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The Methods of a Problem Decision Navier-Stokes for the Incompressible Fluid with Viscosity

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Abstract One of the present problems in mathematics is the Navier-Stokes equation, which describes the motion of viscous Newtonian fluid and which is a basic of a hydrodynamics [1]. Therefore in this work we solve a nonstationary problem Navier-Stokes for incompressible fluid with viscosity.

Keywords Navier-Stokes, Problem, Incompressible, Conditional-smooth, Solution, Fluid, Flow, Viscosity, Convective the acceleration, Algorithm, Newton's potential, Limiting, Hypothesis, Beale-Kato-Majda

1. Introduction

If to designate components of vectors of speed and external force, as

$$\boldsymbol{v}(\boldsymbol{x},t)=[\upsilon_1(\boldsymbol{x},t),\upsilon_2(\boldsymbol{x},t),\upsilon_3(\boldsymbol{x},t)],$$

$$\boldsymbol{f}(\boldsymbol{x},t)=[f_1(\boldsymbol{x},t),f_2(\boldsymbol{x},t),f_3(\boldsymbol{x},t)],$$

that corresponding problem Navier-Stokes is represented in a kind

$$\frac{\partial\upsilon_i}{\partial t}+\sum_{j=1}^{3}\upsilon_j\frac{\partial\upsilon_i}{\partial x_j}=f_i-\frac{1}{\rho}\frac{\partial P}{\partial x_i}+\mu\Delta\upsilon_i, \quad (1.1)$$

$$\operatorname{div}v=0,\forall(x,t)\in T=R^3\times[0,T_0], \quad (1.2)$$

$$\upsilon_i\big|_{t=0}=\upsilon_{i0}(x_1,x_2,x_3),\ \forall(x_1,x_2,x_3)\in R^3, \quad (1.3)$$

$\mu>0$ is kinematic viscosity, ρ is density, Δ is Laplace operator. The additional equation is the condition incompressibility fluid (1.2). Unknown are speed v and pressure *P*.

The decision of many problems of theoretical and mathematical physics leads to use of various special weight space. In works [7, 8] for the first time have offered a method which gives solution of problem Navier-Stokes in $G^2_\lambda(D_0)$.

To answer the brought attention to the question, we offer the following method of the decision of a problem Navier-Stokes. For this purpose (1.1) we will transform to a kind

$$\upsilon_{it}+\theta_i=f_i-\frac{1}{\rho}P_{x_i}-\frac{1}{2}Q_{x_i}+\mu\Delta\upsilon_i,(i=\overline{1,3}), \quad (1.4)$$

$$\theta_i=\sum_{j=1}^{3}(\upsilon_j\upsilon_{ix_j}-\frac{1}{2}Q_{x_i}), \quad (1.5)$$

where

$$\begin{cases}\theta_i\big|_{t=0}=\theta_i^0(x_1,x_2,x_3),\forall(x_1,x_2,x_3)\in R^3,\\ Q(x_1,x_2,x_3,t)\equiv\sum_{i=1}^{3}\upsilon_i^2(x_1,x_2,x_3,t),\\ Q_{x_i}=2\sum_{j=1}^{3}\upsilon_j\upsilon_{jx_i};\ \ Q^0_{x_i}=2\sum_{j=1}^{3}\upsilon_{j0}\upsilon_{j0x_i},(i=\overline{1,3}),\end{cases}$$

without breaking equivalence of system (1.1) and (1.4), (1.5). The received systems (1.4), (1.5) contain unknown functions υ_i, θ_i and pressure *P*. Here θ_i^0 - known functions because are known $\upsilon_{j0},\upsilon_{j0x_i}$.

The developed method of the decision of systems (1.4) and (1.5), is connected with functions $\theta_i,(i=\overline{1,3})$, i.e.

A_1) $\operatorname{rot}\tilde\theta=0,\tilde\theta=(\theta_1,\theta_2,\theta_3);\operatorname{rot}v\neq0,$ or

A_2) $\operatorname{div}\tilde\theta=0,\ \operatorname{rot}v\neq0,$ or

A_3) θ_i is any functions if, accordingly, as necessary conditions, take place: a_{01}) $\operatorname{rot}\tilde\theta^0=0,\tilde\theta^0=(\theta^0_1,\theta^0_2,\theta^0_3),$

a_{02}) $\operatorname{div}\tilde\theta^0=0,$ a_{03}) $\tilde\theta^0$ is any functions.

The work purpose. The main object of this work the proof - existence and singleness the problem decision

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Navier-Stokes for an incompressible fluid with viscosity in cases (A_1)-(A_3).

Theoretical and practical value. The Received decisionson the basis of the developed analytical methods proves in the general applicability of the equations of Navier-Stokes. Thus it is proved that the system (1.1) has the single decision in cases (A_1)-(A_3). The received results prove to legality of the requirement [1].

Remarks: 1. The Beale-Kato-Majda regularity criterion, originally derived for solutions to the 3D Euler equations [2] and holds for solutions to the 3D equations Navier-Stokes [5] and at that is proved an inequality of a criterion Beale-Kato-Majda [5] for an this problem, and the criterion can be viewed as a continuation principle for strong solutions. A further generalization was presented in [7] where the regularity condition is expressed in terms of the time integrability.

Let's note that there are several criteria of a priori estimates. For instance, it is sufficient to prove an estimate [5]

$$\sup_{R^3} \int_0^{T_0} \left|\operatorname{rot} v(x_1,x_2,x_3,t)\right| dt \le M < \infty, \qquad (1.6)$$

where M is a constant as (1.6) for the proof aforesaid the statement.

2. In a case $0<\mu<1$ the current is considered with very small viscosity, i.e. in viscous liquids, when force a friction a very small, than forces of inertia [9, 10]. Here Reynolds's number is very great $(\mathrm{Re} \ge 2300)$ there is an border layer in which viscosity influence is concentrated. Therefore the analytical methods giving the decisions of a problem Navier-Stokes allow to reach full understanding of physics of turbulence [3, 4, 9, 10].

And in a case $0<\mu=\mu_0=\mathrm{const}<+\infty$ the current is considered with average size of viscosity [10]. Therefore in a case when convective acceleration is not equal to zero problems connected with methods of integration of the equations of Navier-Stokes in their general view are arisen.

Our problem does not include a derivation of an equation in a physical meaning, since there is a big amount of works reflecting these questions [3, 4, 6, 9-11].

2. Problem Navier-Stokes for Fluid with a Condition (A1)

In this paragraph in the subsequent points, at the specified restrictions on the entrance data, the strict substantiation of compatibility of systems (1.4), (1.5) will be given with very small viscosity $0<\mu<1$.

2.1. Research with a Condition (A_1)

Let functions $\theta_i^0, (i=\overline{1,3})$ satisfy to a condition (a_{01}). Then relatively θ_i we suppose a condition (A_1) and

$$\operatorname{div} f \ne 0, \qquad (2.1)$$

where from system (1.4) and (1.5), accordingly we will receive following systems

$$\upsilon_{it} + \theta_{x_i} + \frac{1}{2}Q_{x_i} = f_i - \frac{1}{\rho}P_{x_i} + \mu\Delta\upsilon_i, (i=\overline{1,3}), \qquad (2.2)$$

$$\theta_i = \theta_{x_i} : \theta_{x_i} = \sum_{j=1}^{3}(\upsilon_j\upsilon_{ix_j} - \frac{1}{2}Q_{x_i}), (i=\overline{1,3}). \qquad (2.3)$$

Theorem 1. Let conditions (1.2), (1.3), (A_1) and (2.1) are satisfied. Then systems (2.2) and (2.3) it is equivalent will be transformed to a kind

$$\begin{cases} \Delta J = -F_0, (F_0 \equiv -\sum_{i=1}^{3} f_{ix_i}), \\ \upsilon_{it} = f_i + \mu\Delta\upsilon_i - J_{x_i}, i=\overline{1,3}, \\ \Delta\theta = -\psi^0, \ (\psi^0 \equiv -\sum_{i=1}^{3}\psi_{ix_i}), \\ J = \dfrac{1}{4\pi}\int_{R^3} F_0(s_1,s_2,s_3,t)\dfrac{ds_1 ds_2 ds_3}{r}, \\ J(x_1,x_2,x_3,t) \equiv \dfrac{1}{\rho}P + \dfrac{1}{2}Q + \theta, \\ r = \sqrt{(x_1-s_1)^2 + (x_2-s_2)^2 + (x_3-s_3)^2}. \end{cases} \qquad (2.4)$$

Hence, the problem (1.1) - (1.3) has the unique decision which satisfies to a condition (1.2).

Proof. From system (2.2) it is visible, if the 1-equation (2.2, $i=1$) it is differentiated on x_1, 2-equation on x_2 (2.2, $i=2$), 3-equation on x_3 (2.2, $i=3$), and it is summarised

$$\sum_{i=1}^{3}\frac{\partial}{\partial x_i}[\upsilon_{it} + \theta_{x_i} + \frac{1}{2}Q_{x_i}] = \sum_{i=1}^{3}\frac{\partial}{\partial x_i}[f_i - \frac{1}{\rho}P_{x_i} + \mu\Delta\upsilon_i],$$

from here we will receive the equation of Puasson [11]

$$\Delta J = -F_0, \qquad (2.5)$$

as

$$\begin{cases} \operatorname{div} v = 0 : \dfrac{\partial}{\partial t}(\upsilon_{1x_1} + \upsilon_{2x_2} + \upsilon_{3x_3}) = 0, \\ \mu\sum_{i=1}^{3}\dfrac{\partial^2}{\partial x_i^2}(\upsilon_{1x_1} + \upsilon_{2x_2} + \upsilon_{3x_3}) = 0. \end{cases} \qquad (2.6)$$

At that it is proved

$$\begin{cases} J = \frac{1}{4\pi}\int\limits_{R^3} F_0(s_1,s_2,s_3;t)\frac{ds_1ds_2ds_3}{r}, \\ J_{x_i} = \frac{1}{4\pi}\int\limits_{R^3} \frac{\tau_i F_0(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t)}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}d\tau_1 d\tau_2 d\tau_3, (s_i - x_i = \tau_i, i=\overline{1,3}). \end{cases} \tag{2.7}$$

Algorithm when we will receive the equation of Puasson (2.5) for brevity we name «algorithm puassonization systems».

In work of Sobolev [11] it is specified that function (2.7) satisfies to the equation (2.5) and is called Newton's potential. Therefore, if J - the decision of the equation (2.5), then substituting

$$\frac{1}{\rho}P_{x_i} + \frac{1}{2}Q_{x_i} + \theta_{x_i} \equiv J_{x_i}, (i=\overline{1,3}), \tag{2.8}$$

in (2.2), we have

$$\upsilon_{it} = f_i + \mu\Delta\upsilon_i - J_i, (i=\overline{1,3}; J_{x_i} \equiv J_i), \tag{2.9}$$

i.e. system (2.2) it is equivalent by (2.9) will be transformed to a kind linear the nonuniform equation of heat conductivity. Here the equations (2.5), (2.9) is there are first and second equations of system (2.4).

From system (2.9), follows [7]:

$$\begin{aligned} \upsilon_i &= \frac{1}{8(\sqrt{\pi\mu t})^3}\int\limits_{R^3}\exp(-\frac{r^2}{4\mu t})\upsilon_{i0}(s_1,s_2,s_3)\times ds_1ds_2ds_3 + \frac{1}{8\sqrt{\pi^3}}\int\limits_0^t\int\limits_{R^3}\exp(-\frac{r^2}{4\mu(t-s)})\times \\ &\times\frac{1}{\sqrt{(\mu(t-s))^3}}[f_i(s_1,s_2,s_3,s) - J_i(s_1,s_2,s_3,s)]ds_1ds_2ds_3ds \equiv \\ &\equiv \frac{1}{\sqrt{\pi^3}}\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\upsilon_{i0}(x_1+2\tau_1\sqrt{\mu t}, x_2+2\tau_2\sqrt{\mu t}, x_3+2\tau_3\sqrt{\mu t})d\tau_1d\tau_2d\tau_3 + \\ &+\frac{1}{\sqrt{\pi^3}}\int\limits_0^t\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times\big[f_i(x_1+2\tau_1\sqrt{\mu(t-s)}, x_2+2\tau_2\sqrt{\mu(t-s)}, x_3+ \\ &+2\tau_3\sqrt{\mu(t-s)};s) - J_i(x_1+2\tau_1\sqrt{\mu(t-s)}, x_2+2\tau_2\sqrt{\mu(t-s)}, x_3+2\tau_3\sqrt{\mu(t-s)};s)\big]\times \\ &\times d\tau_1d\tau_2d\tau_3ds \equiv H_i(x_1,x_2,x_3,t), (s_i - x_i = 2\tau_i\sqrt{\mu t};\ s_i - x_i = 2\tau_i\sqrt{\mu(t-s)};\ i=\overline{1,3}). \end{aligned} \tag{2.10}$$

All H_i - is known functions and $\upsilon_{ix_j}, (i=\overline{1,3}, j=\overline{1,3})$ are defined from system (2.10)

$$\begin{aligned} \upsilon_{ix_j} &= \frac{1}{\sqrt{\pi^3}}\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\frac{\partial}{\partial x_j}\upsilon_{i0}(x_1+2\tau_1\sqrt{\mu t}, x_2+2\tau_2\sqrt{\mu t}, x_3+2\tau_3\sqrt{\mu t})\times \\ &\times d\tau_1d\tau_2d\tau_3 + \frac{1}{\sqrt{\pi^3}}\int\limits_0^t\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))[\frac{\partial}{\partial x_j}f_i(x_1+2\tau_1\sqrt{\mu(t-s)}, x_2+2\tau_2\sqrt{\mu(t-s)}, \\ &x_3+2\tau_3\sqrt{\mu(t-s)};s) - \frac{\partial}{\partial x_j}J_i(x_1+2\tau_1\sqrt{\mu(t-s)}, x_2+2\tau_2\sqrt{\mu(t-s)}, x_3+2\tau_3\times \\ &\times\sqrt{\mu(t-s)};s)\big]d\tau_1d\tau_2d\tau_3ds \equiv H_{ix_j}(x_1,x_2,x_3,t), (i=\overline{1,3}; j=\overline{1,3}). \end{aligned} \tag{2.11}$$

Then, on the basis of (2.3), (2.10) and (2.11), and their private derivatives on x_i, we find

$$\theta_{x_i} = \sum_{j=1}^{3}(H_j \cdot H_{ix_j} - H_j \cdot H_{jx_i}) \equiv \psi_i, i=\overline{1,3}. \quad (2.12)$$

As ψ_i - is known functions, hence from system (2.12) differentiating 1 equation on x_1 [(2.12): $i=1$], 2 equations on x_2 [(2.12): $i=2$], 3 equations on x_3 [(2.12): $i=3$], and summarising, we will receive

$$\Delta\theta = -\psi^0, (\psi^0 \equiv -\sum_{i=1}^{3}\psi_{ix_i}(x_1,x_2,x_3,t)), \quad (2.13)$$

at that

$$\theta \in C^2(T): \; \theta = \frac{1}{4\pi}\int_{R^3}\psi^0(s_1,s_2,s_3,t)\frac{ds_1 ds_2 ds_3}{r}.$$

The equation (2.13) is the third equation of system (2.4). Therefore, from the received results, taking into account (2.7), follows

$$\frac{1}{\rho}P = -\theta - \frac{1}{2}Q + \frac{1}{4\pi}\int_{R^3}F_0(s_1,s_2,s_3,t)\frac{ds_1 ds_2 ds_3}{r}, \quad (2.14)$$

i.e. functions υ_i, θ, P are defined from systems (2.10), (2.13), (2.14), where (2.14) - the equation of type of Bernoulli.

Singleness is obvious, as a method by contradiction from (2.10) singleness of the decision follows $\upsilon_i \in C^{3,0}(T)$. Results (2.10) with a condition ((A$_1$), (2.1)) are received where smoothness of functions is required only on x_i as the derivative of 1st order is in time has $t>0$. Then taking into account (2.10), (2.13), (2.14) and the system (2.4) has the single continuous decision.

Further, considering private derivatives of 1st order

$$\upsilon_{x_i} = \frac{\partial}{\partial x_i}\{H_i\}, i=\overline{1,3}, \quad (2.15)$$

and summarising (2.15) with taking into account (1.2), we have

$$\begin{cases} 0 = \sum_{i=1}^{3}\upsilon_{x_i} = \frac{1}{\sqrt{\pi^3}}\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\sum_{i=1}^{3}\frac{\partial}{\partial x_i}\upsilon_{i0}(x_1+2\tau_1\sqrt{\mu t}, x_2+2\tau_2\sqrt{\mu t}, x_3+2\tau_3\sqrt{\mu t})\times \\ \times d\tau_1 d\tau_2 d\tau_3 + \frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp[-(\tau_1^2+\tau_2^2+\tau_3^2)]\{-F_0[x_1+2\tau_1\sqrt{\mu(t-s)}, x_2+2\tau_2\sqrt{\mu(t-s)}, x_3+2\tau_3\times \\ \times\sqrt{\mu(t-s)}; s] - \Delta J[x_1+2\tau_1\sqrt{\mu(t-s)}, x_2+2\tau_2\sqrt{\mu(t-s)}, x_3+2\tau_3\sqrt{\mu(t-s)}; s]\}d\tau_1 d\tau_2 d\tau_3 ds = 0, \\ \sum_{i=1}^{3}\frac{\partial}{\partial x_i}\upsilon_{i0} = 0; \quad \sum_{i=1}^{3}\frac{\partial}{\partial x_i}J_i \equiv \sum_{i=1}^{3}\frac{\partial}{\partial x_i}J_{x_i} \equiv \Delta J, (J_i \equiv J_{x_i}), \end{cases}$$

as $\Delta J = -F_0$. Means, the system (2.10) satisfies to the equation (1.2).

