

# Numerical Simulation of the Evolution of Reynolds Number on Laminar Flow in a Rotating Pipe

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**Abstract** Laminar flow in a stationary pipe is destabilized when swirl is introduced due to pipe rotation. This paper employs the practicability of COMSOL Multiphysics, in numerically simulating laminar flow in a straight axially rotating pipe, with a view of examining the influence of varying Reynolds numbers on the flow. The results show that the velocity profiles for low Reynolds numbers are significantly influenced by large rotation rates. The parabolic solution for axial velocity in this case, is approached faster at downstream sections further from the inlet than for higher Reynolds number. With a decrease in shear rate and swirl downstream of the pipe, the tangential velocity component of the flow becomes large at high rotation rates, and the flow approaches a forced vortex type. This becomes apparent further away from the pipe inlet. The overall results suggest that when a fluid is characterized by a Reynolds number in a pipe, the flow behavior is controlled to some large extent by the swirl effect induced by pipe rotation, and this is seen from the description of the velocity components associated with swirl flows.

**Keywords** Reynolds number, Rotating pipe, Axisymmetric, COMSOL, Swirl flow, Forced vortex, Velocity profile

## 1. Introduction

Laminar flow in a non-rotating pipe is characterized by the Poiseuille parabolic solution, such that the pressure decreases in the flow direction, and the resulting velocity profile has a negative curvature everywhere; with maximum axial velocity occurring at the centreline. The influence of pipe rotation on the flow results in a swirl and introduction of turbulence. The effect of turbulence becomes more pronounced with increased disturbance due to increased pipe rotation. Laminar flow entering an axially rotating pipe experiences instability even at Reynolds number less than 2000 [1]. The problem of swirl flows has been a subject of interest over the years, and has been extensively studied via experimental, analytical and numerical methods. Some of such studies are quickly described here. The velocity profiles in the developing region of a rotating pipe at very low Reynolds numbers were analyzed by Lavan et al. [2] and Pedly [3]. The analysis of Pedly, revealed that there is a critical Reynolds number of 82.9 above which flow instabilities in form of growing spiral waves occur in the rotating pipe. The experimental work of Nagib et al. [4] also confirms the study of Pedly by revealing that; at fairly low Reynolds number, there is destabilization of laminar flow due to pipe rotation. Torii et al. [5] numerically investigated

the rotational effect on secondary flow phenomena in an axially rotating flow passage with sudden expansion or contraction, where the Reynolds number and rotation rates were varied to determine their effects on the formation of secondary flows. They disclosed that the stretch of the secondary flow zone is amplified with an increase in rotation rate and Reynolds number when laminar flow enters the rotating pipe with expansion and the flow zone is suppressed for rotating pipe flows with contraction. Yamada et al. [6] observed that when the laminar flow enters the axially rotating pipe, a destabilization of the flow due to the swirl is induced, resulting in an enhancement in the turbulence intensity. Lei et al. [7], through an experimental approach, studied velocity measurements in four different flow regimes of the laminar flow in a rotating straight pipe. Imao et al [8] compared numerical results of velocity profiles for laminar flow in the developing region of an axially rotating pipe with experimental results obtained when a uniform axial flow is introduced with interest in flows where swirl component is large to produce flow reversal near the wall. The laminar flow inlet length was calculated by Mizutani et al. [9] (cited by Imao et al [8]) in a work restricted to small pipe rotation rates. Ayinde [10] showed that swirl decays downstream a pipe and established a generalized relationship for swirl decay as function of swirl number at inlet and other parameters.

This present study is concerned with examining the effect of varying low Reynolds numbers on the laminar flow at various downstream sections of a rotating pipe. The study is motivated by the versatility of the commercial CFD

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modeling tool, Comsol Multiphysics, in numerically simulating laminar flow in a straight axially rotating pipe. The velocity profiles at different downstream sections of a short straight axially rotating pipe section are analyzed at specific pipe Reynolds number, and at varying rotation speed. The application in engineering practice is obvious for example, in rotating heat exchangers, cooling systems of rotors, and in the design of the cooling channels inside the rotating blades of gas turbines. Furthermore, it also occurs in the inlet parts of fluid machines and some pipe flow systems in a rotating frame, such as those on a spinning satellite.

### 2. Theoretical Background

Consider an incompressible fluid flowing in a pipe with a radius R, which is axially rotating about the axis at constant angular velocity,  $\omega$ , the configuration and cylindrical coordinate system of the flow is shown in figure 1.

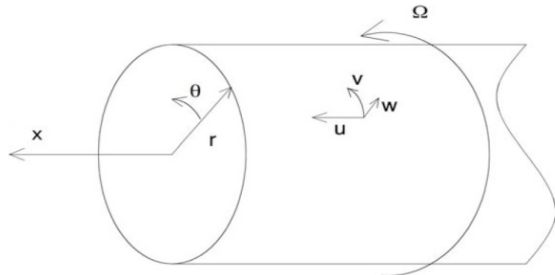


Figure 1. The cylindrical coordinates

The governing differential equations for mass and momentum are expressed as

**Continuity equation:**

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial r} + \frac{V}{r} = 0 \tag{1}$$

**Momentum equations:**

*x-direction (representing axial coordinate z):*

$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial r} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \left( \frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial r^2} + \frac{1}{r} \frac{\partial U}{\partial r} \right) \tag{2}$$

*r-direction:*

$$U \frac{\partial V}{\partial x} + V \frac{\partial V}{\partial r} - \frac{V^2}{r} = -\frac{1}{\rho} \frac{\partial P}{\partial r} + \nu \left( \frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial r^2} + \frac{1}{r} \frac{\partial V}{\partial r} - \frac{V}{r^2} \right) \tag{3}$$

*Theta-direction:*

$$U \frac{\partial W}{\partial x} + V \frac{\partial W}{\partial r} + \frac{VW}{r} = \nu \left( \frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial r^2} + \frac{1}{r} \frac{\partial W}{\partial r} - \frac{W}{r^2} \right) \tag{4}$$

In the axially rotating pipe, the boundary conditions are specified as:

$$U = V = 0, \quad W = W_w \text{ at wall,}$$

$$U = U_{in}, \quad V = W = 0 \text{ at the inlet, that is: } x = 0$$

$$\frac{\partial U}{\partial r} = 0, \quad V = W = 0 \text{ at center (} r = 0 \text{)}$$

### 3. Numerical Method

COMSOL Multiphysics Version 4.0a was employed in simulating the swirl flow so as to determine the velocities at various Reynolds numbers and pipe rotation rates. COMSOL version 4.0a runs the Finite Element Method (FEM) which allows the momentum conservation in the domain. The use of the FEM in package in domain discretisation and in solving all equations associated with the model permits a reduction in ‘numerical loss’ of momentum in the computational domain [12], and as such, this enables investigation of other parameters that characterizes the flow. In this study, the swirling flow is two dimensions even though the model includes all three velocity components, U, V and W. Under the assumption that the flow is symmetric, only half of the pipe cross-section (Figure 2) is analysed. However, this helps to simplify the modeling and also facilitate the concept of rotation applicable to the model, hence, minimizing the calculation time. The domain of interest is discretised using the readily available unstructured grid provided by the solver, and the mesh employed consists of 1956 elements with a refined mesh near the wall as shown in figure 2(b), to capture flow profiles close to the wall. From this figure and in this work, counter-clockwise pipe rotation is considered, where the wall (boundary 4) is defined as a sliding (moving) wall, and the direction is orthogonal to, and out of the r-z plane of the domain.

