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Magneto-hydrodynamic Thermal Instability of Walters' B' Fluid under the Effects of Rotation, Variable Gravity Field and Suspended Particles in a Brinkman Porous Medium

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Abstract The objective of the present paper is to study the stability of a rotating Walters' B' elastico-viscous fluid layer heated from below through a Brinkman porous medium in the presence of magnetic field, variable gravity and suspended particles for the case of free boundaries. After linearizing the governing hydrodynamic equations and applying normal mode analysis method, dispersion relation is obtained and solved numerically. For the case of stationary convection, Walters' B' elastico-viscous fluid behaves like an ordinary Newtonian fluid. The suspended particles have a destabilizing effect on the system whereas rotation has a stabilizing effect on the system. The magnetic field, medium permeability and Darcy number have both stabilizing and destabilizing effects on the system under certain conditions. The effects of suspended particles, rotation, magnetic field, medium permeability and Darcy-Brinkman number on critical Rayleigh number have also been plotted graphically to depict the stability characteristics. It is also observed that suspended particles, rotation, magnetic field and medium permeability introduce oscillatory modes in the system which is non-existent in their absence. Comparisons with the previous published work are also presented.

Keywords Thermal Convection, Suspended Particles, Magnetic Field, Permeability, Rotation, Brinkman Porous Medium

1. Introduction

When a horizontal layer of a viscous fluid is heated from below in a gravitational field, the system becomes unstable and convection sets in form of a regular cellular pattern when the Rayleigh number R exceeds a certain critical value R_c. A detailed account of the onset of thermal instability (Bénard convection) of a Newtonian fluid in a fluid layer under varying assumptions of hydrodynamics has been given by Chandrasekhar[1] in his celebrated monograph. Rayleigh[2] was the first to study the problem theoretically. Scanlon & Segel[3] have investigated the effect of suspended particles on the onset of Bénard convection and found that the critical Rayleigh number is reduced because the heat capacity of the pure gas was supplemented by the particles. Thus the effect of the suspended particles is to destabilize the layer. Thermal instability in a horizontal layer of an electrically conducting fluid heated from below and permeated by a uniform vertical magnetic field has been studied by Thompson[4] and experimentally by Nakagawa[5]. Bhatia & Steiner[6] have studied the problem of thermal instability of a Maxwellian visco-elastic fluid in the presence of a magnetic field and found that the magnetic field has a stabilizing influence on the overstable mode of convection in a visco-elastic fluid layer.

In stellar atmospheres and interiors, the magnetic field may be variable and may alter the nature of the instability. Coriolis forces also play an important role on the stability of a system. Numerous studies have been undertaken in the past to understand the convective instability of Newtonian fluids. The interest for investigations of non-Newtonian fluids (e.g. polymer suspensions, ketchup, paint, shampoo, blood, protein solutions, soap solution, liquid crystals, starch suspensions) is motivated by a wide range of engineering applications which include geophysical fluid dynamics, chemical technology and petroleum industry. There are many elastico-viscous fluids that cannot be characterized either by Oldroyd's constitutive relations or Maxwell's constitutive relations. One such class of viscoelastic fluid is Walters' (model B') fluid. Walters'[7] reported that the mixture of polymethyl methacrylate and pyridine at 25℃

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containing 30.5g of polymer with density 0.98g per litre behaves very closely as the Walters' (model B') fluid. With the growing importance of Walters' (model B') fluids in the manufacture of parts of space crafts, cushions, ropes, tyres, foams, seats, engineering equipments, the investigations on such fluids are desirable. Thermal instability of a Walters' B' elastic-viscous fluid in the presence of a variable gravity field and rotation in a porous medium has been investigated by Sharma & Rana[8]. Rana & Kumar[9] have studied the effect of rotation and suspended particles on the stability of an incompressible Walters' (model B') fluid heated from below under a variable gravity field in a porous medium while the problem of thermal convection in Walter' (model B') rotating fluid permeated with suspended particles and variable gravity field in porous medium in hydromagnetics have been studied by Sharma and Rana [10]. Recently, Rana[11] investigated the effect of suspended particles on thermal instability of Walters' (model B') fluid in a Brinkman porous medium.

Flow through porous medium has gained considerable interest and has been recognized as a problem of fundamental importance for many decades owing to its importance in several branches of engineering and science like ground water hydrology, petroleum engineering, soil science, chemical engineering, geophysics, astrophysics etc. The physical properties of comets and meteorites suggest the importance of porosity in astrophysical context. The investigation in porous medium has been started by Darcy model and gradually was extended to Darcy-Brinkman model. It is found, both theoretically and experimentally, that Darcy's equation provides inadequate results of the hydrodynamic conditions, particularly near the boundaries of a porous medium. To be mathematically compatible with the Navier-Stokes equations, Brinkman proposed the inclusion of the term $\frac{\tilde{\mu}}{\rho_0}\nabla^2 q$ in addition to $-\left(\frac{\mu}{k_1}\right)q$ in the equations of motion. Nield and Bejan[12] have studied the problem of convection in a porous medium. Lapwood[13] and Wooding[14] have investigated the stability of convective flow in a porous medium taking into account the Darcy's resistance. Durlofsky & Brady[15], using a Green's function, showed that the Brinkman model is valid for particles whose porosity $(\in)$> 0.95.

Since the earth's gravity field varies with the vertical distance from its surface, so it is necessary to take into account the gravity as a variable quantity with height from the earth's centre. This variation of gravity plays an important role for large scale flows in ocean, atmosphere and mantle but for laboratory purposes, we usually neglect these variations. Thermal instability of a fluid layer under varying gravity field heated from above and below has been investigated by Pradhan and Samal[16].