2.2. Limitation of Functions $(\upsilon_1,\upsilon_2,\upsilon_3)$ in $G_\lambda^2(D_0)$ or $W_\lambda^2(D_0)$

I. The limiting case which we will consider concerns results of the theorem 1. Then the decision of system (1.1) is representing in the form of (2.10) with conditions (1.2), (1.3), (A$_1$), (2.1) and

$$\begin{cases} \forall(x_1,x_2,x_3,t)\in T; f_i: \sup\limits_T \frac{1}{\sqrt{\pi^3}}\int\limits_0^t\int\limits_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)\left|D^k f_i(l_1,l_2,l_3;s)\right| d\tau_1 d\tau_2 d\tau_3 ds \le \beta_1, \\ \sup\limits_T \frac{1}{\sqrt{\pi^3}}\int\limits_0^t\int\limits_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)\frac{1}{\sqrt{t-s}}(\sum\limits_{j=1}^3 |\tau_j| \times \left|f_{il_j}(l_1,l_2,l_3;s)\right|) d\tau_1 d\tau_2 d\tau_3 ds \le \beta_2, \\ (\sup\limits_{R^3}\int\limits_0^{T_0} \lambda(s)\left|f_i(x_1,x_2,x_3,s)\right|^2 ds)^{\frac{1}{2}} \le \beta_3;\ \ 0\le\lambda(t): \int\limits_0^{T_0}\lambda(t)\frac{1}{t}dt = q_0;\ \ \int\limits_0^{T_0}\lambda(t)dt = q_1, \\ J_{x_i}\equiv J_i: \sup\limits_T \frac{1}{\sqrt{\pi^3}}\int\limits_0^t\int\limits_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)\left|D^k J_i(l_1,l_2,l_3;s)\right| d\tau_1 d\tau_2 d\tau_3 ds \le \beta_4, \\ (\sup\limits_{R^3}\int\limits_0^{T_0} \lambda(s)\left|J_i(x_1,x_2,x_3,s)\right|^2 ds)^{\frac{1}{2}} \le \beta_5, (l_j = x_j + 2\tau_j\sqrt{\mu(t-s)};\ \ i=\overline{1,3}), \\ \sup\limits_T \frac{1}{\sqrt{\pi^3}}\int\limits_0^t\int\limits_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)\frac{1}{\sqrt{t-s}}(\sum\limits_{j=1}^3 |\tau_j| \times \left|J_{il_j}(l_1,l_2,l_3;s)\right|) d\tau_1 d\tau_2 d\tau_3 ds \le \beta_6, \\ \upsilon_{i0}: \sup\limits_{R^3}\left|D^k\upsilon_{i0}\right| \le \beta_7, (i,j=\overline{1,3};\ k=\overline{0,3}),\ \beta = \max\limits_{1\le i\le 7}\beta_i;\ \ \beta_0 = \beta(3\sqrt{\mu q_0}+2+2\sqrt{\mu q_1}). \end{cases} \tag{2.16}$$

Really, estimating (2.10) in $G_\lambda^2(D_0)$, we have [7]:

$$\begin{cases} \|v\|_{G_\lambda^2(D_0)} = \sum\limits_{i=1}^3 \|\upsilon_i\|_{\tilde{G}^2_{(\upsilon_i;\lambda)}(D_0)} \le 3[N_1+\beta_0] = M^*;\ \ D_0 = R^3\times(0,T_0), \\ \|\upsilon_i\|_{\tilde{G}^2_{(\upsilon_i;\lambda)}(D_0)} = \|\upsilon_i\|_{C^{3,0}(T)} + \|\upsilon_{it}\|_{L_\lambda^2}, (i=\overline{1,3}), \\ \|\upsilon_i\|_{C^{3,0}(T)} = \sum\limits_{0\le|k|\le 3}\left\|D^k\upsilon_i\right\|_{C(T)} \le N_1 = 60\beta, \\ \|\upsilon_{it}\|_{L_\lambda^2} = (\sup\limits_{R^3}\int\limits_0^{T_0}\lambda(t)\left|\upsilon_{it}(x_1,x_2,x_3,t)\right|^2 dt)^{\frac{1}{2}} \le \beta(3\sqrt{\mu q_0}+2+2\sqrt{\mu q_1}) = \beta_0, (i=\overline{1,3}), \\ \|\upsilon_i\|_{C(T)} \le 3\beta,\ (i=\overline{1,3}),\quad C^{3,3,3,0}(T)\equiv C^{3,0}(T), \\ k=0: D^0\upsilon_i\equiv\upsilon_i;\ k\ne 0: D^k\upsilon_i = \frac{\partial^{|k|}\upsilon_i}{\partial x_1^{\alpha_1}\partial x_2^{\alpha_2}\partial x_3^{\alpha_3}}, (|k| = \sum\limits_{i=1}^3\alpha_i; \alpha_i = \overline{0,3}). \end{cases} \tag{2.17}$$

Theorem 2. In the conditions of the theorem 1 and (2.16), (2.17) the problem (1.1) - (1.3) has the single decision in $G_\lambda^2(D_0)$.

II. Alternatively, we can consider, e.g., a class of suitable solutions constructed in $W_\lambda^2(D_0)$. As $\upsilon_{i0}\in C^3(R^3)$, the decision of problem Navier-Stokes (1.1) - (1.3) belongs in $G^2_\lambda(D_0)$. Then on the basis of lemma K. Friedrichs [13], it is the decision and belongs to space of weight $W^2_\lambda(D_0)$:

$$\|v\|_{W^2_\lambda} = \sum_{i=1}^{3} \|\upsilon_i\|_{\tilde{W}^2_{(\upsilon_i,\lambda)}}, \quad v = (\upsilon_1, \upsilon_2, \upsilon_3)$$

where

$$\begin{cases} \tilde{W}^2_{(\upsilon_i,\lambda)} = \{(x_1,x_2,x_3,t) \in D_0 : D^k\upsilon_i \in L^2(0,T_0), \upsilon_{it} \in L^2_\lambda(0,T_0)\}, i = \overline{1,3}, \\ \|\upsilon_i\|_{\tilde{W}^2_{(\upsilon_i,\lambda)}} = \{ \sum\limits_{0 \le |k| \le 3} \sup\limits_{R^3} \int\limits_0^{T_0} [D^k \upsilon_i(x_1,x_2,x_3,t)]^2 dt + \sup\limits_{R^3} \int\limits_0^{T_0} \lambda(t) |\upsilon_{it}(x_1,x_2,x_3,t)|^2 dt\}^{\frac{1}{2}}, \\ k = 0 : D^0\upsilon_i \equiv \upsilon_i;\ k \ne 0 : D^k\upsilon_i = \dfrac{\partial^{|k|}\upsilon_i}{\partial x_1^{\alpha_1} \partial x_2^{\alpha_2} \partial x_3^{\alpha_3}}, (|k| = \sum\limits_{i=1}^{3} \alpha_i; \alpha_i = \overline{0,3}). \end{cases}$$

The limiting case which we will consider concerns results of the theorem 1. Then the decision of system (1.1) is representing in the form of (2.10). Then estimating (2.10) in $W^2_\lambda(D_0)$, we have

$$\begin{cases} \|v\|_{W^2_\lambda(D_0)} \le 3[N_1\sqrt{T_0} + \beta_0] = M^*, \\ \|\upsilon_i\|_{\tilde{W}^2_{(\upsilon_i,\lambda)}} \le N_1\sqrt{T_0} + \beta_0, (N_1 = 60\beta; i = \overline{1,3}), \end{cases} \tag{2.18}$$

i.e. in the conditions of (1.2), (1.3), (A_1) and (2.16), (2.18) the problem (1.1) - (1.3) has a single decision in $W^2_\lambda(D_0)$.

Remarks:

1. From the received results follows that on the basis of the developed methods of equation Navier-Stokes it is led to the linear equations of a kind of heat conductivity with a condition of Koshi and for

$$\{(x_1,x_2,x_3,t) : 0 \le t \le T_0, -\infty < x_i < \infty, i = \overline{1,3}\}$$

in a class of the limited functions with smooth enough initial data at $t = 0$ is correctly put [11, 12]. Accordingly there is a unique, is conditional-smooth decision of problem Navier-Stokes in $G^2_\lambda(D_0)$ or $W^2_\lambda(D_0)$.

2. Definition 1. The generalised decision a problems (1.1)-(1.3), (A_1) in area D_0 we name any continuous in T equation decision (2.10). Therefore, the nonstationary problem of Navier-Stokes (1.1) - (1.3) has the single decision.

If the problem (1.1) - (1.3), (A_1) has the exact classical decision the generalised decision coincides with it [11].

2.3. Inequality Beale-Kato-Majda

Inequality Beale-Kato-Majda. The criterion can be viewed as a continuation principle for strong solutions.

On the basis of results of the theorem 1 the of decision a systems (1.1) it is presented in of a kind (2.10), where global existence of decisions is received in a class $G^2_\lambda(D_0)$ (or $W^2_\lambda(D_0)$) from the point of view of the initial data satisfying (2.10). It is pleasant that results of this theorem leads to such global classical decision Navier-Stokes, besides it is known that in [5] classical decision is received, if the criterion of Beale-Kato-Majda is executed. There are several criteria of a priori estimates. For instance, it is sufficient to prove an estimate (1.6).

Really at performance of conditions of the theorem 1 takes place

$$\begin{cases}\sup\limits_T \frac{1}{\sqrt{\pi^3}} \int\limits_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)\Big| \ \frac{\partial}{\partial x_2}\upsilon_{30}(x_1+2\tau_1\sqrt{\mu t}, x_2+2\tau_2\sqrt{\mu t}, x_3+2\tau_3\sqrt{\mu t}) - \\ -\frac{\partial}{\partial x_3}\upsilon_{20}(x_1+2\tau_1\sqrt{\mu t}, x_2+2\tau_2\sqrt{\mu t}, x_3+2\tau_3\sqrt{\mu t})\Big| d\tau_1 d\tau_2 d\tau_3 \le h_1^0, \\ \sup\limits_T \frac{1}{\sqrt{\pi^3}} \int\limits_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)\Big| \ \frac{\partial}{\partial x_3}\upsilon_{10}(x_1+2\tau_1\sqrt{\mu t}, x_2+2\tau_2\sqrt{\mu t}, x_3+2\tau_3\sqrt{\mu t}) - \\ -\frac{\partial}{\partial x_1}\upsilon_{30}(x_1+2\tau_1\sqrt{\mu t}, x_2+2\tau_2\sqrt{\mu t}, x_3+2\tau_3\sqrt{\mu t})\Big| d\tau_1 d\tau_2 d\tau_3 \le h_2^0, \\ \sup\limits_T \frac{1}{\sqrt{\pi^3}} \int\limits_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)\Big| \ \frac{\partial}{\partial x_1}\upsilon_{20}(x_1+2\tau_1\sqrt{\mu t}, x_2+2\tau_2\sqrt{\mu t}, x_3+2\tau_3\sqrt{\mu t}) - \\ -\frac{\partial}{\partial x_2}\upsilon_{10}(x_1+2\tau_1\sqrt{\mu t}, x_2+2\tau_2\sqrt{\mu t}, x_3+2\tau_3\sqrt{\mu t})\Big| d\tau_1 d\tau_2 d\tau_3 \le h_3^0, \\ \sup\limits_T \frac{1}{\sqrt{\pi^3}} \int\limits_0^t \int\limits_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)\Big| \ \frac{\partial}{\partial x_2}[f_3(x_1+2\tau_1\sqrt{\mu(t-s)}, x_2+2\tau_2\sqrt{\mu(t-s)}, x_3 + \\ +2\tau_3\sqrt{\mu(t-s)};s) - J_3(x_1+2\tau_1\sqrt{\mu(t-s)}, x_2+2\tau_2\sqrt{\mu(t-s)}, x_3+2\tau_3\sqrt{\mu(t-s)};s)] - \\ -\frac{\partial}{\partial x_3}[f_2(x_1+2\tau_1\sqrt{\mu(t-s)}, x_2+2\tau_2\sqrt{\mu(t-s)}, x_3+2\tau_3\sqrt{\mu(t-s)};s) - J_2(x_1 + \\ +2\tau_1\sqrt{\mu(t-s)}, x_2+2\tau_2\sqrt{\mu(t-s)}, x_3+2\tau_3\sqrt{\mu(t-s)};s)]\Big| d\tau_1 d\tau_2 d\tau_3 ds \le h_1, \\ \sup\limits_T \frac{1}{\sqrt{\pi^3}} \int\limits_0^t \int\limits_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)\Big| \ \frac{\partial}{\partial x_3}[f_1(x_1+2\tau_1\sqrt{\mu(t-s)}, x_2+2\tau_2\sqrt{\mu(t-s)}, x_3 + \\ +2\tau_3\sqrt{\mu(t-s)};s) - J_1(x_1+2\tau_1\sqrt{\mu(t-s)}, x_2+2\tau_2\sqrt{\mu(t-s)}, x_3+2\tau_3\sqrt{\mu(t-s)};s)] - \\ -\frac{\partial}{\partial x_1}[f_3(x_1+2\tau_1\sqrt{\mu(t-s)}, x_2+2\tau_2\sqrt{\mu(t-s)}, x_3+2\tau_3\sqrt{\mu(t-s)};s) - J_3(x_1+2\tau_1\sqrt{\mu(t-s)}, \\ x_2+2\tau_2\sqrt{\mu(t-s)}, x_3+2\tau_3\sqrt{\mu(t-s)};s)]\Big| d\tau_1 d\tau_2 d\tau_3 ds \le h_2, \\ \sup\limits_T \frac{1}{\sqrt{\pi^3}} \int\limits_0^t \int\limits_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)\Big| \ \frac{\partial}{\partial x_1}[f_2(x_1+2\tau_1\sqrt{\mu(t-s)}, x_2+2\tau_2\sqrt{\mu(t-s)}, x_3 + \\ +2\tau_3\sqrt{\mu(t-s)};s) - J_2(x_1+2\tau_1\sqrt{\mu(t-s)}, x_2+2\tau_2\sqrt{\mu(t-s)}, x_3+2\tau_3\sqrt{\mu(t-s)};s)] - \\ -\frac{\partial}{\partial x_2}[f_1(x_1+2\tau_1\sqrt{\mu(t-s)}, x_2+2\tau_2\sqrt{\mu(t-s)}, x_3+2\tau_3\sqrt{\mu(t-s)};s) - J_1(x_1+2\tau_1\sqrt{\mu(t-s)}, \\ x_2+2\tau_2\sqrt{\mu(t-s)}, x_3+2\tau_3\sqrt{\mu(t-s)};s)]\Big| d\tau_1 d\tau_2 d\tau_3 ds \le h_3; \ \sum\limits_{i=1}^3 (h_i^0 + h_i) = N_0. \end{cases}$$

Therefore we will receive an estimation of type Beale-Kato-Majda

$$\sup_{R^3} \int_0^{T_0} \left|\operatorname{rot} v(x_1,x_2,x_3,t)\right| dt \le [\sum_{i=1}^3 (h_i^0 + h_i)] T_0 = N_0 T_0 = M < \infty. \tag{2.19}$$

Let's note that the similar inequality turns out and in $W_{\lambda}^{2}(D_0)$. Really in conditions, when the decision of system (2.10) belongs in we will receive

$$\sup_{R^3}\int_0^{T_0}\left|\operatorname{rot}v(x_1,x_2,x_3,t)\right|^2 dt \le \sup_{R^3}\int_0^{T_0}\{\left|\upsilon_{3x_2}(x_1,x_2,x_3,s)-\upsilon_{2x_3}(x_1,x_2,x_3,s)\right|+$$
$$+\left|\upsilon_{1x_3}(x_1,x_2,x_3,s)-\upsilon_{3x_1}(x_1,x_2,x_3,s)\right|+\left|\upsilon_{2x_1}(x_1,x_2,x_3,s)-\upsilon_{1x_2}(x_1,x_2,x_3,s)\right|\}^2 ds \le \tag{2.20}$$
$$\le [\sum_{i=1}^{3}(h_i^0+h_i)]^2 T_0 = N_0^2 T_0 = M_0 < \infty.$$

3. Fluid with Average Viscosity with a Condition (A_2)

Area of the fluid with average viscosity where all inertial members contain in the equations of Navier-Stokes theoretically it is not investigated till now [10].

The decision method, from where follows of equations integration of Navier-Stokes in a case (A_2), is a major factor of this point. The developed method of the decision of system (1.1) is connected with θ_i, where these functions will transform (1.1) to systems (1.4), (1.5) with conditions (a_{02}) and

$$(1.3):\quad \upsilon_i\big|_{t=0}=0,(i=\overline{1,3}),(\forall(x_1,x_2,x_3)\in R^3:\upsilon_{i0}(x_1,x_2,x_3)\equiv 0), \tag{1.3*}$$

$$\begin{cases}\theta_i\big|_{t=0}=0,\forall(x_1,x_2,x_3)\in R^3,(i=\overline{1,3}),\\ 0<\mu=\mu_0=\text{const}<\infty;\ \ \operatorname{div}f\ne 0,\end{cases} \tag{3.1}$$

where the current is considered with average size of viscosity.

