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A Sixth Order Multiderivative Method for Direct Integration of Non – linear Third Order Ordinary Differential Equations

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Abstract This study developed, investigated and implemented a novel Implicit Two – step Third derivative method of order six for direct solution of initial value problems of third order ordinary differential equations. The method was developed through Taylors series expansion, incorporating third derivatives (*l*=3) to enhance higher accuracy while maintaining computational efficiency through a two – step (*k*=2) formulation thus eliminating the need to reducing the third order ordinary differential equation into a system of first order equations. Key properties of the method which include accuracy, consistency, stability and convergence were investigated. Numerical implementation demonstrated the method's efficiency and effectiveness across diverse test problems with error analysis revealing high order accuracy when compared with exact solution and some existing block methods of same and higher order of accuracy thus establishing the method as an efficient and accurate tool for solving third order ordinary differential equations in scientific and engineering applications.

Keywords Third order, Differential equation, Implicit, Two – step, Multiderivative, Direct, Integration, Non – linear

1. Introduction

Due to the limitations of analytical methods particularly when solving nonlinear and higher order systems in modern science, numerical methods are found to be indispensable option and tool for theoretical exploration and simulation. Rigorous and extensive research are devoted to the development of effective and efficient numerical schemes which are aimed at obtaining reliable approximate solutions where exact solutions are impractical or unavailable. It is note – worthy that ordinary differential equations serve as crucial tools for modelling phenomena across diverse domains such as Biology, Economics, Physics, Chemistry and Engineering. These models give rise to various forms of Initial Value Problems (IVPs) of Ordinary Differential Equations (ODEs) of different orders that have the general form:

$$y^{\omega} = f(t, y, y', \dots, y^{\omega-1}), y(t_0) = y_0,$$
$$y'(t_0) = y_1, \dots, y^{(\omega-1)}(t_0) = y_{\omega-1} \qquad (1.1)$$

where ω is the order of the problem, $y \epsilon C^{\omega}[a,b]$ and f is a continuous function of t, y, and the derivatives of y. This study considered the solution to (1.1) when $\omega = 3$, that is

$$y''' = f(t, y, y', y''), y(t_0) = y_0, y'(t_0) = y_1, y''(t_0) = y_2 \qquad (1.2)$$

IVPs of higher order ODEs are conventionally reduced to a system of first order ODEs for solution but literature revealed that several researchers have developed direct approaches for the solution. [1] developed a three – step implicit block method for direct solution of IVPs of second order ODEs using interpolation and collocation techniques with Chebyshev polynomials as basis function, the scheme enabled the simultaneous evaluation of solutions at step and off – step points with numerical results that demonstrated better performance over some existing methods. [2] developed an efficient numerical method for solving IVPs of second order ODEs using the method of interpolation of the combination of Chebyshev and Legendre polynomials approximate solution and collocation of differential system which was found to give a better approximation when compared with some existing methods. [3] developed a one - step block hybrid method based on third derivative for direct solution of IVPs of second order ODEs using power series approximation function, the method was interpolated at two points and collocated using both second and third derivatives at five strategically chosen off – step points. The derived method was found to be effective and efficient for solving stiff and non – stiff equations. [4] developed an efficient implicit two - step second derivative method for direct solution of second order ODEs by making an extension to the

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implicit second derivative methods for the solution of first order ODEs in [5] and [6], to mention a few. [7] listed some researchers that developed different block methods using the approach of interpolation and collocation for solving (1.2). Improving on the studies in [7] and advancing the field, this study developed an efficient and effective Implicit two – step third derivative (k=2, l=3) method tailored towards direct solution of IVPs of third order ODEs, using predictor - corrector approach that utilized Taylor's series expansion as the basis function. The development of the method was found to be easier than block method and processed better accuracy than those in [7].