We have performed a linear stability analysis as a first step towards investigating this problem. Keeping in mind the importance and various applications of non-Newtonian fluids, our main objective, in the present paper, is to study the effects of rotation, magnetic field and suspended particles on thermal instability of a Walters' (model B') fluid in a Brinkman porous medium in the presence of a variable gravity field. The interest for studying this problem is to control the magneto-hydrodynamic thermal instability under the influences of variable gravity, rotation, suspended particles in a Brinkman porous medium.

2. Mathematical Formulation and Linear Stability Analysis

We consider a horizontal layer of an incompressible Walters' B' visco-elastic fluid between two plates at a distance d apart in the presence of a porous medium of porosity $\in$ and permeability k. The fluid layer is acted upon by a uniform rotation **Ω** (0, 0, Ω), vertical magnetic field **H** (0, 0, H) and variable gravity field $\mathbf{g} = \lambda g_0$, where λ can be positive or negative according as the gravity increases or decreases upward from its value g_0 (>0). An adverse temperature gradient $\beta = \left|\frac{dT}{dz}\right|$ is maintained by underside heating. The pressure p, density ρ, viscosity μ and visco-elasticity μ' depend upon the vertical co-ordinate z-only.

The basic equations of hydromagnetic in unsteady state relevant to the problem are defined as:

$$\frac{\rho_0}{\in}\left[\frac{\partial q}{\partial t}+\frac{1}{\in}(q.\nabla)q\right]=-\nabla p+\rho_0 X_i-\frac{1}{k_1}\left(\mu-\mu'\frac{\partial}{\partial t}\right)q$$
$$+\tilde{\mu}\nabla^2 q+\frac{2\rho_0}{\in}(q\times\Omega)+\frac{K'N}{\in}(q_d-q)+\frac{\mu_e}{4\pi}(\nabla\times H)\times H \qquad (1)$$

$$\nabla.q = 0, \qquad (2)$$

Where in the above equations ρ_0 is the fluid density, $\in$ is porosity, μ denotes viscosity, μ' denotes visco-elasticity, $\tilde{\mu}$ is the effective viscosity of the porous medium, υ is the kinematic viscosity, υ' is the kinematic visco-elasticity, p is the pressure, μ_e is magnetic permeability, X_i is the external force due to gravity variations, ∇^2 is the three dimensional Laplacian operator, $K' = 6\pi\rho\upsilon\delta$ where δ being particle radius, is the Stokes' drag coefficient, α is the co-efficient of volume expansion and q is the velocity of pure fluid. The last term on the right side of equation (1) incorporates the effect of the magnetic field on the fluid motion and also known as Lorentz force. Here $q_d(l,r,s)$ and $N(\bar{x},t)$ denote the velocity and number density of the particles respectively. The presence of suspended particles adds an extra force term, in equation of motion, proportional to velocity difference between particles and fluid. Since the force exerted by the fluid on the particles is equal and opposite to that exerted by the particles on the fluid, there must be an extra force term, equal in magnitude but opposite in sign, in the equations of motion for the particles. The distances between the particles are assumed to be quite large compared with their diameters thereby neglecting the effect of inter-particle reactions. The effects of pressure, magnetic field and gravity on the particles are very small and hence ignored.

If mN is the mass of particles per unit volume, then the equations of motion and continuity for the particles are:

$$mN\left[\frac{\partial q_d}{\partial t}+\frac{1}{\in}\left(q_d.\nabla\right)q_d\right]=K'N\left(q-q_d\right), \quad (3)$$

$$\in\frac{\partial N}{\partial t}+\nabla.\left(Nq_d\right)=0, \quad (4)$$

Assuming that the suspended particles and the fluid particles are in thermal equilibrium then the equation of heat conduction gives:

$$\left[\in\rho_0 c_v+\rho_s c_s\left(1-\in\right)\right]\frac{\partial T}{\partial t}+\rho_0 c_v\left(q.\nabla\right)T$$
$$+mNc_{pt}\left(\in\frac{\partial}{\partial t}+q_d.\nabla\right)T=k\nabla^2 T, \quad (5)$$

In the above equation $\rho_s, c_v, c_s, c_{pt}, T\ and\ k$ denote, respectively, the density of solid material, specific heat at constant volume, heat capacity of solid material, heat capacity of particles, the temperature and the effective thermal conductivity of the pure fluid.

The Maxwell's electromagnetic equation yields

$$\in\frac{\partial H}{\partial t}=\left(H.\nabla\right)q-\left(q.\nabla\right)H+\in\eta\nabla^2 H, \quad (6)$$

$$\text{and } \nabla.H=0 \quad (7)$$

Where η denote the resistivity.

The equation of state of the system is

$$\rho=\rho_0\left[1+\alpha\left(T_0-T\right)\right] \quad (8)$$

3. Perturbation Equations

Let the perturbations in fluid velocity q(0,0,0), particle velocity **q_d** (0,0,0), temperature T, pressure p, density ρ and magnetic field H be denoted by **q** (u,v,w), **q_d** (l,r,s), $\theta, \delta p$, $\delta\rho$, $h\left(h_x, h_y, h_z\right)$, respectively.