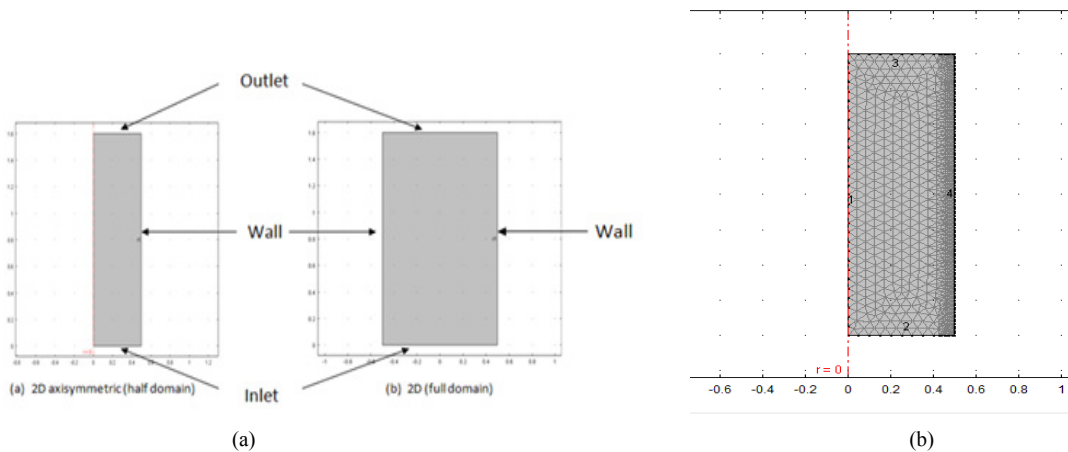
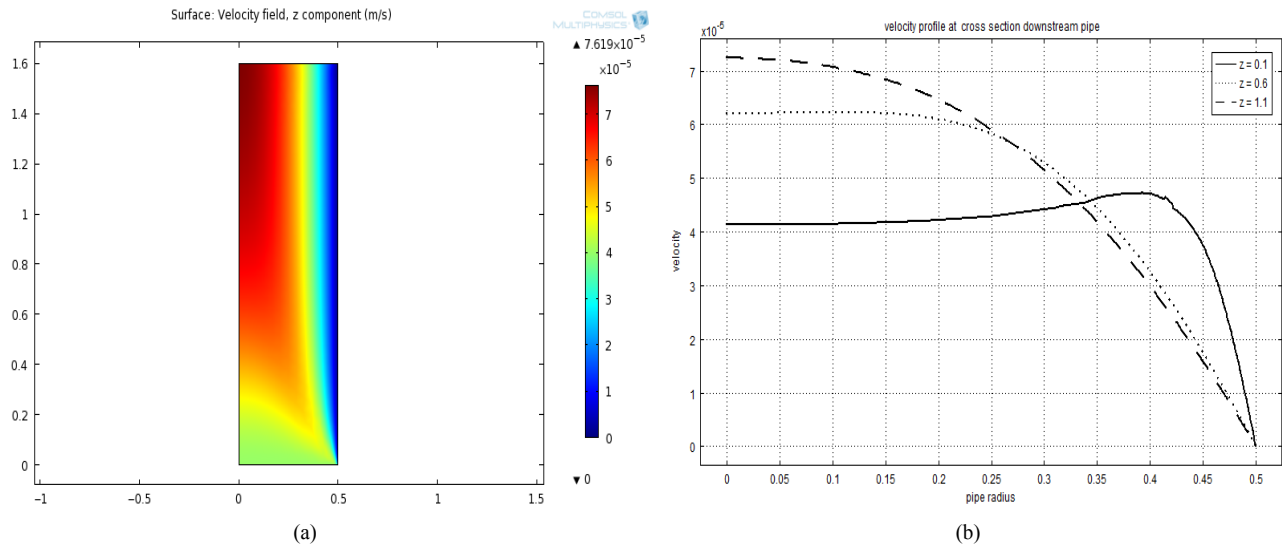


Figure 2. (a) Boundary settings for domain. (b) Mesh used for the axisymmetric domain - 1956 elements

## 4. Results and Discussion



**Figure 3.** Reynolds number of 40 (a) Comsol Display for axial velocity; z component (b) Axial velocity plot at three downstream sections

A short pipe section is used to examine the flow behavior in the developing region of a rotating pipe. In order to establish the rationality of this analysis, and with respect to utilizing the axisymmetric model case on COMSOL, a case of flow considering a non-axisymmetric model – complete domain (full radial distance) in the solver is modeled and then, flow in a stationary pipe for an axisymmetric – half domain is considered. A uniform entry flow is assumed by the inlet boundary condition specified using the solver. The resulting velocity profile at varying downstream sections of the pipe is analyzed. The extracted data from the model is subjected to the boundary conditions at the wall for axial velocity and at the pipe center for the radial velocity. No significant differences are seen in the resulting profiles and results for this flow at a Reynolds number of 40. Hence, the boundary condition employed at the entrance and the exit is accepted as being reasonable. The axial velocity profile for three downstream sections,  $z = 0.1, 0.6$  and  $1.1$  of a stationary pipe are shown in figure 3. It is quickly seen that at  $z = 0.1$ , the flow is developing and a parabolic profile with maximum velocity at the center is obtained further downstream the pipe,  $z = 1.1$ .

In order to further validate the rationality behind this study; for rotation rates of  $N=0, 2, 4$  and  $6$ , and Reynolds numbers =  $20, 30, 40, 50$  and  $60$ , we compared the results of the velocity components at  $z = 0.2, 0.4$  and  $1$ , with those obtained from the solution to the numerical procedure presented by Imao et al. [8], which we implemented using MATLAB codes. The standard errors were observed to fall within the range of  $0.0423 - 0.2357$  ( $N = 0, 2, 4$  and  $6$ ) for axial velocity,  $0.0225-0.04$  ( $N = 0, 2, 4$  and  $6$ ) for the radial velocity, and  $0.006-0.06$  ( $N = 2, 4$  and  $6$ ) for the tangential velocity. For all these velocities, the increase in error is proportional to increasing rotation rates.

The axial velocity profiles of the four downstream cross-sections of the pipe ( $z = 0.1, 0.2, 0.5$  and  $1.2$ ), and at

various Reynolds numbers ( $20, 30, 40, 50$  and  $60$ ) are shown in Figures 4, 5, 6 and 7 for rotation rate of  $N=0, 2, 4$  and  $6$ , respectively.

In all the Reynolds numbers considered for the case of a stationary pipe,  $N = 0$  (Figure 4), the axial velocity profile approaches a parabolic profile further downstream of the pipe as the shear rate and pressure gradient decreases. Meanwhile, when the rotation rate increase to  $N = 2$ , no significant changes are noted at  $z = 0.1$  and  $z = 0.2$  relative to a stationary pipe flow. This is not surprising since the swirl is not strong enough to induce a significant disturbance on the flow. However, the changes become more pronounced at  $z = 0.5$  and  $1.2$  indicating that the fluid is gradually been accelerated. At higher rotation rate,  $N = 4$ , the profile at  $z = 0.1$  is similar to those of  $N = 0$ . For all the Reynolds numbers, the maximum axial velocity at  $N = 4$ , shifts away from the pipe wall due to the increasing rate of shear and a larger tangential velocity component given to the fluid. At a Reynolds number of  $60$ , the rate of shear is high. Interestingly, flow reversal is noticed to occur at  $N = 6$ , and at  $z = 0.2$  as shown in Figure 7b. This is largely due to high rotation rate, strong swirl and adverse pressure gradient imposed on the fluid within this vicinity, and this effect is stronger as Reynolds number increases. This implies that an asymptotic critical rotation rate has been exceeded.

It becomes apparent that, when laminar flow is introduced into an axially rotating pipe, an adverse or large pressure gradient in the axial direction is generated and this causes a flow reversal to occur near the pipe wall at a section near the inlet, and it does not occur at the pipe entrance [8], [12]. Due to pipe rotation and the ensuing swirl flow, the pressure is minimum towards the center and maximum at the wall. Using this model, this phenomenon becomes obvious at downstream sections in the developing region for axial location  $z$  between  $0.12$  and  $0.43$ . Furthermore, its occurrence results in an extension of the downstream section

required for the flow to become developed. However, the parabolic axial velocity profile is recovered further downstream of the pipe. The trend of recovering of the parabolic axial velocity profile is amplified with an increase in the rotation rate and Reynolds number. Ayinde [10] showed that the original Poiseuille axial flow parabolic

profile is gradually recovered as the strength of swirl decays downstream of the pipe. This is attributed to viscous dissipation. Similar to stationary pipe, the axial velocity is increased towards the center and decrease close to the wall. This is as a result of contributions from friction and shear stress at the solid boundary.

