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Tomonaga-Luttinger Anomalous Exponents of k_F, $3k_F$, $5k_F$ and $7k_F$ Momentum Distribution in the *t-J* Model

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Abstract Contrary to the usual Fermi liquids, where the exponent of the momentum distribution at $k=k_F$ is fixed to an integer, for the Tomonaga-Luttinger (TL) liquids the anomalous exponent of the momentum distribution changes continuously, resulting in a power-law singularity of the momentum distribution function. It has been said that this power-law singularity appears at k_F and $3k_F$ for the Hubbard model as well as at k_F for the t-J model. Using the conformal field theory (CFT) technique, we present an exact calculation of the anomalous exponent of the t-J model at k_F, $3k_F$, $5k_F$ and $7k_F$ and compare it with the results for the Hubbard model.

Keywords Tomonaga-Luttinger liquid, Momentum distribution, Power-law singularity

1. Introduction

Since the original work on high-T_c superconductivity in the 80's by Bednorz and Muller [1], the interest in studying highly correlated electron systems in low-dimension has greatly increased. It is usually believed that one-dimensional (1D) systems are the simplest to discuss electron correlation problem [2]. Haldane [3] introduced the theory of TL liquid that is valid in understanding the low-energy behaviour of a large class of 1D correlated models. The role of Fermi liquid (FL) in three dimensions is replaced by TL liquid in 1D. The remarkable feature of TL liquids is the absence of a finite jump discontinuity in the momentum distribution function $\tilde{G}(k)$ at the Fermi momentum, k_F and the presence of power-law singularity near the Fermi point, which corresponds to collective motion of Fermions instead of quasi-particle excitations.

The efforts to find appropriate model to clarify the non-Fermi liquid behaviour of low-dimensional highly correlated systems, led to series of numerical and analytical investigations. Significant progress was made by Parola and Sorella [4] and Ogata and Shiba [5] for the Hubbard model. This also motivated Kawakami and Yang [6] to calculate the correlation exponents in 1D correlated systems and to clarify their TL liquid nature. In fact, Kawakami and Yang calculated the long-distant behaviour of correlation functions at k_F in the *t-J* model at $t=J$. The critical exponents ($\theta=\frac{1}{8}$ at k_F and $\frac{9}{8}$ at $3k_F$) obtained numerically by these authors is in agreement with analytical predictions for the Hubbard model. However, it is hard to determine numerically the exact nature of the singularity at $k=3k_F$. Since the anomalous exponents around $3k_F$, $5k_F$ and $7k_F$ has not been calculated yet for the *t-J* model, our aim in this paper is to follow the work of Kawakami and Yang [6], and extend their work by calculating the anomalous exponents of the momentum distribution function $\tilde{G}^{\uparrow}(k)$ around the Fermi points k_F, $3k_F$, $5k_F$ and $7k_F$ using the CFT technique.

2. Finite-Size Scaling in Conformal Field Theory

The conformal dimensions are obtained from the Hamiltonian [7] of 1D *t-J* model defined by

$$\begin{aligned} H = & -t\sum_{i,\sigma}\left(c_{i\sigma}^{\dagger}c_{i+1,\sigma}+c_{i+1,\sigma}^{\dagger}c_{i\sigma}\right) \\ & +2J\sum_{i}\left(S_i.S_{i+1}-\tfrac{1}{4}n_in_{i+1}\right) \\ & -\mu\sum_{i}n_i-\frac{1}{2}\mathrm{H}\sum_{i}\left(n_{i\uparrow}-n_{i\downarrow}\right) \end{aligned} \qquad (1)$$

Where $c_{i\sigma}^{\dagger}$, $c_{i\sigma}$ is the spin- σ ($\uparrow$ or $\downarrow$) electron creation, annihilation operators at the i^{th} site,

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$S_i = c_{i\sigma}^{\dagger} S_{\sigma\sigma} c_{i\sigma}$ is the spin $-1/2$ matrix *S*, $n_i = n_{i\uparrow} + n_{i\downarrow}$ is the number operator with $n_{i\sigma} = c_{i\sigma}^{\dagger} c_{i\sigma}$, and μ and *H* are the chemical potential and the external magnetic field, respectively. Equation (1) has been solved exactly by Kawakami and Yang to obtain the Bethe Ansatz equations

$$2N \tan^{-1}(2k_j) = 2\pi I_j + 2\sum_{\beta=1}^{M} \tan^{-1}\left(2(k_j - \Lambda_\beta)\right), \quad j = 1,\ 2,\ \ldots, N_c - 2M \tag{2}$$

and

$$2N \tan^{-1}(\Lambda_\alpha) = 2\pi J_\alpha + 2\sum_{j=1}^{N_c - 2M} \tan^{-1}\left(2(\Lambda_\beta - k_j)\right) + 2\sum_{\beta=1}^{M} \tan^{-1}\left(\Lambda_\alpha - \Lambda_\beta\right),\ \alpha = 1,\ \ldots, M \tag{3}$$

with k_j and Λ_α as spin rapidities

$$I_j = \frac{M}{2} \bmod 1 \text{ and } J_\alpha = \frac{N_c + M + 1}{2} \bmod 1 \tag{4}$$

The state corresponding to the solution of the Bethe Ansatz equations has energy and momentum given by

$$E_n\left(I,\ D\right) - E_0 = \frac{2\pi}{N} v_c \left(\Delta_c^+ + \Delta_c^-\right) + \frac{2\pi}{N} v_s \left(\Delta_s^+ + \Delta_s^-\right) + O\left(N^{-1}\right) \tag{5}$$

$$P\left(I,\ D\right) - P_0 = \left(2\pi - 2k_{F,\uparrow} - k_{F,\downarrow}\right) D_c + \left(2\pi - 2k_{F,\uparrow}\right) D_s + \frac{2\pi}{N}\left(\Delta_c^+ - \Delta_c^- + \Delta_s^+ - \Delta_s^-\right) \tag{6}$$

Where the conformal dimensions are given by

$$\Delta_c^{\pm}\left(I, D\right) = \frac{1}{2}\left(Z_{cc} D_c + Z_{sc} D_s \pm \frac{Z_{ss} I_c - Z_{cs} I_s}{2 \det Z}\right)^2 + N_c^{\pm} \tag{7}$$

$$\Delta_s^{\pm}\left(I, D\right) = \frac{1}{2}\left(Z_{cs} D_c + Z_{ss} D_s \pm \frac{Z_{cc} I_s - Z_{sc} I_c}{2 \det Z}\right)^2 + N_s^{\pm} \tag{8}$$

The non-negative integers $N_\alpha^{\pm}$, where $\alpha = c$ (holon) and $\alpha = s$ (spinon) describes particle-hole excitations, with $N_\alpha^{+}(N_\alpha^{-})$ being the number of occupancies that a particle at the right (left) Fermi level jumps to, $I_c(I_s)$ represents the change in the number of electrons (down-spin) with respect to the ground state, D_c represents the number of particles which transfer from one Fermi level of the holon to the other and D_s represents the number of particles which transfer from one Fermi level of the spinon to the other, and both D_c and D_s takes integer or half-odd integer values. Lastly, Z is the dressed charge matrix defined by

$$Z = \begin{pmatrix} Z_{cc} & Z_{cs} \\ Z_{sc} & Z_{ss} \end{pmatrix} \tag{9}$$

The elements of the dressed charge matrix are given by the coupled integral equations

$$Z_{cc}(k) = 1 + \int_{-\lambda_0}^{\lambda_0} a_1(k - \lambda)\ Z_{cs}(\lambda) d\lambda \tag{10}$$

$$Z_{cs}(\lambda) = \int_{-k_0}^{k_0} a_1(\lambda - k)\ Z_{cc}(k) dk - \int_{-\lambda_0}^{\lambda_0} a_2(\lambda - \mu)\ Z_{cs}(\mu) d\mu \tag{11}$$

$$Z_{sc}(k) = \int_{-\lambda_0}^{\lambda_0} a_1(k - \lambda)\ Z_{ss}(\lambda) d\lambda \tag{12}$$

$$Z_{ss}(\lambda) = 1 + \int_{-k_0}^{k_0} a_1(\lambda - k)\, Z_{sc}(k) dk - \int_{-\lambda_0}^{\lambda_0} a_2(\lambda - \mu)\, Z_{ss}(\mu) d\mu \tag{13}$$

The kernels are defined by

$$a_1(\lambda) = \frac{1}{\pi} \frac{2u}{u^2 + (2\lambda)^2}, \quad a_2(\lambda) = \frac{1}{\pi} \frac{u}{4u^2 + \lambda^2} \tag{14}$$

Usually, the CFT expression for two-point correlation function of the scaling fields $\varphi_{\Delta^\pm}(x,t)$ with conformal dimensions $\Delta^\pm$ for the *t-J* model takes the form

$$\begin{aligned} &\left\langle \varphi_{\Delta^\pm}(x,t) \varphi_{\Delta^\pm}(0,0) \right\rangle \equiv G(x,t) \\ &= \frac{\exp\left(i\left(2\pi - 2k_{F,\uparrow} - 2k_{F,\downarrow}\right) D_c x\right) \exp\left(i\left(2\pi - 2k_{F,\uparrow}\right) D_s x\right)}{\left(x - iv_c t\right)^{2\Delta_c^+} \left(x + iv_c t\right)^{2\Delta_c^-} \left(x - iv_s t\right)^{2\Delta_s^+} \left(x + iv_s t\right)^{2\Delta_s^-}} \end{aligned} \tag{15}$$

Where $k_{F,\uparrow}$ and $k_{F,\downarrow}$ are the Fermi momentum for electrons with spin up and down, respectively. v_c and v_s are the Fermi velocities of charge and spin density waves and A_k are constants.