After linearizing the system of equations (1) - (8), we have the perturbation equations:

$$\frac{1}{\in}\frac{\partial q}{\partial t}=-\frac{1}{\rho_0}\nabla\delta p+\mathbf{g}\alpha\theta\lambda_i-\frac{1}{k_1}\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)q$$
$$+\frac{\tilde{\mu}}{\rho_0}\nabla^2 q+\frac{2}{\in}\left(q\times\Omega\right)+\frac{K'N_0}{\rho_0\in}\left(q_d-q\right) \quad (9)$$
$$+\frac{\mu_e}{4\pi\rho_0}\left(\nabla\times h\right)\times H,$$

$$\nabla.q=0, \quad (10)$$

$$\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)q_d=q, \quad (11)$$

$$\left(E+b\in\right)\frac{\partial\theta}{\partial t}=\beta\left(w+bs\right)+\kappa\nabla^2\theta, \quad (12)$$

$$\in\frac{\partial N}{\partial t}+\nabla.\left(N.q_d\right)=0, \quad (13)$$

$$\nabla.h=0, \quad (14)$$

$$\in\frac{\partial h}{\partial t}=\left(H.\nabla\right)q+\in\eta\nabla^2 h, \quad (15)$$

where $E=\in+\left(1-\in\right)\left(\frac{\rho_s c_s}{\rho_0 c_v}\right)$, $b=\frac{mNc_{pt}}{\rho_0 c_v}$, $\kappa=\frac{k}{\rho_0 c_v}$, $\lambda_i=\left(0,0,1\right)$ and w, s be the vertical fluid and particles velocity, respectively.

In the Cartesian form, equations (9)-(15) can be expressed as

$$\frac{1}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial u}{\partial t}=-\frac{1}{\rho_0}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial x}\left(\delta p\right)-$$
$$\frac{1}{k_1}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)u+\frac{\tilde{\mu}}{\rho_0}\nabla^2 u-\frac{mN}{\in\rho_0}\frac{\partial u}{\partial t}+ \quad (16)$$
$$\frac{\mu_e H}{4\pi\rho_0}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left(\frac{\partial h_x}{\partial z}-\frac{\partial h_z}{\partial x}\right)+\frac{2}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\Omega v,$$

$$\frac{1}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial v}{\partial t}=-\frac{1}{\rho_0}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial y}\left(\delta p\right)-$$
$$\frac{1}{k_1}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)v+\frac{\tilde{\mu}}{\rho_0}\nabla^2 v-\frac{mN}{\in\rho_0}\frac{\partial v}{\partial t}+ \quad (17)$$
$$\frac{\mu_e H}{4\pi\rho_0}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left(\frac{\partial h_y}{\partial z}-\frac{\partial h_z}{\partial y}\right)-\frac{2}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\Omega u,$$

$$\frac{1}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial w}{\partial t}=-\frac{1}{\rho_0}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial z}\left(\delta p\right)$$
$$-\frac{1}{k_1}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)w+\frac{\tilde{\mu}}{\rho_0}\nabla^2 w \quad (18)$$
$$-\frac{mN}{\in\rho_0}\frac{\partial w}{\partial t}+g\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\alpha\theta,$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0, \quad (19)$$

$$\left(E+b\in\right)\frac{\partial\theta}{\partial t}=\beta\left(w+bs\right)+\kappa\nabla^2\theta, \quad (20)$$

$$\frac{\partial M}{\partial t}+\frac{\partial l}{\partial x}+\frac{\partial r}{\partial y}+\frac{\partial s}{\partial z}=0, \quad (21)$$

$$\frac{\partial h_x}{\partial x}+\frac{\partial h_y}{\partial y}+\frac{\partial h_z}{\partial z}=0, \quad (22)$$

$$\in \frac{\partial h_x}{\partial t} = H\frac{\partial u}{\partial z} + \in \eta\nabla^2 h_x, \tag{23}$$

$$\in \frac{\partial h_y}{\partial t} = H\frac{\partial v}{\partial z} + \in \eta\nabla^2 h_y, \tag{24}$$

$$\in \frac{\partial h_z}{\partial t} = H\frac{\partial w}{\partial z} + \in \eta\nabla^2 h_z, \tag{25}$$

Operating equations (16) and (17) by $\frac{\partial}{\partial x}$ and $\frac{\partial}{\partial y}$ respectively, adding them and using eq. (19), we get:

$$\begin{aligned}&\frac{1}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial z}\right) = \frac{1}{\rho_0}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\\ &\left(\nabla^2 - \frac{\partial^2}{\partial z^2}\right)\delta p - \frac{1}{k_1}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)\\ &\left(\frac{\partial w}{\partial z}\right)+\frac{\tilde{\mu}}{\rho_0}\nabla^2\left(\frac{\partial w}{\partial z}\right)-\frac{mN}{\in\rho_0}\frac{\partial}{\partial t}\left(\frac{\partial w}{\partial z}\right)+\\ &\frac{\mu_e H}{4\pi\rho_0}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\nabla^2 h_z - \frac{2\Omega}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\zeta\end{aligned} \tag{26}$$

Where $\zeta = \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right)$ denotes the z-component of the vorticity. Operating equations (18) and (26) by $\left(\nabla^2 - \frac{\partial^2}{\partial z^2}\right)$ and $\frac{\partial}{\partial z}$ respectively and adding them to eliminate δp, we get:

$$\begin{aligned}&\frac{1}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial t}\left(\nabla^2 w\right) = -\frac{1}{k_1}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\\ &\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)\left(\nabla^2 w\right)+\frac{\tilde{\mu}}{\rho_0}\nabla^4 w-\frac{mN}{\in\rho_0}\frac{\partial}{\partial t}\left(\nabla^2 w\right)\\ &+g\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\alpha\theta+\frac{\mu_e H}{4\pi\rho_0}\\ &\left(\frac{m}{k'}\frac{\partial}{\partial t}+1\right)\frac{\partial}{\partial z}\left(\nabla^2 h_z\right)-\frac{2\Omega}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial\zeta}{\partial z}\end{aligned} \tag{27}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$.