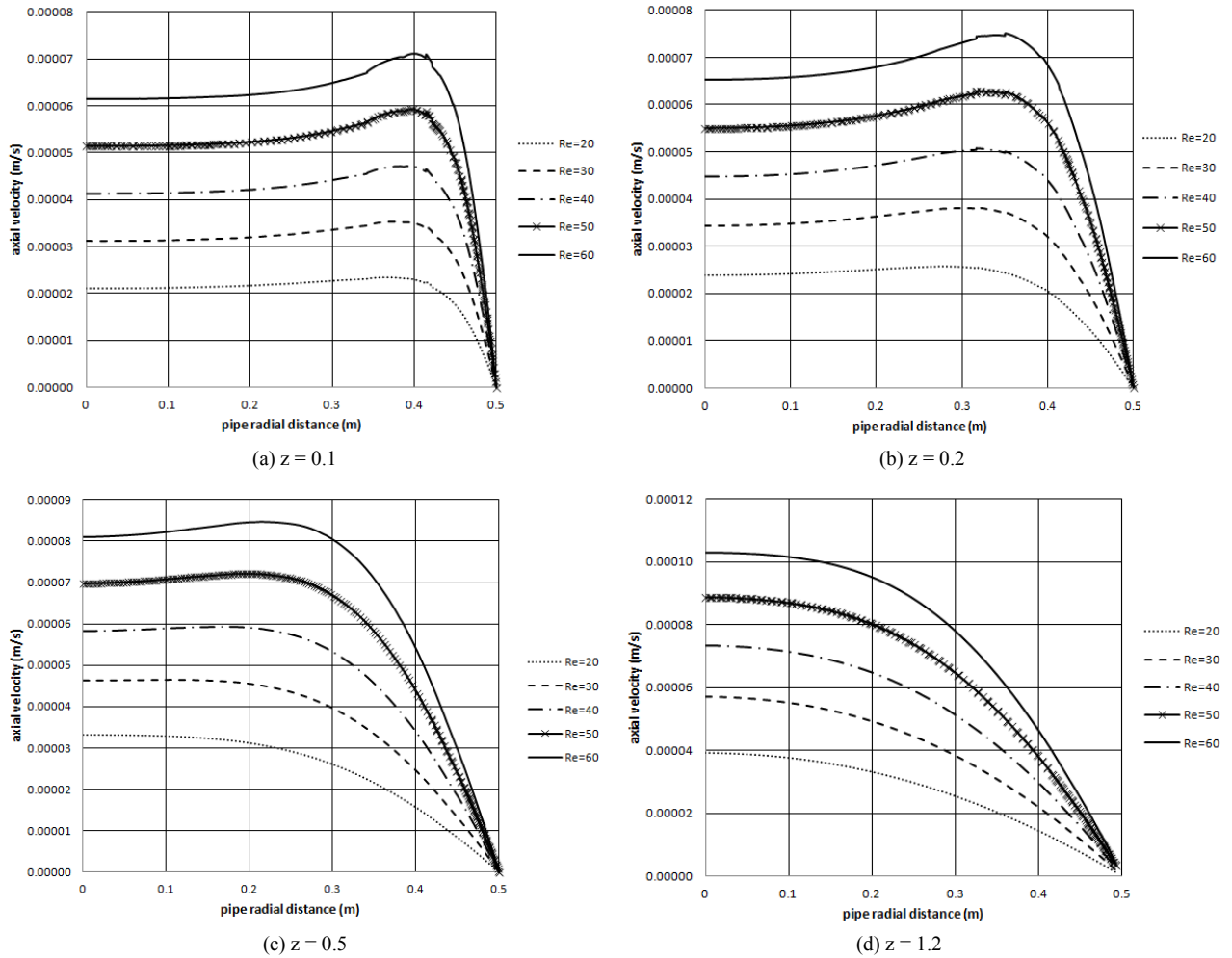
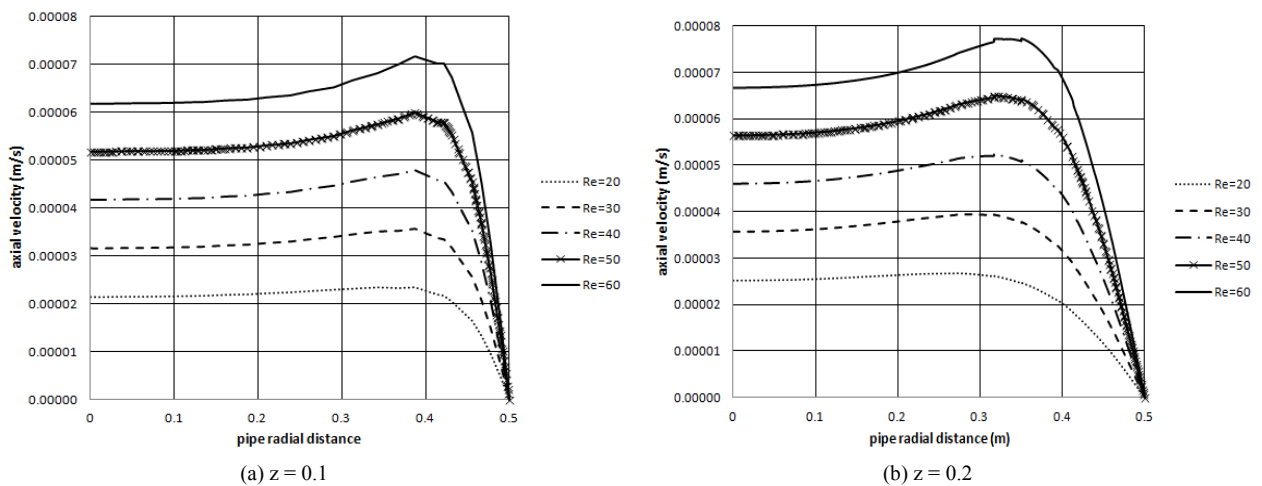