3. Correlation Function

For small magnetic field $H << 1$, we solve the dressed charge matrix equations (10) to (13) by Wiener-Hopf technique [7, 8] and obtain (see Appendix A)

$$Z_{cc} = 1 \tag{16}$$

$$Z_{cs} = 0 \tag{17}$$

$$Z_{sc} = \frac{1}{2}\left(1 - \frac{4}{\pi^2} \frac{H}{H_c}\right) \tag{18}$$

$$Z_{sc} = \sqrt{2}\left(\frac{1}{2} + \frac{1}{8 \ln(H_0/H)}\right) \tag{19}$$

Using the dressed charge matrix elements Eqns. (16) to (19) on the conformal dimension Eqns. (7) and (8), we obtain [see (A72) and (A73)]

$$2\Delta_c^\pm = \left((D_c + \tfrac{1}{2} D_s) \pm \tfrac{1}{2} I_c - \frac{2D_s}{\pi^2} \frac{H}{H_c}\right)^2 + 2N_c^\pm \tag{20}$$

$$2\Delta_s^\pm = \frac{1}{2}\left\{D_s \pm \left(I_s - I_c\left(\frac{1}{2} - \frac{2}{\pi^2} \frac{H}{H_c}\right)\right)\right\}^2 + \frac{1}{4 \ln(H_0/H)}\left\{D_s^2 + \left(I_s - I_c\left(\frac{1}{2} - \frac{2}{\pi^2} \frac{H}{H_c}\right)\right)^2\right\} + 2N_s^\pm \tag{21}$$

Therefore, the long-distance behaviour of the electron field correlation function with up-spin is obtained from the set of quantum numbers $I_c = I_s = 1$, $(D_c,\ D_s) = (0,\ 1/2)$, $(1,\ -1/2)$, $(2,\ -3/2)$, $(3,\ -5/2)$ and $N_{c,s}^\pm = 0$. For $(D_c,\ D_s) = (0,\ 1/2)$, the corresponding conformal dimensions are

$$2\Delta_c^\pm = \left(\frac{1}{4} \pm \frac{1}{2} - \frac{1}{\pi^2} \frac{H}{H_c}\right)^2 \tag{22}$$

$$2\Delta_c^+ = \frac{9}{16} - \frac{3}{2\pi^2}\frac{H}{H_c}$$
$$2\Delta_c^- = \frac{1}{16} + \frac{1}{2\pi^2}\frac{H}{H_c} \tag{23}$$

$$2\Delta_s^\pm = \frac{1}{2}\left(\frac{1}{2} \pm \left(\frac{1}{2} + \frac{2}{\pi^2}\frac{H}{H_c}\right)\right)^2 + \frac{1}{4\ln(H_0/H)}\left\{\frac{1}{4} + \left(\frac{1}{2} + \frac{2}{\pi^2}\frac{H}{H_c}\right)^2\right\} \tag{24}$$

$$2\Delta_s^+ = \frac{1}{2} + \frac{2}{\pi^2}\frac{H}{H_c} + \frac{1}{8\ln(H_0/H)}$$
$$2\Delta_s^- = \frac{1}{8\ln(H_0/H)} \tag{25}$$

Here, we have neglected contributions from $\left(H/H_c\right)^2$ and terms of order $O\left(H/H_c \ln(H_0/H)\right)$. Using Eqns. (23) and (25) on (15), we obtain

$$\frac{A_1 \cos(k_{F,\uparrow} x)}{\left|x + iv_c t\right|^{\vartheta_{c1}} \left|x + iv_s t\right|^{\vartheta_{s1}}} \tag{26}$$

The critical exponent is given by

$$\vartheta = 2\Delta_{c,s}^+ + 2\Delta_{c,s}^- \tag{27}$$

This implies that

$$\vartheta_{c1} = 2\Delta_c^+ + 2\Delta_c^- = \frac{5}{8} - \frac{1}{\pi^2}\frac{H}{H_c} \tag{28}$$

and

$$\vartheta_{s1} = 2\Delta_s^+ + 2\Delta_s^- = \frac{1}{2} + \frac{2}{\pi^2}\frac{H}{H_c} + \frac{1}{4\ln(H_0/H)} \tag{29}$$

Next, for $(D_c,\, D_s) = \left(1,\ -1/2\right)$, we obtain the conformal dimensions as

$$2\Delta_c^\pm = \left(\frac{3}{4} \pm \frac{1}{2} - \frac{1}{\pi^2}\frac{H}{H_c}\right)^2 \tag{30}$$

$$2\Delta_c^+ = \frac{25}{16} + \frac{5}{2\pi^2}\frac{H}{H_c}$$
$$2\Delta_c^- = \frac{1}{16} + \frac{1}{2\pi^2}\frac{H}{H_c} \tag{31}$$

$$2\Delta_s^\pm = \frac{1}{2}\left(-\frac{1}{2} \pm \left(\frac{1}{2} + \frac{2}{\pi^2}\frac{H}{H_c}\right)\right)^2 + \frac{1}{4\ln(H_0/H)}\left\{\frac{1}{4} + \left(\frac{1}{2} + \frac{2}{\pi^2}\frac{H}{H_c}\right)^2\right\} \tag{32}$$

$$\begin{aligned} 2\Delta_s^+ &= \frac{1}{8\ln(H_0/H)} \\ 2\Delta_s^- &= \frac{1}{2} + \frac{2}{\pi^2}\frac{H}{H_c} + \frac{1}{8\ln(H_0/H)} \end{aligned} \tag{33}$$

Using Eqns. (33) and (31) on (15), we obtain

$$\frac{A_2 \cos[(k_{F,\uparrow} + 2k_{F,\downarrow})x]}{|x+iv_c t|^{\vartheta_{c2}}\,|x+iv_s t|^{\vartheta_{s2}}} \tag{34}$$

Where the critical exponents are given by

$$\vartheta_{c2} = \frac{13}{8} + \frac{3}{\pi^2}\frac{H}{H_c} \tag{35}$$

$$\vartheta_{s2} = \frac{1}{2} + \frac{2}{\pi^2}\frac{H}{H_c} + \frac{1}{4\ln(H_0/H)} \tag{36}$$

Also, the conformal dimensions for $(D_c,\, D_s) = (2,\ -3/2)$, are

$$2\Delta_c^{\pm} = \left(\frac{5}{4} \pm \frac{1}{2} + \frac{3}{\pi^2}\frac{H}{H_c}\right)^2 \tag{37}$$

$$\begin{aligned} 2\Delta_c^+ &= \frac{49}{16} + \frac{21}{2\pi^2}\frac{H}{H_c} \\ 2\Delta_c^- &= \frac{9}{16} + \frac{9}{2\pi^2}\frac{H}{H_c} \end{aligned} \tag{38}$$

$$2\Delta_s^{\pm} = \frac{1}{2}\left(-\frac{3}{2} \pm \left(\frac{1}{2} + \frac{2}{\pi^2}\frac{H}{H_c}\right)\right)^2 + \frac{1}{4\ln(H_0/H)}\left\{\frac{9}{4} + \left(\frac{1}{2} + \frac{2}{\pi^2}\frac{H}{H_c}\right)^2\right\} \tag{39}$$

$$\begin{aligned} 2\Delta_s^+ &= \frac{1}{2} - \frac{2}{\pi^2}\frac{H}{H_c} + \frac{5}{8\ln(H_0/H)} \\ 2\Delta_s^- &= 2 + \frac{4}{\pi^2}\frac{H}{H_c} + \frac{5}{8\ln(H_0/H)} \end{aligned} \tag{40}$$

Using Eqns. (40) and (38) on (15), we obtain

$$\frac{A_3 \cos[(k_{F,\uparrow} + 4k_{F,\downarrow})x]}{|x+iv_c t|^{\vartheta_{c3}}\,|x+iv_s t|^{\vartheta_{s3}}} \tag{41}$$

$$\vartheta_{c3} = \frac{29}{8} + \frac{15}{\pi^2}\frac{H}{H_c} \tag{42}$$

$$\vartheta_{s3} = \frac{5}{2} + \frac{2}{\pi^2}\frac{H}{H_c} + \frac{5}{4\ln(H_0/H)} \tag{43}$$

Finally, the conformal dimensions for $(D_c,\, D_s) = (3,\ -5/2)$ are

$$2\Delta_c^{\pm} = \left(\frac{7}{4} \pm \frac{1}{2} + \frac{5}{\pi^2}\frac{H}{H_c}\right)^2 \tag{44}$$

$$\begin{aligned} 2\Delta_c^{+} &= \frac{81}{16} + \frac{45}{2\pi^2}\frac{H}{H_c} \\ 2\Delta_c^{-} &= \frac{25}{16} + \frac{25}{2\pi^2}\frac{H}{H_c} \end{aligned} \tag{45}$$

$$2\Delta_s^{\pm} = \frac{1}{2}\left(-\frac{5}{2} \pm \left(\frac{1}{2} + \frac{2}{\pi^2}\frac{H}{H_c}\right)\right)^2 + \frac{1}{4\ln(H_0/H)}\left\{\frac{25}{4} + \left(\frac{1}{2} + \frac{2}{\pi^2}\frac{H}{H_c}\right)^2\right\} \tag{46}$$

$$\begin{aligned} 2\Delta_s^{+} &= 2 - \frac{4}{\pi^2}\frac{H}{H_c} + \frac{13}{8\ln(H_0/H)} \\ 2\Delta_s^{-} &= \frac{9}{2} + \frac{6}{\pi^2}\frac{H}{H_c} + \frac{13}{8\ln(H_0/H)} \end{aligned} \tag{47}$$