Now, operating equation (16) and (17) by $-\frac{\partial}{\partial y}$ and $\frac{\partial}{\partial x}$ respectively and adding them, we get:

$$\begin{aligned}&\frac{1}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial\zeta}{\partial t} = -\frac{1}{k_1}\left(\upsilon-\upsilon'\frac{\partial}{\partial t}\right)\\ &\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\zeta - \frac{mN}{\in\rho_0}\frac{\partial\zeta}{\partial t}+\frac{\tilde{\mu}}{\rho_0}\nabla^2\zeta+\\ &\frac{\mu_e H}{4\pi\rho_0}\left(\frac{m}{k'}\frac{\partial}{\partial t}+1\right)\frac{\partial\xi}{\partial z}+\frac{2\Omega}{\in}\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\frac{\partial w}{\partial z},\end{aligned} \tag{28}$$

Where $\xi = \frac{\partial h_y}{\partial x} - \frac{\partial h_x}{\partial y}$ is the z-component of current density.

From equation (20), we get

$$\begin{aligned}&\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)\left[\left(E+b\in\right)\frac{\partial}{\partial t}-\kappa\nabla^2\right]\theta =\\ &\beta\left[\left(\frac{m}{K'}\frac{\partial}{\partial t}+1\right)+b\right]w\end{aligned} \tag{29}$$

Now, operating equations (23) and (24) by $-\frac{\partial}{\partial y}$ and $\frac{\partial}{\partial x}$ respectively, adding them and using equations (19) & (22), we get

$$\in \frac{\partial \xi}{\partial t} = H\frac{\partial \zeta}{\partial z} + \in \eta\nabla^2 \xi, \tag{30}$$

From equation (25), we get,

$$\in \frac{\partial h_z}{\partial t} = H\frac{\partial w}{\partial z} + \in \eta\nabla^2 h_z, \tag{31}$$

Equations (27)-(31) are the basic equations governing the motion of a Walters' B' visco-elastic fluid under the influences of rotation, magnetic field, suspended particles and Brinkman porous medium.

4. Normal Mode Method and Dispersion Relation

By supposing that the perturbations in various physical quantities have a solution with a dependence on x, y and t of the form:

$$\left[w,s,\theta,\zeta,h_z,\xi\right] = \begin{bmatrix}W(z),S(z),\Theta(z),\\ Z(z),K(z),X(z)\end{bmatrix}\exp\left(ik_x x+ik_y y+nt\right), \tag{32}$$

Where $k^2 = \left(k_x{}^2 + k_y{}^2\right)$, is the resultant wave number of

the disturbance and n is the frequency of the harmonic disturbance, which is, in general, a complex constant.
Using expression (32) in equations (27)-(31), we get:

$$\frac{n}{\in}\left(\frac{d^2}{dz^2}-k^2\right)W=-\frac{1}{k_1}\left(\upsilon-\upsilon' n\right)\left(\frac{d^2}{dz^2}-k^2\right)W$$

$$-\frac{mNn}{\in\rho_0\left(\frac{m}{K'}n+1\right)}\left(\frac{d^2}{dz^2}-k^2\right)W+\frac{\tilde{\mu}}{\rho_0}\left(\frac{d^2}{dz^2}-k^2\right)^2 w \quad (33)$$

$$-gk^2\alpha\Theta+\frac{\mu_e H}{4\pi\rho_0}\left(\frac{d^2}{dz^2}-k^2\right)\frac{dK}{dz}-\frac{2\Omega}{\in}\frac{dZ}{dz},$$

$$\frac{n}{\in}Z=-\frac{1}{k_1}\left(\upsilon-\upsilon' n\right)Z-\frac{mNn}{\in\rho_0\left(\frac{m}{K'}n+1\right)}Z \quad (34)$$

$$+\frac{\tilde{\mu}}{\rho_0}\left(\frac{d^2}{dz^2}-k^2\right)Z+\frac{\mu_e H}{4\pi\rho_0}\frac{dX}{dz}+\frac{2\Omega}{\in}\frac{dW}{dz},$$

$$\left[\left(E+b\in\right)n-\kappa\left(\frac{d^2}{dz^2}-k^2\right)\right]\theta=\beta\left[1+\frac{b}{\left(\frac{m}{K'}n+1\right)}\right]w \quad (35)$$

$$n\in K=H\frac{dW}{dz}+\in\eta\left(\frac{d^2}{dz^2}-k^2\right)K, \quad (36)$$

$$n\in X=H\frac{dZ}{dz}+\in\eta\left(\frac{d^2}{dz^2}-k^2\right)X, \quad (37)$$

Equations (33) – (37) in non-dimensional form becomes

$$\left[\frac{P_l\sigma}{\in}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\left(1-F\sigma\right)-D_A\left(D^2-a^2\right)\right]$$

$$\left(D^2-a^2\right)W(z)+\frac{g\alpha a^2 d^2 P_l\Theta}{\upsilon}+\frac{2P_l\Omega d^3}{\in\upsilon}DZ \quad (38)$$

$$-\frac{\mu_e P_l Hd}{4\pi\rho_0\upsilon}\left(D^2-a^2\right)DK=0,$$

$$\left[\frac{P_l\sigma}{\in}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\left(1-F\sigma\right)-D_A\left(D^2-a^2\right)\right]Z \quad (39)$$

$$=\frac{\mu_e P_l Hd}{4\pi\rho_0\upsilon}DX+\left(\frac{2P_l\Omega d}{\in\upsilon}\right)DW,$$

$$\left[\left(\left(D^2-a^2\right)-E_1 p_1\sigma\right)\right]\Theta=-\left(\frac{\beta d^2}{\kappa}\right)\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)W, \quad (40)$$

$$\left[\sigma\in-\frac{\left(D^2-a^2\right)\in}{p_2}\right]K=\left(\frac{Hd}{\upsilon}\right)DW, \quad (41)$$

$$\left[\sigma\in-\frac{\left(D^2-a^2\right)\in}{p_2}\right]X=\left(\frac{Hd}{\upsilon}\right)DZ, \quad (42)$$

where we have used the following non-dimensional quantities with the scaling:

$$z=z^*d,\ a=kd,\ \sigma=\frac{nd^2}{\upsilon},\tau=\frac{m}{K'},\tau_1=\frac{\tau\upsilon}{d^2},$$

$$F=\frac{\upsilon'}{d^2},\ N=\frac{\rho_0 M}{m},\ E_1=E+b\in,\ \mathrm{B}=\mathrm{b}+1$$

