the cylinders rotate in opposite directions.

Some calculations have been performed to investigate successively the influence of an axial Poiseuille flow then a radial temperature gradient on the stability of the circular Couette flow regime. These preliminary simulations confirm the results of Altmeyer et *al.* [14] and Viazzo and Poncet [8] respectively: both effects destabilize the CCF regime leading to 3D instability patterns even for weak values of the parameters, such that the TVF may be observed in a narrow range of the parameters. The present solver will be then extended into three-dimensions in a very close future.

REFERENCES

[1] G.I. Taylor, 1923, Stability of a Viscous Liquid contained between Two Rotating Cylinders, Philosophical Transactions of the Royal Society A, 223, 289–343.

[2] C.D. Andereck, S.S. Liu and H.L. Swinney, 1986, Flow regimes in a circular Couette system with independently rotating cylinders, Journal of Fluid Mechanics, 164, 155-183.

[3] A. Schulz and G. Pfister, 2000, Bifurcation and mode interaction with independently rotating cylinders in Taylor-Couette flow, In Physics of Rotating Fluids, Springer, Berlin, 37-54.

[4] A. Schulz, G. Pfister and S.J. Tavener, 2003, The effect of outer cylinder rotation on Taylor-Couette flow at small aspect ratio, Physics of Fluids, 15 (2), 417-425.

[5] C. Hoffmann, M. Luke and A. Pinter, 2000, Spiral vortices and Taylor vortices in the annulus between rotating cylinders, In Physics of Rotating Fluids, Springer, Berlin, 55-65.

[6] C. Hoffmann, M. Luke and A. Pinter, 2005, Spiral vortices travelling between two rotating defects in the Taylor -Couette system, Physical Review E, 69, 056311.

[7] S. Khali, R. Nebbali and K. Bouhadef, 2013, Numerical investigation of non-Newtonians fluids flows between two rotating cylinders using Lattice Boltzmann method, International Journal of Mechanical, Aerospace, Industrial and Mechatronics Engineering, 7 (10), 98-104.

[8] S. Viazzo and S. Poncet, 2014, Stability of a high aspect ratio Taylor-Couette flow submitted to a radial temperature gradient, Computers and Fluids, 101, 15-26.

[9] M. Fénot, Y. Bertin, E. Dorignac and G. Lalizel, 2011, A review of heat transfer between concentric rotating cylinders with or without axial flow, International Journal of Thermal Sciences, 50 (7), 1138-1155.

[10] L. Elena, 1994, Modélisation de la turbulence inhomogène en présence de la rotation, PhD thesis, Université d'Aix-Marseille I-II.

[11] R. C. Di Prima and H. L. Swinney, 1985, Instabilities and transition in flow between concentric rotating cylinders, in Hydrodynamic Instabilities and the Transition to Turbulence, eds. H. L. Swinney and J. P. Gollub, Springer-Verlag, Berlin.

[12] T. Gebhardt and S. Grossmann, 1993, The Taylor-Couette eigenvalue problem with independently rotatingcylinders, Zeitschrift für Physik B Condensed Matter, 90, 475-490.

[13] R.M. Lueptow, 2000, Stability and experimental velocity field in Taylor–Couette flow with axial and radial flow, In Physics of Rotating Fluids, Lecture Notes in Physics, 59, Springer, Berlin, 137-155.

[14] S. Altmeyer, C. Hoffmann and M. Lücke, 2011, Islands of instability for growth of spiral vortices in the Taylor–Couette system with and without axial through flow, Physical Review E, 84, 046308.