2. Methodology

2.1. Derivation of the Method

According to [8], Obrechkoff's Multiderivative Linear Multistep Method of the form;

$$\sum_{j=0}^{k} \alpha_j\, y_{n+j} = \sum_{i=1}^{l} h^i \sum_{j=0}^{k} \beta_{ij}\; {y^i}_{n+j}\,, \quad \alpha_k = +1 \qquad (2.1)$$

has local truncation error:

$$T_{n+k} = \sum_{j=0}^{k} \alpha_j y_{n+j} - \sum_{i=1}^{l} h^i \sum_{j=0}^{k} \beta_{ij} {y^i}_{n+j}\ ;\ \alpha_k = +1 \qquad (2.2)$$

(where *l* is the order of derivative and *k* is the step - number),

Taylor series expansion of the dependent variable, that is; ${y^i}_{n+j};\ i = 0(1)l \ and\ j = 0(1)k$ given as;

$${y^i}_{n+j} = \sum_{r=0}^{\infty} \frac{(jh)^r y^{r+i}}{r!}, \qquad (2.3)$$

was adopted. Accuracy of order P was imposed on the local truncation error T_{n+k} and the resulting equations were solved for parameters α_{js} and $\beta_{i,js}$ to generate the required scheme. Order of accuracy and error constant of the scheme was determined by substituting the results of parameters α_{js} and $\beta_{i,js}$ into the original equation. The scheme was implemented by using it to solve some sampled non - linear IVPs of third order ODEs.

Setting *k=2, l=3* in (2.1) gives

$$\sum_{j=0}^{2} \alpha_j\, y_{n+j} = \sum_{i=1}^{3} h^i \sum_{j=0}^{2} \beta_{ij}\; {y^i}_{n+j}\,, \quad \alpha_k = +1 \qquad (2.4)$$

that is

$$\begin{aligned} &\alpha_0 y_n + \alpha_1 y_{n+1} + \alpha_2 y_{n+2} \\ &= h(\beta_{10} y'_n + \beta_{11} y'_{n+1} + \beta_{12} y'_{n+2}) \\ &+ h^2(\beta_{20} y''_n + \beta_{21} y''_{n+1} + \beta_{22} y''_{n+2}) \\ &+ h^3(\beta_{30} y'''_n + \beta_{31} y'''_{n+1} + \beta_{32} y'''_{n+2}). \end{aligned} \qquad (2.5)$$

with local truncation error

$$\begin{aligned} T_{n+2} &= \alpha_0 y_n + \alpha_1 y_{n+1} + \alpha_2 y_{n+2} \\ &- h({\beta_{10}}' + \beta_{11} y'_{n+1} + \beta_{12} y'_{n+2}) \\ &- h^2 h^2(\beta_{20} y''_n + \beta_{21} y''_{n+1} + \beta_{22} y''_{n+2}) \\ &- h^3(\beta_{30} y'''_n + \beta_{31} y'''_{n+1} + \beta_{32} y'''_{n+2}). \end{aligned} \qquad (2.6)$$

adopting Taylor's series expansion of y_{n+1}, y_{n+2}, y'_{n+1}, y'_{n+2}, y''_{n+1}, y''_{n+2}, y'''_{n+1} and y'''_{n+2} in (2.6) and combining terms in equal powers of h gives

$$T_{n+2} = C_0 y_n + C_1 h y'_n + C_2 h^2 y''_n + C_3 h^3 y''_n + \cdots + 0(h^{11}) \qquad (2.7)$$