Using Eqns. (47) and (45) on (15), we obtain

$$\frac{A_4 \cos[(k_{F,\uparrow} + 6k_{F,\downarrow})x]}{\left|x + iv_c t\right|^{\vartheta_{c4}} \left|x + iv_s t\right|^{\vartheta_{s4}}} \tag{48}$$

The critical exponents are given by

$$\vartheta_{c4} = \frac{53}{8} + \frac{35}{\pi^2}\frac{H}{H_c} \tag{49}$$

$$\vartheta_{s4} = \frac{13}{2} + \frac{2}{\pi^2}\frac{H}{H_c} + \frac{13}{4\ln(H_0/H)} \tag{50}$$

Combining Eqns. (26), (34), (41) and (48), we obtain the asymptotic form of the electron field correlator with up-spin as

$$\begin{aligned} \tilde{G}^{\uparrow}(x,t) \approx & \frac{A_1 \cos(k_{F,\uparrow}x)}{\left|x + iv_c t\right|^{\vartheta_{c1}} \left|x + iv_s t\right|^{\vartheta_{s1}}} + \frac{A_2 \cos[(k_{F,\uparrow} + 2k_{F,\downarrow})x]}{\left|x + iv_c t\right|^{\vartheta_{c2}} \left|x + iv_s t\right|^{\vartheta_{s2}}} \\ & + \frac{A_3 \cos[(k_{F,\uparrow} + 4k_{F,\downarrow})x]}{\left|x + iv_c t\right|^{\vartheta_{c3}} \left|x + iv_s t\right|^{\vartheta_{s3}}} + \frac{A_4 \cos[(k_{F,\uparrow} + 6k_{F,\downarrow})x]}{\left|x + iv_c t\right|^{\vartheta_{c4}} \left|x + iv_s t\right|^{\vartheta_{s4}}} \end{aligned} \tag{51}$$

From Eqn. (51) it is clear that in the strong-coupling limit, the $3k_F$ singularity manifests itself as $(k_{F,\uparrow} + 2k_{F,\downarrow})$, $5k_F$ manifest as $(k_{F,\uparrow} + 4k_{F,\downarrow})$ and $7k_F$ as $(k_{F,\uparrow} + 6k_{F,\downarrow})$. Therefore, the asymptotic form of the equal-time correlator of the electron field (with $t \to 0$, $x \to r$) is given by

$$\begin{aligned} G^{\uparrow}(r,0) \approx & \frac{A_1 \cos(k_{F,\uparrow}r)}{r^{(\vartheta_{c1}+\vartheta_{s1})}} + \frac{A_2 \cos[(k_{F,\uparrow} + 2k_{F,\downarrow})r]}{r^{(\vartheta_{c2}+\vartheta_{s2})}} \\ & + \frac{A_3 \cos[(k_{F,\uparrow} + 4k_{F,\downarrow})r]}{r^{(\vartheta_{c3}+\vartheta_{s3})}} + \frac{A_4 \cos[(k_{F,\uparrow} + 6k_{F,\downarrow})r]}{r^{(\vartheta_{c4}+\vartheta_{s4})}} \end{aligned} \tag{52}$$

3.1. Correlation Function in Momentum Space

It is well known that the asymptotics of the two-point correlation function determines the singularities of the spectral functions [9] near $\omega \approx \pm v_{c,s}(k-k_F)$. The electron field correlation function in momentum space is obtained by Fourier transforming the asymptotic form obtained above. In fact, the result Eqn. (52) has singularities near the Fermi point $k_{F,\uparrow}$, $k_{F,\uparrow}+2k_{F,\downarrow}$, $k_{F,\uparrow}+4k_{F,\downarrow}$ and $k_{F,\uparrow}+6k_{F,\downarrow}$ respectively. Therefore, at the Fermi point $k_{F,\uparrow}$, the momentum distribution takes the form (see Appendix B)

$$\tilde{G}^{\uparrow}(k \approx k_{F,\uparrow}) \approx \mathrm{sgn}(k-k_{F,\uparrow})\left|k-k_{F,\uparrow}\right|^{\zeta} \tag{53}$$

Where the anomalous exponent

$$\begin{aligned}\zeta &= \vartheta_{c1}+\vartheta_{s1}-1 \\ &\approx \frac{1}{8}+\frac{1}{\pi^2}\frac{H}{H_c}, \qquad \text{as } H \to 0\end{aligned} \tag{54}$$

and

$$2s = 1 \tag{55}$$

Here and in what follows, we neglect the logarithmic field dependence of the anomalous exponent. Therefore, the nature of singularities for the contributions with Fermi wave number $k_{F,\uparrow}$ are obtained as

$$\tilde{G}^{\uparrow}(k,\omega) \approx [\omega - v_c(k-k_{F,\uparrow})]^{\zeta}, \text{ for } \omega \to v_c(k-k_{F,\uparrow}) \tag{56}$$

with

$$\zeta = \vartheta_{s1}+2\Delta_c^{+}-1 \approx \frac{1}{16}+\frac{1}{2\pi^2}\frac{H}{H_c}, \text{ as } H \to 0 \tag{57}$$

$$\tilde{G}^{\uparrow}(k,\omega) \approx [\omega + v_c(k-k_{F,\uparrow})]^{\zeta}, \text{for } \omega \to -v_c(k-k_{F,\uparrow}) \tag{58}$$

with

$$\zeta = \vartheta_{s1}+2\Delta_c^{-}-1 \approx -\frac{7}{16}+\frac{5}{2\pi^2}\frac{H}{H_c}, \text{ as } H \to 0 \tag{59}$$

$$\tilde{G}^{\uparrow}(k,\omega) \approx [\omega - v_s(k-k_{F,\uparrow})]^{\zeta}, \text{for } \omega \to v_s(k-k_{F,\uparrow}) \tag{60}$$

with

$$\zeta = \vartheta_{c1}+2\Delta_s^{+}-1 \approx \frac{1}{8}+\frac{1}{\pi^2}\frac{H}{H_c}, \text{ as } H \to 0 \tag{61}$$

$$\tilde{G}^{\uparrow}(k,\omega) \approx [\omega + v_s(k-k_{F,\uparrow})]^{\zeta}, \text{ for } \omega \to -v_s(k-k_{F,\uparrow}) \tag{62}$$

with

$$\zeta = \vartheta_{c1}+2\Delta_s^{-}-1 \approx \frac{5}{8}-\frac{1}{\pi^2}\frac{H}{H_c}, \text{ as } H \to 0 \tag{63}$$

Eqn. (53) represents the momentum distribution function around the Fermi point $k_{F,\uparrow}$ for the electron field correlation function. It exhibits a characteristic power-law singularity of the TL liquid, with exponent Eqn. (54). This anomalous exponent ζ, for $k_{F,\uparrow}$ grows monotonically with increasing magnetic field. i.e. $\zeta \to \frac{1}{8}$ as $\frac{H}{H_c} \to 0$ and $\zeta \to 0.226$ as $\frac{H}{H_c} \to 1$, hence the momentum distribution function in the presence of magnetic field exhibits a rapid change around $k_{F,\uparrow}$ as shown in figure 1.

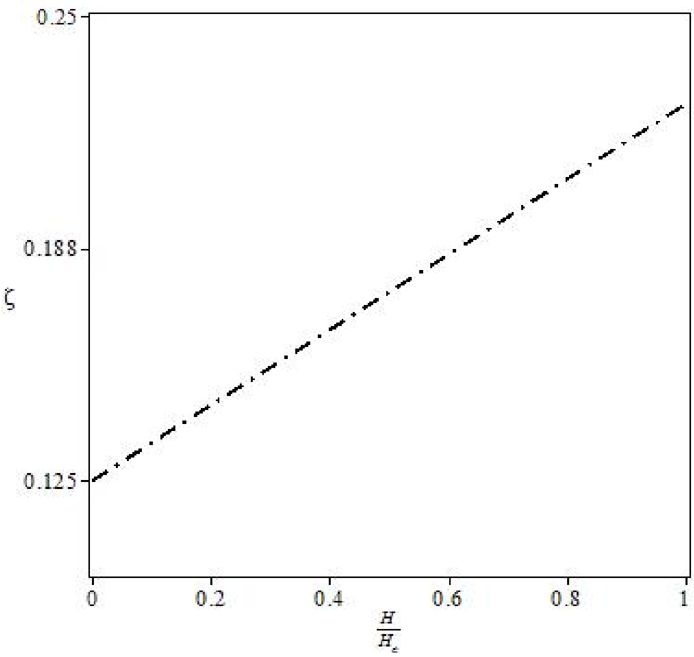

Figure 1. The anomalous exponent ζ for the momentum distribution around $k_{F,\uparrow}$ as a function of $\frac{H}{H_c}$ in the t-J model

Next, at $k_{F,\uparrow}+2k_{F,\downarrow}$, the momentum distribution is obtained as

$$\tilde{G}^{\uparrow}(k\approx k_{F,\uparrow}+2k_{F,\downarrow})\approx \mathrm{sgn}(k-k_{F,\uparrow}-2k_{F,\downarrow})\left|k-k_{F,\uparrow}-2k_{F,\downarrow}\right|^{\zeta} \tag{64}$$