Theorem 3. Systems (1.4), (1.5) it is equivalent will be transformed to a kind

$$\begin{cases}\Delta J_0=-F_0,(J_0\equiv\dfrac{1}{\rho}P+\dfrac{1}{2}Q;F_0=-\sum\limits_{i=1}^{3}f_{ix_i}),\\ \upsilon_{it}=f_i+\mu\Delta\upsilon_i-J_{0x_i}-\theta_i,(i=\overline{1,3}),\\ \theta_i=D_i[\theta_1,\theta_2,\theta_3],(i=\overline{1,3}),\\ \dfrac{1}{\rho}P=-\dfrac{1}{2}Q+\dfrac{1}{4\pi}\int\limits_{R^3}F_0(s_1,s_2,s_3,t)\dfrac{ds_1ds_2ds_3}{r},(r=\sqrt{(x_1-s_1)^2+(x_2-s_2)^2+(x_3-s_3)^2}),\end{cases} \tag{3.2}$$

when conditions (1.2), (1.3)*, (3.1), (A_2) are satisfied. Hence, the nonstationary problem of Navier-Stokes (1.1)-(1.3)* has the single continuous decision.

Proof. Really, from system (1.4), considering conditions (1.2), (1.3)*, (3.1) and having entered «algorithm puassonization systems», i.e. differentiating the equations of system (1.4) accordingly on x_i and, then summarising, we have the equation

$$\begin{cases}\Delta J_0=-F_0,\quad (J_0=\dfrac{1}{4\pi}\int\limits_{R^3}F_0(s_1,s_2,s_3,t)\dfrac{ds_1ds_2ds_3}{r}),\\ J_{0x_i}=\dfrac{1}{4\pi}\int\limits_{R^3}\dfrac{\tau_iF_0(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t)}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}d\tau_1d\tau_2d\tau_3,(s_i-x_i=\tau_i;i=\overline{1,3}).\end{cases} \tag{3.3}$$

If J_0 - the decision of the equation (3.3), then substituting

$$\frac{1}{\rho}P_{x_i}+\frac{1}{2}Q_{x_i}\equiv J_{0x_i},(i=\overline{1,3}),$$

in system (1.4), we have

$$\upsilon_{it} = f_i + \mu\Delta\upsilon_i - J_i - \theta_i, (J_{0x_i} \equiv J_i, i = \overline{1,3}). \tag{3.4}$$

The decision of a problem (1.1)-(1.3)* is represented in a kind

$$\begin{cases} \upsilon_i = H_i^0 - \dfrac{1}{8\sqrt{\pi^3}}\int_0^t\int_{R^3} \exp(-\dfrac{r^2}{4\mu(t-\tau)})\dfrac{1}{(\sqrt{\mu(t-\tau)})^3}\theta_i(s_1,s_2,s_3,\tau)ds_1ds_2ds_3d\tau \equiv \\ \equiv (\Phi_i\theta_i)(x_1,x_2,x_3,t),(i=\overline{1,3}), \\ H_i^0(x_1,x_2,x_3,t) \equiv \dfrac{1}{8\sqrt{\pi^3}}\int_0^t\int_{R^3} \exp(-\dfrac{r^2}{4\mu(t-\tau)})\dfrac{1}{(\sqrt{\mu(t-\tau)})^3}[f_i(s_1,s_2,s_3,\tau) - \\ -J_i(s_1,s_2,s_3,\tau)]ds_1ds_2ds_3d\tau, \end{cases} \tag{3.5}$$

where concerning functions $\theta_i, (i = 1,2,3)$, we will receive

$$\begin{aligned} \theta_i = \sum_{j=1}^{3}\{[H_j^0 - \frac{1}{\sqrt{\pi}^3}\int_0^t\int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\theta_j(x_1+2\tau_1\sqrt{\mu s}, x_2+2\tau_2\sqrt{\mu s}, x_3+2\tau_3\times \\ \times\sqrt{\mu s}; t-s)d\tau_1d\tau_2d\tau_3ds]\times[H_{ix_j}^0 - \frac{1}{\sqrt{\pi}^3}\int_0^t\int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\frac{\tau_j}{\sqrt{\mu s}}\theta_i(x_1+2\tau_1\sqrt{\mu s}, \\ x_2+2\tau_2\sqrt{\mu s}, x_3+2\tau_3\sqrt{\mu s}; t-s)d\tau_1d\tau_2d\tau_3ds] - [H_j^0 - \frac{1}{\sqrt{\pi}^3}\int_0^t\int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times \\ \times\theta_j(x_1+2\tau_1\sqrt{\mu s}, x_2+2\tau_2\sqrt{\mu s}, x_3+2\tau_3\sqrt{\mu s}; t-s)d\tau_1d\tau_2d\tau_3ds]\times[H_{jx_i}^0 - \frac{1}{\sqrt{\pi}^3}\int_0^t\int_{R^3}\frac{\tau_i}{\sqrt{\mu s}}\times \\ \times\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\theta_j(x_1+2\tau_1\sqrt{\mu s}, x_2+2\tau_2\sqrt{\mu s}, x_3+2\tau_3\sqrt{\mu s}; t-s)d\tau_1d\tau_2d\tau_3ds]\} \equiv \\ \equiv D_i[\theta_1,\theta_2,\theta_3],\ (s_i - x_i = 2\tau_i\sqrt{\mu(t-\tau)};\ t-\tau = s;\ i=\overline{1,3}). \end{aligned} \tag{3.6}$$

Here for example, private derivatives of functions υ_i are defined:

$$\begin{cases} \upsilon_{ix_j} = H_{ix_j}^0(x_1,x_2,x_3,t) - \dfrac{1}{8\sqrt{\pi^3}}\int_0^t\int_{R^3}\dfrac{-(x_j-s_j)}{2\mu(t-\tau)}\exp(-\dfrac{r^2}{4\mu(t-\tau)})\dfrac{1}{(\sqrt{\mu(t-\tau)})^3}\times \\ \times\theta_i(s_1,s_2,s_3,\tau)ds_1ds_2ds_3d\tau = H_{ix_j}^0 - \dfrac{1}{\sqrt{\pi}^3}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\dfrac{\tau_j}{\sqrt{\mu s}}\theta_i(x_1 + \\ +2\tau_1\sqrt{\mu s}, x_2+2\tau_2\sqrt{\mu s}, x_3+2\tau_3\sqrt{\mu s}; t-s)d\tau_1d\tau_2d\tau_3ds, (i,j=\overline{1,3}), \\ H_{ix_j}^0 \equiv \dfrac{1}{\sqrt{\pi}^3}\int_0^t\int_{R^3}\dfrac{\tau_j}{\sqrt{\mu s}}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))[f_i(x_1+2\tau_1\sqrt{\mu s}, x_2+2\tau_2\sqrt{\mu s}, x_3+2\tau_3\times \\ \times\sqrt{\mu s}; t-s) - J_i(x_1+2\tau_1\sqrt{\mu s}, x_2+2\tau_2\sqrt{\mu s}, x_3+2\tau_3\sqrt{\mu s}; t-s)]d\tau_1d\tau_2d\tau_3ds, \\ s_i - x_i = 2\tau_i\sqrt{\mu(t-\tau)};\ t-\tau = s,\ (i,j=\overline{1,3}). \end{cases} \tag{3.7}$$

Here (3.6) – system of the nonlinear integrated equations of Volterr-Abel of the second sort concerning θ_i on a variable *t*. Therefore, if operators: D_i compressing with a compression factor d_i,

$$\begin{cases} D_i : \sum_{i=1}^{3} d_i \le d = 12(\sqrt{\mu})^{-1}\gamma_4 < 1, \\ 0 < 144\gamma_4^2 < \mu = \mu_0 < \infty, \\ \gamma_4 = \gamma_0\gamma_1\sqrt{T_0} + \gamma_3 T_0 + r_1\gamma_1\sqrt{T_0^3}; \quad \gamma_1 = \sqrt[4]{2^5}, \\ d_i = 4(\sqrt{\mu})^{-1}[\gamma_0\gamma_1\sqrt{T_0} + \gamma_3 T_0 + r_1\gamma_1\sqrt{T_0^3}] \le 4(\sqrt{\mu})^{-1}\gamma_4 < 1, (i = \overline{1,3}), \\ S_{r_1}(\theta_i^0 = 0) = \{\theta_i : |\theta_i| \le r_1, \forall(x_1, x_2, x_3, t) \in T\}, \\ \frac{1}{\sqrt{\pi^3}} \sup_{[0,T_0]} \int_0^t \int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2)) d\tau_1 d\tau_2 d\tau_3 ds \le T_0, \\ \frac{1}{\sqrt{\pi^3}} \sup_{[0,T_0]} \int_0^t \int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2)) \frac{|\tau_i|}{\sqrt{\mu s}} d\tau_1 d\tau_2 d\tau_3 ds \le (\sqrt{\mu})^{-1}\gamma_1\sqrt{T_0}, \\ |H_i^0| \le \gamma_0; \quad |H_{ix_j}^0| \le \frac{1}{\sqrt{\pi^3}} \sup_T \int_0^t \int_{R^3} \frac{1}{\sqrt{\mu s}} |\tau_j| \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2))[|f_i(x_1 + \\ +2\tau_1\sqrt{\mu s}, x_2 + 2\tau_2\sqrt{\mu s}, x_3 + 2\tau_3\sqrt{\mu s}; t - s)| + |J_i(x_1 + 2\tau_1\sqrt{\mu s}, x_2 + 2\tau_2\sqrt{\mu s}, x_3 + \\ +2\tau_3\sqrt{\mu s}; t - s)|] d\tau_1 d\tau_2 d\tau_3 ds \le (\sqrt{\mu})^{-1}\gamma_3, (i, j = \overline{1,3}), \end{cases} \tag{3.8}$$

then the system (3.6) is solvable at $C^{2,0}(T)$. Hence the solution of this system we can find on the basis of Picard's method:

$$\theta_{i,n+1} \equiv D_i[\theta_{1,n}, \theta_{2,n}, \theta_{3,n}], (n = 0, 1, \dots; i = \overline{1,3}),$$

where $0 = \theta_{1,0}, 0 = \theta_{2,0}, 0 = \theta_{3,0}$ – initial estimates. Thus we will receive

$$\theta_{i,n+1} \xrightarrow[n \to \infty]{d<1} \theta_i \equiv \omega_i, \forall (x_1, x_2, x_3, t) \in T. \tag{3.9}$$

Then according to results of the theorem 3, functions $\upsilon_i, i = 1, 2, 3$ are defined from system (3.5)

$$\begin{cases} \upsilon_i = \frac{1}{\sqrt{\pi}^3} \int_0^t \int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2))[f_i(x_1 + 2\tau_1\sqrt{\mu(t-\tau)}, x_2 + 2\tau_2\sqrt{\mu(t-\tau)}, x_3 + \\ +2\tau_3\sqrt{\mu(t-\tau)}; \tau) - J_i(x_1 + 2\tau_1\sqrt{\mu(t-\tau)}, x_2 + 2\tau_2\sqrt{\mu(t-\tau)}, x_3 + 2\tau_3\sqrt{\mu(t-\tau)}; \tau)] \times \\ \times d\tau_1 d\tau_2 d\tau_3 d\tau - \frac{1}{\sqrt{\pi}^3} \int_0^t \int_{R^3} \exp(-(\tau_1^2 + \tau_2^2 + \tau_3^2))\omega_i(x_1 + 2\tau_1\sqrt{\mu(t-\tau)}, x_2 + 2\tau_2\sqrt{\mu(t-\tau)}, \\ x_3 + 2\tau_3\sqrt{\mu(t-\tau)}; \tau) d\tau_1 d\tau_2 d\tau_3 d\tau \equiv H_i(x_1, x_2, x_3, t), (s_i - x_i = 2\tau_i\sqrt{\mu(t-\tau)}; i = \overline{1,3}), \end{cases} \tag{3.5*}$$

here ω_i, H_i - known functions and

$$\begin{cases} |\upsilon_i| = \dfrac{1}{\sqrt{\pi^3}}\sup\limits_T \int\limits_0^t \int\limits_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))[|f_i(x_1+2\tau_1\sqrt{\mu(t-\tau)}, x_2+2\tau_2\sqrt{\mu(t-\tau)}, \\ x_3+2\tau_3\sqrt{\mu(t-\tau)};\tau)| + |J_i(x_1+2\tau_1\sqrt{\mu(t-\tau)}, x_2+2\tau_2\sqrt{\mu(t-\tau)}, x_3+2\tau_3\sqrt{\mu(t-\tau)};\tau)|]\times \\ \times d\tau_1 d\tau_2 d\tau_3 d\tau + \dfrac{1}{\sqrt{\pi^3}}\sup\limits_T \int\limits_0^t \int\limits_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))|\omega_i(x_1+2\tau_1\sqrt{\mu(t-\tau)}, x_2+2\tau_2\sqrt{\mu(t-\tau)}, \\ x_3+2\tau_3\sqrt{\mu(t-\tau)};\tau)| d\tau_1 d\tau_2 d\tau_3 d\tau \le \bar\beta_1+\bar\beta_2+\bar\beta_3 \le 3\beta,\ \forall(x_1,x_2,x_3,t)\in T, (i=\overline{1,3}), \\ \forall(x_1,x_2,x_3,t)\in T; f_i:\ \sup\limits_T \dfrac{1}{\sqrt{\pi^3}} \int\limits_0^t \int\limits_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)|f_i(l_1,l_2,l_3;\tau)| d\tau_1 d\tau_2 d\tau_3 d\tau \le \bar\beta_1, \\ J_{0x_i} \equiv J_i: \sup\limits_T \dfrac{1}{\sqrt{\pi^3}} \int\limits_0^t \int\limits_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)|J_i(l_1,l_2,l_3;\tau)| d\tau_1 d\tau_2 d\tau_3 d\tau \le \bar\beta_2, \\ \omega_i: \sup\limits_T \dfrac{1}{\sqrt{\pi^3}} \int\limits_0^t \int\limits_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)|\omega_i(l_1,l_2,l_3;\tau)| d\tau_1 d\tau_2 d\tau_3 d\tau \le \bar\beta_3, (i=\overline{1,3}), \\ l_i = x_i + 2\tau_i\sqrt{\mu(t-\tau)};\ \beta = \max\limits_{1\le i\le 3} \bar\beta_i, \end{cases}$$

i.e.

$$\|\upsilon_i\|_{C(T)} \le 3\beta, (i=\overline{1,3}).$$

Thus the equation (3.5) * satisfies conditions (1.2).

Really considering private derivatives of 1st order the equations (3.5)* and summing, we have:

$$\sum_{i=1}^{3} \upsilon_{ix_i} = \sum_{i=1}^{3} H_{ix_i} = 0,$$

as

$$\begin{cases} \operatorname{div}\tilde\theta = 0, (\tilde\theta = (\omega_1,\omega_2,\omega_3); \theta_i \equiv \omega_i), \\ \Delta J_0 = -F_0, (J_{0x_i} \equiv J_i; F_0 = -\sum\limits_{i=1}^{3} f_{ix_i}). \end{cases}$$

From the received results, on the basis of (3.3) follows

$$\frac{1}{\rho}P = -\frac{1}{2}Q + \frac{1}{4\pi}\int\limits_{R^3} \frac{F_0(s_1,s_2,s_3,t)ds_1 ds_2 ds_3}{r}. \tag{3.10}$$

Then according to results of the theorem 3, functions $\upsilon_i, i=1,2,3$ are defined from system (3.5)* and satisfies the equation (1.2). The theorem is proved.

Remark 1. For a problem Navier-Stokes (1.1)-(1.3)*, (A_2), are proved: existence and singleness of the decision in the field of T, and we will notice that the received decision (3.5)* continuously depends on the initial data.

Let's notice that the generalised decision a problems (1.1)-(1.3)*, (A_2) in area D_0 we name any continuous in T equation decision (3.5)*, when $0 < \mu = \mu_0$.

As the problem (1.1)-(1.3)*, (A_2) has the single usual decision the generalised decision coincides with this decision [11].

4. Fluid with Very Small Viscosity with a Condition (A_3)

From the received results of this point follows that system Navier-Stokes (1.1) in the conditions of (1.2), (1.3), (A_3) can have the analytical unique, is conditional-smooth decision. At least, such decision answers a mathematical question, and possibility to construct the decision on a problem Navier-Stokes (1.1)-(1.3) for an incompressible liquid with viscosity with a condition (A_3).

4.1. Fluid with Viscosity with a Condition (A_3)

I. Let υ_{i0} initial components of a vector of speed ν at the moment of time $t=0$ it is set in a kind (1.3):

$$\upsilon_i\big|_{t=0} = \upsilon_{i0}(x_1,x_2,x_3) \equiv \lambda_i V_0(x_1,x_2,x_3), i=\overline{1,3}, \tag{4.1}$$

where $0<\lambda_i -$ known constants. Then speed components ν are defined by a rule

$$\begin{cases} \upsilon_i = \lambda_i V(x_1,x_2,x_3,t), i=\overline{1,3}, \\ V\big|_{t=0} = V_0(x_1,x_2,x_3), \\ \operatorname{div}\nu = 0: \sum_{j=1}^{3} \lambda_j V_{x_j} = 0, \\ \operatorname{div} f \neq 0, f=(f_1,f_2,f_3); \sum_{j=1}^{3} \upsilon_j \upsilon_{ix_j} = \lambda_i V \sum_{j=1}^{3} \lambda_j V_{x_j} = 0. \end{cases} \tag{4.2}$$

Hence, the system (1.1) will be transformed to a kind

$$\lambda_i V_t = (f_i - \frac{1}{\rho} P_{x_i}) + \mu\lambda_i \Delta V, i=\overline{1,3}, \tag{4.3}$$

where $V(x_1,x_2,x_3,t)$ new unknown function which defines the decision on problem Navier-Stokes. Here substitution (4.2) it is equivalent will transform system (1.1) to the linear nonuniform equation of heat conductivity of a kind (4.3).