Figure 4.  $N = 0$



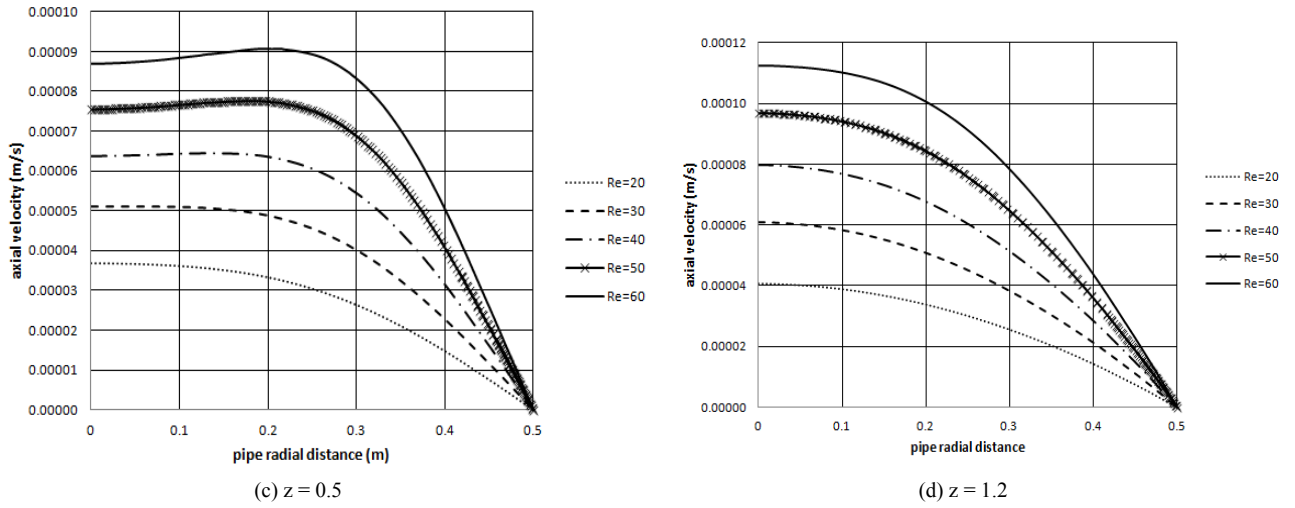


Figure 5.  $N = 2$

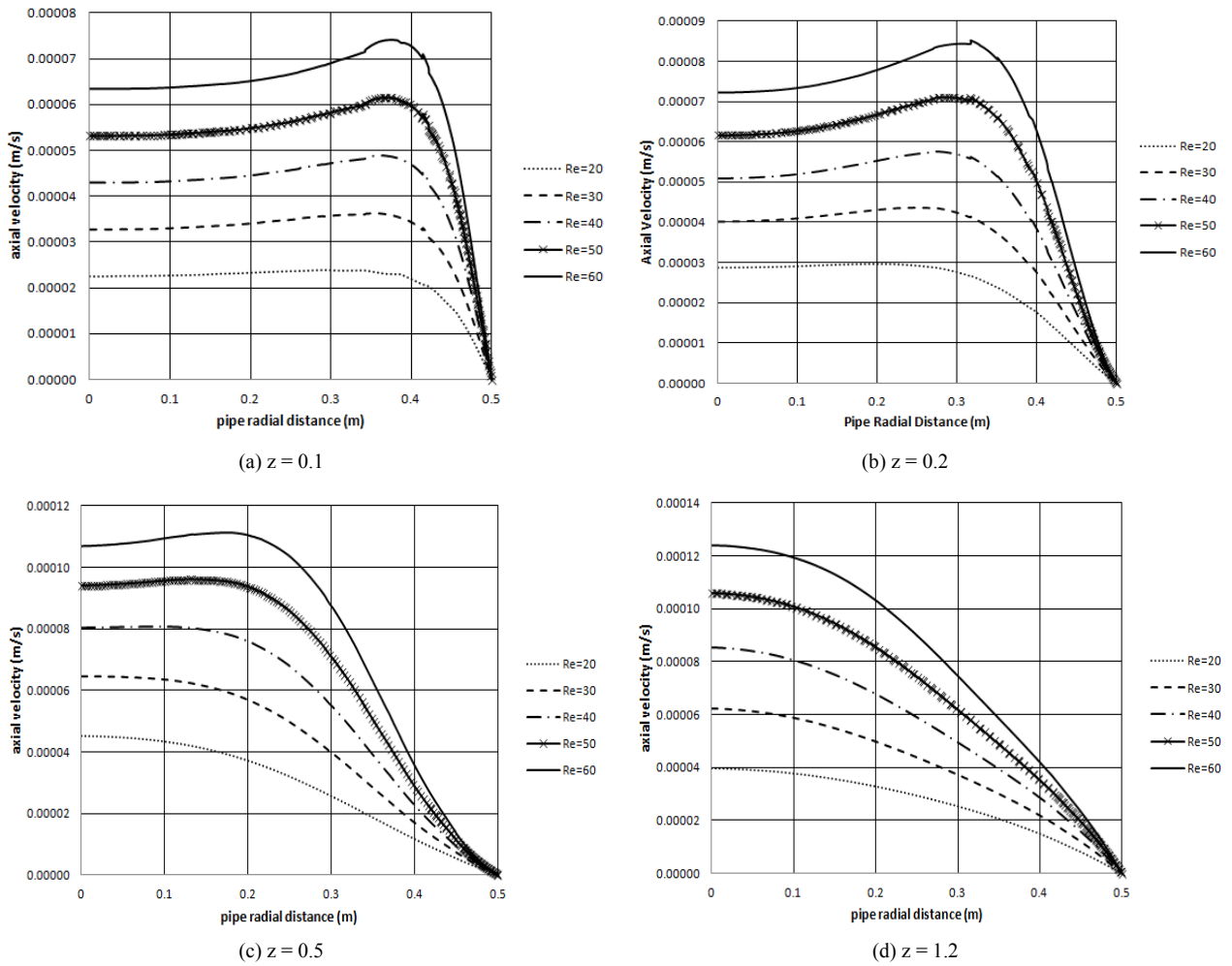
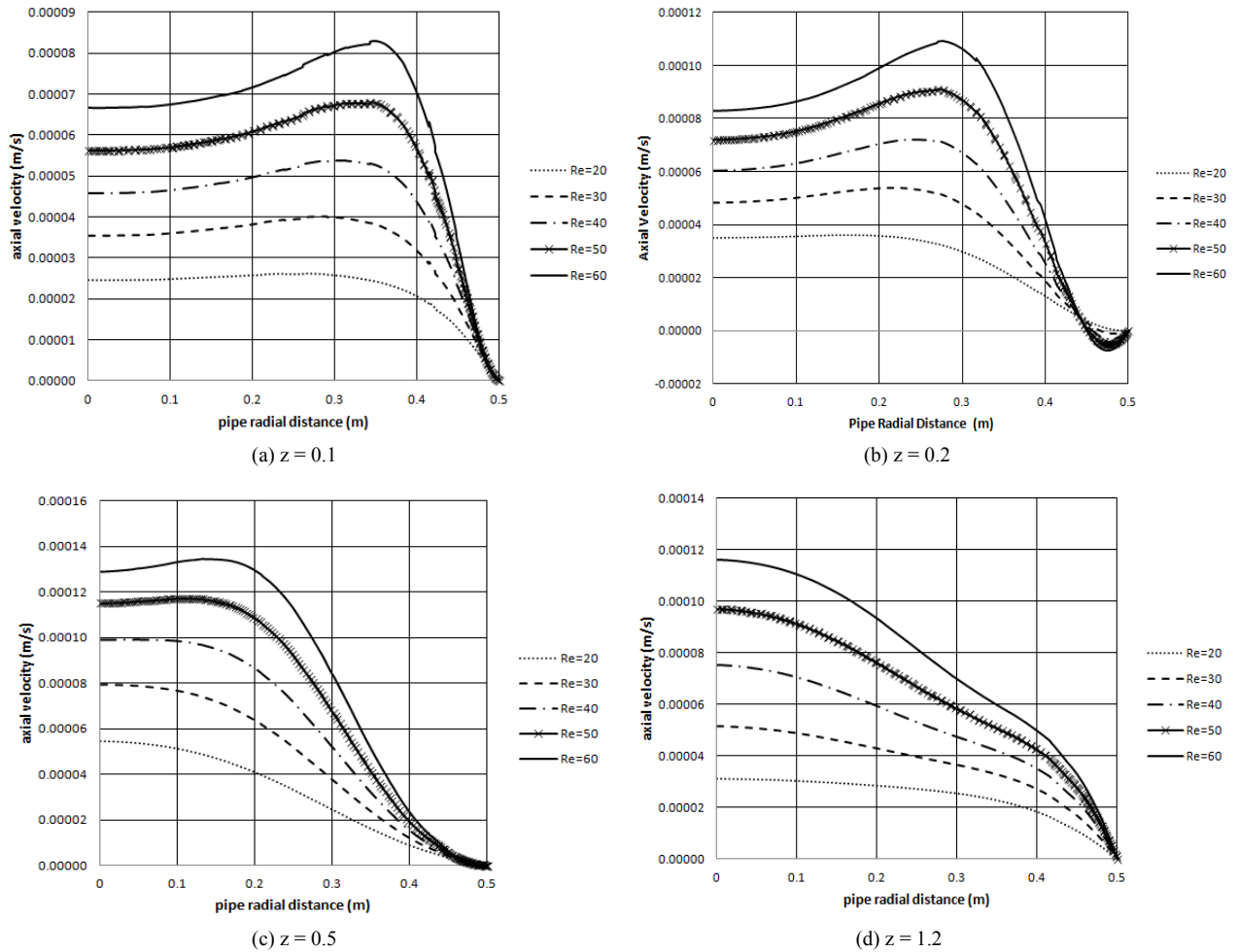


Figure 6.  $N = 4$

Figure 7.  $N = 6$ 

For each of the Reynolds number considered, increase in the rotation rate results in an increase in axial velocity,  $U$ , at the center and this even becomes more significant further downstream at  $z = 0.5$  and  $1.2$  of the pipe where it approaches a parabolic profile. For  $N = 6$  and at  $z = 0.5$ , the profile depicts that the flow being accelerated at the pipe center as the swirl imposed on the flow decays downstream. This indicates that the effect of back flow wears off as the flow progresses away from the pipe inlet, and there is a translation to an accelerated flow at the pipe center due to recovery from the backflow.

At  $N = 4$  and  $6$ , an interesting phenomenon occurs further downstream at  $z = 1.2$  as the parabolic profile is maintained but the influence of rotation rate significantly affects the flow characteristics. For  $N = 4$ , this is quickly noticed that at low Reynolds numbers, 20 and 30, the flow decelerates between  $z = 0.5$  and  $1.2$ . This shows that at low Reynolds numbers, increase in pipe rotation causes a large flow displacement effect. However, this signifies that the influence of swirl at this rotation rate is strong, and is true for all Reynolds numbers at  $N = 6$  considered. The associated effect, results in the reduction of maximum axial velocity between  $z = 0.5$  and  $1.2$ , thereby suggesting that a forced vortex condition on the flow is being approached (Figure 7c

and 7d). The flow approaches a forced vortex type due to large tangential and radial velocity experienced by the fluid but with an increase in Reynolds number. It can be observed that the tendency is significant for low Reynolds number flow in a rotating pipe at increasing rotation rate. Furthermore, it is seen from figure 7(b)–(c) that at  $N = 6$ , for all Reynolds number considered in this investigation, the forced vortex condition in the flow as a result of pipe rotation is apparent as the radial velocity becomes significantly large.

The tangential velocity of the fluid close to the wall equals the tangential speed ( $W_w = N.U_{in}$ ) of the wall. Flow in a stationary pipe has no tangential velocity component, figure (8) ~ (10) but upon rotation, a tangential velocity component is given to the flow by the moving wall and this causes the shearing stress to be large within a region of the pipe entrance. The tangential velocity,  $W$ , increases in the radial direction-towards the wall-for every rotation rate, and for each Reynolds number. At the downstream sections there is flow reversal, and at  $N = 6$ , the tangential velocity at the wall is high owing to the large rate of shear and swirl at this section, which is the developing region. In general, as the pipe rotation rate increases, the tangential velocity component,  $W$ , for the Reynolds numbers considered, at  $z = 0.5$  and  $1.2$ , becomes significant.