The anomalous exponent

$$\begin{aligned}\zeta &= \vartheta_{c2}+\vartheta_{s2}-1\\ &\approx \frac{9}{8}+\frac{1}{\pi^2}\frac{H}{H_c}\ , \qquad \text{as } H\to 0\end{aligned} \tag{65}$$

and

$$2s=1 \tag{66}$$

The nature of singularities for $k_{F,\uparrow}+2k_{F,\downarrow}$ are obtained as

$$\tilde{G}^{\uparrow}(k,\omega)\approx[\omega-v_c(k-k_{F,\uparrow}-2k_{F,\downarrow})]^{\zeta},\quad \text{for}\quad \omega\to v_c(k-k_{F,\uparrow}-2k_{F,\downarrow}) \tag{67}$$

with

$$\zeta\ =\vartheta_{s2}+2\Delta_c^{+}-1\approx\frac{17}{16}+\frac{9}{2\pi^2}\frac{H}{H_c}\ ,\qquad \text{as } H\to 0 \tag{68}$$

$$\tilde{G}^{\uparrow}(k,\omega)\approx[\omega+v_c(k-k_{F,\uparrow}-2k_{F,\downarrow})]^{\zeta},\quad \text{for}\quad \omega\to -v_c(k-k_{F,\uparrow}-2k_{F,\downarrow}) \tag{69}$$

with

$$\zeta\ =\vartheta_{s2}+2\Delta_c^{-}-1\approx-\frac{7}{16}+\frac{5}{2\pi^2}\frac{H}{H_c}\ ,\qquad \text{as } H\to 0 \tag{70}$$

$$\tilde{G}^{\uparrow}(k,\omega)\approx[\omega-v_s(k-k_{F,\uparrow}-2k_{F,\downarrow})]^{\zeta},\qquad \text{for}\quad \omega\to v_s(k-k_{F,\uparrow}-2k_{F,\downarrow}) \tag{71}$$

with

$$\zeta = \vartheta_{c2} + 2\Delta_s^+ - 1 \approx \frac{5}{8} + \frac{3}{\pi^2}\frac{H}{H_c}, \qquad \text{as } H \to 0 \tag{72}$$

$$\tilde{G}^{\uparrow}(k,\omega) \approx [\omega + v_s(k - k_{F,\uparrow} - 2k_{F,\downarrow})]^{\zeta}, \quad \text{for} \quad \omega \to -v_s(k - k_{F,\uparrow} - 2k_{F,\downarrow}) \tag{73}$$

with

$$\zeta = \vartheta_{c2} + 2\Delta_s^- - 1 \approx \frac{9}{8} + \frac{5}{\pi^2}\frac{H}{H_c}, \qquad \text{as } H \to 0 \tag{74}$$

Also, Eqn. (64) represents the momentum distribution around the Fermi wave number $k_{F,\uparrow} + 2k_{F,\downarrow}$ for the electron field correlator. It exhibits a characteristic power-law singularity of the TL liquid with exponent Eqn. (65). This anomalous exponent ζ, grows monotonically from $\frac{9}{8}$ to 1.226 as the magnetic field goes from 0 to 1, and hence the momentum distribution function in the presence of magnetic field exhibits a rapid change around $k_{F,\uparrow} + 2k_{F,\downarrow}$ as shown in figure 2.

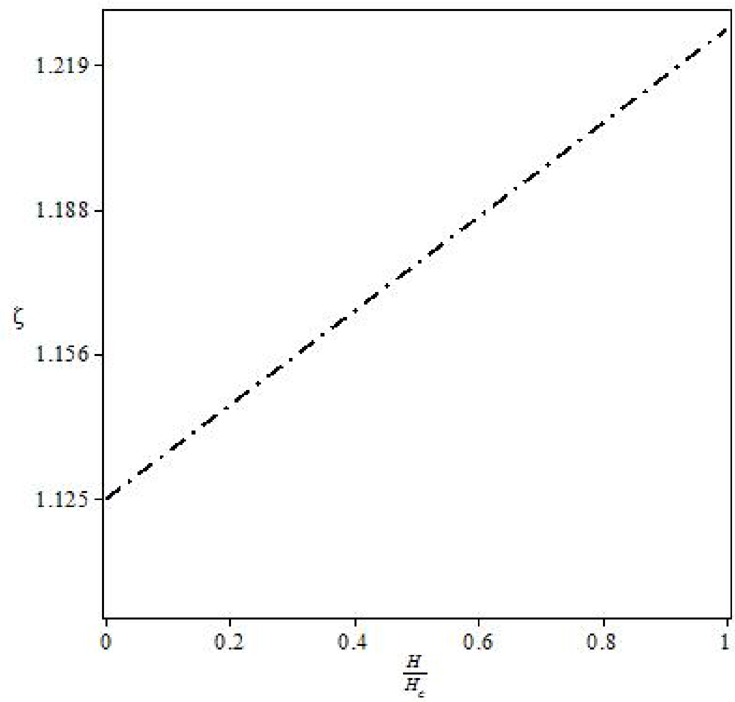

Figure 2. The anomalous exponent ζ for the momentum distribution around $k_{F,\uparrow} + 2k_{F,\downarrow}$ as a function of $\frac{H}{H_c}$ in the t-J model

At $k_{F,\uparrow} + 4k_{F,\downarrow}$, the momentum distribution is obtained as

$$\tilde{G}^{\uparrow}(k \approx k_{F,\uparrow} + 4k_{F,\downarrow}) \approx \mathrm{sgn}(k - k_{F,\uparrow} - 4k_{F,\downarrow})\left|k - k_{F,\uparrow} - 4k_{F,\downarrow}\right|^{\zeta} \tag{75}$$

Where the anomalous exponent

$$\begin{aligned}\zeta &= \vartheta_{c3} + \vartheta_{s3} - 1 \\ &\approx \frac{41}{8} + \frac{17}{\pi^2}\frac{H}{H_c}, \qquad \text{as } H \to 0\end{aligned} \tag{76}$$

and

$$2s = 1 \tag{77}$$

The nature of singularities for $k_{F,\uparrow} + 4k_{F,\downarrow}$ are

$$\tilde{G}^{\uparrow}(k,\omega) \approx [\omega - v_c(k - k_{F,\uparrow} - 4k_{F,\downarrow})]^{\zeta}, \quad \text{for} \quad \omega \to v_c(k - k_{F,\uparrow} - 4k_{F,\downarrow}) \tag{78}$$

with

$$\zeta = \vartheta_{s3} + 2\Delta_c^{+} - 1 \approx \frac{73}{16} + \frac{25}{2\pi^2}\frac{H}{H_c}, \qquad \text{as } H \to 0 \tag{79}$$

$$\tilde{G}^{\uparrow}(k,\omega) \approx [\omega + v_c(k - k_{F,\uparrow} - 4k_{F,\downarrow})]^{\zeta}, \quad \text{for} \quad \omega \to -v_c(k - k_{F,\uparrow} - 4k_{F,\downarrow}) \tag{80}$$

with

$$\zeta = \vartheta_{s3} + 2\Delta_c^{-} - 1 \approx \frac{33}{16} + \frac{13}{2\pi^2}\frac{H}{H_c}, \qquad \text{as } H \to 0 \tag{81}$$

$$\tilde{G}^{\uparrow}(k,\omega) \approx [\omega - v_s(k - k_{F,\uparrow} - 4k_{F,\downarrow})]^{\zeta}, \ \text{for} \ \omega \to v_s(k - k_{F,\uparrow} - 4k_{F,\downarrow}) \tag{82}$$

with

$$\zeta = \vartheta_{c3} + 2\Delta_s^{+} - 1 \approx \frac{25}{8} + \frac{13}{\pi^2}\frac{H}{H_c}, \qquad \text{as } H \to 0 \tag{83}$$

$$\tilde{G}^{\uparrow}(k,\omega) \approx [\omega + v_s(k - k_{F,\uparrow} - 4k_{F,\downarrow})]^{\zeta}, \quad \text{for} \quad \omega \to -v_s(k - k_{F,\uparrow} - 4k_{F,\downarrow}) \tag{84}$$

with

$$\zeta = \vartheta_{c3} + 2\Delta_s^{-} - 1 \approx \frac{37}{8} + \frac{19}{\pi^2}\frac{H}{H_c}, \qquad \text{as } H \to 0 \tag{85}$$

Eqn. (75) represents the momentum distribution function around the Fermi point $k_{F,\uparrow} + 4k_{F,\downarrow}$. It exhibits a characteristic power-law singularity of the TL liquid with exponent Eqn. (76). The anomalous exponent ζ, grows monotonically from 5.125 to 6.847 as the magnetic field goes from 0 to 1, and hence the momentum distribution function in the presence of magnetic field exhibits a rapid change around $k_{F,\uparrow} + 4k_{F,\downarrow}$ as shown in figure 3.