From system (4.3), considering conditions (4.1), (4.2), and having entered «algorithm puassonization systems», i.e. differentiating the equations of system (4.3) accordingly on x_i and, then summarising, we have the equation

$$\begin{cases} \frac{1}{\rho}\Delta P = -F_0, (F_0 \equiv -\sum_{i=1}^{3} f_{ix_i}(x_1,x_2,x_3,t)), \\ \frac{1}{\rho} P = \frac{1}{4\pi} \int_{R^3} F_0(s_1,s_2,s_3,t) \frac{ds_1 ds_2 ds_3}{r}, \\ \frac{1}{\rho} P_{x_i} = \frac{1}{4\pi} \int_{R^3} F_0(x_1+\tau_1, x_2+\tau_2, x_3+\tau_3; t) \frac{\tau_i d\tau_1 d\tau_2 d\tau_3}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}, (s_i - x_i = \tau_i, i=\overline{1,3}). \end{cases} \tag{4.4}$$

Then the decision of a problem (4.2), (4.3) is represented in a kind

$$V = \frac{1}{8\sqrt{(\mu\pi t)^3}} \int_{R^3} \exp(-\frac{r^2}{4\mu t}) V_0(s_1,s_2,s_3) ds_1 ds_2 ds_3 + \frac{1}{8\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-\frac{r^2}{4\mu(t-s)}) \times$$

$$\times \frac{1}{(\sqrt{\mu(t-s)})^3} \Phi_0(s_1,s_2,s_3,s) ds_1 ds_2 ds_3 ds = \frac{1}{\sqrt{\pi^3}} \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)) V_0(x_1 +$$

$$+2\tau_1\sqrt{\mu t},x_2+2\tau_2\sqrt{\mu t},x_3+2\tau_3\sqrt{\mu t})d\tau_1 d\tau_2 d\tau_3+\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\times$$

$$\times\Phi_0(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2 d\tau_3 ds\equiv \tag{4.5}$$

$$\equiv H(x_1,x_2,x_3,t),\forall(x_1,x_2,x_3,t)\in T,\ (s_i-x_i=2\tau_i\sqrt{\mu t};s_i-x_i=2\tau_i\sqrt{\mu(t-s)};i=\overline{1,3}),$$

here H - known function and

$$\begin{cases}\sum_{i=1}^{3}\lambda_i H_{x_i}=0,\\ (\lambda_1)^{-1}(f_1-\rho^{-1}P_{x_1})=(\lambda_2)^{-1}(f_2-\rho^{-1}P_{x_2})=(\lambda_3)^{-1}(f_3-\rho^{-1}P_{x_3})\equiv\Phi_0(x_1,x_2,x_3,t).\end{cases} \tag{4.6}$$

From the received results follows that functions υ_i are defined on the basis of (4.2), i.e.

$$\upsilon_i=\lambda_i H(x_1,x_2,x_3,t),i=\overline{1,3}. \tag{4.7}$$

Remark 2. Further, considering private derivatives of 1st order systems (4.7) and summing up with acceptance in attention (1.2), (4.2) we have, that the system (4.7) satisfies to a condition (1.2).

II. Let's notice that, as $\upsilon_{i0}\in C^3(R^3)$, the decision of problem Navier-Stokes (1.1), (1.2), (4.1) belongs in $v\in W_\lambda^2(D_0)$. For this purpose it is enough to show function accessories V in $\tilde{W}_\lambda^2(D_0)$:

$$\begin{cases}\|V\|_{\tilde{W}_\lambda^2}=\{\sum_{0\le|k|\le 3}\sup_{R^3}\int_0^{T_0}[D^kV(x_1,x_2,x_3,t)]^2dt+\sup_{R^3}\int_0^{T_0}\lambda(t)|V_t(x_1,x_2,x_3,t)|^2dt\}^{\frac{1}{2}},\\ k=0:D^0V\equiv V;k\ne 0:D^kV=\dfrac{\partial^{|k|}V}{\partial x_1^{\alpha_1}\partial x_2^{\alpha_2}\partial x_3^{\alpha_3}},|k|=\sum_{i=1}^{3}\alpha_i,(\alpha_i=\overline{0,3}).\end{cases} \tag{4.8}$$

Really, if

$$\begin{cases}\Phi_0:\sup_T\dfrac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\left|D^k\Phi_0(l_1,l_2,l_3;s)\right|d\tau_1d\tau_2d\tau_3ds\le\beta_1,\\ \sup_T\dfrac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\dfrac{1}{\sqrt{t-s}}(\sum_{j=1}^{3}|\tau_j|\times|\ \Phi_{0l_j^k}^{(k)}(l_1,l_2,l_3;s)|)d\tau_1d\tau_2d\tau_3ds\le\beta_2,\\ l_j=x_j+2\tau_j\sqrt{\mu(t-s)},(j=\overline{1,3}),\ (\sup_{R^3}\int_0^{T_0}\lambda(s)|\Phi_0(x_1,x_2,x_3,s)|^2ds)^{\frac{1}{2}}\le\beta_3,\\ V_0:\sup_{R^3}\left|D^kV_0\right|\le\beta_4,(k=\overline{0,3}),\ 0\le\lambda:\int_0^{T_0}\dfrac{1}{t}\lambda(t)dt=q_0;\ \int_0^{T_0}\lambda(t)dt=q_1,\\ \beta=\max_{1\le i\le 4}\beta_i;\ \beta_0=\beta(3\sqrt{\mu q_0}+1+\sqrt{\mu q_1}),\end{cases} \tag{4.9}$$

that

$$\|V\|_{\tilde{W}_{\lambda}^{2}(D_0)} \le N_*\sqrt{T_0} + \beta_0 = M^*, (N_* = 40\beta). \quad (4.10)$$

Therefore in the conditions of (4.2), (4.6) and (4.9) the equation (4.5) has a single decision in $\tilde{W}_{\lambda}^{2}(D_0)$.

Theorem 4. In the conditions of (1.2), (4.1), (A_3), (4.2), (4.6) and (4.9) the problem (1.1), (1.2), (4.1) has a single decision in $W_{\lambda}^{2}(D_0)$, which is defined by a rule (4.7).

Remark 3. At performance of conditions of remark 2 and

$$\begin{cases}
\sup\limits_{T} \dfrac{1}{\sqrt{\pi^3}} \int\limits_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)) \Big| \lambda_3 \dfrac{\partial}{\partial x_2} V_0(x_1+2\tau_1\sqrt{\mu t}, x_2+2\tau_2\sqrt{\mu t}, x_3+ \\
+2\tau_3\sqrt{\mu t}) - \lambda_2 \dfrac{\partial}{\partial x_3} V_0(x_1+2\tau_1\sqrt{\mu t}, x_2+2\tau_2\sqrt{\mu t}, x_3+2\tau_3\sqrt{\mu t}) \Big| d\tau_1 d\tau_2 d\tau_3 \le h_1^0, \\
\sup\limits_{T} \dfrac{1}{\sqrt{\pi^3}} \int\limits_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)) \Big| \lambda_1 \dfrac{\partial}{\partial x_3} V_0(x_1+2\tau_1\sqrt{\mu t}, x_2+2\tau_2\sqrt{\mu t}, x_3+ \\
+2\tau_3\sqrt{\mu t}) - \lambda_1 \dfrac{\partial}{\partial x_1} V_0(x_1+2\tau_1\sqrt{\mu t}, x_2+2\tau_2\sqrt{\mu t}, x_3+2\tau_3\sqrt{\mu t}) \Big| d\tau_1 d\tau_2 d\tau_3 \le h_2^0, \\
\sup\limits_{T} \dfrac{1}{\sqrt{\pi^3}} \int\limits_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)) \Big| \lambda_2 \dfrac{\partial}{\partial x_1} V_0(x_1+2\tau_1\sqrt{\mu t}, x_2+2\tau_2\sqrt{\mu t}, x_3+ \\
+2\tau_3\sqrt{\mu t}) - \lambda_1 \dfrac{\partial}{\partial x_2} V_0(x_1+2\tau_1\sqrt{\mu t}, x_2+2\tau_2\sqrt{\mu t}, x_3+2\tau_3\sqrt{\mu t}) \Big| d\tau_1 d\tau_2 d\tau_3 \le h_3^0, \\
\sup\limits_{T} \dfrac{1}{\sqrt{\pi^3}} \int\limits_0^t \int\limits_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)) \Big| \lambda_3 \Phi_{0l_2}(l_1,l_2,l_3;s) - \lambda_2 \Phi_{0l_3}(l_1,l_2,l_3;s) \Big| d\tau_1 d\tau_2 d\tau_3 ds \le h_1, \\
\sup\limits_{T} \dfrac{1}{\sqrt{\pi^3}} \int\limits_0^t \int\limits_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)) \Big| \lambda_1 \Phi_{0l_3}(l_1,l_2,l_3;s) - \lambda_3 \Phi_{0l_1}(l_1,l_2,l_3;s) \Big| d\tau_1 d\tau_2 d\tau_3 ds \le h_2, \\
\sup\limits_{T} \dfrac{1}{\sqrt{\pi^3}} \int\limits_0^t \int\limits_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2)) \Big| \lambda_2 \Phi_{0l_1}(l_1,l_2,l_3;s) - \lambda_1 \Phi_{0l_2}(l_1,l_2,l_3;s) \Big| d\tau_1 d\tau_2 d\tau_3 ds \le h_3, \\
l_i = x_i + 2\tau_i\sqrt{\mu(t-s)},\ (i=\overline{1,3});\ \ N_0 = \sum\limits_{i=1}^{3} (h_i^0 + h_i),
\end{cases}$$

takes place

$$\begin{cases}
\nu = (\upsilon_1, \upsilon_2, \upsilon_3), \upsilon_i \equiv \lambda_i V, (i=\overline{1,3}), \\
\sup\limits_{R^3} \int\limits_0^{T_0} |\operatorname{rot} \nu(x_1,x_2,x_3,t)| dt \le \sup\limits_{R^3} \int\limits_0^{T_0} \{ |\upsilon_{3x_2}(x_1,x_2,x_3,s) - \upsilon_{2x_3}(x_1,x_2,x_3,s)| + \\
+ |\upsilon_{1x_3}(x_1,x_2,x_3,s) - \upsilon_{3x_1}(x_1,x_2,x_3,s)| + |\upsilon_{2x_1}(x_1,x_2,x_3,s) - \upsilon_{1x_2}(x_1,x_2,x_3,s)| \} ds \le \\
\le \sum\limits_{i=1}^{3} (h_i^0 + h_i) T_0 = N_0 T_0 = M < \infty.
\end{cases} \quad (4.11)$$

And in a case $\nu \in W_\lambda^2(D_0)$, we will receive a inequality

$$\sup_{R^3}\int_0^{T_0}\left|\mathrm{rot}\,\nu(x_1,x_2,x_3,t)\right|^2 dt \le T_0[\sum_{i=1}^{3}(h_i + h_i^0)]^2 = M_0 < \infty,$$

i.e. we will receive an estimation of type Beale-Kato-Majda [2, 5].

4.2. Modified Variant of a Method (4.2) with the Condition (A$_3$), When $\mathrm{div} f = 0$

Here we will show that at certain mathematical transformations of the equation Navier-Stokes is led to a linear kind. Thus once again visually confirming [10], i.e. that really equations Navier-Stokes in a case of very small viscosity will have of the decision the obvious form really turns out.

Let's consider updating of the basic method §4.2: (4.2). Here we will see that components of speed the more any, than in rules (4.2). Therefore in this case the problem (1.1) - (1.3) does not contain in itself restriction in kinds (A$_1$), (A$_2$). And in it urgency of research in a case (A$_3$) consists.

Let's for incompressible streams with a friction with a condition (A$_3$) also it is prospective

$$\begin{cases} \upsilon_i\big|_{t=0} = \upsilon_{i0}(x_1,x_2,x_3) \equiv \lambda_i V_0(x_1,x_2,x_3), (i=\overline{1,3}), \\ \sum_{i=1}^{3}\lambda_i V_{0x_i} = 0;\ f_i \equiv \varphi_i + \Omega_{it};\ \mathrm{div} f = 0: \\ \mathrm{div}\varphi = 0;\ \sum_{i=1}^{3}\frac{\partial}{\partial x_i}\Omega_{it} = 0,\ \ (\Omega_{it} = \mu\overline{f}_i(x_1,x_2,x_3,t);\ i=\overline{1,3}), \\ \mathrm{div}\overline{f} = 0;\ \mathrm{rot}\,\overline{f} = 0: \Delta\overline{f}_i = 0, (\Delta\Omega_i = 0), \\ U = \sum_{j=1}^{3}\Omega_j^2;\ \ \sum_{j=1}^{3}\Omega_j\Omega_{ix_j} = \frac{1}{2}(\sum_{j=1}^{3}\Omega_j^2)_{x_i} = \frac{1}{2}U_{x_i}, \\ \Omega_i(x_1,x_2,x_3,t) \equiv \mu\int_0^t \overline{f}_i(x_1,x_2,x_3,s')ds';\ \ \varphi = (\varphi_1,\varphi_2,\varphi_3); \overline{f} = (\overline{f}_1,\overline{f}_2,\overline{f}_3). \end{cases} \tag{4.12}$$

Then functions $\upsilon_i, i=\overline{1,3}$ is represented in a kind

$$\begin{cases} \upsilon_i = \lambda_i V(x_1,x_2,x_3,t) + \Omega_i(x_1,x_2,x_3,t), (i=\overline{1,3}),\ \ V\big|_{t=0} = V_0(x_1,x_2,x_3), \\ \mathrm{div}\,\nu = 0: \sum_{i=1}^{3}\lambda_i V_{x_i} = 0;\ \ \sum_{i=1}^{3}\Omega_{ix_i} \equiv \mu\int_0^t(\sum_{i=1}^{3}\overline{f}_{ix_i}(x_1,x_2,x_3,s))ds = 0, \end{cases} \tag{4.13}$$

where $0 < \lambda_i -$ known constants.

Further, supposing (4.12), (4.13) and

$$\begin{cases} \sum_{j=1}^{3}\upsilon_j\upsilon_{ix_j} \equiv V\sum_{j=1}^{3}\lambda_j\Omega_{ix_j} + \lambda_i\sum_{j=1}^{3}V_{x_j}\Omega_j + \frac{1}{2}U_{x_i}, (\lambda_i V\sum_{j=1}^{3}\lambda_j V_{x_j} = 0), \\ \upsilon_{it} \equiv \lambda_i V_t + \Omega_{it},\ \ \mu\Delta\upsilon_i \equiv \mu[\lambda_i\Delta V + \Delta\Omega_i] = \mu\lambda_i\Delta V, (i=\overline{1,3}). \end{cases} \tag{4.14}$$

Then for incompressible currents with a friction the equations of Navier-Stokes (1.1) become simpler as take place (4.12)-(4.14). Therefore the problem (1.1)-(1.3), is led to a kind

$$\lambda_i V_t + V\sum_{j=1}^{3}\lambda_j\Omega_{ix_j} + \lambda_i\sum_{j=1}^{3}V_{x_j}\Omega_j + \frac{1}{2}U_{x_i} = \varphi_i - \frac{1}{\rho}P_{x_i} + \mu\lambda_i\Delta V, (i=\overline{1,3}). \quad (4.15)$$