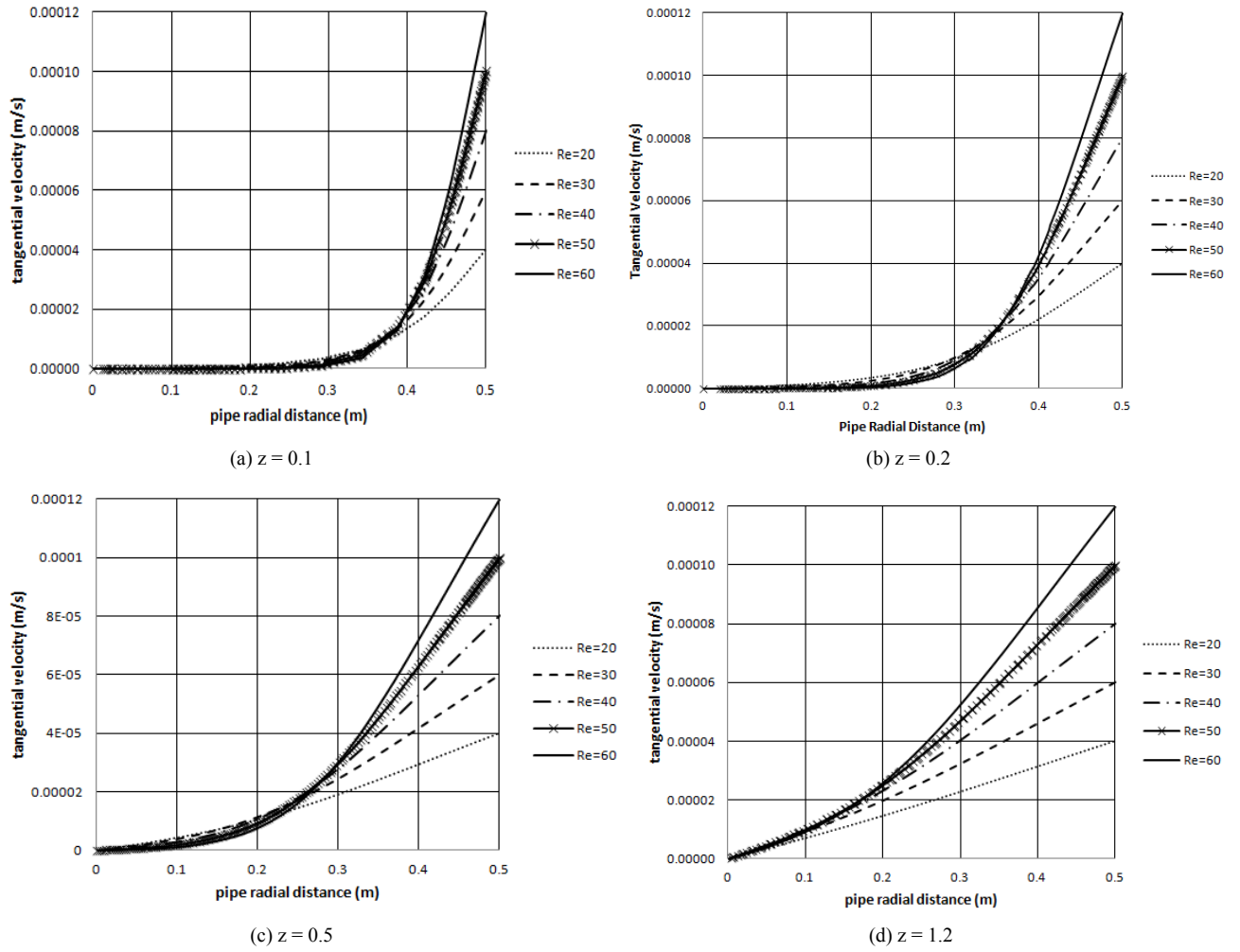
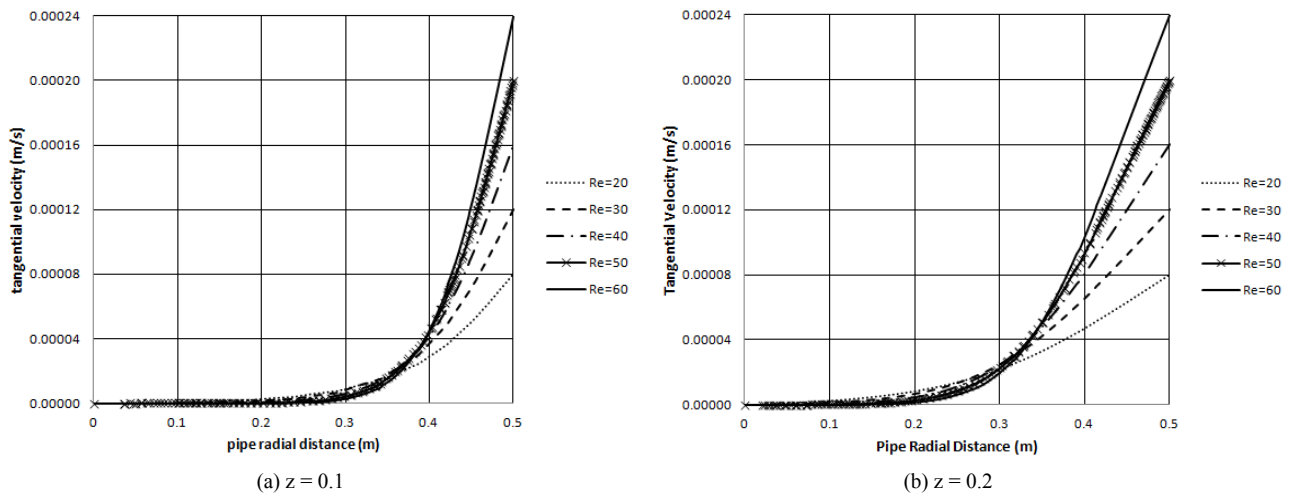