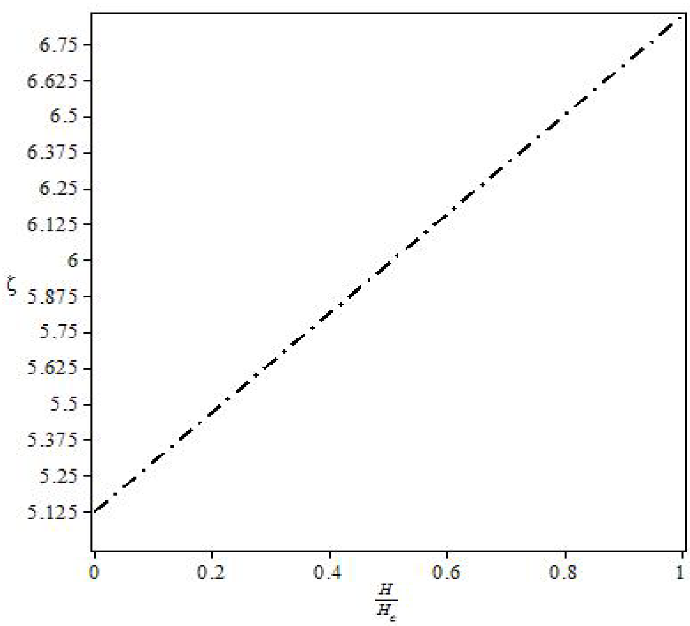

Figure 3. The anomalous exponent ζ for the momentum distribution around $k_{F,\uparrow} + 4k_{F,\downarrow}$ as a function of $\frac{H}{H_c}$ in the t-J model

Finally, at $k_{F,\uparrow} + 6k_{F,\downarrow}$, the momentum distribution is obtained as

$$\tilde{G}^{\uparrow}(k \approx k_{F,\uparrow} + 6k_{F,\downarrow}) \approx \mathrm{sgn}(k - k_{F,\uparrow} - 6k_{F,\downarrow})\left|k - k_{F,\uparrow} - 6k_{F,\downarrow}\right|^{\zeta} \tag{86}$$

The anomalous exponent

$$\begin{aligned}\zeta &= \vartheta_{c4} + \vartheta_{s4} - 1 \\ &\approx \frac{97}{8} + \frac{37}{\pi^2}\frac{H}{H_c}\ , \qquad \text{as } H \to 0\end{aligned} \tag{87}$$

and

$$2s = 1 \tag{88}$$

The nature of singularities for $k_{F,\uparrow} + 6k_{F,\downarrow}$ are

$$\tilde{G}^{\uparrow}(k,\omega) \approx [\omega - v_c(k - k_{F,\uparrow} - 6k_{F,\downarrow})]^{\zeta},\ \text{for } \omega \to v_c(k - k_{F,\uparrow} - 6k_{F,\downarrow}) \tag{89}$$

$$\tilde{G}^{\uparrow}(k,\omega) \approx [\omega - v_c(k - k_{F,\uparrow} - 6k_{F,\downarrow})]^{\zeta},\ \text{for } \omega \to v_c(k - k_{F,\uparrow} - 6k_{F,\downarrow}) \tag{90}$$

with

$$\zeta = \vartheta_{s4} + 2\Delta_c^{+} - 1 \approx \frac{169}{16} + \frac{49}{2\pi^2}\frac{H}{H_c}\ , \qquad \text{as } H \to 0 \tag{91}$$

$$\tilde{G}^{\uparrow}(k,\omega) \approx [\omega + v_c(k - k_{F,\uparrow} - 6k_{F,\downarrow})]^{\zeta},\ \text{for } \omega \to -v_c(k - k_{F,\uparrow} - 6k_{F,\downarrow}) \tag{92}$$

with

$$\zeta = \vartheta_{s4} + 2\Delta_c^{-} - 1 \approx \frac{113}{16} + \frac{29}{2\pi^2}\frac{H}{H_c}\ , \qquad \text{as } H \to 0 \tag{93}$$

$$\tilde{G}^{\uparrow}(k,\omega) \approx [\omega - v_s(k - k_{F,\uparrow} - 6k_{F,\downarrow})]^{\zeta},\ \text{for } \omega \to v_s(k - k_{F,\uparrow} - 6k_{F,\downarrow}) \tag{94}$$

with

$$\zeta = \vartheta_{c4} + 2\Delta_s^{+} - 1 \approx \frac{61}{8} + \frac{31}{\pi^2}\frac{H}{H_c}\ , \qquad \text{as } H \to 0 \tag{95}$$

$$\tilde{G}^{\uparrow}(k,\omega) \approx [\omega + v_s(k - k_{F,\uparrow} - 6k_{F,\downarrow})]^{\zeta},\ \text{for } \omega \to -v_s(k - k_{F,\uparrow} - 6k_{F,\downarrow}) \tag{96}$$

with

$$\zeta = \vartheta_{c4} + 2\Delta_s^{-} - 1 \approx \frac{81}{8} + \frac{41}{\pi^2}\frac{H}{H_c}\ , \qquad \text{as } H \to 0 \tag{97}$$

The momentum distribution Eqn. (86) around the Fermi point $k_{F,\uparrow} + 6k_{F,\downarrow}$ also exhibits a power-law singularity of the TL liquid with exponent Eqn. (87). This anomalous exponent ζ, grows monotonically from 12.125 to 15.874 as the magnetic field goes from 0 to 1, and hence the momentum distribution function in the presence of magnetic field exhibits a rapid change around $k_{F,\uparrow} + 6k_{F,\downarrow}$ as shown in figure 4.

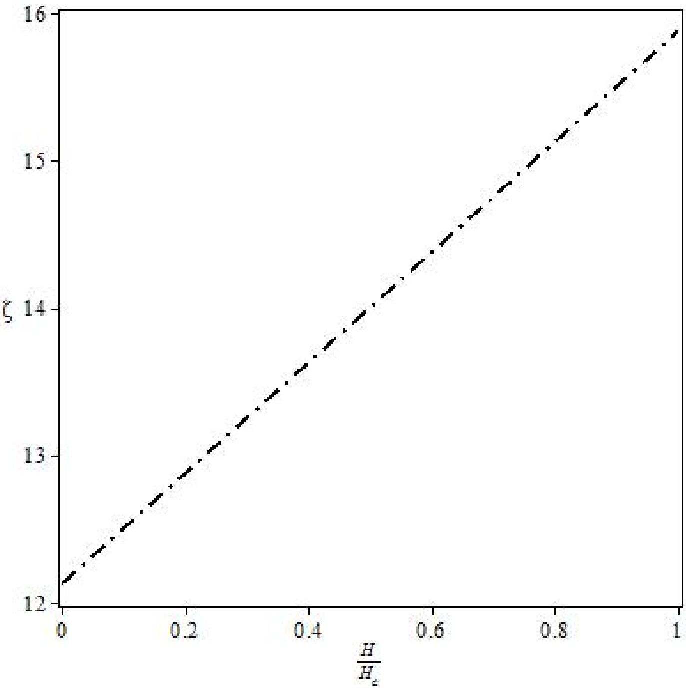

Figure 4. The anomalous exponent ζ for the momentum distribution around $k_{F,\uparrow}+6k_{F,\downarrow}$ as a function of $\frac{H}{H_c}$ in the t-J model

4. Discussion

In conclusion, the result Eqn. (52) has singularities near the Fermi points $k_{F,\uparrow}$, $k_{F,\uparrow}+2k_{F,\downarrow}$, $k_{F,\uparrow}+4k_{F,\downarrow}$ and $k_{F,\uparrow}+6k_{F,\downarrow}$ respectively. The $k_{F,\uparrow}$ part arises from the excitation of $(I_c, I_s, D_c, D_s)=(1,1,0,\pm 1/2)$, the $k_{F,\uparrow}+2k_{F,\downarrow}$ part from $(I_c, I_s, D_c, D_s)=(1,1,\pm 1,\mp 1/2)$, the $k_{F,\uparrow}+4k_{F,\downarrow}$ part from $(I_c, I_s, D_c, D_s)=(1,1,\pm 2,\mp 3/2)$ and $k_{F,\uparrow}+6k_{F,\downarrow}$ from $(I_c, I_s, D_c, D_s)=(1,1,\pm 3,\mp 5/2)$. This implies that the $k_{F,\uparrow}$ part is dominated by spinon excitation alone. On the other hand both the holon and spinon excitations are responsible for $k_{F,\uparrow}+2k_{F,\downarrow}$, $k_{F,\uparrow}+4k_{F,\downarrow}$ and $k_{F,\uparrow}+6k_{F,\downarrow}$ oscillation parts respectively. We observed that the anomalous exponent, ζ grows monotonically with increasing magnetic field. However, from Eqns. (54), (65), (76) and (87), at vanishing magnetic field the exponent, ζ goes to $\frac{1}{8}$ for k_F, $\frac{9}{8}$ for $3k_F$, 5.125 for $5k_F$ and 12.125 for $7k_F$ respectively. The values $\frac{1}{8}$ and $\frac{9}{8}$ for k_F and $3k_F$ is in agreement with the evaluation for the Hubbard model by Ogata and others [5, 10, 11]. However, our exponents for the momentum distribution function at $3k_F$ and $5k_F$ disagrees with the values of Shaojin and Lu [12] for the Hubbard model they obtained as $\frac{3}{4}$ and 1, respectively. Finally, the exponent for the momentum distribution function around $7k_F$ is quite new. It indicates the presence of singularity and shows characteristic TL liquid property at this Fermi point.

Appendix

A. Wiener-Hopf technique for dressed charge matrix

Some useful relations are

$$n_c = \int_{-k_0}^{k_0} \frac{Z_{cc}(k)}{2\pi} dk = \int_{-k_0}^{k_0} \rho_c(k) dk \tag{A1}$$

and

$$n_s = \int_{-k_0}^{k_0} \frac{Z_{sc}(k)}{2\pi} dk = \int_{-\lambda_0}^{\lambda_0} \frac{Z_{cs}(\lambda)}{2\pi} d\lambda = \int_{-\lambda_0}^{\lambda_0} \rho_s(\lambda) d\lambda \tag{A2}$$

Where, n_s is the density of down-spin electrons, n_c is density of electrons, $\rho_c(k)$ is the charge distribution function with holon momentum k and $\rho_s(\lambda)$ is down-spin distribution function with spinon rapidity λ.