$P_l=\frac{k_1}{d^2}$, is the dimensionless medium permeability, $p_1=\frac{\upsilon}{\kappa}$, is the thermal Prandtl number, $p_2=\frac{\upsilon}{\eta}$, is the magnetic Prandtl number and $D_A=\frac{\tilde{\mu}P_l}{\mu}$ is the Darcy-Brinkman number modified by the viscosity ratio.

Eliminating K, Θ, X and Z between equations (38) – (42), we obtain:

$$\left\{\begin{aligned}&\left[\frac{P_l\sigma}{\in}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\left(1-F\sigma\right)-D_A\left(D^2-a^2\right)\right]\\&\left(D^2-a^2\right)\left(D^2-a^2-E_1p_1\sigma\right)\left[p_2\sigma-\left(D^2-a^2\right)\right]\end{aligned}\right\}W(z)$$

$$-Ra^2\lambda P_l\left(\frac{B+\tau_1\sigma}{1+\tau_1\sigma}\right)\left[p_2\sigma-\left(D^2-a^2\right)\right]W(z)-$$

$$\frac{QP_l}{\in}\left(D^2-a^2\right)\left(D^2-a^2-E_1p_1\sigma\right)D^2W+ \quad (43)$$

$$\frac{T_A P_l^2}{\in^2}\left(D^2-a^2-E_1p_1\sigma\right)$$

$$\frac{\left[p_2\sigma-\left(D^2-a^2\right)\right]^2 D^2W(z)}{\left[\frac{P_l\sigma}{\in}\left(1+\frac{M}{1+\tau_1\sigma}\right)+\left(1-F\sigma\right)-D_A\left(D^2-a^2\right)\right]\left[p_2\sigma-\left(D^2-a^2\right)\right]-\frac{QP_lD^2}{\in}}=0,$$

Where $R=\frac{g_0\alpha\beta d^4}{\upsilon\kappa}$, is the thermal Rayleigh number, $Q=\frac{\mu_e H^2 d^2}{4\pi\rho_0\upsilon\eta}$, is the Chandrasekhar number and $T_A=\left(\frac{2\Omega d^2}{\upsilon}\right)^2$, is the Taylor number.

For the case of two free boundaries, the boundary conditions appropriate to the problem are defined as:

$W = D^2W = 0$, $DZ = 0$ and $\Theta = 0$ at $z = 0$ and 1. (44)

and the components of h are continuous.

Since the components of the magnetic field are continuous and the tangential components are zero outside the fluid, we have

$DK = 0$, on the boundaries. (45)

The boundary conditions (44) and (45) imply that all even order derivatives of W must vanish at the boundaries. So a suitable solution of W characterizing the lowest mode is

$$W = W_0 \sin \pi z, \quad (46)$$

where W_0 is a constant. Using solution (46) in equation (43), we obtain the dispersion relation

$$R_1 x\lambda P = \left\{ \begin{array}{l} \left[\dfrac{iP\sigma_1}{\in}\left(1+\dfrac{M}{1+i\tau_1\pi^2\sigma_1}\right)+\left(1-iF\pi^2\sigma_1\right) \right. \\ \left. +D_{A_1}(1+x) \right] \\ (1+x)(1+x+iE_1p_1\sigma_1)\left(\dfrac{1+i\tau_1\pi^2\sigma_1}{B+i\tau_1\pi^2\sigma_1}\right) \end{array} \right\}$$

$$+\frac{Q_1P}{\in}\frac{(1+x)(1+x+iE_1p_1\sigma_1)}{(1+x+ip_2\sigma_1)}\left(\frac{1+i\tau_1\pi^2\sigma_1}{B+i\tau_1\pi^2\sigma_1}\right)+ \quad (47)$$

$$\frac{\dfrac{T_{A_1}P^2}{\in^2}(1+x+E_1p_1i\sigma_1)(1+x+ip_2\sigma_1)\left(\dfrac{1+\tau_1\pi^2 i\sigma_1}{B+\tau_1\pi^2 i\sigma_1}\right)}{\left[\dfrac{iP\sigma_1}{\in}\left(1+\dfrac{M}{1+i\tau_1\pi^2\sigma_1}\right)+\left(1-iF\pi^2\sigma_1\right)+D_{A_1}(1+x)\right](1+x+ip_2\sigma_1)+\dfrac{Q_1P}{\in}},$$

where $R_1 = \dfrac{R}{\pi^4},\ T_{A_1} = \dfrac{T_A}{\pi^4},\ x = \dfrac{a^2}{\pi^2},\ i\sigma_1 = \dfrac{\sigma}{\pi^4},$ $P = \pi^2 P_l,\ Q_1 = \dfrac{Q}{\pi^2},\ D_A = \dfrac{D_{A_1}}{\pi^2}.$ Equation (47) is required dispersion relation accounting the effects of rotation, medium permeability, variable gravity and magnetic field on thermal instability of Walters' (Model B') elastico-viscous fluid permeated with suspended particles in a Brinkman porous medium.