where

$$\begin{aligned}
C_0 &= \alpha_0 + \alpha_1 + \alpha_2 \\
C_1 &= \alpha_1 + 2\alpha_2 - \beta_{10} - \beta_{11} - \beta_{12} \\
C_2 &= \frac{\alpha_1}{2} + 2\alpha_2 - \beta_{11} - 2\beta_{12} - \beta_{20} - \beta_{22} \\
C_3 &= \frac{\alpha_1}{3!} + \frac{4\alpha_2}{3} - \tfrac{1}{2}\beta_{11} - 2\beta_{12} - \beta_{21} - 2\beta_{22} - \beta_{30} - \beta_{31} - \beta_{32} \\
C_4 &= \frac{\alpha_1}{4!} + \frac{2\alpha_2}{3} - \tfrac{1}{3!}\beta_{11} - \frac{4}{3}\beta_{12} - \tfrac{1}{2}\beta_{21} - 2\beta_{22} - \beta_{31} - 2\beta_{32} \\
C_5 &= \frac{\alpha_1}{5!} + \frac{4\alpha_2}{15} - \tfrac{1}{4!}\beta_{11} - \frac{2}{3}\beta_{12} - \tfrac{1}{3!}\beta_{21} - \frac{4}{3}\beta_{22} - \tfrac{1}{2}\beta_{31} - 2\beta_{32} \\
C_6 &= \frac{\alpha_1}{6!} + \frac{4\alpha_2}{45} - \tfrac{1}{5!}\beta_{11} - \frac{4}{15}\beta_{12} - \tfrac{1}{4!}\beta_{21} - \frac{2}{3}\beta_{22} - \tfrac{1}{3!}\beta_{31} - \frac{4}{3}\beta_{32} \\
C_7 &= \frac{\alpha_1}{7!} + \frac{4\alpha_2}{315} - \tfrac{1}{6!}\beta_{11} - \frac{4}{45}\beta_{12} - \tfrac{1}{5!}\beta_{21} - \frac{4}{15}\beta_{22} - \tfrac{1}{4!}\beta_{31} - \frac{2}{3}\beta_{32} \\
C_8 &= \frac{\alpha_1}{8!} + \frac{2\alpha_2}{315} - \tfrac{1}{7!}\beta_{11} - \frac{8}{315}\beta_{12} - \tfrac{1}{6!}\beta_{21} - \frac{4}{45}\beta_{22} - \tfrac{1}{5!}\beta_{31} - \frac{4}{45}\beta_{32} \\
C_9 &= \frac{\alpha_1}{9!} + \frac{4\alpha_2}{2835} - \tfrac{1}{8!}\beta_{11} - \frac{2}{315}\beta_{12} - \tfrac{1}{7!}\beta_{21} - \frac{8}{315}\beta_{22} - \tfrac{1}{6!}\beta_{31} - \frac{4}{45}\beta_{32} \\
C_{10} &= \frac{\alpha_1}{10!} + \frac{4\alpha_2}{14175} - \tfrac{1}{9!}\beta_{11} - \frac{4}{2835}\beta_{12} - \tfrac{1}{8!}\beta_{21} - \frac{4}{315}\beta_{22} - \tfrac{1}{7!}\beta_{31} - \frac{8}{315}\beta_{32}
\end{aligned} \qquad (2.8)$$

imposing accuracy of order 10 on T_{n+2} to have

$C_0 = C_1 = C_2 = \ldots = C_9 = C_{10} = 0$ and $T_{n+2} = 0(h^{11})$

solving gives;

$$\alpha_0 = -1, \alpha_1 = 0, \beta_{10} = \tfrac{41}{105}, \beta_{11} = \tfrac{128}{105},$$
$$\beta_{12} = \tfrac{41}{105}, \beta_{11} = \tfrac{142}{35}, \beta_{21} = 0, \beta_{22} = \tfrac{-2}{35},$$
$$\beta_{30} = \tfrac{2}{35}, \beta_{31} = \tfrac{16}{315}, \beta_{32} = \tfrac{1}{315}$$

putting these values into equation (2.5) gives two – step, third derivative method of the form:

$$\begin{aligned} y_{n+2} &= y_n + \frac{h}{105}(41y'_n + 128y'_{n+1} + 41y'_{n+2}) \\ &+ \frac{h^2}{35}(142y''_n - 2y''_{n+2}) + \frac{h^3}{315}(18y'''_n + 16y'''_{n+1} + y'''_{n+2}) \end{aligned} \qquad (2.9)$$

which resolves into:

$$\begin{aligned} y'''_{n+2} &= \frac{315}{h^2}(2y_{n+2} - y_{n)} \\ &- \frac{3}{h^2}(41y'_n + 128y'_{n+1} + 41y'_{n+2}) \\ &- \frac{9}{h}(142y''_n - 2y''_{n+2}) - (18y'''_n + 16y'''_{n+1}) \end{aligned} \qquad (2.10)$$

2.2. Verification of Basic Properties

2.2.1. Accuracy

According to [9], a numerical method is accurate if and only if its result tends to the exact solution and order P of the method is greater than or equal to one ($P \geq 1$). A linear multistep method (L.M.M) is said to be of order P, if the order of the local truncation error T_{n+k} is P, that is, the method is of order P if $C_0 = C_1 = C_2 = C_3 = \cdots = C_p = 0$ *and* $C_{p+1} \neq 0$.