For small magnetic field $H << 1$, we solve the dressed charge matrix Eqns. (10) to (13) by Wiener-Hopf technique for only terms up to order $1/u$ in the strong coupling limit. With Eqn. (A2), we write Eqn. (13) as

$$\begin{aligned} Z_{ss}(\lambda) &= 1 + a_1(\lambda) \int_{-k_0}^{k_0} Z_{sc}(k)\, dk - \int_{-\lambda_0}^{\lambda_0} a_2(\lambda-\mu)\, Z_{ss}(\mu) d\mu \\ &= 1 + 2\pi n_s a_1(\lambda) - \int_{-\lambda_0}^{\lambda_0} a_2(\lambda-\mu)\, Z_{ss}(\mu) d\mu \end{aligned} \tag{A3}$$

Fourier transforming (A3), we obtain

$$Z_{ss}(\lambda) = \tfrac{1}{2} + 2\pi n_s a_1(\lambda) - \int_{|\mu| \geq \lambda_0} K(\lambda-\mu) Z_{ss}(\mu) d\mu \tag{A4}$$

Where the kernels are given by

$$\begin{aligned} s(\lambda) &= \frac{1}{2u \cosh(\pi\lambda/u)} \\ K(\lambda) &= \frac{1}{2} \int_{-\infty}^{\infty} \frac{\exp(-i\omega\lambda)}{1+\exp(\omega u)} d\omega \end{aligned} \tag{A5}$$

We solve Eqn. (A4) by introducing the function

$$y(\lambda) = Z_{ss}(\lambda + \lambda_0) \tag{A6}$$

and expanding it as

$$y(\lambda) = \sum_{n=o}^{\infty} y_n(\lambda) \tag{A7}$$

Where $y_n(\lambda)$ are defined as the solutions of the Wiener-Hopf equations

$$y_n(\lambda) = g_n(\lambda) + \int_0^{\infty} K(\lambda-\mu) y_n(\mu) d\mu \tag{A8}$$

$$\begin{aligned} g_n(\lambda) &= \int_0^{\infty} K(\lambda+\mu+2\lambda_0) y_{n-1}(\mu) d\mu, \qquad n \geq 1 \\ g_0(\lambda) &= \tfrac{1}{2} + 2\pi n_s s(\lambda+\lambda_0) \end{aligned} \tag{A9}$$

The driving terms $g_n(\lambda)$ and the solutions $y_n(\lambda)$ becomes smaller as n as increases because λ is large. Our procedure follows Fabian et al. [9]. Assuming the function $y_{n-1}(\lambda)$ and $g_n(\lambda)$ are known. We define

$$\tilde{y}_n^+(\omega) = \int_0^\infty \exp(i\omega\lambda) y_n(\lambda) d\lambda$$
$$\tilde{y}_n^-(\omega) = \int_{-\infty}^0 \exp(i\omega\lambda) y_n(\lambda) d\lambda \tag{A10}$$

Where the functions $\tilde{y}_n^\pm(\omega)$ are analytic on the upper and lower planes respectively, with

$$\tilde{y}(\omega) = \tilde{y}_n^+(\omega) + \tilde{y}_n^-(\omega) \tag{A11}$$

Also we assume

$$\tilde{y}_n^\pm(\infty) = \tilde{g}(\infty) = 0 \tag{A12}$$

In terms of these functions we express the Fourier transform of Eqn. (A9) as

$$\tilde{g}_n(\omega) = \frac{\tilde{y}_n^+(\omega)}{1+\exp(-2u|\omega|)} + \tilde{y}_n^-(\omega) \tag{A13}$$

Where $\tilde{g}_n(\omega)$ is the Fourier transform of $g_n(\lambda)$. Now we split (A9) into the sum of two parts that are analytical and non-zero in the upper and lower half planes. To obtain this we use the factorization

$$1+\exp(-2u|\omega|) = G^+(\omega)G^-(\omega) \tag{A14}$$

$$G^+(\omega) = G^-(-\omega)\frac{\sqrt{2\pi}}{\Gamma(\frac{1}{2}-\frac{iu\omega}{\pi})}\left(-\frac{iu\omega}{\pi}\right)^{-\frac{iu\omega}{\pi}} e^{\frac{iu\omega}{\pi}} \tag{A15}$$

Where $G^\pm(\omega)$ are analytic and non-zero in the upper and lower half planes respectively and are normalized as

$$\lim_{\omega\to\infty} G^\pm(\omega) = 1 \tag{A16}$$

Useful special function of $G^\pm(\omega)$ are

$$G^\pm(0) = \sqrt{2}$$
$$G^\pm\left(\frac{i\pi}{2u}\right) = G^\pm\left(-\frac{i\omega}{2u}\right) = \sqrt{\frac{\pi}{e}} \tag{A17}$$

Using (A13) and (A14), we obtain

$$\frac{\tilde{y}_n^+(\omega)}{G^+(\omega)} + G^-(\omega)\tilde{y}_n^-(\omega) = G^-(\omega)\tilde{g}_n(\omega) \tag{A18}$$

Decompose the right hand side of (A18) into the sum of two functions

$$G^-(\omega)\tilde{g}_n(\omega) = Q_n^+(\omega) + Q_n^-(\omega) \tag{A19}$$

This implies that

$$\tilde{y}_n^+(\omega) = G^+(\omega)Q_n^+(\omega)$$
$$\tilde{y}_n^-(\omega) = \frac{Q_n^-(\omega)}{G^-(\omega)} \tag{A20}$$

To obtain the solution of (A8) for $y_0(\lambda)$, we set the driving term to be

$$\tilde{g}(\omega) = 2\pi\delta(\omega) + \frac{b\ e^{-i\lambda_0\omega}}{2\cosh(u\omega)} \tag{A21}$$

We decompose the first term by using

$$2\pi\delta(\omega) = i\left(\frac{1}{\omega+i\varepsilon} - \frac{1}{\omega-i\varepsilon}\right), \qquad (\varepsilon \to 0) \tag{A22}$$

The second term of (A21) is meromorphic function of ω with simple poles located at

$$\begin{aligned} \omega_n &= \frac{i\pi}{2u}(2n+1) \\ \omega_0 &= \frac{i\pi}{2u}, \quad \omega_1 = \frac{3i\pi}{2u}, \quad \omega_2 = \frac{5i\pi}{2u}, \ldots \end{aligned} \tag{A23}$$

Note, there is no pole at $\omega = 0$. The decomposition of the factor $1/\cosh(u\omega)$ gives

$$\frac{1}{\cosh(u\omega)} = A^+(\omega) + A^-(\omega) \tag{A24}$$

$$\begin{aligned} A^+(\omega) &= \frac{i}{u}\sum_{n=0}^{\infty}\frac{(-1)^n}{\omega+\omega_n} \\ A^+(\omega) &= \frac{1}{\cosh(u\omega)} - \frac{i}{u}\sum_{n=0}^{\infty}\frac{(-1)^n}{\omega+\omega_n} \end{aligned} \tag{A25}$$

Using (A25) we can express the function $f^-(\omega)/\cosh(u\omega)$, for any function $f^-(x)$ that is analytic and bounded in the lower half-plane as the sum of two functions $\chi^{\pm}(\omega)$ analytic in the upper/lower half-plane

$$\frac{f^-(\omega)}{\cosh(u\omega)} = \chi^+(\omega) + \chi^-(\omega) \tag{A26}$$

$$\begin{aligned} \chi^+(\omega) &= \frac{i}{u}\sum_{n=0}^{\infty}\frac{(-1)^n f^-(-\omega_n)}{\omega+\omega_n} \\ \chi^-(\omega) &= \frac{f^-(-\omega_n)}{\cosh(u\omega)} - \frac{i}{u}\sum_{n=0}^{\infty}\frac{(-1)^n f^-(-\omega_n)}{\omega+\omega_n} \end{aligned} \tag{A27}$$

Applying the formula (A27) to (A21) and (A19), we obtain

$$\tilde{g}(\omega) = ai\left(\frac{1}{\omega+i\varepsilon} - \frac{1}{\omega-i\varepsilon}\right) + \frac{b}{2}[\chi^+(\omega) + \chi^-(\omega)] \tag{A28}$$

$$\tilde{g}(\omega) = \frac{ai}{\omega+i\varepsilon} + \frac{bi}{2u}\sum_{n=0}^{\infty}\frac{(-1)^n e^{-i\lambda_0\omega}}{\omega+\omega_n} - \frac{ai}{\omega-i\varepsilon} + \frac{b}{2}\frac{e^{-i\lambda_0\omega}}{\cosh(u\omega)} - \frac{bi}{2u}\sum_{n=0}^{\infty}\frac{(-1)^n e^{-i\lambda_0\omega}}{\omega+\omega_n} \tag{A29}$$