From system (4.15), considering conditions (4.12)-(4.14) and having entered «algorithm puassonization systems», i.e. differentiating the equations of system (4.15) accordingly on $x_i, i=1,2,3$ and, then summarising, we have the equation:

$$\begin{cases} \sum_{i=1}^{3}\frac{\partial}{\partial x_i}(4.15): \quad \Delta(\frac{1}{\rho}P+\frac{1}{2}U) = -\{\sum_{i=1}^{3}V_{x_i}(\sum_{j=1}^{3}\lambda_j\Omega_{ix_j}) + \sum_{i=1}^{3}(\sum_{j=1}^{3}\Omega_{jx_i}V_{x_j})\lambda_i\}, \\ \frac{1}{\rho}P+\frac{1}{2}U = \frac{1}{4\pi}\int_{R^3}\frac{1}{r}\{\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\Omega_{is_j}(s_1,s_2,s_3,t))V_{s_i}(s_1,s_2,s_3,t) + \\ +\sum_{i=1}^{3}(\sum_{j=1}^{3}\Omega_{js_i}(s_1,s_2,s_3,t)V_{s_j}(s_1,s_2,s_3,t))\lambda_i\}ds_1ds_2ds_3, \\ \frac{1}{\rho}P_{x_i}+\frac{1}{2}U_{x_i} = \frac{1}{4\pi}\int_{R^3}\frac{\tau_i}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}\{\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\Omega_{ih_j}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t))\times \\ \times V_{h_i}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t) + \sum_{i=1}^{3}(\sum_{j=1}^{3}\Omega_{jh_i}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t)\times \\ \times V_{h_j}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t))\lambda_i\}d\tau_1d\tau_2d\tau_3, (s_i-x_i=\tau_i; h_i=x_i+\tau_i), \end{cases} \quad (4.16)$$

as takes place

$$\begin{cases} \frac{\partial}{\partial t}[\sum_{i=1}^{3}\lambda_i V_{x_i}] = 0; \ \mathrm{div}\varphi = 0, \ \ \mu\sum_{i=1}^{3}\frac{\partial}{\partial x_i}(\lambda_i\Delta V) = 0, \\ \sum_{i=1}^{3}\frac{\partial}{\partial x_i}(\frac{1}{2}U_{x_i}) \equiv \frac{1}{2}\Delta U; \ \ \sum_{i=1}^{3}\frac{\partial}{\partial x_i}(-\frac{1}{\rho}P_{x_i}) \equiv -\frac{1}{\rho}\Delta P, \\ \sum_{i=1}^{3}\frac{\partial}{\partial x_i}(V\sum_{j=1}^{3}\lambda_j\Omega_{ix_j}) \equiv \sum_{i=1}^{3}\lambda_i(\sum_{j=1}^{3}V_{x_j}\Omega_{jx_i}) + \sum_{j=1}^{3}\Omega_j\frac{\partial}{\partial x_j}(\sum_{i=1}^{3}\lambda_i V_{x_i})_{x_j} = \sum_{i=1}^{3}\lambda_i(\sum_{j=1}^{3}V_{x_j}\Omega_{jx_i}), \\ \sum_{i=1}^{3}\frac{\partial}{\partial x_i}(\lambda_i\sum_{j=1}^{3}V_{x_j}\Omega_j) \equiv \sum_{i=1}^{3}V_{x_i}(\sum_{j=1}^{3}\lambda_j\Omega_{ix_j}) + V\sum_{j=1}^{3}\lambda_j(\sum_{i=1}^{3}\Omega_{ix_i})_{x_j} = \sum_{i=1}^{3}V_{x_i}(\sum_{j=1}^{3}\lambda_j\Omega_{ix_j}), \\ \sum_{j=1}^{3}\Omega_j\frac{\partial}{\partial x_j}(\sum_{i=1}^{3}\lambda_i V_{x_i})_{x_j} = 0; \ \ V\sum_{j=1}^{3}\lambda_j(\sum_{i=1}^{3}\Omega_{ix_i})_{x_j} = 0. \end{cases}$$

Then on a basis (4.16) system (4.15) it is equivalent, will be transformed to a kind:

$$\begin{cases} V_t + d^{-1}V[\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\Omega_{ix_j})] + \sum_{j=1}^{3}V_{x_j}\Omega_j = \Phi_0 - d^{-1}[\frac{1}{4\pi}\int_{R^3}(\sum_{i=1}^{3}\frac{\tau_i}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}\times \\ \times\{\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\Omega_{ih_j}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t))V_{h_i}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t)+ \\ +\sum_{i=1}^{3}(\sum_{j=1}^{3}\Omega_{jh_i}(x_1+\tau_1,x_2+\tau_2 x_3+\tau_3;t)V_{h_j}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t))\lambda_i\})\times \\ \times d\tau_1 d\tau_2 d\tau_3] + \mu\Delta V, (i=\overline{1,3}), \\ \Phi_0(x_1,x_2,x_3,t) \equiv d^{-1}\sum_{i=1}^{3}\varphi_i(x_1,x_2,x_3,t), \\ d = \sum_{i=1}^{3}\lambda_i > 0, \end{cases} \quad (4.17)$$

or from (4.17), follows:

$$\begin{cases} V = M_1(x_1,x_2,x_3,t) - \frac{1}{8\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-\frac{r^2}{4\mu(t-s)})\frac{1}{(\sqrt{\mu(t-s)})^3}\times \\ \times\{d^{-1}[V(s_1,s_2,s_3,s)\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\Omega_{is_j}(s_1,s_2,s_3,s))] + \sum_{j=1}^{3}V_{s_j}(s_1,s_2,s_3,s)\times \\ \times\Omega_j(s_1,s_2,s_3,s) + d^{-1}[\frac{1}{4\pi}\int_{R^3}(\sum_{i=1}^{3}\frac{\overline{\tau}_i}{\sqrt{(\overline{\tau}_1^2+\overline{\tau}_2^2+\overline{\tau}_3^2)^3}}\{\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\Omega_{il_j}(s_1+\overline{\tau}_1,s_2+\overline{\tau}_2, \\ s_3+\overline{\tau}_3;s))V_{l_i}(s_1+\overline{\tau}_1,s_2+\overline{\tau}_2,s_3+\overline{\tau}_3;s) + \sum_{i=1}^{3}(\sum_{j=1}^{3}\Omega_{jl_i}(s_1+\overline{\tau}_1,s_2+\overline{\tau}_2,s_3+\overline{\tau}_3;s)\times \\ \times V_{l_j}(s_1+\overline{\tau}_1,s_2+\overline{\tau}_2,s_3+\overline{\tau}_3;s))\lambda_i\})d\overline{\tau}_1 d\overline{\tau}_2 d\overline{\tau}_3]\}ds_1 ds_2 ds_3 ds, \\ l_j = s_j + \overline{\tau}_j,\ (j=\overline{1,3}), \\ M_1(x_1,x_2,x_3,t) \equiv \frac{1}{\sqrt{\pi^3}}\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))V_0(x_1+2\tau_1\sqrt{\mu t},x_2+2\tau_2\sqrt{\mu t},x_3+ \\ +2\tau_3\sqrt{\mu t})d\tau_1 d\tau_2 d\tau_3 + \frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_0(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+ \\ +2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2 d\tau_3 ds, \\ s_j - x_j = 2\tau_j\sqrt{\mu t};\quad s_j - x_j = 2\tau_j\sqrt{\mu(t-s)},(j=\overline{1,3}). \end{cases} \quad (4.18)$$

Further, differentiating (4.18) on $x_i,(i=1,2,3)$ and having entered designations

$$\begin{cases} V_{x_i} = W_i(x_1,x_2,x_3,t), \forall (x_1,x_2,x_3,t) \in T, (i=\overline{1,3}), \\ (Q[V,V_{s_1},V_{s_2},V_{s_3}])(s_1,s_2,s_3,s) \equiv -\{d^{-1}[V(s_1,s_2,s_3,s)\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j \Omega_{is_j}(s_1,s_2,s_3,s))] + \\ +\sum_{j=1}^{3} V_{s_j}(s_1,s_2,s_3,s)\Omega_j(s_1,s_2,s_3,s) + d^{-1}[\frac{1}{4\pi}\int_{R^3}(\sum_{i=1}^{3}\overline{\tau}_i \frac{1}{\sqrt{(\overline{\tau}_1^2+\overline{\tau}_2^2+\overline{\tau}_3^2)^3}} \times \\ \times\{\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j \Omega_{il_j}(s_1+\overline{\tau}_1,s_2+\overline{\tau}_2,s_3+\overline{\tau}_3;s))V_{l_i}(s_1+\overline{\tau}_1,s_2+\overline{\tau}_2,s_3+\overline{\tau}_3;s) + \\ +\sum_{i=1}^{3}(\sum_{j=1}^{3}\Omega_{jl_i}(s_1+\overline{\tau}_1,s_2+\overline{\tau}_2,s_3+\overline{\tau}_3;s)V_{l_j}(s_1+\overline{\tau}_1,s_2+\overline{\tau}_2,s_3+\overline{\tau}_3;s))\lambda_i\})d\overline{\tau}_1 d\overline{\tau}_2 d\overline{\tau}_3]\}, \\ l_j = s_j + \overline{\tau}_j,\ (j=\overline{1,3}), \end{cases} \tag{4.19}$$

from (4.18) we will receive

$$\begin{cases} V = M_1(x_1,x_2,x_3,t) + \frac{1}{8\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-\frac{r^2}{4\mu(t-s)})\frac{1}{(\sqrt{\mu(t-s)})^3}\times \\ \times(Q[V,W_1,W_2,W_3])(s_1,s_2,s_3,s)ds_1 ds_2 ds_3 ds, \\ W_i = M_{1x_i} + \frac{1}{8\sqrt{\pi^3}}\int_0^t\int_{R^3}\frac{-(x_i-s_i)}{2\mu(t-s)}(\exp(-\frac{r^2}{4\mu(t-s)}))\frac{1}{(\sqrt{\mu(t-s)})^3}\times \\ \times(Q[V,W_1,W_2,W_3])(s_1,s_2,s_3,s)ds_1 ds_2 ds_3 ds. \end{cases} \tag{4.20}$$

Or we will transform (4.20) to a kind

$$\begin{cases} V = M_1 + \frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))(Q[V,W_1,W_2,W_3])(x_1 + \\ +2\tau_1\sqrt{\mu\tau}, x_2+2\tau_2\sqrt{\mu\tau}, x_3+2\tau_3\sqrt{\mu\tau}; t-\tau)d\tau_1 d\tau_2 d\tau_3 d\tau \equiv \\ \equiv \Psi_0[V,W_1,W_2,W_3], \\ W_i = M_{1x_i} + \frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\frac{\tau_i}{\sqrt{\mu\tau}}(Q[V,W_1,W_2,W_3])(x_1 + \\ +2\tau_1\sqrt{\mu\tau}, x_2+2\tau_2\sqrt{\mu\tau}, x_3+2\tau_3\sqrt{\mu\tau}; t-\tau)d\tau_1 d\tau_2 d\tau_3 d\tau \equiv \Psi_i[V,W_1,W_2,W_3], \\ s_i - x_i = 2\tau_i\sqrt{\mu(t-s)}; t-s=\tau,\ \ (i=\overline{1,3}). \end{cases} \tag{4.21}$$

Here (4.21) – system of the nonlinear integrated equations of Volterr-Abel of the second sort concerning V, W_i on a variable *t*. Therefore, if takes place:

$$\begin{cases} \forall(x_1,x_2,x_3,t)\in T; M_1,\Pi,\Omega_i:\ \sup\limits_T\left|D^k M_1(x_1,x_2,x_3,t)\right|\le\beta_1,\ (k=\overline{0,3}), \\ \sup\limits_{T\times T}\Pi(x_1,x_2,x_3,\tau_1,\tau_2,\tau_3;t,\tau)\equiv\sup\limits_{T\times T}\{d^{-1}\sum\limits_{i=1}^{3}(\sum\limits_{j=1}^{3}\lambda_j\left|\Omega_{is_j}(x_1+2\tau_1\sqrt{\mu\tau},x_2+\right. \\ \left.+2\tau_2\sqrt{\mu\tau},x_3+2\tau_3\sqrt{\mu\tau};t-\tau)\right|)+\sum\limits_{j=1}^{3}\left|\Omega_j(x_1+2\tau_1\sqrt{\mu\tau},x_2+2\tau_2\sqrt{\mu\tau},x_3+\right. \\ \left.+2\tau_3\sqrt{\mu\tau};t-\tau)\right|d^{-1}[\frac{1}{4\pi}\int\limits_{R^3}(\sum\limits_{i=1}^{3}\left|\overline{\tau}_i\right|\frac{1}{\sqrt{(\overline{\tau}_1^2+\overline{\tau}_2^2+\overline{\tau}_3^2)^3}}\{\sum\limits_{i=1}^{3}(\sum\limits_{j=1}^{3}\lambda_j\left|\Omega_{il_j}(x_1+\right. \\ \left.+2\tau_1\sqrt{\mu\tau}+\overline{\tau}_1,x_2+2\tau_2\sqrt{\mu\tau}+\overline{\tau}_2,x_3+2\tau_3\sqrt{\mu\tau}+\overline{\tau}_3;t-\tau)\right|)+\sum\limits_{i=1}^{3}(\sum\limits_{j=1}^{3}\left|\Omega_{jl_i}(x_1+\right. \\ \left.+2\tau_1\sqrt{\mu\tau}+\overline{\tau}_1,x_2+2\tau_2\sqrt{\mu\tau}+\overline{\tau}_2,x_3+2\tau_3\sqrt{\mu\tau}+\overline{\tau}_3;t-\tau)\right|)\lambda_i\})d\overline{\tau}_1d\overline{\tau}_2d\overline{\tau}_3]\}\le\beta_2\mu, \\ k_i=\frac{1}{\sqrt{\pi^3}}\sup\limits_T\int\limits_0^t\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Pi(x_1,x_2,x_3,\tau_1,\tau_2,\tau_3;t,\tau)\frac{|\tau_i|}{\sqrt{\mu\tau}}d\tau_1d\tau_2d\tau_3d\tau\le \\ \le\gamma_1\sqrt{T_0}\beta_2\sqrt{\mu},(i=\overline{1,3}), \\ k_0=\frac{1}{\sqrt{\pi^3}}\sup\limits_T\int\limits_0^t\int\limits_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Pi(x_1,x_2,x_3,\tau_1,\tau_2,\tau_3;t,\tau)d\tau_1d\tau_2d\tau_3d\tau\le\beta_2\mu T_0, \\ \gamma_1=\sqrt[4]{2^5};\ \ \beta=\max(\beta_2T_0;3\gamma_1\sqrt{T_0}\beta_2), \\ \Psi_i,(i=\overline{0,3}): \\ h=\sum\limits_{i=0}^{3}k_i\le\sqrt{\mu}(\beta_2T_0\sqrt{\mu}+3\gamma_1\sqrt{T_0}\beta_2)\le\sqrt{\mu}(\sqrt{\mu}+1)\beta<1, \end{cases} \tag{4.22}$$

that on a basis (4.22) and (4.21) it is had

$$\begin{cases} |V|=|\Psi_0[V,W_1,W_2,W_3]|\le\beta_1+k_0[\|V\|_C+\sum\limits_{i=1}^{3}\|W_i\|_C], \\ |W_i|=|\Psi_i[V,W_1,W_2,W_3]|\le\beta_1+k_i[\|V\|_C+\sum\limits_{i=1}^{3}\|W_i\|_C], \end{cases}$$

i.e.:

$$\begin{cases} V,W_i\in C(T): E=\|V\|_C+\sum\limits_{i=1}^{3}\|W_i\|_C, \\ E\le(1-h)^{-1}4\beta_1. \end{cases} \tag{4.23}$$

Therefore the solution of this system we can find on the basis of Pikard's method

$$\begin{cases} \forall(x_1,x_2,x_3,t)\in T:\ \ V_{n+1}=\Psi_0[V_n,W_{1,n},W_{2,n},W_{3,n}], \\ W_{i,n+1}=\Psi_i[V_n,W_{1,n},W_{2,n},W_{3,n}],(i=\overline{1,3};\ n=0,1,...), \end{cases} \tag{4.24}$$

at that

$$\begin{cases} E_0 = \|V - V_0\|_C + \sum_{i=1}^{3} \|W_i - W_{i,0}\|_C; \ E_{n+1} \equiv \|V_{n+1} - V\|_C + \sum_{i=1}^{3} \|W_{i,n+1} - W_i\|_C, \\ E_{n+1} \le h^{n+1} E_0 \xrightarrow[n\to\infty]{h<1} 0. \end{cases} \tag{4.25}$$

From here follows

$$\begin{cases} V_n \xrightarrow[n\to\infty]{h<1} V \equiv H(x_1,x_2,x_3,t), \\ W_{i,n} \xrightarrow[n\to\infty]{h<1} W_i, \forall (x_1,x_2,x_3,t) \in T, (i = \overline{1,3}). \end{cases} \tag{4.26}$$

Theorem 5. Under conditions (1.2), (1.3), (A_3), (4.12), (4.13), (4.22), (4.26) problem Navier-Stokes has the single decision in $C(T)$ in a kind:

$$\begin{cases} \upsilon_i = \lambda_i H + \Omega_i \equiv H_i(x_1,x_2,x_3,t), \forall (x_1,x_2,x_3,t) \in T, \\ \Omega_i(x_1,x_2,x_3,t) \equiv \mu \int_0^t \overline{f}_i(x_1,x_2,x_3,s')ds', (i = \overline{1,3}). \end{cases} \tag{4.27}$$

Remarks:

I. If takes place $0 < \beta \le 2^{-1}$, that $0 < \mu < 1$. In a case $\beta > 2^{-1}$, then

$$0 < \mu < 2^{-1}[\sqrt{1 + 4\beta^{-1}} - 1] < 1. \tag{4.28}$$

Let's note, as (4.22) Concerns is to the equations of Volterr-Abel on a variable $t \in [0, T_0]$, discussing in language of the Volterrov equations we can find the decision. But such way from the practical point of view when $t \in R_+$ it is not applicable. Therefore, an offered variant more universal in sense of the theory of operators [11, 12].

II. From the received results follows that a problem Navier-Stokes (1.1)-(1.3) in a case (4.12), (A_3) with smooth enough initial data has the conditional-smooth and single decision in a kind (4.27).