Figure 8.  $N = 2$  – Tangential velocity



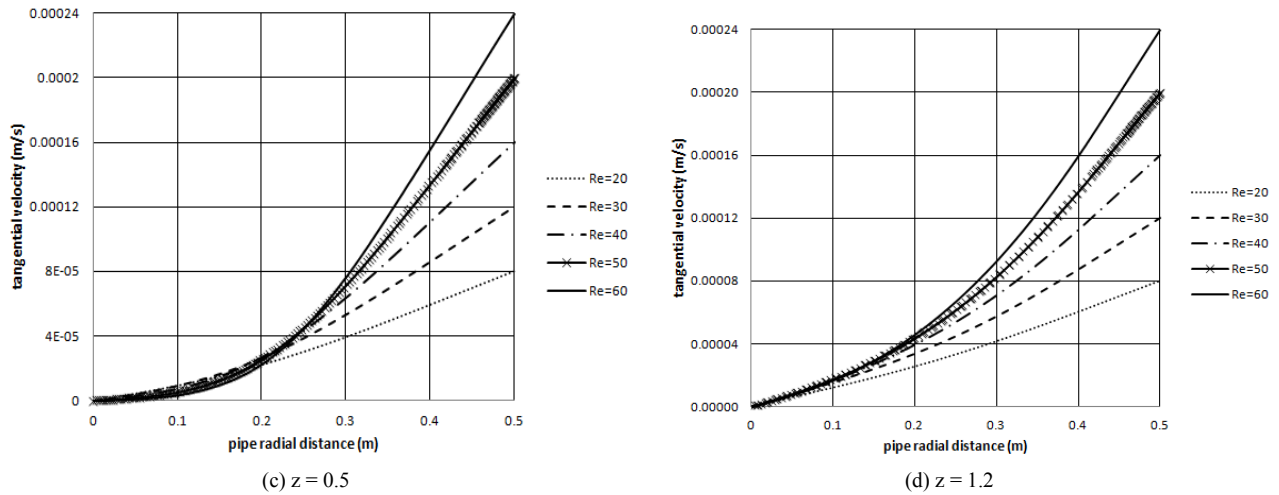


Figure 9.  $N = 4$  – Tangential velocity

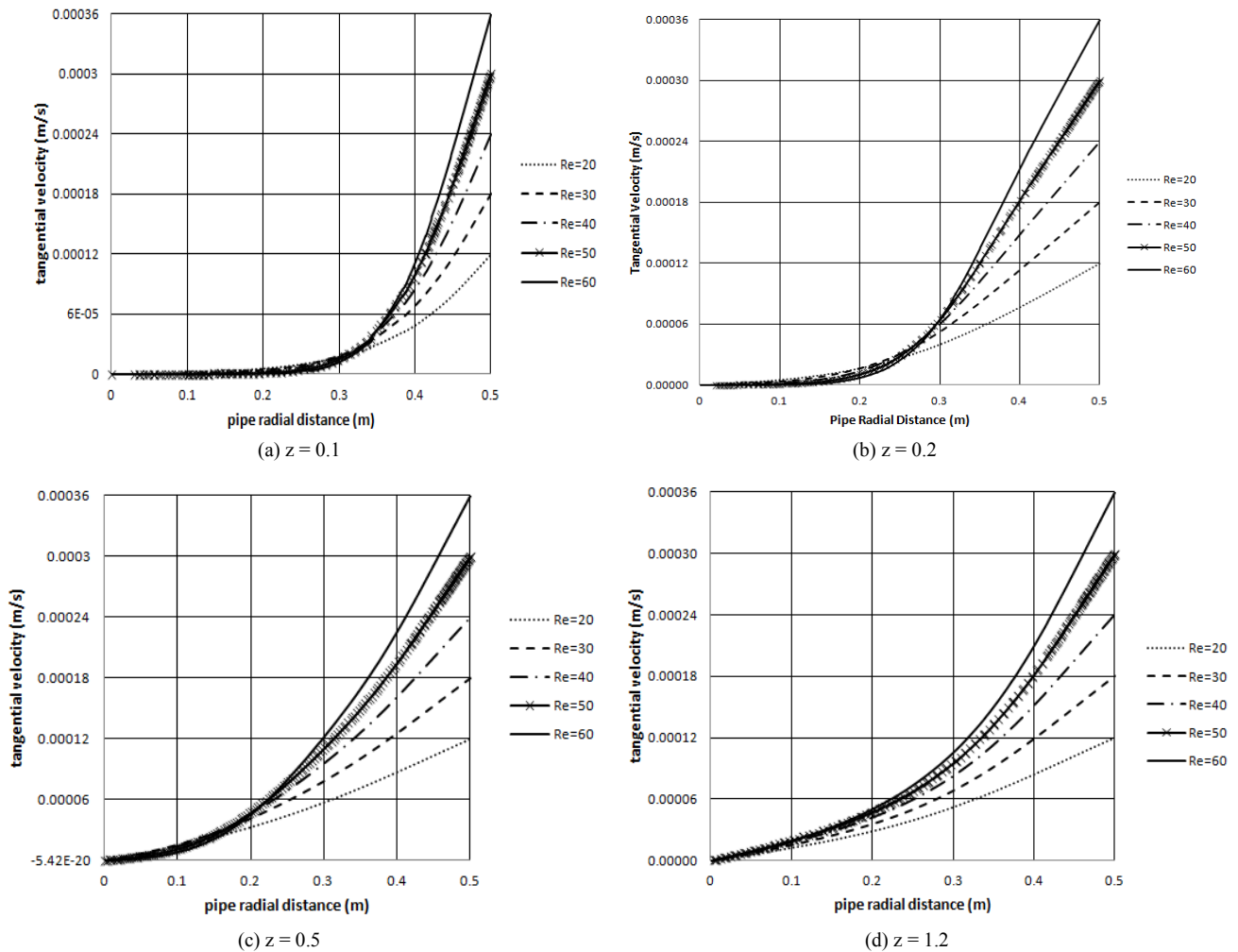


Figure 10.  $N = 6$  – Tangential velocity

For the location  $z = 0.1$  and  $0.2$ , at the Reynolds numbers considered and at low rotation rate, there is a sudden increase in the tangential velocity component given to the flow occurs close to the wall, and away from the pipe center (between  $0.3 - 0.5$ ), as the shear rate and swirl is small. This change is seen to wear off and proceed to the pipe center upon an

increase in rotation rate. But further downstream the pipe,  $z = 0.5$  and  $1.2$ , with decreasing shear rate, the profile is seen to approach a forced vortex type [8] and the tangential velocity profile assumes a linear profile (for low Reynolds numbers). This becomes significant with increasing rotation rates. The rate of shear in a case of low Reynolds number of 20 is small,

hence the linear profile of the tangential velocity at  $z = 0.5$  and  $1.2$  is indicative of large fluid displacement owing to the swirl imposed on it by reason of rotation. At large rotation rate,  $N=6$ , where the rate of deformation imposed on the flow by the rotating wall is high and the shear rate of the flowing fluid is reduced at a downstream section greater than  $z = 1.2$ , the fluid is displaced towards the wall of the rotating pipe.

It can be quickly inferred from figures (11) ~ (14) that the radial velocity is greatly influenced by varying Reynolds numbers, and rotation rates. For example, at low Reynolds numbers, the radial velocity is large and this tendency decreases as Reynolds number increases. A larger radial velocity component is however noticeable at downstream sections in the pipe. For all the Reynolds numbers considered, the radial velocity is negative everywhere for  $N = 0$ , and at low rotation rate of  $N = 2$ . However, a variation of the profile (transition from negative to positive values) is seen downstream of the pipe ( $z = 1.2$ ) with increase in Reynolds numbers and at  $N = 2$ . Downstream of the pipe, the radial velocity increases for all rotation rates (and stationary pipe) investigated. The radial velocity reduces as the rotation rate increases for the region close to the pipe entrance (up to  $z = 0.2$ ) and is negative everywhere suggesting that fluid is

transferred away from the wall, and the axial velocity is retarded due to growing viscous boundary layers associated with the flow progression. It is easily noticed from figures (12)~ (14), that there exist downstream sections, where the transition the radial velocity profile from being negative to positive occurs, and is influenced by rotation rates. At higher rotation rates, the influence of pipe rotation is seen to affect flows with higher Reynolds numbers, as the radial velocity component is higher compared to those of low Reynolds numbers. This suggests that the fluid undergoes deceleration at the downstream location  $z = 1.2$ , owing to fluid displacement effects associated with forced vortex motion. Consequently, the transition of the radial velocity from being negative to being positive gradually shifts to the upstream section, towards the pipe inlet as the rotation rate increases, and is pronounced at higher rotation rates. To corroborate this, at  $N = 2$  and  $z = 1.2$ , a transition is observed as shown in figure 12(d) and this also occurs at  $N = 4$  at a downstream location between  $z = 0.5$  and  $1.2$ . Meanwhile, upon an increase in  $N$ , this phenomenon is apparent at  $z = 0.5$  and beyond which the radial velocity becomes very large as evidenced at a downstream section of  $z = 1.2$  suggesting a large displacement effect on the flow towards the wall.