For this two – step, third derivative method $(k = 2, l = 3)$ with

$\alpha_0 = -1, \alpha_1 = 0, \beta_{10} = \frac{41}{105}, \beta_{11} = \frac{128}{105}$,

$\beta_{12} = \frac{41}{105}, \beta_{20} = \frac{142}{35}, \beta_{21} = 0, \beta_{22} = \frac{-2}{35}$,

$\beta_{30} = \frac{2}{35}, \beta_{31} = \frac{16}{315}, \beta_{32} = \frac{1}{315}$.

Substituting the values of these parameters in equation (2.8) gives

$C_0 = C_1 = C_2 = C_3 = C_4 = C_5 = C_6 = C_7 = 0$ and $C_8 \neq 0$.

Since the method is a two - step method i.e. (k=2) then $C_{p+2} = C_8$ thus $P = 6$ with error constant $C_8 = \frac{-16}{1575}$.

2.2.2. Consistency

According to [10], a linear two - step method is consistent if the parameters α_{js} and β_{ijs} satisfy the following conditions:

(i) order $P \geq 1$;

(ii) $\sum_{j=0}^{k} \alpha_j = 0$;

(iii) $\sum_{j=0}^{k} j\alpha_j = \sum_{j=0}^{k} \beta_{1j}$.

(iv) $\sum_{j=0}^{k} j^2\alpha_j = 2\sum_{j=0}^{k} \beta_{2j}$.

For the two - step third derivative method $(k = 2, l = 3)$,

(i) Order P=6.

(ii) $\sum_{j=0}^{2} \alpha_j = \alpha_0 + \alpha_1 + \alpha_2$

$= -1 + 0 + 1$

$= 0$

(iii) $\sum_{j=0}^{1} j\alpha_j = 0(\alpha_0) + 1\alpha_1 + 2\alpha_2$

= 0(-1) + 1(0) + 2(1)

= 2

$\sum_{j=0}^{k} \beta_{1j} = \beta_{10} + \beta_{11} + \beta_{12}$

$= \frac{41}{105} + \frac{128}{105} + \frac{41}{105}$

= 2

(iv) $0(\alpha_0) + 1\alpha_1 + 4\alpha_2$

= 0(-1) + 1(0) + 4(1)

= 4

$2(\beta_{10} + \beta_{11} + \beta_{12})$

$= 2(\frac{41}{105} + \frac{128}{105} + \frac{41}{105})$

= 4

Having satisfied the given conditions, the method is consistent.

2.2.3. Zero-stability

According to [11], a L.M.M of the form:

$$y_{n+k} = \alpha_0 y_n + h(\beta_1 y^1{}_{n+k} + \beta_0 y^1{}_n)$$

with first characteristic polynomial

$$\rho(r) = r^{n+k} - \alpha_0 r^n$$

is said to be zero-stable if the root of the first characteristic polynomial $\rho(r)$ has modulus less than or equal to 1; $(|r| \leq 1)$ and must be simple.

For the two – step, third derivative method $(k = 2, l = 3)$ of the form:

$$y_{n+2} = y_n + \frac{h}{105}(41y'_n + 128y'_{n+1} + 41y'_{n+2})$$
$$+ \frac{h^2}{35}(142y''_n - 2y''_{n+2}) + \frac{h^3}{315}(18y'''_n + 16y'''_{n+1} + y'''_{n+2})$$

whose first characteristic polynomial $\rho(r)\ is$

$$r^{n+2} - r^n = 0$$
$$r^n(r^2 - 1) = 0$$

and its roots are $r = 0, r = -1\ or\ r = 1$ [roots are within a unit circle and simple]

Hence, the method is zero-stable.