4.3. Method (4.13), When $\operatorname{div} f \ne 0$

The algorithm (4.13) also is applicable in a case, if

$$\begin{cases} f_i \equiv \varphi_i + \Omega_{it}, \ (i = \overline{1,3}), \ \operatorname{div} f \ne 0: \ \operatorname{div}\varphi \ne 0; \ \sum_{i=1}^{3} \frac{\partial}{\partial x_i} \Omega_{it} = 0, \\ \operatorname{div}\overline{f} = 0; \ \operatorname{rot}\overline{f} = 0: \ \Delta \overline{f}_i = 0, (\Delta\Omega_i = 0; i = \overline{1,3}), \\ U = \sum_{j=1}^{3} \Omega_j^2; \ \sum_{j=1}^{3} \Omega_j \Omega_{ix_j} = \frac{1}{2} U_{x_i}, (i = \overline{1,3}), \\ \upsilon_i \big|_{t=0} = \upsilon_{i0}(x_1,x_2,x_3) \equiv \lambda_i V_0(x_1,x_2,x_3), \ (\sum_{i=1}^{3} \lambda_i V_{0x_i} = 0), \end{cases} \tag{4.29}$$

That on a basis (4.14) we will receive (4.15). Hence, we have

$$\begin{cases}\sum_{i=1}^{3}\frac{\partial}{\partial x_i}(4.15): \quad \Delta(\frac{1}{\rho}P+\frac{1}{2}U)=-\{\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\Omega_{ix_j})V_{x_i}+\sum_{i=1}^{3}(\sum_{j=1}^{3}\Omega_{jx_i}V_{x_j})\lambda_i-\mathrm{div}\varphi\},\\ \frac{1}{\rho}P+\frac{1}{2}U=\frac{1}{4\pi}\int_{R^3}\frac{1}{r}\{\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\Omega_{is_j}(s_1,s_2,s_3,t))V_{s_i}(s_1,s_2,s_3,t)+\\ +\sum_{i=1}^{3}(\sum_{j=1}^{3}\Omega_{js_i}(s_1,s_2,s_3,t)V_{s_j}(s_1,s_2,s_3,t))\lambda_i-\sum_{i=1}^{3}\varphi_{is_i}(s_1,s_2,s_3,t)\}ds_1ds_2ds_3,\\ \frac{1}{\rho}P_{x_i}+\frac{1}{2}U_{x_i}=\frac{1}{4\pi}\int_{R^3}\frac{\tau_i}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}\{\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\Omega_{ih_j}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t))\times\\ \times V_{h_i}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t)+\sum_{i=1}^{3}(\sum_{j=1}^{3}\Omega_{jh_i}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t)V_{h_j}(x_1+\tau_1,x_2+\\ +\tau_2,x_3+\tau_3;t))\lambda_i-\sum_{i=1}^{3}\varphi_{ih_i}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t)\}d\tau_1d\tau_2d\tau_3,(s_i-x_i=\tau_i;h_i=x_i+\tau_i).\end{cases} \quad (4.30)$$

Then on a basis (4.30) system (4.15) it is equivalent, will be transformed to a kind:

$$\begin{cases}V_t+d^{-1}V[\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\Omega_{ix_j})]+\sum_{j=1}^{3}V_{x_j}\Omega_j=\Phi_0-d^{-1}[\frac{1}{4\pi}\int_{R^3}(\sum_{i=1}^{3}\frac{\tau_i}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}\times\\ \times\{\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\Omega_{ih_j}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t))V_{h_i}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t)+\\ +\sum_{i=1}^{3}(\sum_{j=1}^{3}\Omega_{jh_i}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t)V_{h_j}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t))\lambda_i\})d\tau_1d\tau_2d\tau_3]+\\ +\mu\Delta V,(i=\overline{1,3}),\\ \Phi_0(x_1,x_2,x_3,t)\equiv d^{-1}[\sum_{i=1}^{3}\varphi_i+\frac{1}{4\pi}\int_{R^3}\{\sum_{i=1}^{3}\frac{\tau_i}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}(\sum_{i=1}^{3}\varphi_{ih_i}(x_1+\tau_1,x_2+\tau_2,x_3+\\ +\tau_3;t))\}d\tau_1d\tau_2d\tau_3]; \quad d=\sum_{i=1}^{3}\lambda_i>0; \quad \mathrm{div}\varphi\neq 0,\end{cases} \quad (4.31)$$

or

$$\begin{cases}V=M_1+\frac{1}{8\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-\frac{r^2}{4\mu(t-s)})\frac{1}{(\sqrt{\mu(t-s)})^3}(Q[V,V_{s_1},V_{s_2},V_{s_3}])(s_1,s_2,s_3,s)ds_1ds_2ds_3ds,\\ M_1(x_1,x_2,x_3,t)\equiv\frac{1}{\sqrt{\pi^3}}\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))V_0(x_1+2\tau_1\sqrt{\mu t},x_2+2\tau_2\sqrt{\mu t},x_3+2\tau_3\sqrt{\mu t})\times\\ \times d\tau_1d\tau_2d\tau_3+\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_0(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+\end{cases}$$

$$\begin{cases} +2\tau_3\sqrt{\mu(t-s)};s)d\tau_1 d\tau_2 d\tau_3 ds, \\ (Q[V,V_{s_1},V_{s_2},V_{s_3}])(s_1,s_2,s_3,s) \equiv -\{d^{-1}[V(s_1,s_2,s_3,s)\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\Omega_{is_j}(s_1,s_2,s_3,s))]+ \\ +\sum_{j=1}^{3}V_{s_j}(s_1,s_2,s_3,s)\Omega_j(s_1,s_2,s_3,s)+d^{-1}[\frac{1}{4\pi}\int_{R^3}(\sum_{i=1}^{3}\overline{\tau}_i\frac{1}{\sqrt{(\overline{\tau}_1^2+\overline{\tau}_2^2+\overline{\tau}_3^2)^3}}\times \\ \times\{\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\Omega_{il_j}(s_1+\overline{\tau}_1,s_2+\overline{\tau}_2,s_3+\overline{\tau}_3;s))V_{l_i}(s_1+\overline{\tau}_1,s_2+\overline{\tau}_2,s_3+\overline{\tau}_3;s)+ \\ +\sum_{i=1}^{3}(\sum_{j=1}^{3}\Omega_{jl_i}(s_1+\overline{\tau}_1,s_2+\overline{\tau}_2,s_3+\overline{\tau}_3;s)V_{l_j}(s_1+\overline{\tau}_1,s_2+\overline{\tau}_2,s_3+\overline{\tau}_3;s))\lambda_i\})d\overline{\tau}_1 d\overline{\tau}_2 d\overline{\tau}_3]\}, \\ s_j-x_j=2\tau_j\sqrt{\mu t}, s_j-x_j=2\tau_j\sqrt{\mu(t-s)},(j=\overline{1,3}). \end{cases} \quad (4.32)$$

Further, differentiating (4.32) on x_i and having entered designation:

$$V_{x_i}=W_i,\forall(x_1,x_2,x_3,t)\in T,(i=\overline{1,3}),$$

from (4.32) we will receive (4.21):

$$\begin{cases} V=\Psi_0[V,W_1,W_2,W_3], \\ W_i=\Psi_i[V,W_1,W_2,W_3],(i=\overline{1,3}). \end{cases} \quad (4.33)$$

Hence, as takes place (4.22), that the solution of system (4.33) we can find on the basis of Pikard's method. Then on a basis (4.24), (4.25) we have

$$\begin{cases} V_n \xrightarrow[n\to\infty]{h<1} V\equiv H(x_1,x_2,x_3,t), \\ W_{i,n} \xrightarrow[n\to\infty]{h<1} W_i, \ \ \forall(x_1,x_2,x_3,t)\in T,(i=\overline{1,3}). \end{cases} \quad (4.34)$$

Further, we will receive similar results in the conditions of the theorem 5, i.e. under conditions (1.2), (1.3), (A_3), (4.29), (4.34) problem Navier-Stokes has the single decision in $C(T)$ in a kind (4.27).

4.4. Method (4.13), When $(\mathrm{div}\overline{f}=0;\ \mathrm{rot}\,\overline{f}\neq 0)$

Results §4.2, §4.3 it is modified in a case when condtions:

$$\begin{cases} \upsilon_i\big|_{t=0}=\upsilon_{i0}(x_1,x_2,x_3)\equiv\lambda_i V_0(x_1,x_2,x_3),(i=\overline{1,3}), \\ f_i\equiv\varphi_i+\Omega_{it};\ \ \mathrm{div}f\neq 0: \\ \mathrm{div}\varphi\neq 0;\ \ \sum_{i=1}^{3}\frac{\partial}{\partial x_i}\Omega_{it}=0,\ \ (\Omega_{it}\equiv\mu\overline{f}_i(x_1,x_2,x_3,t);\ i=\overline{1,3}), \\ \mathrm{div}\overline{f}=0;\ \mathrm{rot}\,\overline{f}\neq 0: \\ \Delta\overline{f}_i\neq 0,(\Delta\Omega_i\neq 0;\ \ 0<\mu<1),\ \varphi=(\varphi_1,\varphi_2,\varphi_3);\ \overline{f}=(\overline{f}_1,\overline{f}_2,\overline{f}_3), \end{cases} \quad (4.35)$$

are satisfied. Then considering results §4.2, §4.3 we will receive similar conclusions of theorems 5, accordingly.

Allow a condition (4.35) it is satisfied. Then from (4.13) follows

$$\begin{cases} \upsilon_i = \lambda_i V(x_1,x_2,x_3,t) + \Omega_i(x_1,x_2,x_3,t),(i=\overline{1,3}), \\ V\big|_{t=0} = V_0(x_1,x_2,x_3), \\ \operatorname{div} v = 0: \sum_{i=1}^{3}\lambda_i V_{x_i} = 0;\ \sum_{i=1}^{3}\Omega_{ix_i} = 0, \\ \upsilon_{it} \equiv \lambda_i V_t + \Omega_{it},\ (i=\overline{1,3}), \\ \mu\Delta\upsilon_i \equiv \mu[\lambda_i \Delta V + \Delta\Omega_i],\ (i=\overline{1,3}), \\ \sum_{j=1}^{3}\upsilon_j\upsilon_{ix_j} \equiv V\sum_{j=1}^{3}\lambda_j\Omega_{ix_j} + \lambda_i\sum_{j=1}^{3}V_{x_j}\Omega_j + \sum_{j=1}^{3}\Omega_j\Omega_{ix_j} = 0,\ (\lambda_i V\sum_{j=1}^{3}\lambda_j V_{x_j} = 0). \end{cases} \tag{4.36}$$

Therefore from system (1.1) follows

$$\lambda_i V_t + V\sum_{j=1}^{3}\lambda_j\Omega_{ix_j} + \lambda_i\sum_{j=1}^{3}V_{x_j}\Omega_j + \sum_{j=1}^{3}\Omega_j\Omega_{ix_j} = \varphi_i - \frac{1}{\rho}P_{x_i} + \mu[\lambda_i\Delta V + \Delta\Omega_i],(i=\overline{1,3}). \tag{4.37}$$

From system (4.37), considering conditions (4.35), (4.36) and having entered «algorithm puassonization systems», as a result we will receive

$$\begin{cases} \sum_{i=1}^{3}\frac{\partial}{\partial x_i}(4.37):\quad \Delta\frac{1}{\rho}P = -\{F_0(x_1,x_2,x_3,t) + \sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\Omega_{ix_j})V_{x_i} + \sum_{i=1}^{3}(\sum_{j=1}^{3}\Omega_{jx_i}V_{x_j})\lambda_i\}, \\ \frac{1}{\rho}P = \frac{1}{4\pi}\int_{R^3}\frac{1}{r}\{F_0(s_1,s_2,s_3,t) + \sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\Omega_{is_j}(s_1,s_2,s_3,t))V_{s_i}(s_1,s_2,s_3,t) + \\ +\sum_{i=1}^{3}(\sum_{j=1}^{3}\Omega_{js_i}(s_1,s_2,s_3,t)V_{s_j}(s_1,s_2,s_3,t))\lambda_i\}ds_1ds_2ds_3, \\ \frac{1}{\rho}P_{x_i} = \frac{1}{4\pi}\int_{R^3}\frac{\tau_i}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}\{F_0(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t) + \sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\Omega_{ih_j}(x_1+\tau_1, \\ x_2+\tau_2,x_3+\tau_3;t))V_{h_i}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t) + \sum_{i=1}^{3}(\sum_{j=1}^{3}\Omega_{jh_i}(x_1+\tau_1,x_2+\tau_2,x_3+ \\ +\tau_3;t)V_{h_j}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t))\lambda_i\}d\tau_1d\tau_2d\tau_3,(s_i - x_i = \tau_i; h_i = x_i+\tau_i), \end{cases} \tag{4.38}$$

as takes place

$$\begin{cases} F_0(x_1,x_2,x_3,t) \equiv \sum_{i=1}^{3}(\frac{\partial}{\partial x_i}(\sum_{j=1}^{3}\Omega_j\Omega_{ix_j}) - \varphi_{ix_i});\quad \frac{\partial}{\partial t}[\sum_{i=1}^{3}\lambda_i V_{x_i}] = 0; \\ \sum_{i=1}^{3}\frac{\partial}{\partial x_i}(-\frac{1}{\rho}P_{x_i}) \equiv -\frac{1}{\rho}\Delta P;\ \ \mu\sum_{i=1}^{3}\frac{\partial}{\partial x_i}(\lambda_i\Delta V) = 0;\ \ \mu\sum_{i=1}^{3}\frac{\partial}{\partial x_i}(\Delta\Omega_i) = 0. \end{cases}$$

Then on a basis (4.38) system (4.37) it is equivalent, will be transformed to a kind:

$$\begin{cases} V_t + d^{-1}V[\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j \Omega_{ix_j})] + \sum_{j=1}^{3} V_{x_j}\Omega_j = \Phi_0 - d^{-1}[\frac{1}{4\pi}\int_{R^3}(\sum_{i=1}^{3}\frac{\tau_i}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}\times \\ \times\{\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j \Omega_{ih_j}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t))V_{h_i}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t)+ \\ +\sum_{i=1}^{3}(\sum_{j=1}^{3}\Omega_{jh_i}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t)V_{h_j}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t))\lambda_i\})\times \\ \times d\tau_1 d\tau_2 d\tau_3] + \mu\Delta V, (i=\overline{1,3}), \\ \Phi_0(x_1,x_2,x_3,t) \equiv d^{-1}[\sum_{i=1}^{3}\varphi_i - \frac{1}{4\pi}\int_{R^3}\{\sum_{i=1}^{3}\tau_i\frac{1}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}(F_0(x_1+\tau_1,x_2+\tau_2, \\ x_3+\tau_3;t))\}d\tau_1 d\tau_2 d\tau_3 + \mu\sum_{i=1}^{3}\Delta\Omega_i];\ \ d=\sum_{i=1}^{3}\lambda_i > 0;\ \ \mathrm{div}\varphi \neq 0. \end{cases} \tag{4.39}$$

Hence

$$\begin{cases} V = M_1 + \frac{1}{8\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-\frac{r^2}{4\mu(t-s)})(Q[V,V_{s_1},V_{s_2},V_{s_3}])(s_1,s_2,s_3,s)\times \\ \times\frac{1}{(\sqrt{\mu(t-s)})^3}ds_1 ds_2 ds_3 ds, \\ M_1(x_1,x_2,x_3,t) \equiv \frac{1}{\sqrt{\pi^3}}\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))V_0(x_1+2\tau_1\sqrt{\mu t}, x_2+2\tau_2\sqrt{\mu t}, \\ x_3+2\tau_3\sqrt{\mu t})d\tau_1 d\tau_2 d\tau_3 + \frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_0(x_1+2\tau_1\sqrt{\mu(t-s)}, \\ x_2+2\tau_2\sqrt{\mu(t-s)}, x_3+2\tau_3\sqrt{\mu(t-s)}; s)d\tau_1 d\tau_2 d\tau_3 ds, \\ s_j - x_j = 2\tau_j\sqrt{\mu t};\ \ s_j - x_j = 2\tau_j\sqrt{\mu(t-s)}, (j=\overline{1,3}), \\ (Q[V,V_{s_1},V_{s_2},V_{s_3}])(s_1,s_2,s_3,s) \equiv -\{d^{-1}[V(s_1,s_2,s_3,s)\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\Omega_{is_j}(s_1,s_2,s_3,s))]+ \\ +\sum_{j=1}^{3}V_{s_j}(s_1,s_2,s_3,s)\Omega_j(s_1,s_2,s_3,s) + d^{-1}[\frac{1}{4\pi}\int_{R^3}(\sum_{i=1}^{3}\overline{\tau}_i\frac{1}{\sqrt{(\overline{\tau}_1^2+\overline{\tau}_2^2+\overline{\tau}_3^2)^3}}\times \\ \times\{\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\Omega_{il_j}(s_1+\overline{\tau}_1,s_2+\overline{\tau}_2,s_3+\overline{\tau}_3;s))V_{l_i}(s_1+\overline{\tau}_1,s_2+\overline{\tau}_2,s_3+\overline{\tau}_3;s)+ \\ +\sum_{i=1}^{3}(\sum_{j=1}^{3}\Omega_{jl_i}(s_1+\overline{\tau}_1,s_2+\overline{\tau}_2,s_3+\overline{\tau}_3;s)V_{l_j}(s_1+\overline{\tau}_1,s_2+\overline{\tau}_2,s_3+\overline{\tau}_3;s))\lambda_i\})d\overline{\tau}_1 d\overline{\tau}_2 d\overline{\tau}_3]\}, \end{cases}$$

or

$$\begin{cases} V_{x_i} = W_i, \forall(x_1,x_2,x_3,t) \in T, (i=\overline{1,3}), \\ V = M_1 + \frac{1}{\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))(Q[V,W_1,W_2,W_3])(x_1+2\tau_1\sqrt{\mu\tau}, x_2 + \\ +2\tau_2\sqrt{\mu\tau}, x_3+2\tau_3\sqrt{\mu\tau}; t-\tau)d\tau_1 d\tau_2 d\tau_3 d\tau \equiv \Psi_0[V,W_1,W_2,W_3], \\ W_i = M_{1x_i} + \frac{1}{\sqrt{\pi^3}} \int_0^t \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\frac{\tau_i}{\sqrt{\mu\tau}}(Q[V,W_1,W_2,W_3])(x_1+2\tau_1\sqrt{\mu\tau}, \\ x_2+2\tau_2\sqrt{\mu\tau}, x_3+2\tau_3\sqrt{\mu\tau}; t-\tau)d\tau_1 d\tau_2 d\tau_3 d\tau \equiv \Psi_i[V,W_1,W_2,W_3], \\ s_i - x_i = 2\tau_i\sqrt{\mu(t-s)};\ \ t-s=\tau, (i=\overline{1,3}). \end{cases} \tag{4.40}$$

If concerning system (4.40) takes place (4.22), that the solution of system (4.40) we can find on the basis of Pkard's method

$$\begin{cases} V_{n+1} = \Psi_0[V_n, W_{1,n}, W_{2,n}, W_{3,n}], \\ W_{i,n+1} = \Psi_i[V_n, W_{1,n}, W_{2,n}, W_{3,n}], (i=\overline{1,3};\ n=0,1,\ldots), \\ E_0 = \|V-V_0\|_C + \sum_{i=1}^{3}\|W_i - W_{i,0}\|_C;\ \ E_{n+1} \equiv \|V_{n+1}-V\|_C + \sum_{i=1}^{3}\|W_{i,n+1}-W_i\|_C, \\ E_{n+1} \le h^{n+1}E_0 \xrightarrow[n\to\infty]{h<1} 0, \\ V_n \xrightarrow[n\to\infty]{h<1} V \equiv H(x_1,x_2,x_3,t), \\ W_{i,n} \xrightarrow[n\to\infty]{h<1} W_i,\ \forall(x_1,x_2,x_3,t)\in T, (i=\overline{1,3}). \end{cases} \tag{4.41}$$

Further, we will receive similar results in the conditions of the theorem 5.