5. The Stationary Convection

When instability sets in as stationary convection, the marginal state will be characterized by $\sigma = 0$. So, we put $\sigma = 0$ in equation (47) and get:

$$R_1 = \frac{(1+x)}{x\lambda B}\left\{ \frac{(1+x)\left[1+D_{A_1}(1+x)\right]}{P}+\frac{Q_1}{\in}+\frac{T_{A_1}P(1+x)}{\in\left\{\in(1+x)\left[1+D_{A_1}(1+x)\right]+Q_1P\right\}} \right\}, \quad (48)$$

which expresses the modified Rayleigh number R_1 as a function of the dimensionless wave number x and the parameters T_{A_1}, B, P, Q_1, D_{A_1}. Hence, it is clear that for stationary convection Walters' (Model B') elastico-viscous fluid behave like an ordinary Newtonian fluid since elastico-viscous parameter F vanishes with σ.

In the absence of rotation, magnetic field and variable gravity field $(i.e.\ T_A = 0,\ Q = 0,\ \lambda = 1)$, expression (48) reduces to:

$$R_1 = \frac{1}{xPB}(1+x)^2\left[1+D_{A_1}(1+x)\right], \quad (49)$$

which is identical with the expression derived by Rana[10].

To study the effects of suspended particles, rotation, magnetic field, medium permeability and Darcy number, we examine the behaviour of $\dfrac{dR_1}{dB}, \dfrac{dR_1}{dT_{A_1}}, \dfrac{dR_1}{dQ_1}, \dfrac{dR_1}{dP}, \dfrac{dR_1}{dD_{A_1}}$ analytically.

Equation (48) yields

$$\frac{dR_1}{dB} = -\frac{(1+x)}{x\lambda B^2}\left\{ \frac{(1+x)\left[1+D_{A_1}(1+x)\right]}{P}+\frac{Q_1}{\in}+\frac{T_{A_1}P(1+x)}{\in\left\{\in(1+x)\left[1+D_{A_1}(1+x)\right]+Q_1P\right\}} \right\}, \quad (50)$$

which is negative implying thereby that the effect of suspended particles is to destabilize the system. This result is a good agreement of the earlier work of Scanlon & Segel[3].

From equation (48), we get

$$\frac{dR_1}{dT_{A_1}} = \frac{(1+x)}{x\lambda B}\left\{ \frac{P(1+x)}{\in\left\{\in(1+x)\left[1+D_{A_1}(1+x)\right]+Q_1P\right\}} \right\}, \quad (51)$$

which shows that rotation has stabilizing effect on the system. This stabilizing effect is an agreement of the earlier work of Sharma & Rana[8].

From equation (48), we get

$$\frac{dR_1}{dQ_1} = \frac{(1+x)}{x\lambda B}\left\{ \frac{1}{\in}-\frac{T_{A_1}P^2(1+x)}{\in^2\left\{\in(1+x)\left[1+D_{A_1}(1+x)\right]+Q_1P\right\}^2} \right\}, \quad (52)$$

which shows that magnetic field stabilizes the system if $\in\left\{\in(1+x)\left[1+D_{A_1}(1+x)\right]+Q_1P\right\}^2 > T_A(1+x)P^2$ and destabilizes the system if $\in\left\{\in(1+x)\left[1+D_{A_1}(1+x)\right]+Q_1P\right\}^2 < T_A(1+x)P^2$

Equation (48) yields

$$\frac{dR_1}{dP}=\frac{(1+x)^2}{x\lambda B}\left\{\begin{array}{l}-\dfrac{\left[1+D_{A_1}(1+x)\right]}{P^2}+\\ \dfrac{T_{A_1}(1+x)^2\left[1+D_{A_1}(1+x)\right]}{\left\{\begin{array}{l}\in(1+x)\left[1+D_{A_1}(1+x)\right]\\+Q_1P\end{array}\right\}^2}\end{array}\right\}, \quad (53)$$

From equation (53), we observe that medium permeability has destabilizing effect when $T_A(1+x)^2P^2 < \left\{\in(1+x)\left[1+D_{A_1}(1+x)\right]+Q_1P\right\}^2$ and medium permeability has a stabilizing effect when $T_A(1+x)^2P^2 > \left\{\in(1+x)\left[1+D_{A_1}(1+x)\right]+Q_1P\right\}^2$

Equation (48) yields

$$\frac{dR_1}{dD_{A_1}}=\frac{(1+x)}{x\lambda B}\left\{\begin{array}{l}\dfrac{(1+x)^2}{P}-\\ \dfrac{T_{A_1}P(1+x)^3}{\left\{\begin{array}{l}\in(1+x)\left[1+D_{A_1}(1+x)\right]\\+Q_1P\end{array}\right\}^2}\end{array}\right\}, \quad (54)$$

From equation (54), we observe that Darcy-Brinkman number has destabilizing effect when $T_A(1+x)P^2 > \left\{\in(1+x)\left[1+D_{A_1}(1+x)\right]+Q_1P\right\}^2$ and has a stabilizing effect when $T_A(1+x)P^2 < \left\{\in(1+x)\left[1+D_{A_1}(1+x)\right]+Q_1P\right\}^2$

6. Results and Discussion

The dispersion relation (48) is analyzed numerically. Graphs have been plotted for various values of the parameters to determine the region of stability.

Figure 1 shows the decrease in critical Rayleigh number R_1 with the increase in suspended particles B. This shows that the effect of suspended particles is to destabilize the system.

Figure 2 shows that the critical Rayleigh number R_1 increases with the increase in rotation T_A, thereby depicting the stabilizing effects of rotation T_A.

Figure 3, 4 and **5** shows that the magnetic field, medium permeability and Darcy number have a destabilizing effect for Q=1 to Q=2, P=1 to P=5, D_A=1 to D_A=3 and stabilizing effect for Q=2.5 to Q=5.5, P=6 to P=10, D_A=4 to D_A=10 respectively. Therefore, magnetic field, medium permeability and Darcy-Brinkman number have both destabilizing and stabilizing influences on the system.