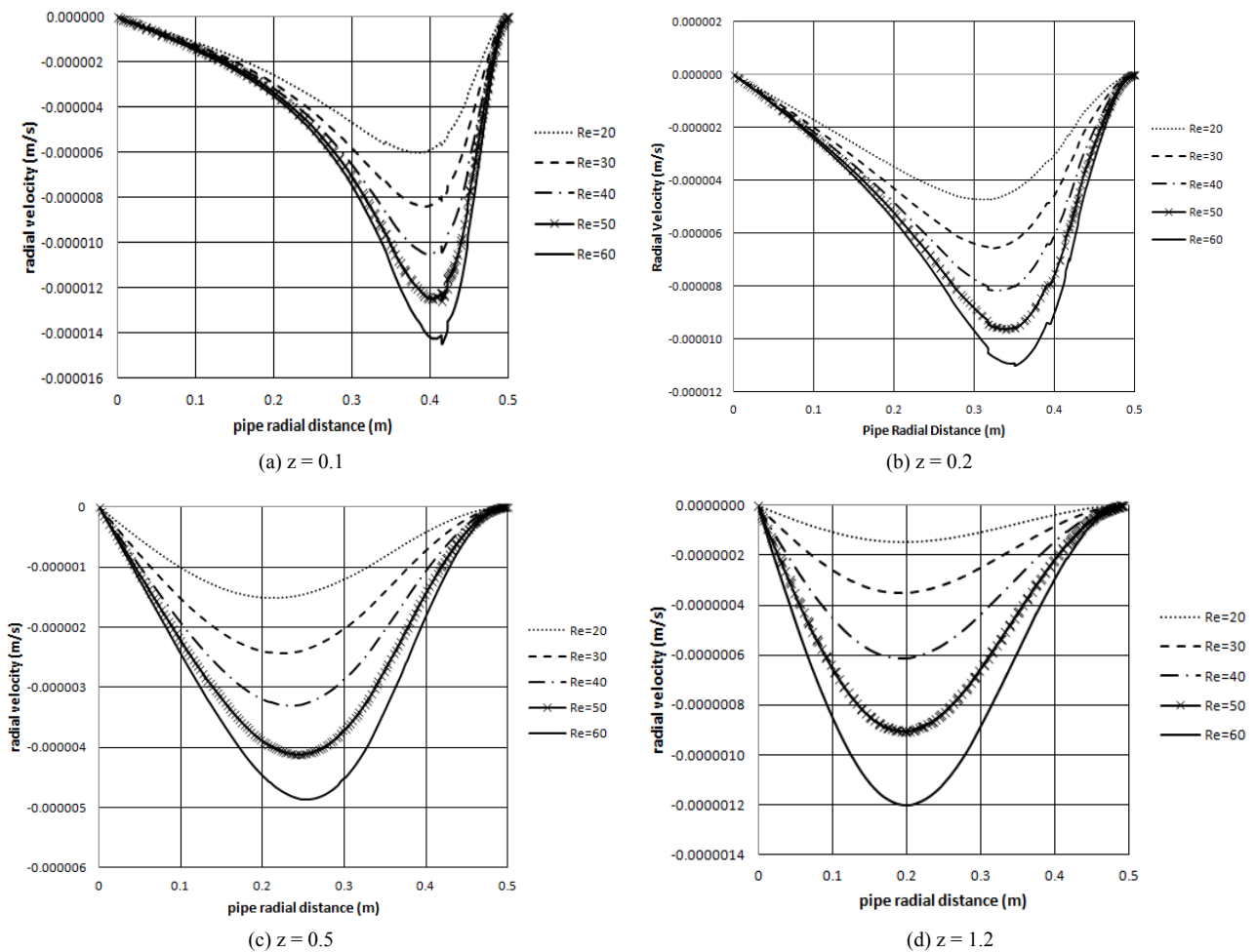


Figure 11.  $N = 0$  – Radial velocity

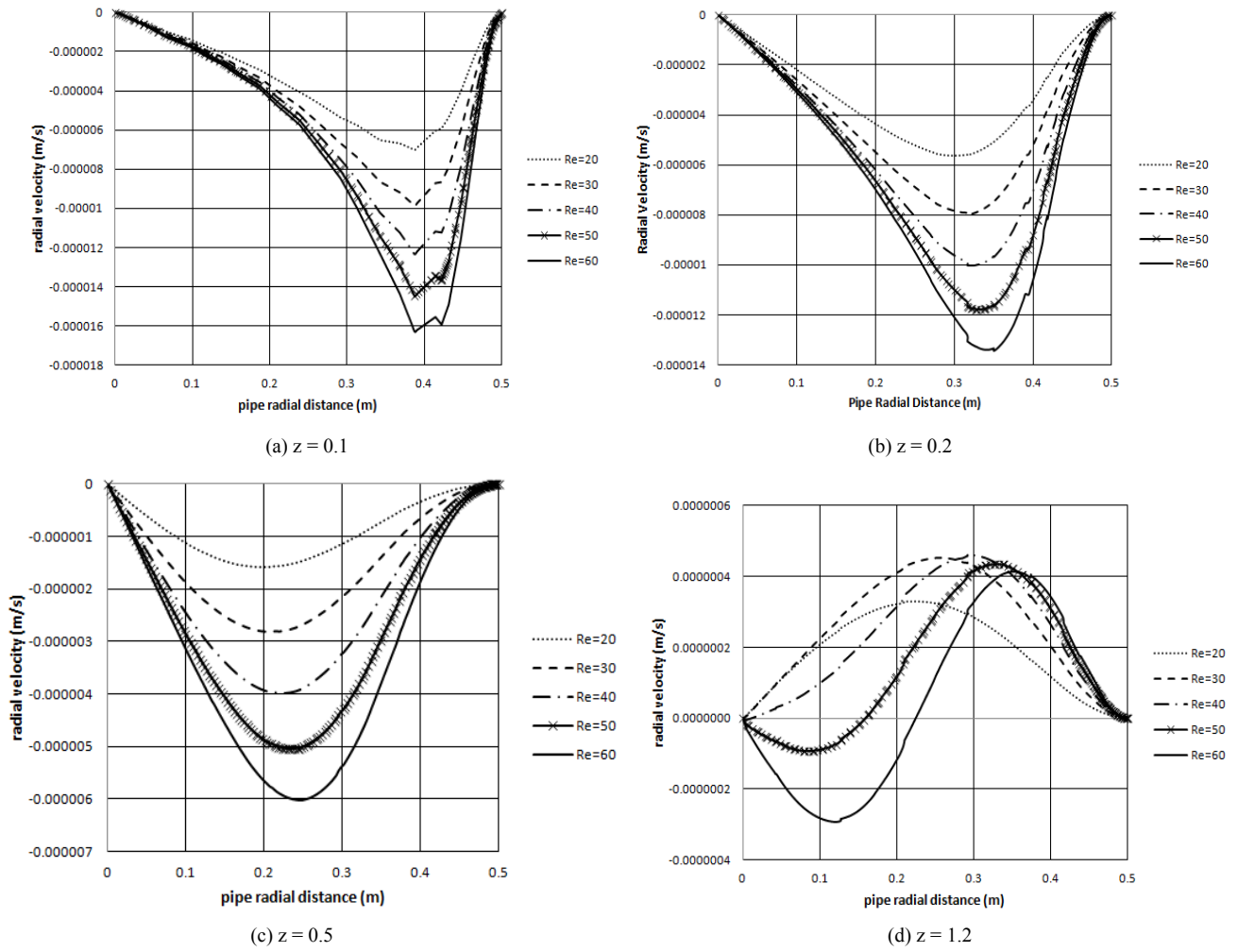
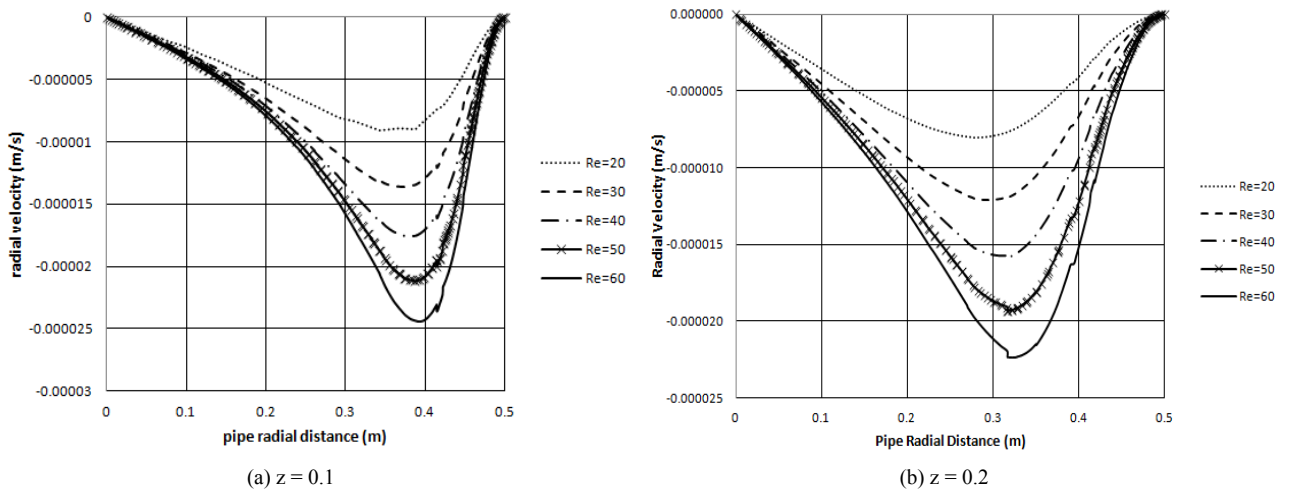