Remark 4. Actually at use of offered transformations occurs linearization of equations Navier-Stokes in the integrated form without the requirement of additional conditions. Thus, the decision of the received integrated equations possesses that and properties, as the decision of initial problems. The received obvious analytical decision is regular concerning viscosity factor $0 < \mu < 1$ and in many respects simplifies carrying out of the analysis and in mathematical and physical sense. Therefore there is no necessity to copy known results of fundamental works, and it is enough to refer to them.

5. Problem Navier - Stokes with Average Viscosity with a Condition (A_3)

Area of the fluid with average viscosity with condition (A_2), when $\mathrm{div} f \neq 0$ it is studied in point 3. The decision method, from where follows of equations integration of Navier-Stokes (1.1)-(1.3) in a case (A_2), it is not applicable to a problem (1.1) - (1.3) with a condition (A_3). Hence, here we will consider a methods of the equations integration of Navier-Stokes with a condition (A_3), when: $\mathrm{div} f = 0; 0 < \mu = \mu_0 < \infty$.

5.1. Fluid with Average Viscosity with a Condition (A_3)

Therefore, in this case, if the initial data $(\upsilon_{i0}; f_i)$ is set in a kind:

$$\begin{cases} \upsilon_i\big|_{t=0} = \upsilon_{i0}(x_1,x_2,x_3) \equiv \lambda_i V_0(x_1,x_2,x_3), (0 < \lambda_i = \mathrm{const}; i=\overline{1,3}), \\ \sum_{i=1}^{3}\lambda_i V_{0x_i} = 0;\ f_i \equiv \varphi_i + K_{it},\ [K_{it} \equiv \frac{1}{\sqrt{\mu}}\bar{f}_i(x_1,x_2,x_3,t), i=\overline{1,3}], \end{cases}$$

$$\begin{cases} \mathrm{div} f = 0: \ \mathrm{div}\varphi = 0; \ \sum_{i=1}^{3} \frac{\partial}{\partial x_i} K_{it} = 0, \\ \mathrm{div}\overline{f} = 0; \ \mathrm{rot}\,\overline{f} = 0: \ \Delta \overline{f}_i = 0, (\Delta K_i = 0; i = \overline{1,3}), \\ U = \sum_{j=1}^{3} K_j^2; \ \sum_{j=1}^{3} K_j K_{ix_j} = \frac{1}{2}(\sum_{j=1}^{3} K_j^2)_{x_i} = \frac{1}{2} U_{x_i}, (i = \overline{1,3}), \\ K_i \equiv \frac{1}{\sqrt{\mu}} \int_0^t \overline{f}_i(x_1, x_2, x_3, s) ds; \ \varphi = (\varphi_1, \varphi_2, \varphi_3); \ \overline{f} = (\overline{f}_1, \overline{f}_2, \overline{f}_3), \end{cases} \quad (5.1)$$

we enter for definition a component of speeds:

$$\begin{cases} \upsilon_i = \lambda_i V(x_1, x_2, x_3, t) + K_i(x_1, x_2, x_3, t), (i = \overline{1,3}), \\ V\big|_{t=0} = V_0(x_1, x_2, x_3); \ \mathrm{div}\, v = 0: \sum_{i=1}^{3} \lambda_i V_{x_i} = 0; \ \sum_{i=1}^{3} K_{ix_i} = 0, \end{cases} \quad (5.2)$$

at that

$$\begin{cases} \sum_{j=1}^{3} \upsilon_j \upsilon_{ix_j} \equiv V \sum_{j=1}^{3} \lambda_j K_{ix_j} + \lambda_i \sum_{j=1}^{3} V_{x_j} K_j + \frac{1}{2} U_{x_i}, (\lambda_i V \sum_{j=1}^{3} \lambda_j V_{x_j} = 0), \\ \upsilon_{it} \equiv \lambda_i V_t + K_{it}; \ \mu \Delta \upsilon_i \equiv \mu\{\lambda_i \Delta V + \Delta K_i\} = \mu \lambda_i \Delta V, (i = \overline{1,3}). \end{cases} \quad (5.3)$$

Then on the basis (5.2), (5.3) we will receive

$$\lambda_i V_t + V \sum_{j=1}^{3} \lambda_j K_{ix_j} + \lambda_i \sum_{j=1}^{3} V_{x_j} K_j + \frac{1}{2} U_{x_i} = \varphi_i - \frac{1}{\rho} P_{x_i} + \mu \lambda_i \Delta V, (i = \overline{1,3}). \quad (5.4)$$

Hence on a basis (5.4), we have

$$\begin{cases} \Delta(\frac{1}{\rho} P + \frac{1}{2} U) = -\{\sum_{i=1}^{3} (\sum_{j=1}^{3} \lambda_j K_{ix_j}) V_{x_i} + \sum_{i=1}^{3} (\sum_{j=1}^{3} K_{jx_i} V_{x_j}) \lambda_i\}, \\ \frac{1}{\rho} P + \frac{1}{2} U = \frac{1}{4\pi} \int_{R^3} \frac{1}{r} \{\sum_{i=1}^{3} (\sum_{j=1}^{3} \lambda_j K_{is_j}(s_1, s_2, s_3, t)) V_{s_i}(s_1, s_2, s_3, t) + \\ + \sum_{i=1}^{3} (\sum_{j=1}^{3} K_{js_i}(s_1, s_2, s_3, t) V_{s_j}(s_1, s_2, s_3, t)) \lambda_i\} ds_1 ds_2 ds_3, \\ \frac{1}{\rho} P_{x_i} + \frac{1}{2} U_{x_i} = \frac{1}{4\pi} \int_{R^3} \frac{\tau_i}{\sqrt{(\tau_1^2 + \tau_2^2 + \tau_3^2)^3}} \{\sum_{i=1}^{3} (\sum_{j=1}^{3} \lambda_j K_{ih_j}(x_1 + \tau_1, x_2 + \tau_2, x_3 + \\ + \tau_3; t)) V_{h_i}(x_1 + \tau_1, x_2 + \tau_2, x_3 + \tau_3; t) + \sum_{i=1}^{3} (\sum_{j=1}^{3} K_{jh_i}(x_1 + \tau_1, x_2 + \tau_2, x_3 + \tau_3; t) \times \\ \times V_{h_j}(x_1 + \tau_1, x_2 + \tau_2, x_3 + \tau_3; t)) \lambda_i\} d\tau_1 d\tau_2 d\tau_3, (s_i - x_i = \tau_i; h_i = x_i + \tau_i; i = \overline{1,3}), \end{cases} \quad (5.5)$$

as

$$
\begin{cases}
\sum_{i=1}^{3}\frac{\partial}{\partial x_i}(5.4): \ \frac{\partial}{\partial t}[\sum_{i=1}^{3}\lambda_i V_{x_i}(x_1,x_2,x_3,t)]=0, \\
\sum_{i=1}^{3}\frac{\partial}{\partial x_i}\frac{1}{2}U_{x_i}\equiv \Delta U; \ \sum_{i=1}^{3}\frac{\partial}{\partial x_i}(-\frac{1}{\rho}P_{x_i})\equiv -\frac{1}{\rho}\Delta P; \ \ \mu\sum_{i=1}^{3}\frac{\partial}{\partial x_i}(\lambda_i\Delta V)=0, \\
\sum_{i=1}^{3}\frac{\partial}{\partial x_i}(V\sum_{j=1}^{3}\lambda_j K_{ix_j})\equiv \sum_{i=1}^{3}\lambda_i(\sum_{j=1}^{3}V_{x_j}K_{jx_i})+\sum_{j=1}^{3}K_j\frac{\partial}{\partial x_j}(\sum_{i=1}^{3}\lambda_i V_{x_i})_{x_j}=\sum_{i=1}^{3}\lambda_i(\sum_{j=1}^{3}V_{x_j}K_{jx_i}), \\
\sum_{i=1}^{3}\frac{\partial}{\partial x_i}(\lambda_i\sum_{j=1}^{3}V_{x_j}K_j)\equiv \sum_{i=1}^{3}V_{x_i}(\sum_{j=1}^{3}\lambda_j K_{ix_j})+V\sum_{j=1}^{3}\lambda_j(\sum_{i=1}^{3}K_{ix_i})_{x_j}=\sum_{i=1}^{3}V_{x_i}(\sum_{j=1}^{3}\lambda_j K_{ix_j}), \\
\sum_{j=1}^{3}K_j\frac{\partial}{\partial x_j}(\sum_{i=1}^{3}\lambda_i V_{x_i})_{x_j}=0; \ \ V\sum_{j=1}^{3}\lambda_j(\sum_{i=1}^{3}K_{ix_i})_{x_j}=0; \ \ \mathrm{div}\varphi=0.
\end{cases}
$$

Then system (5.4) it is equivalent will be transformed to a kind

$$
\begin{cases}
V_t+d^{-1}V[\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j K_{ix_j})]+\sum_{j=1}^{3}V_{x_j}K_j=\Phi_0-d^{-1}[\frac{1}{4\pi}\int_{R^3}(\sum_{i=1}^{3}\frac{\tau_i}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}\times \\
\times\{\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j K_{ih_j}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t))V_{h_i}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t)+\sum_{i=1}^{3}(\sum_{j=1}^{3}K_{jh_i}(x_1+ \\
+\tau_1,x_2+\tau_2,x_3+\tau_3;t)V_{h_j}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t))\lambda_i\})d\tau_1 d\tau_2 d\tau_3]+\mu\Delta V,(i=\overline{1,3}), \\
\Phi_0\equiv d^{-1}\sum_{i=1}^{3}\varphi_i; \ \ d=\sum_{i=1}^{3}\lambda_i>0.
\end{cases} \tag{5.6}
$$

Or for consideration of unknown function V we have

$$
\begin{cases}
V=M_1(x_1,x_2,x_3,t)+\frac{1}{8\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-\frac{r^2}{4\mu(t-s)})(Q[V,V_{s_1},V_{s_2},V_{s_3}])(s_1,s_2,s_3,s)\times \\
\times\frac{1}{(\sqrt{\mu(t-s)})^3}ds_1 ds_2 ds_3 ds, \\
M_1\equiv\frac{1}{\sqrt{\pi^3}}\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))V_0(x_1+2\tau_1\sqrt{\mu t},x_2+2\tau_2\sqrt{\mu t},x_3+2\tau_3\sqrt{\mu t})d\tau_1 d\tau_2 d\tau_3+ \\
+\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\Phi_0(x_1+2\tau_1\sqrt{\mu(t-s)},x_2+2\tau_2\sqrt{\mu(t-s)},x_3+2\tau_3\sqrt{\mu(t-s)};s)\times \\
\times d\tau_1 d\tau_2 d\tau_3 ds, \\
(Q[V,V_{s_1},V_{s_2},V_{s_3}])(s_1,s_2,s_3,s)\equiv-\{d^{-1}[V(s_1,s_2,s_3,s)\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j K_{is_j}(s_1,s_2,s_3,s))]+
\end{cases}
$$

$$\begin{cases} +\sum_{j=1}^{3} V_{s_j}(s_1,s_2,s_3,s)\times K_j(s_1,s_2,s_3,s)+d^{-1}[\frac{1}{4\pi}\int_{R^3}(\sum_{i=1}^{3}\overline{\tau}_i\frac{1}{\sqrt{(\overline{\tau}_1^2+\overline{\tau}_2^2+\overline{\tau}_3^2)^3}}\times \\ \times\{\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j K_{il_j}(s_1+\overline{\tau}_1,s_2+\overline{\tau}_2,s_3+\overline{\tau}_3;s))V_{l_i}(s_1+\overline{\tau}_1,s_2+\overline{\tau}_2,s_3+\overline{\tau}_3;s)+ \\ +\sum_{i=1}^{3}(\sum_{j=1}^{3}K_{jl_i}(s_1+\overline{\tau}_1,s_2+\overline{\tau}_2,s_3+\overline{\tau}_3;s)V_{l_j}(s_1+\overline{\tau}_1,s_2+\overline{\tau}_2,s_3+\overline{\tau}_3;s))\lambda_i\})d\overline{\tau}_1 d\overline{\tau}_2 d\overline{\tau}_3]\}, \\ s_j-x_j=2\tau_j\sqrt{\mu t};\ \ s_j-x_j=2\tau_j\sqrt{\mu(t-s)},(j=\overline{1,3}). \end{cases} \tag{5.7}$$

Hence, differentiating (5.7) on x_i and having entered designation:

$$V_{x_i}=W_i,\forall(x_1,x_2,x_3,t)\in T,(i=\overline{1,3}), \tag{5.8}$$

from (5.7) we will receive

$$\begin{cases} V=M_1+\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))(Q[V,W_1,W_2,W_3])(x_1+2\tau_1\sqrt{\mu\tau},x_2+ \\ +2\tau_2\sqrt{\mu\tau},x_3+2\tau_3\sqrt{\mu\tau};t-\tau)d\tau_1 d\tau_2 d\tau_3 d\tau\equiv Z_0[V,W_1,W_2,W_3], \\ W_i=M_{1x_i}+\frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\frac{\tau_i}{\sqrt{\mu\tau}}(Q[V,W_1,W_2,W_3])(x_1+2\tau_1\sqrt{\mu\tau}, \\ x_2+2\tau_2\sqrt{\mu\tau},x_3+2\tau_3\sqrt{\mu\tau};t-\tau)d\tau_1 d\tau_2 d\tau_3 d\tau\equiv Z_i[V,W_1,W_2,W_3], \\ s_i-x_i=2\tau_i\sqrt{\mu(t-s)};t-s=\tau,(i=\overline{1,3}). \end{cases} \tag{5.9}$$