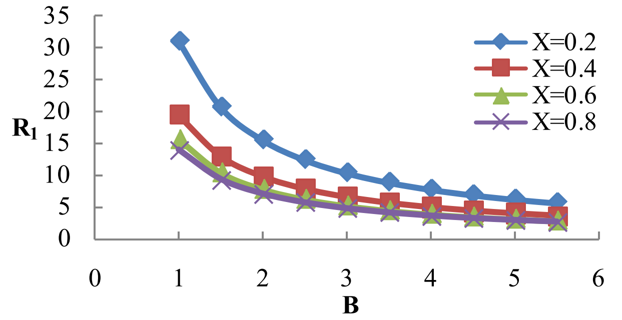

Figure 1. Variation of Rayleigh number R_1 with suspended particles B for fixed values of $T_A=5$, $M_1=5$, $D_A=2$, $Q=3$, $\in=0.5$, $\lambda=2$, $P_l=5$ and x = 0.2, 0.4, 0.6, 0.8.

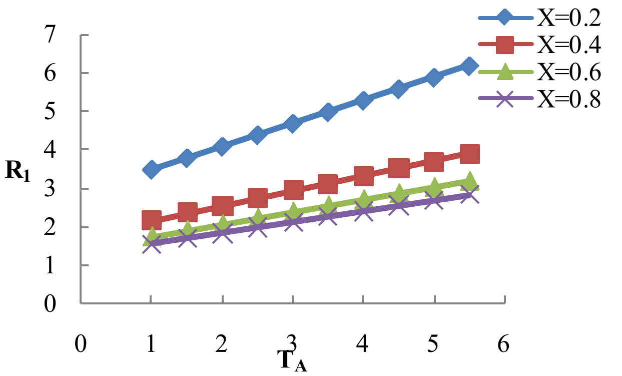

Figure 2. Variation of Rayleigh number R_1 with rotation T_A for fixed values of $B=5$, $M_1=5$, $D_A=2$, $Q=2$, $\in=0.5$, $\lambda=2$, $P_l=5$ and x = 0.2, 0.4, 0.6, 0.8.

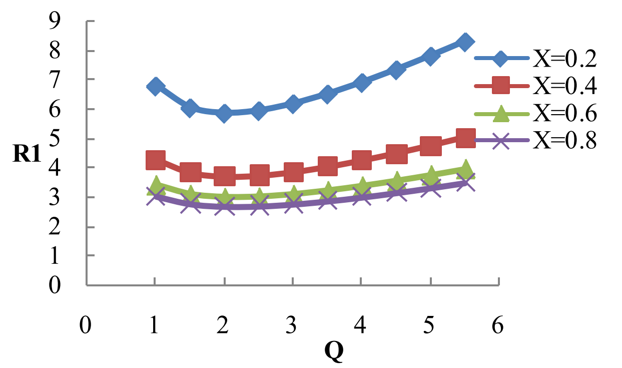

Figure 3. Variation of Rayleigh number R_1 with magnetic field Q for fixed values of $B=5$, $T_A=5$, $M_1=5$, $D_A=2$, $\in=0.5$, $\lambda=2$, $P_l=5$ and x = 0.2, 0.4, 0.6, 0.8.

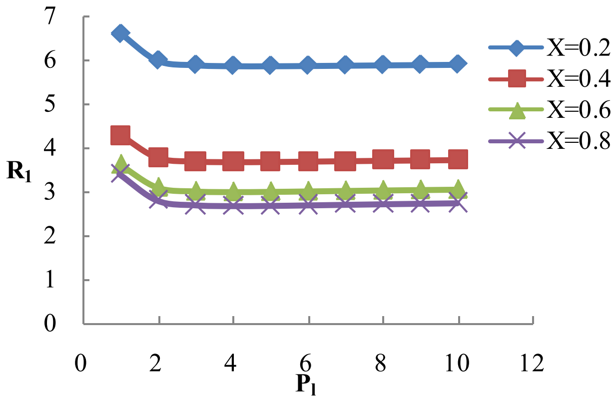

Figure 4. Variation of Rayleigh number R_1 with medium permeability P_l for fixed values of $B=5$, $T_A=5$, $M_1=5$, $D_A=2$, $\in=0.5$, $\lambda=2$, $P_l=5$, $Q=2$, and x = 0.2, 0.4, 0.6, 0.8.

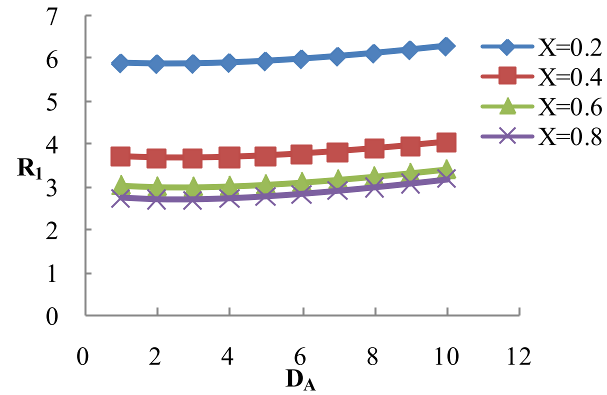

Figure 5. Variation of Rayleigh number R_1 with Darcy –Brinkman number D_{A_1} for fixed values of $B=5$, $T_A=5$, $M_1=5$, $\in=0.5$, $\lambda=2$, $P_l=5$, $Q=2$,, and x = 0.2, 0.4, 0.6, 0.8.