2.2.4. Convergence

According to [12], a necessary and sufficient condition for a L.M.M to be convergent is that it must be consistent and zero-stable.

Since this method is consistent and zero-stable, then it is convergent.

3. Implementation

The developed method was used to solve three non - linear IVPs of third order ODEs with exact solutions to test its effectiveness and accuracy. Numerical result of Problems 1 was compared with the result of 4PHBM in [13] while the result of Problem 2 was compared with S2S4PHM in [7]. The results are as presented in Tables 1 - 3.

Problem 1

$y''' = 3sinx, y(0) = 1, y^1(0) = 0, y''(0) = -2, h = 0.1$

Exact solution: $y(x) = 3cosx + \frac{x^2}{2} - 2$

Source: [13] and [14]

Problem 2

$y''' = -\exp(x) = 0, y(0) = 1, y^1(0) = -1, y''(0) = 3,$ $h = 0.1$

Exact solution: $y(x) = 2x^2 - \exp(x) + 2$

Source: [7]

Problem 3

$y''' = -y', y(0) = 0, y'(0) = 1, y''(0) = 2, h = 0.1$

Exact solution: $y(x) = 2(1 - \cos x) + sinx$

Table 1. Numerical result for Problem 1 using the new method in relation with the method in [13]

x	y_{exact}	$y_{computed}$	Error in [13]	Errors in $y_{computed}$
0.1	0.99001249583407729110	0.9900124958340773021	6.11595 x 10^{-16}	1.01 x 10^{-17}
0.2	0.96019973352372485952	0.9601997335237248968	3.91441 x 10^{-15}	3.73 x 10^{-17}
0.3	0.91100946737681797886	0.9110094673768180921	3.46333 x 10^{-14}	1.13 x 10^{-16}
0.4	0.84318298200865510264	0.8431829820086552634	1.09373 x 10^{-13}	1.61 x 10^{-16}
0.5	0.75774768567111792351	0.7577476856711182216	2.89056 x 10^{-13}	2.98 x 10^{-16}
0.6	0.65600684472903457111	0.6560068447290350285	6.40742 x 10^{-13}	4.57 x 10^{-16}
0.7	0.53952656185346484563	0.5395265618534655771	6.24976 x 10^{-12}	7.31 x 10^{-16}
0.8	0.41012012804149570040	0.4101201280414968234	2.18703 x 10^{-12}	1.12 x 10^{-15}
0.9	0.26982990481199266729	0.2698299048119940135	3.60282 x 10^{-12}	1.35 x 10^{-15}
1.0	0.12090691760441829866	0.1209069176044200317	5.57843 x 10^{-12}	1.73 x 10^{-15}

Table 2. Numerical result for Problem 2 using the new method in relation with the method in [7]

x	y_{exact}	$y_{computed}$	Error in [7]	Errors in $y_{computed}$
0.1	0.91482908192435237812	0.9148290819243523766	1.82410 x 10^{-13}	1.52 x 10^{-16}
0.2	0.85859724183983017986	0.8585972418398301942	1.67078 x 10^{-12}	1.44 x 10^{-15}
0.3	0.83014119242399692860	0.8301411924239969584	6.00142 x 10^{-12}	2.98 x 10^{-15}
0.4	0.82817530235872974149	0.8281753023587298225	1.48598 x 10^{-11}	8.10 x 10^{-15}
0.5	0.85127872929987194629	0.8512787292998720891	3.01205 x 10^{-11}	1.43 x 10^{-14}
0.6	0.89788119960949116362	0.8978811996094913151	5.38418 x 10^{-11}	1.51 x 10^{-14}
0.7	0.96624729252952367378	0.9662472925295238715	8.83157 x 10^{-11}	1.98 x 10^{-14}
0.8	1.05445907150753265934	1.0544590715075329212	1.36060 x 10^{-10}	2.62 x 10^{-14}
0.9	1.16039688884305067883	1.1603968888430510344	1.99870 x 10^{-10}	3.56 x 10^{-14}
1.0	1.28171817154095520294	1.2817181715409556882	2.82814 x 10^{-10}	4.85 x 10^{-14}