If takes place:

$$\begin{cases} \forall(x_1,x_2,x_3,t)\in T;M_1,Y,K_i:\ \sup_T\left|D^k M_1(x_1,x_2,x_3,t)\right|\le\beta_1,\ (k=\overline{0,3}), \\ \sup_{T\times T}Y(x_1,x_2,x_3,\tau_1,\tau_2,\tau_3;t,\tau)\equiv\sup_{T\times T}\{d^{-1}\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\left|K_{is_j}(x_1+2\tau_1\sqrt{\mu\tau},\right. \\ \left.x_2+2\tau_2\sqrt{\mu\tau},x_3+2\tau_3\sqrt{\mu\tau};t-\tau)\right|)+\sum_{j=1}^{3}\left|K_j(x_1+2\tau_1\sqrt{\mu\tau},x_2+2\tau_2\sqrt{\mu\tau},x_3+\right. \\ \left.+2\tau_3\sqrt{\mu\tau};t-\tau)\right|+d^{-1}[\frac{1}{4\pi}\int_{R^3}(\sum_{i=1}^{3}\left|\overline{\tau}_i\right|\frac{1}{\sqrt{(\overline{\tau}_1^2+\overline{\tau}_2^2+\overline{\tau}_3^2)^3}}\{\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j\left|K_{il_j}(x_1+\right. \\ \left.+2\tau_1\sqrt{\mu\tau}+\overline{\tau}_1,x_2+2\tau_2\sqrt{\mu\tau}+\overline{\tau}_2,x_3+2\tau_3\sqrt{\mu\tau}+\overline{\tau}_3;t-\tau)\right|)+\sum_{i=1}^{3}(\sum_{j=1}^{3}\left|K_{jl_i}(x_1+\right. \\ \left.+2\tau_1\sqrt{\mu\tau}+\overline{\tau}_1,x_2+2\tau_2\sqrt{\mu\tau}+\overline{\tau}_2,x_3+2\tau_3\sqrt{\mu\tau}+\overline{\tau}_3;t-\tau)\right|)\lambda_i\})d\overline{\tau}_1 d\overline{\tau}_2 d\overline{\tau}_3]\}\le\frac{1}{\sqrt{\mu}}\beta_2, \end{cases}$$

$$\begin{cases} k_i = \frac{1}{\sqrt{\pi^3}} \sup_T \int_0^t \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))Y(x_1,x_2,x_3,\tau_1,\tau_2,\tau_3;t,\tau)\frac{|\tau_i|}{\sqrt{\mu\tau}} \times \\ \times d\tau_1 d\tau_2 d\tau_3 d\tau \le \gamma_1\sqrt{T_0}\beta_2\frac{1}{\mu}, (i=\overline{1,3}), \\ k_0 = \frac{1}{\sqrt{\pi^3}} \sup_T \int_0^t \int_{R^3} \exp(-(\tau_1^2+\tau_2^2+\tau_3^2))Y(x_1,x_2,x_3,\tau_1,\tau_2,\tau_3;t,\tau)d\tau_1 d\tau_2 d\tau_3 d\tau \le \\ \le \frac{1}{\sqrt{\mu}}\beta_2 T_0, (\gamma_1 = \sqrt[4]{2^5}),\ \beta = \max(\beta_2 T_0; 3\gamma_1\sqrt{T_0}\beta_2), \\ Z_i, (i=\overline{0,3}): \\ h = \sum_{i=0}^{3} k_i \le \frac{1}{\sqrt{\mu}}(\beta_2 T_0 + 3\gamma_1\sqrt{T_0}\beta_2\frac{1}{\sqrt{\mu}}) \le \frac{1}{\sqrt{\mu}}(1+\frac{1}{\sqrt{\mu}})\beta < 1, (0<\mu=\mu_0<\infty), \end{cases} \tag{5.10}$$

that

$$\begin{cases} V, W_i \in C(T): E = \|V\|_C + \sum_{i=1}^{3}\|W_i\|_C, \\ E \le (1-h)^{-1}4\beta_1. \end{cases} \tag{5.11}$$

Then the solution of this system we can find on the basis of Pikard's method

$$\begin{cases} V_{n+1} = Z_0[V_n, W_{1,n}, W_{2,n}, W_{3,n}], \\ W_{i,n+1} = Z_i[V_n, W_{1,n}, W_{2,n}, W_{3,n}], (i=\overline{1,3}; n=0,1,...), \end{cases} \tag{5.12}$$

at that

$$\begin{cases} E_0 = \|V-V_0\|_C + \sum_{i=1}^{3}\|W_i - W_{i,0}\|_C;\ E_{n+1} \equiv \|V_{n+1}-V\|_C + \sum_{i=1}^{3}\|W_{i,n+1}-W_i\|_C, \\ E_{n+1} \le h^{n+1}E_0 \xrightarrow[n\to\infty]{h<1} 0. \end{cases} \tag{5.13}$$

From here follows

$$\begin{cases} V_n \xrightarrow[n\to\infty]{h<1} V \equiv H(x_1,x_2,x_3,t), \\ W_{i,n} \xrightarrow[n\to\infty]{h<1} W_i,\ \forall(x_1,x_2,x_3,t)\in T, (i=\overline{1,3}). \end{cases} \tag{5.14}$$

Theorem 6. Under conditions (1.2), (1.3), (A_3), (5.1), (5.10) problem Navier-Stokes has the single continuous decision, which it is found by a rule (5.2).

Remarks:

I. Singleness is obvious, as a method by contradiction. Results (5.14) with a condition ((A_3), (5.1), (5.9)) are received where smoothness of functions is required only on x_i as the derivative of 1st order is in time has $t>0$. Then taking into account (5.14) the system (5.9) has the single continuous decision $V \in C^{3,0}(T)$.

II. The algorithm (5.2) also is applicable in a case, if

$$\begin{cases} f_i \equiv \varphi_i + K_{it},\ \mathrm{div} f \neq 0:\ \mathrm{div}\varphi \neq 0;\ \sum_{i=1}^{3} \frac{\partial}{\partial x_i} K_{it} = 0, \\ \mathrm{div}\overline{f} = 0;\ \mathrm{rot}\,\overline{f} = 0:\ \Delta \overline{f}_i = 0, (\Delta K_i = 0; i = \overline{1,3}),\ U = \sum_{j=1}^{3} K_j^2;\ \sum_{j=1}^{3} K_j K_{ix_j} = \frac{1}{2} U_{x_i}, \\ \upsilon_i \big|_{t=0} = \upsilon_{i0}(x_1, x_2, x_3) \equiv \lambda_i V_0(x_1, x_2, x_3), (i = \overline{1,3}). \end{cases} \tag{5.15}$$

That on a basis (5.15), (5.2) we will receive (5.4). Hence, we have

$$\begin{cases} \sum_{i=1}^{3} \frac{\partial}{\partial x_i}(5.4):\ \ \Delta(\frac{1}{\rho}P + \frac{1}{2}U) = -\{\sum_{i=1}^{3}(\sum_{j=1}^{3} \lambda_j K_{ix_j}) V_{x_i} + \sum_{i=1}^{3}(\sum_{j=1}^{3} K_{jx_i} V_{x_j})\lambda_i - \mathrm{div}\varphi\}, \\ \frac{1}{\rho}P + \frac{1}{2}U = \frac{1}{4\pi} \int_{R^3} \frac{1}{r}\{\sum_{i=1}^{3}(\sum_{j=1}^{3} \lambda_j K_{is_j}(s_1, s_2, s_3, t)) V_{s_i}(s_1, s_2, s_3, t) + \\ + \sum_{i=1}^{3}(\sum_{j=1}^{3} K_{js_i}(s_1, s_2, s_3, t) V_{s_j}(s_1, s_2, s_3, t))\lambda_i - \sum_{i=1}^{3} \varphi_{is_i}(s_1, s_2, s_3, t)\} ds_1 ds_2 ds_3, \\ \frac{1}{\rho}P_{x_i} + \frac{1}{2}U_{x_i} = \frac{1}{4\pi} \int_{R^3} \frac{\tau_i}{\sqrt{(\tau_1^2 + \tau_2^2 + \tau_3^2)^3}} \{\sum_{i=1}^{3}(\sum_{j=1}^{3} \lambda_j K_{ih_j}(x_1 + \tau_1, x_2 + \tau_2, x_3 + \tau_3; t)) \times \\ \times V_{h_i}(x_1 + \tau_1, x_2 + \tau_2, x_3 + \tau_3; t) + \sum_{i=1}^{3}(\sum_{j=1}^{3} K_{jh_i}(x_1 + \tau_1, x_2 + \tau_2, x_3 + \tau_3; t) V_{h_j}(x_1 + \tau_1, x_2 + \tau_2, \\ x_3 + \tau_3; t))\lambda_i - \sum_{i=1}^{3} \varphi_{ih_i}(x_1 + \tau_1, x_2 + \tau_2, x_3 + \tau_3; t)\} d\tau_1 d\tau_2 d\tau_3, (s_i - x_i = \tau_i; h_i = x_i + \tau_i), \end{cases} \tag{5.16}$$

here (5.16) differs from (5.5), as $\mathrm{div}\varphi \neq 0.$ Then considering (5.6)-(5.8) we will receive system (5.9):

$$\begin{cases} V = Z_0[V, W_1, W_2, W_3], \\ W_i = Z_i[V, W_1, W_2, W_3], (i = \overline{1,3}). \end{cases} \tag{5.17}$$

Therefore, as takes place (5.10) in a consequence (5.12), (5.13) we will receive

$$V_n \xrightarrow[n \to \infty]{h<1} V \equiv H(x_1, x_2, x_3, t);\ W_{i,n} \xrightarrow[n \to \infty]{h<1} W_i,\ \ \forall (x_1, x_2, x_3, t) \in T, (i = \overline{1,3}). \tag{5.18}$$

Further, we will receive similar results in the conditions of the theorem 6.

5.2. Method (5.2), When $\mathrm{div} f \neq 0, (\mathrm{div}\overline{f} = 0;\ \mathrm{rot}\,\overline{f} \neq 0)$

If the initial data $\upsilon_{i0}(x_1, x_2, x_3), f_i$ is set in a kind

$$\begin{cases} \upsilon_i \big|_{t=0} = \upsilon_{i0}(x_1, x_2, x_3) \equiv \lambda_i V_0(x_1, x_2, x_3), f_i \equiv \varphi_i + K_{it}, (K_{it} \equiv \frac{1}{\sqrt{\mu}} \overline{f}_i(x_1, x_2, x_3, t), i = \overline{1,3}), \\ \mathrm{div} f \neq 0:\ \mathrm{div}\varphi \neq 0; \sum_{i=1}^{3} \frac{\partial}{\partial x_i} K_{it} = 0;\ \ \mathrm{div}\overline{f} = 0;\ \ \mathrm{rot}\,\overline{f} \neq 0, (\Delta K_i \neq 0, i = \overline{1,3}), \end{cases} \tag{5.19}$$

that on a basis (5.2) and (5.19), we have

$$\begin{cases} \upsilon_i = \lambda_i V(x_1,x_2,x_3,t) + K_i(x_1,x_2,x_3,t),(i=\overline{1,3}), \\ V\big|_{t=0} = V_0(x_1,x_2,x_3); \ \operatorname{div} v = 0: \sum_{i=1}^{3}\lambda_i V_{x_i} = 0; \ \sum_{i=1}^{3} K_{ix_i} = 0, \\ \sum_{j=1}^{3}\upsilon_j \upsilon_{ix_j} \equiv V\sum_{j=1}^{3}\lambda_j K_{ix_j} + \lambda_i \sum_{j=1}^{3} V_{x_j} K_j + \sum_{j=1}^{3} K_j K_{ix_j},(\lambda_i V \sum_{j=1}^{3}\lambda_j V_{x_j} = 0), \\ \upsilon_{it} \equiv \lambda_i V_t + K_{it}; \ \mu\Delta\upsilon_i \equiv \mu[\lambda_i \Delta V + \Delta K_i],(i=\overline{1,3}). \end{cases} \tag{5.20}$$

Then from (1.1) follows

$$\lambda_i V_t + V\sum_{j=1}^{3}\lambda_j K_{ix_j} + \lambda_i \sum_{j=1}^{3} V_{x_j} K_j + \sum_{j=1}^{3} K_j K_{ix_j} = \varphi_i - \frac{1}{\rho}P_{x_i} + \mu[\lambda_i \Delta V + \Delta K_i],(i=\overline{1,3}). \tag{5.21}$$

Hence on a basis (5.21), we have

$$\begin{cases} \sum_{i=1}^{3}\frac{\partial}{\partial x_i}(5.21): \ \Delta\frac{1}{\rho}P = -\{F_0(x_1,x_2,x_3,t) + \sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j K_{ix_j})V_{x_i} + \\ +\sum_{i=1}^{3}(\sum_{j=1}^{3} K_{jx_i} V_{x_j})\lambda_i\}, \\ \frac{1}{\rho}P = \frac{1}{4\pi}\int_{R^3}\frac{1}{r}\{F_0(s_1,s_2,s_3,t) + \sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j K_{is_j}(s_1,s_2,s_3,t))V_{s_i}(s_1,s_2,s_3,t) + \\ +\sum_{i=1}^{3}(\sum_{j=1}^{3} K_{js_i}(s_1,s_2,s_3,t)V_{s_j}(s_1,s_2,s_3,t))\lambda_i\}ds_1 ds_2 ds_3, \\ \frac{1}{\rho}P_{x_i} = \frac{1}{4\pi}\int_{R^3}\frac{\tau_i}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}\{F_0(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t) + \sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j K_{ih_j}(x_1 + \\ +\tau_1,x_2+\tau_2,x_3+\tau_3;t))V_{h_i}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t) + \sum_{i=1}^{3}(\sum_{j=1}^{3} K_{jh_i}(x_1+\tau_1,x_2+\tau_2, \\ x_3+\tau_3;t)V_{h_j}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t))\lambda_i\}d\tau_1 d\tau_2 d\tau_3, \\ s_i - x_i = \tau_i; \ h_i = x_i + \tau_i,(i=\overline{1,3}), \end{cases} \tag{5.22}$$

as

$$\begin{cases} F_0(x_1,x_2,x_3,t) \equiv \sum_{i=1}^{3}(\frac{\partial}{\partial x_i}(\sum_{j=1}^{3} K_j K_{ix_j}) - \varphi_{ix_i}), \\ \frac{\partial}{\partial t}[\sum_{i=1}^{3}\lambda_i V_{x_i}] = 0; \ \mu\sum_{i=1}^{3}\frac{\partial}{\partial x_i}(\lambda_i \Delta V) = 0; \ \sum_{i=1}^{3}\frac{\partial}{\partial x_i}(-\frac{1}{\rho}P_{x_i}) \equiv -\frac{1}{\rho}\Delta P; \ \mu\sum_{i=1}^{3}\frac{\partial}{\partial x_i}(\Delta K_i) = 0. \end{cases}$$

Therefore system (5.21) it is equivalent will be transformed to a kind

$$\begin{cases} V_t + d^{-1}V[\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j K_{ix_j})] + \sum_{j=1}^{3}V_{x_j}K_j = \Phi_0 - d^{-1}[\frac{1}{4\pi}\int_{R^3}(\sum_{i=1}^{3}\frac{\tau_i}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}\times \\ \times\{\sum_{i=1}^{3}(\sum_{j=1}^{3}\lambda_j K_{ih_j}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t))V_{h_i}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t)+ \\ +\sum_{i=1}^{3}(\sum_{j=1}^{3}K_{jh_i}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t)V_{h_j}(x_1+\tau_1,x_2+\tau_2,x_3+\tau_3;t))\lambda_i\})\times \\ \times d\tau_1 d\tau_2 d\tau_3] + \mu\Delta V,(i=\overline{1,3}), \\ \Phi_0(x_1,x_2,x_3,t) \equiv d^{-1}[\sum_{i=1}^{3}\varphi_i - \frac{1}{4\pi}\int_{R^3}\{\sum_{i=1}^{3}\tau_i\frac{1}{\sqrt{(\tau_1^2+\tau_2^2+\tau_3^2)^3}}(F_0(x_1+\tau_1,x_2+\tau_2, \\ x_3+\tau_3;t))\}d\tau_1 d\tau_2 d\tau_3 + \mu\sum_{i=1}^{3}\Delta K_i], \end{cases} \tag{5.23}$$

where $d = \sum_{i=1}^{3}\lambda_i > 0;\ \operatorname{div}\varphi \neq 0.$

Or considering (5.7) - (5.9) for of unknown functions

$$V(x_1,x_2,x_3,t),\ W_i = V_{x_i}(x_1,x_2,x_3,t),(i=\overline{1,3}),$$

we have

$$\begin{cases} V = M_1(x_1,x_2,x_3,t) + \frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))(Q[V,W_1,W_2,W_3])(x_1+ \\ +2\tau_1\sqrt{\mu\tau},x_2+2\tau_2\sqrt{\mu\tau},x_3+2\tau_3\sqrt{\mu\tau};t-\tau)d\tau_1 d\tau_2 d\tau_3 d\tau \equiv Z_0[V,W_1,W_2,W_3], \\ W_i = M_{1x_i}(x_1,x_2,x_3,t) + \frac{1}{\sqrt{\pi^3}}\int_0^t\int_{R^3}\exp(-(\tau_1^2+\tau_2^2+\tau_3^2))\frac{\tau_i}{\sqrt{\mu\tau}}\times \\ \times(Q[V,W_1,W_2,W_3])(x_1+2\tau_1\sqrt{\mu\tau},x_2+2\tau_2\sqrt{\mu\tau},x_3+2\tau_3\sqrt{\mu\tau};t-\tau)d\tau_1 d\tau_2 d\tau_3 d\tau \equiv \\ \equiv Z_i[V,W_1,W_2,W_3], \\ s_i - x_i = 2\tau_i\sqrt{\mu(t-s)};t-s=\tau,(i=\overline{1,3}). \end{cases} \tag{5.24}$$

Further, at the condition (5.10) in a consequence (5.11)-(5.13) takes place

$$V_n \xrightarrow[n\to\infty]{h<1} V \equiv H(x_1,x_2,x_3,t);\ W_{i,n} \xrightarrow[n\to\infty]{h<1} W_i,\ \forall(x_1,x_2,x_3,t)\in T,(i=\overline{1,3}).$$

Hence, we will receive similar results in the conditions of the theorem 6.

6. Conclusions

1. From the received results follows that system Navier-Stokes (1.1) in the conditions of (1.2), (1.3), (A_1)-(A_3) can have single analytical is conditional-smooth decision, i.e. we will receive the solution of a millennium problem [1]. At least, such decision answers to a question, and possibility to construct the decision of a problem of Navier-Stokes (1.1)-(1.3) for an incompressible fluid with viscosity [1].

2. The received results is analogous to the celebrated Beale-Kato-Majda type criterion for the inviscid equations of incompressible fluids [2, 5].

3. Results of the theorems 1, 2 and 4 are applicable in a case $\nu \in R^3, x \in R^3, T \in R_+$. And results of the theorems 3, 5, 6 can be applied to a problem of Navier-Stokes of an incompressible fluid with viscosity, when

$$\nu \in R^n, x \in R^n, t \in [0, T_0].$$

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