7. Stability of the system and Oscillatory modes

Here we examine the possibility of oscillatory modes, if any, in Walters' (Model B') elastico-viscous fluid due to the presence of suspended particles, rotation, magnetic field and variable gravity field in a Brinkman porous medium. Multiply equation (38) by W* (the complex conjugate of W), integrating over the range of z and making use of equations (39)-(42) with the help of boundary conditions (44) and (45), we obtain

$$\left[\frac{P_l\sigma}{\in}\left(1+\frac{M}{1+\tau_1\sigma}\right)+(1-F\sigma)\right]I_1-D_AI_2$$
$$-\frac{g\alpha a^2P_l\lambda\kappa}{\upsilon\beta}\left(\frac{1+\tau_1\sigma^*}{B+\tau_1\sigma^*}\right)\left(I_3+E_1p_1\sigma^*I_4\right)$$
$$-d^2\left[\begin{array}{l}\frac{P_l\sigma^*}{\in}\left(1+\frac{M}{1+\tau_1\sigma^*}\right)\\+\left(1-F\sigma^*\right)\end{array}\right]I_5+d^2D_AI_6$$
$$+\frac{\mu_e\in P_l\eta}{4\pi\rho_0\upsilon}\left(I_8+p_2\sigma^*I_7\right)-\frac{\mu_e\in P_l\eta d^2}{4\pi\rho_0\upsilon}\left(I_{10}+p_2\sigma I_9\right)=0, \quad (55)$$

where

$$I_1=\int_0^1\left(|DW|^2+a^2|W|^2\right)dz,$$
$$I_2=\int_0^1\left(|D^2W|^2+a^4|W|^2+2a^2|DW|^2\right)dz,$$
$$I_3=\int_0^1\left(|D\Theta|^2+a^2|\Theta|^2\right)dz,\ I_4=\int_0^1\left(|\Theta|^2\right)dz,$$
$$I_5=\int_0^1\left(|Z|^2\right)dz,\ I_6=\int_0^1\left(|DZ|^2+a^2|Z|^2\right)dz,$$
$$I_7=\int_0^1\left(|DK|^2+a^2|K|^2\right)dz,$$
$$I_8=\int_0^1\left(|D^2K|^2+a^4|K|^2+2a^2|DK|^2\right)dz,$$
$$I_9=\int_0^1\left(|X|^2\right)dz,\ I_{10}=\int_0^1\left(|DX|^2+a^2|X|^2\right)dz,$$

The integral part I_1-I_{10} are all positive definite. Putting $\sigma=i\sigma_i$ in equation (55), where σ_i is real and equating the imaginary parts, we obtain

$$\sigma_i\left\{\begin{array}{l}\left[\frac{P_l}{\in}\left(1+\frac{M}{1+\tau_1^2\sigma_i^2}\right)-F\right]\left(I_1+d^2I_5\right)-\frac{\mu_eP_l\in\eta}{4\pi\rho_0\upsilon}\\ \left[\left(I_7-d^2I_9\right)p_2\right]+\frac{g\alpha a^2P_l\lambda\kappa}{\beta\upsilon}\left[\begin{array}{l}\frac{\tau_1(B-1)}{\left(B^2+\tau_1^2\sigma_i^2\right)}I_3+\\ \frac{\left(B+\tau_1^2\sigma_i^2\right)}{\left(B^2+\tau_1^2\sigma_i^2\right)}E_1p_1I_4\end{array}\right]\end{array}\right\}=0. (56)$$

Equation (56) implies that $\sigma_i=0$ or $\sigma_i\neq0$ which mean that modes may be non oscillatory or oscillatory. The oscillatory modes introduced due to presence of rotation, magnetic field, suspended particles, visco-elasticity and medium permeability.

8. Conclusions

The problem of thermal convection in Walters' (Model B') visco-elastic fluid permitted with suspended particles in the presence of rotation, variable gravity and magnetic field has been investigated in a Brinkman porous medium. The main conclusions of the paper are as follows:

(a). For the case of stationary convection, Walters' (B') elastico-viscous fluid behaves like an ordinary Newtonian fluid.

(b).The expressions for $\frac{dR_1}{dB}, \frac{dR_1}{dT_{A_1}}, \frac{dR_1}{dQ_1}, \frac{dR_1}{dP}\ and\ \frac{dR_1}{dD_{A_1}}$ are examined analytically and it has been found that the effect of suspended particles is to destabilize the system as gravity increases upward from its value $g_0\ (i.e.\ \lambda > 0)$ whereas the effect of rotation is to stabilize the system.

(c). The magnetic field has both stabilizing and destabilizing effects on the system under the conditions $\in\left\{\in(1+x)\left[1+D_{A_1}(1+x)\right]+Q_1P\right\}^2 > T_A(1+x)P^2$ and $\in\left\{\in(1+x)\left[1+D_{A_1}(1+x)\right]+Q_1P\right\}^2 < T_A(1+x)P^2$ respectively.

(d). The medium permeability has both stabilizing and destabilizing effects on the system under the conditions $T_A(1+x)^2P^2 > \left\{\in(1+x)\left[1+D_{A_1}(1+x)\right]+Q_1P\right\}^2$ and $T_A(1+x)^2P^2 < \left\{\in(1+x)\left[1+D_{A_1}(1+x)\right]+Q_1P\right\}^2$ respectively.

(e). The Darcy-Brinkman number has both stabilizing and destabilizing effects on the system under the conditions

$$T_A(1+x)P^2 < \left\{\in(1+x)\left[1+D_{A_1}(1+x)\right]+Q_1P\right\}^2$$

and

$$T_A(1+x)P^2 > \left\{\in(1+x)\left[1+D_{A_1}(1+x)\right]+Q_1P\right\}^2$$

respectively.

(f). The effects of suspended particles, rotation, magnetic field and medium permeability and Darcy-Brinkman number on thermal instability of Walters' (model B') fluids in a Brinkman porous medium have also been shown graphically in fig. 1, 2, 3, 4, and 5.

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