Figure 12.  $N = 2$  – Radial velocity



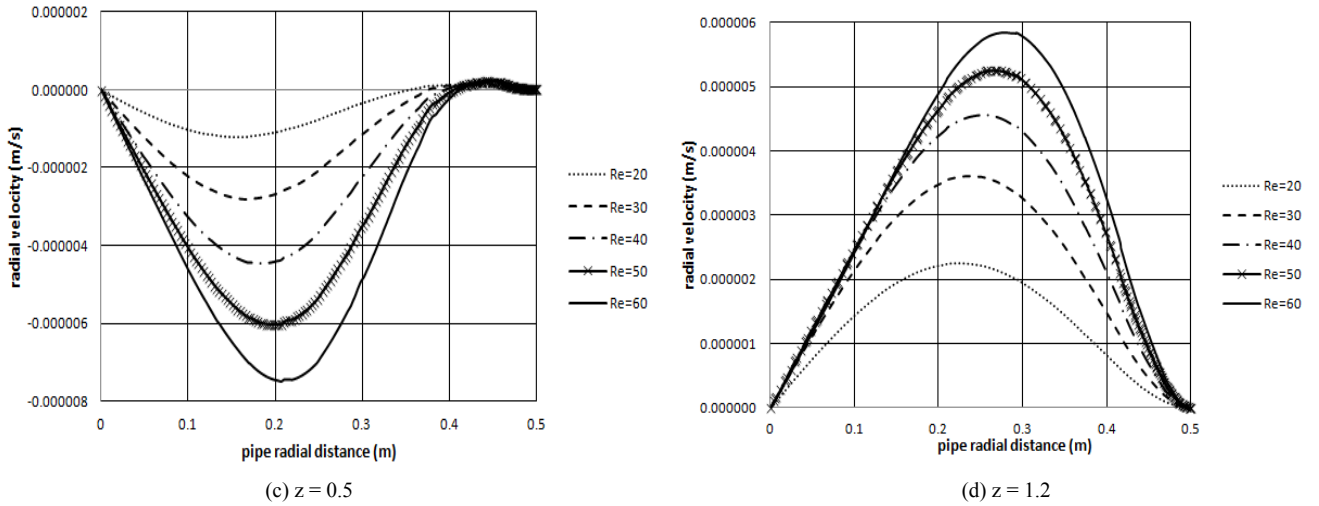


Figure 13. N = 4 – Radial velocity

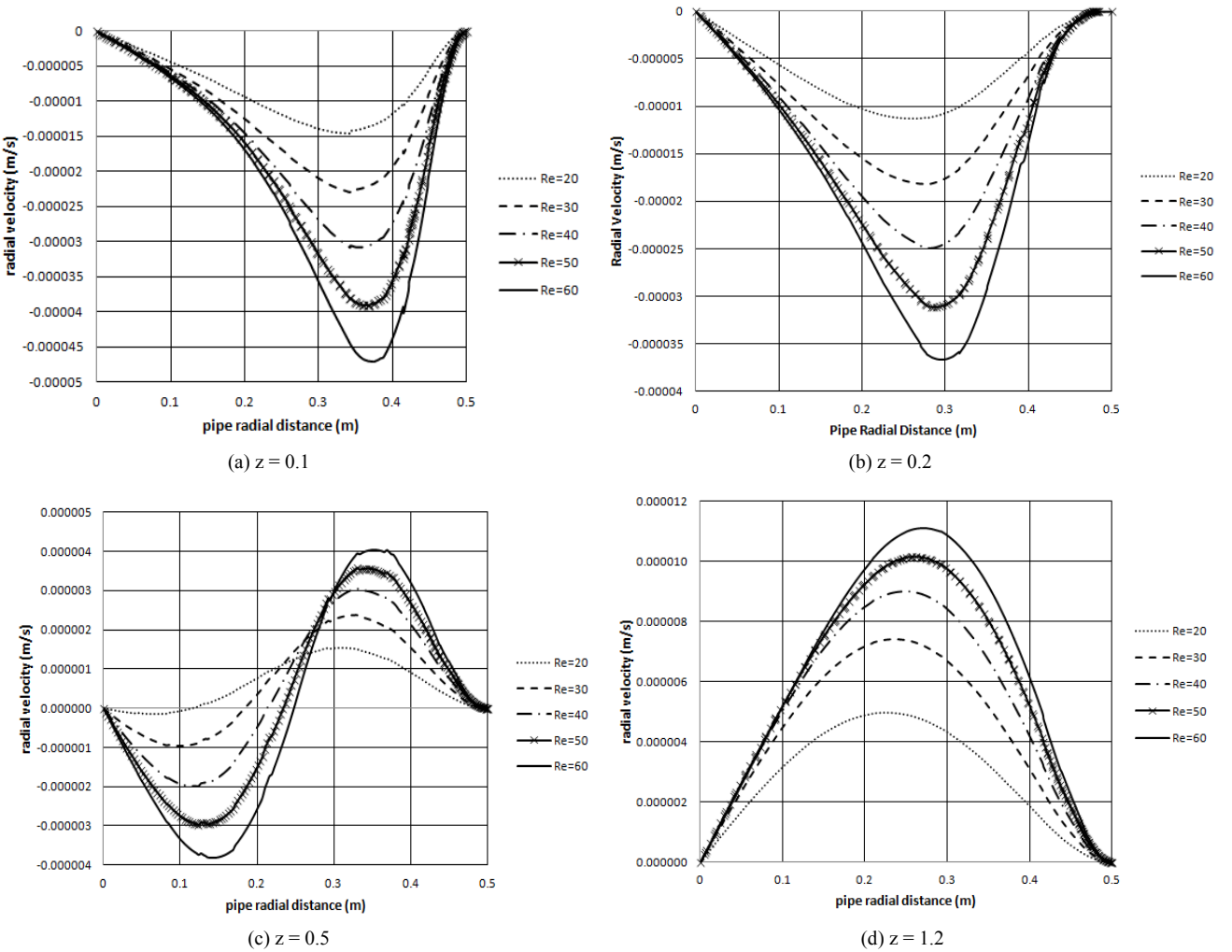


Figure 14. N = 6 – Radial velocity

### 5. Conclusions

The evolution of Reynolds number on the laminar flow in an axially rotating pipe has been investigated numerically using COMSOL Multiphysics software for various rotation

rates at different downstream sections. The conclusions can be summarized as follows:

- i. Flow reversal occurs downstream but near the pipe entrance and this trend is amplified with increase in rotation rates and Reynolds numbers. In the downstream

section of the pipe, the axial velocity approaches a parabolic profile and increases at the center with increased rotation rates and Reynolds numbers as the influence of flow reversal contributes to this acceleration at the center. For low Reynolds number, the tendency to approach a forced vortex condition increases and this condition is noticeable at a downstream section away from the pipe inlet.

ii. With increasing Reynolds number, the rotation rate to cause a forced vortex condition increases and as the swirl given to the flow decays further downstream the pipe, the rate of shear impacted upon the flow decreases, hence the flow experiences deceleration at axial locations downstream the pipe at higher rotation rates.

iii. In the downstream section of a rotating pipe, a transition from negative values for radial velocity to positive values is observed and this occurs with increasing rotation rates. The transition from being negative values to positive values shift upstream with increasing rotation rates. But further downstream the positive values become significant upon higher rotation rate.

The present results showed that the COMSOL Multiphysics software is a versatile tool in flow modeling as it gives an approximate solution to experimental, and other numerical simulation methods that pertain to flow of engineering interests especially pipe flow.

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## REFERENCES

- [1] Yamada, Y and Imao, S., 1980. Swirling flow in an axially rotating pipe. *Trans. Japan of Mechanical Engineers*, (in Japanese), Vol. 46, No.409, P.1662.
- [2] Lavan, Z., Nielsen, H., and Fejer A.A., 1969. Separation and flow reversal in swirling flows in circular ducts. *Phys. Fluids*, Vol. 12, No 9, p1747.
- [3] Pedly, T.J, 1969. On the stability of viscous flow in a rapidly rotating pipe, *J. Fluid Mech.*, vol. 35.No.1,p. 97.
- [4] Najib, H. M., Wolf, L., Lavan, Z. and Feger, A.A., 1969. On the Stability of the Flow in Rotating Pipes. *Aerospace Research Laboratories Report*, No.69-9176. P.1.
- [5] Torii, S and Yang, W.J, 1999, Secondary flow phenomena in an axially rotating flow passage with sudden expansion or contraction. *International Journal of Rotating Machinery*, 1999, Vol. 5 No 2, pp,117-122.
- [6] Yamada, Y and Imao, S., 1985. Turbulence and its structure of the flow in an axially rotating pipe. *Trans. Japan of Mechanical Engineers*, (in Japanese), Vol. 51, No.461, P.34.
- [7] Lei U., Lin M. J., Sheen., H.J and Lin C. M., 1994. Velocity measurements of the Laminar flow through a rotating straight pipe, *Phys. Fluids*, vol. 6, No 6.
- [8] Imao, S, Zhang, Q and Yamada, Y, 1989. The Laminar Flow in the developing region of a Rotating Pipe. *Japan Society of Mechanical Engineers International Journal*, Series II, Vol 32 Page 317-323.
- [9] Mizutani, M., Nishibori, K., Kikuyama, K and Murakami, M., 1987. Inlet Length for Laminar flow in an axially rotating pipe. *Turbomachinery*, vol. 15 No.3, p33.
- [10] Ayinde, T. F., 2010. A generalized relationship for swirl decay in laminar pipe flow, *Indian Academy of Sciences, Sadhana* Vol. 35, Part 2, pp.129-137.
- [11] Kikuyama, K., Nishibori, K., and Murakami, M.,1983. Development of three dimensional turbulent boundary layer in an axially rotating pipe, *trans. ASME, J. fluid Eng.*, Vol. 105, p. 154.
- [12] Siavash, D., Johan, V., Jan, D., and Bart., V., 2010. Computational fluid dynamic simulation of the membrane filtration module, 20th European Symposium on Computer Aided Process Engineering – ESCAPE20, Elsevier B.V.