Now,

$$\begin{aligned} G^{-}(\omega)\tilde{g}(\omega) &= \frac{aiG^{-}(\omega)}{\omega+i\varepsilon} + \frac{bi}{2u}\sum_{n=0}^{\infty}\frac{(-1)^n G^{-}(\omega)e^{-i\lambda_0\omega}}{\omega+\omega_n} - \frac{aiG^{-}(\omega)}{\omega-i\varepsilon} \\ &\quad + \frac{b}{2}\frac{G^{-}(\omega)e^{-i\lambda_0\omega}}{\cosh(u\omega)} - \frac{bi}{2u}\sum_{n=0}^{\infty}\frac{(-1)^n G^{-}(\omega)e^{-i\lambda_0\omega}}{\omega+\omega_n} \\ &\equiv Q_n^{+}(\omega) + Q_n^{-}(\omega) \end{aligned} \tag{A30}$$

Therefore,

$$\begin{aligned} Q_n^{-}(\omega) &= -\frac{aiG^{-}(-\omega_n)}{\omega-i\varepsilon} + \frac{b}{2}\frac{G^{-}(-\omega_n)e^{-i\lambda_0\omega}}{\cosh(u\omega)} \\ &\quad - \frac{bi}{2u}\sum_{n=0}^{\infty}\frac{(-1)^n G^{-}(-\omega_n)e^{-i\lambda_0\omega_n}}{\omega+\omega_n} \end{aligned} \tag{A31}$$

$$Q_n^{+}(\omega) = \frac{aiG^{-}(-\omega_n)}{\omega+i\varepsilon} + \frac{bi}{2u}\sum_{n=0}^{\infty}\frac{(-1)^n G^{-}(-\omega_n)e^{-i\lambda_0\omega}}{\omega+\omega_n} \tag{A32}$$

For $n = 0$

$$Q_0^{+}(\omega) = \frac{aiG^{-}(0)}{\omega+i\varepsilon} + \frac{bi}{2u}\frac{G^{-}\left(\frac{-\pi i}{2u}\right)e^{-\frac{\lambda_0\pi}{2u}}}{\omega+\frac{\pi i}{2u}} \tag{A33}$$

The functions $y_0^{\pm}(\omega)$ are obtained by using (A20)

$$\tilde{y}_0^{+}(\omega) = G^{+}(\omega)\left(\frac{aiG^{-}(0)}{\omega+i\varepsilon} + \frac{bi}{2u}\frac{G^{-}\left(\frac{-\pi i}{2u}\right)e^{-\frac{\lambda_0\pi}{2u}}}{\omega+\frac{\pi i}{2u}}\right) \tag{A34}$$

From (A9) for $n = 0$, by setting $a = 1/2$, $b = 2\pi n_s$ in (A34), we obtain

$$\tilde{y}_0^{+}(\omega) = G^{+}(\omega)\left(\frac{1}{2}\frac{iG^{-}(0)}{\omega+i\varepsilon} + \frac{\pi n_s i}{u}\frac{G^{-}\left(\frac{-\pi i}{2u}\right)e^{-\frac{\lambda_0\pi}{2u}}}{\omega+\frac{\pi i}{2u}}\right) + O\left(H^2\right) \tag{A35}$$

By definition

$$y(0) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\tilde{y}^{+}(\omega)e^{-i\omega\varepsilon}d\omega = -i\lim_{\omega\to\infty}\omega\tilde{y}^{+}(\omega) \tag{A36}$$

$$\begin{aligned} \lambda_0 &\approx \frac{2u}{\pi}\ln\left(\frac{H_0}{H}\right) \\ H_0 &= \sqrt{\frac{\pi^3}{2e}}H_c \\ H_c &= \frac{4\pi^2 n_c^3}{3u}\left(1-\frac{\pi^2 n_c^2}{5u^2}\right); \quad u >> 1 \end{aligned} \tag{A37}$$

Where, H is magnetic field, H_c is critical field, u strong coupling, H_0 magnetic field at zero temperature and λ_0

corresponds to Fermi points.

Combining the result (A35) with (A36), we obtain the first the first order contribution to Z_{ss} as follows

$$-i\lim_{\omega\to\infty}\omega\tilde{y}^{+}(\omega)=-i\lim_{\omega\to\infty}\omega G^{+}(\omega)\left(\frac{1}{2}\frac{iG^{-}(0)}{\omega+i\varepsilon}+\frac{\pi n_s i}{u}\frac{G^{-}\left(\frac{-\pi i}{2u}\right)e^{-\frac{\lambda_0\pi}{2u}}}{\omega+\frac{\pi i}{2u}}\right) \tag{A38}$$

As $\varepsilon\to 0$, we use Eqns. (A16) and (A17) on (A38) to obtain

$$y_0(0)=-i\omega\left(\frac{1}{2}\frac{i\sqrt{2}}{\omega}+\frac{\pi n_s i}{u}\frac{\sqrt{\frac{\pi}{e}}e^{-\frac{\lambda_0\pi}{2u}}}{\omega+\frac{\pi i}{2u}}\right) \tag{A39}$$

Simplifying further, we obtain

$$\begin{aligned} y_0(0)&=\frac{\sqrt{2}}{2}+\frac{\pi n_s}{u}\frac{\sqrt{\frac{\pi}{e}}}{1+\frac{\pi i}{2u\omega}}\exp\left(-\frac{\pi}{2u}\frac{2u}{\pi}\ln(H_0/H)\right)\\ &=\frac{\sqrt{2}}{2}+\frac{\pi n_s}{u}\frac{\sqrt{\frac{\pi}{e}}}{1+\frac{\pi i}{\infty}}\frac{H}{H_0},\quad \text{since } e^{\ln x}=x\\ &=\frac{\sqrt{2}}{2}+\frac{\pi n_s\sqrt{\frac{\pi}{e}}}{u}\left(\frac{H}{H_0}\right)\end{aligned} \tag{A40}$$

Using (A37), we obtain

$$y_0(0)=\frac{\sqrt{2}}{2}+\frac{\pi n_s\sqrt{\frac{\pi}{e}}}{u}\left(\frac{H}{\sqrt{\frac{\pi^3}{2e}}H_c}\right)=\sqrt{2}\left(\frac{1}{2}+\frac{n_s}{u}\frac{H}{H_c}\right)+O\left(H^2\right) \tag{A41}$$

Next, the second order contribution to $y(0)=Z_{ss}(\lambda_0)$ is obtained by taking the Fourier transform of (A9) for $n=1$.

$$\begin{aligned} g_1(\lambda)&=\int_0^{\infty}K(\lambda+\mu+2\lambda_0)y_0(\mu)d\mu\\ \tilde{g}_1(\omega)&=\frac{\exp(-2i\lambda_0\omega)\tilde{y}_0^{+}(-\omega)}{1+\exp(2u|\omega|)}\end{aligned} \tag{A42}$$

From (A14)

$$\frac{1}{1+\exp(2u|\omega|)}=1-\frac{1}{G^{+}(\omega)G^{-}(\omega)} \tag{A43}$$

$$\tilde{g}_1(\omega)=\exp(-2i\lambda_0\omega)\tilde{y}_0^{+}(-\omega)\left(1-\frac{1}{G^{+}(\omega)G^{-}(\omega)}\right) \tag{A44}$$

From Eqn. (A19),

$$G^{-}(\omega)\tilde{g}_1(\omega)=Q_1^{+}(\omega)+Q_1^{-}(\omega) \tag{A45}$$

We have decomposed $G^{-}(\omega)\tilde{g}_1(\omega)$ into $Q_1^{\pm}(\omega)$ which is analytic in the upper and lower half-planes. $Q_1^{+}(\omega)$ is given by

$$Q_1^+(\omega)=\frac{1}{2\pi i}\int_{-\infty}^{\infty}\frac{\exp(-2i\lambda_0 x)\tilde{y}_0^+(-x)}{x-\omega-i\varepsilon}\frac{1}{G^+(x)}dx \tag{A46}$$

Where ε is a small positive constant. $G^+(x)$ has a branch cut along the negative imaginary axis and by deforming the contour of integration we rewrite (A44) as

$$Q_1^+(\omega)=\frac{1}{2\pi i}\int_0^{\infty}\frac{\exp(-2i\lambda_0 x)\tilde{y}_0^+(ix)}{x-i\omega}\left(\frac{1}{G^+(-ix-\varepsilon)}-\frac{1}{G^+(-ix+\varepsilon)}\right)dx \tag{A47}$$

From (A15), as $\omega\to ix$

$$\frac{1}{G^+(-ix-\varepsilon)}=\frac{\Gamma(\frac{1}{2}-\frac{ux}{\pi})}{\sqrt{2\pi}}\left(\frac{ux}{\pi}\right)^{\frac{iux}{\pi}}e^{-\frac{iux}{\pi}} \tag{A48}$$

$$Q_1^+(\omega)=\frac{1}{(2\pi)^{\frac{3}{2}}i}\int_0^{\infty}\frac{e^{-2\lambda_0 x}\tilde{y}_0^+(ix)}{x-i\omega}\Gamma(\tfrac{1}{2}-\tfrac{ux}{\pi})\left(\frac{ux}{\pi}\right)^{\frac{iux}{\pi}}\left(e^{-\frac{iux}{\pi}}-e^{-\frac{iux}{\pi}}\right)dx \tag{A49}$$

Since, $\sin x=(e^x-e^{-x})/2i$

$$Q_1^+(\omega)=\frac{2i}{(2\pi)^{\frac{3}{2}}i}\int_0^{\infty}\frac{e^{-2\lambda_0 x}\tilde{y}_0^+(ix)}{x-i\omega}\left(\frac{ux}{\pi}\right)^{\frac{iux}{\pi}}\Gamma(\tfrac{1}{2}-\tfrac{ux}{\pi})\sin(ux)dx \tag{A50}$$

For $x>0$ the integrand rapidly decrease because $\lambda_0\gg 1$, and hence the integral is approximated by expanding the terms other than $\exp(-2\lambda_0 x)$ around $x=0$. Therefore, we obtain

$$\begin{aligned}Q_1^+(\omega)&\approx\frac{1}{2\pi}\int_0^{\infty}\frac{e^{-2\lambda_0 x}}{-i\omega}\left(\frac{2u}{\sqrt{2}}+O(x)\right)dx\\&=\frac{1}{-i\omega}\left(\frac{u}{2\sqrt{2}\pi\lambda_0}+O\left(\frac{1}{\lambda_0^2}\right)\right)\end{aligned} \tag{A51}$$

From (A20), we obtain

$$\tilde{y}_1^+(\omega)=\frac{G^+(\omega)}{-i\omega}\left(\frac{u}{2\sqrt{2}\pi\lambda_0}+O\left(\frac{1}{\lambda_0^2}\right)\right) \tag{A52}$$

Using

$$y_1(0)=-i\lim_{\omega\to\infty}\omega\,\tilde{y}_1^+(\omega)\quad\text{and}\quad\lim_{\omega\to\infty}G^{\pm}(\omega)=1, \tag{A53}$$

we obtain

$$y_1(0)=\frac{u}{2\sqrt{2}\pi\lambda_0}+O\left(\frac{1}{\lambda_0^2}\right) \tag{A54}$$

From (A37)

$$y_1(0) = \frac{1}{4\sqrt{2}}\frac{1}{\ln(H_0/H)} + O\left(\frac{1}{(\ln H_0/H)^2}\right)$$
$$y_1(0) = \frac{\sqrt{2}}{8\ln(H_0/H)} + O\left(\frac{1}{(\ln H_0/H)^2}\right) \tag{A55}$$

Therefore, with (A39) and (A52), we obtain

$$Z_{ss}(\lambda_0) = \sqrt{2}\left(\frac{1}{2} + \frac{n_s}{u}\frac{H}{H_c} + \frac{1}{8\ln(H_0/H)}\right) + O\left(\frac{1}{(\ln H_0/H)^2}\right) \tag{A56}$$

Now to evaluate the dressed charge matrix element $Z_{sc}(k_0)$, we take the Fourier transform of Eqn. (12) and (A4) and obtain

$$Z_{sc}(k) = \tfrac{1}{2} + 2\pi n_s K(k) - \int_{|\mu|\geq\lambda_0} K(k-\lambda) Z_{ss}(\lambda)\, d\lambda \tag{A57}$$

Applying the same process in the determination of Eqn. (61), we obtain

$$Z_{sc}(k_0) = \frac{1}{2} + \frac{n_s \ln 2}{u} - \frac{2}{\pi^2}\frac{H}{H_c} + O\left(\frac{H}{H_c \ln(H_0/H)}\right) \tag{A58}$$

Similarly, with the same process, we obtain the other two elements of the dressed charge matrix as

$$Z_{cc}(k_0) = 1 + \frac{n_c \ln 2}{u} - \frac{2n_c}{\pi^2 u}\left(\frac{H}{H_c}\right)^2 + O\left(\frac{H^2}{H_c^2[\ln(H_0/H)]^2}\right) \tag{A59}$$

and

$$Z_{cs}(\lambda_0) = \frac{\sqrt{2}n_c}{u}\frac{H}{H_c} + O\left(\frac{H}{H_c[\ln(H_0/H)]^2}\right) \tag{A60}$$

From Eqn. (A2) together with the property that $Z_{sc}(k) \approx Z_{sc}(k_0) + O(\frac{1}{u^2})$ for $u >> 1$ and $k_0 \approx \pi n_c \big/ (1 + \frac{n_c \ln 2}{u})$, the down-spin density n_s is obtained as

$$n_s = \frac{n_c}{2} - \frac{2n_c}{\pi^2}\frac{H}{H_c} \tag{A61}$$

Using Eqn. (A58) on (A53) to (A56), we obtain the dressed charge matrix equations as

$$Z_{cc}(k_0) = 1 + \frac{n_c}{u}\left(\ln 2 - \frac{2}{\pi^2}\left(\frac{H}{H_c}\right)^2\right) + O\left(\frac{H^2}{H_c^2[\ln(H_0/H)]^2}\right) \tag{A62}$$

$$Z_{cs}(\lambda_0) = \frac{\sqrt{2}n_c}{u}\frac{H}{H_c} + O\left(\frac{H}{H_c[\ln(H_0/H)]^2}\right) \tag{A63}$$

$$Z_{sc}(k_0) = \frac{1}{2} - \frac{2}{\pi^2}\frac{H}{H_c} + \frac{n_c \ln 2}{u}\left(\frac{1}{2} - \frac{2}{\pi^2}\frac{H}{H_c}\right) + O\left(\frac{H}{H_c \ln(H_0/H)}\right) \tag{A64}$$

$$Z_{ss}(\lambda_0)=\sqrt{2}\left(\frac{1}{2}+\frac{n_c}{u}\left(\frac{1}{2}\frac{H}{H_c}-\frac{2}{\pi^2}\left(\frac{H}{H_c}\right)^2\right)+\frac{1}{8\ln(H_0/H)}\right)+O\left(\frac{1}{(\ln H_0/H)^2}\right) \quad \text{(A65)}$$

At half-filling $n_c=1$, and by neglecting corrections to order $(1/u)$, the elements of the dressed charge become

$$Z_{cc}(k_0)=1 \quad \text{(A66)}$$

$$Z_{cs}(\lambda_0)=0 \quad \text{(A67)}$$

$$Z_{sc}(k_0)=\frac{1}{2}-\frac{2}{\pi^2}\frac{H}{H_c} \quad \text{(A68)}$$

$$Z_{ss}(\lambda_0)=\sqrt{2}\left(\frac{1}{2}+\frac{1}{8\ln(H_0/H)}\right) \quad \text{(A69)}$$

Derivation of conformal dimensions in terms of small magnetic field
Note that,

$$(\det Z)^2=\left(Z_{cc}^2+Z_{cs}^2\right)\left(Z_{ss}^2+Z_{sc}^2\right)-\left(Z_{cc}Z_{sc}+Z_{cs}Z_{ss}\right)^2 \quad \text{(A70)}$$

Using (A66) to (A69) on (A70) gives

$$\det Z=Z_{ss} \quad \text{(A71)}$$

Now, with Eqns. (A66) to (A69) on Eqns. (7) and (8), we obtain the magnetic field dependence of the conformal dimensions as

$$\begin{aligned}2\Delta_c^{\pm}(I,D)&=\left(Z_{cc}D_c+D_s\left(\frac{1}{2}-\frac{2}{\pi^2}\frac{H}{H_c}\right)\pm\frac{Z_{ss}I_c}{2\,Z_{ss}}\right)^2+2N_c^{\pm}\\&=\left((D_c+\tfrac{1}{2}D_s)\pm\tfrac{1}{2}I_c-\frac{2D_s}{\pi^2}\frac{H}{H_c}\right)^2+2N_c^{\pm}\end{aligned} \quad \text{(A72)}$$

$$2\Delta_s^{\pm}=\frac{1}{2}\left\{D_s\pm\left(I_s-I_c\left(\frac{1}{2}-\frac{2}{\pi^2}\frac{H}{H_c}\right)\right)\right\}^2+\frac{1}{4\ln(H_0/H)}\left\{D_s^2+\left(I_s-I_c\left(\frac{1}{2}-\frac{2}{\pi^2}\frac{H}{H_c}\right)\right)^2\right\}+2N_s^{\pm} \quad \text{(A73)}$$

B. Correlation function in momentum space

The long-distance behaviour of the two point correlation function usually determines the singularities of spectral functions near $\omega\approx\pm v_{c,s}(k-k_F)$ and the Fourier transform $\tilde{G}(k,\omega)$ produces

$$\tilde{G}(k,\omega)=\begin{cases}\text{const}[\omega-v_c(k-k_c)]^{2(\Delta_s^{+}+\Delta_s^{-}+\Delta_c^{+})-1}, & \text{for } \omega\to v_c(k-k_c)\\ \text{const}[\omega-v_c(k-k_c)]^{2(\Delta_s^{+}+\Delta_s^{-}+\Delta_c^{-})-1}, & \text{for } \omega\to -v_c(k-k_c)\\ \text{const}[\omega-v_c(k-k_c)]^{2(\Delta_c^{+}+\Delta_c^{-}+\Delta_s^{+})-1}, & \text{for } \omega\to v_s(k-k_c)\\ \text{const}[\omega-v_c(k-k_c)]^{2(\Delta_c^{+}+\Delta_c^{-}+\Delta_s^{+})-1}, & \text{for } \omega\to -v_s(k-k_c)\end{cases}, \quad \text{(B1)}$$

and the Fourier transform of equal-time correlator is given by

$$\tilde{G}(k\approx k_F)\approx[\text{sgn}(k-k_F)]^{2s}\left|k-k_F\right|^{\zeta} \quad \text{(B2)}$$

Where the anomalous exponent

$$\zeta = 2(\Delta_c^+ + \Delta_c^- + \Delta_s^+ + \Delta_s^-) - 1 \tag{B3}$$

and the conformal spin is

$$2s = 2(\Delta_c^+ - \Delta_c^- + \Delta_s^+ - \Delta_s^-) \tag{B4}$$

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