Table 3. Numerical solution for Problem 3 using the new method

x	Exact solution	Computed Solution	Errors
0.1	0.10982508609077662447	0.10982508609077662903	3.57 x 10^{-16}
0.2	0.23853617511257797365	0.23853617511257799511	1.15 x 10^{-16}
0.3	0.38484722841012758397	0.38484722841012763453	2.06 x 10^{-16}
0.4	0.54729635430288041324	0.54729635430288050476	9.15 x 10^{-15}
0.5	0.72426041482345770065	0.72426041482345783989	6.39 x 10^{-15}
0.6	0.91397124357567894718	0.91397124357567914087	1.94 x 10^{-15}
0.7	1.11453331266871444341	1.11453331266871469777	2.54 x 10^{-15}
0.8	1.32394267220519222514	1.32394267220519254573	2.21 x 10^{-14}
0.9	1.54010697308615484496	1.54010697308615523290	5.88 x 10^{-14}
1.0	1.76086637307161750441	1.76086637307161795860	5.54 x 10^{-14}

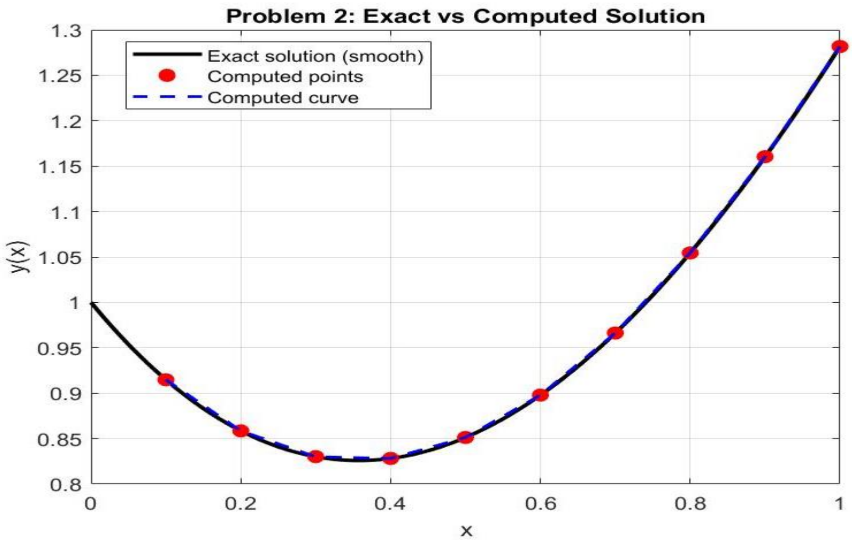

Figure 1. Graph of Numerical result and Exact solution for Problem 1

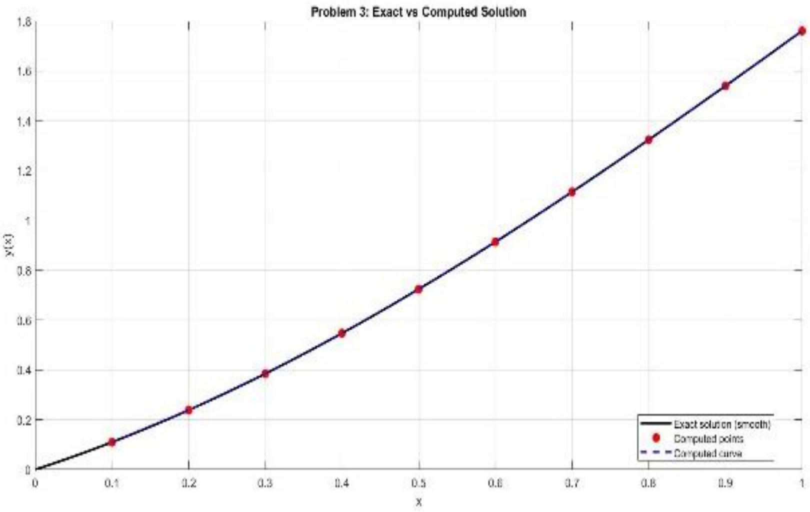

Figure 2. Graph of Numerical result and Exact solution for Problem 2

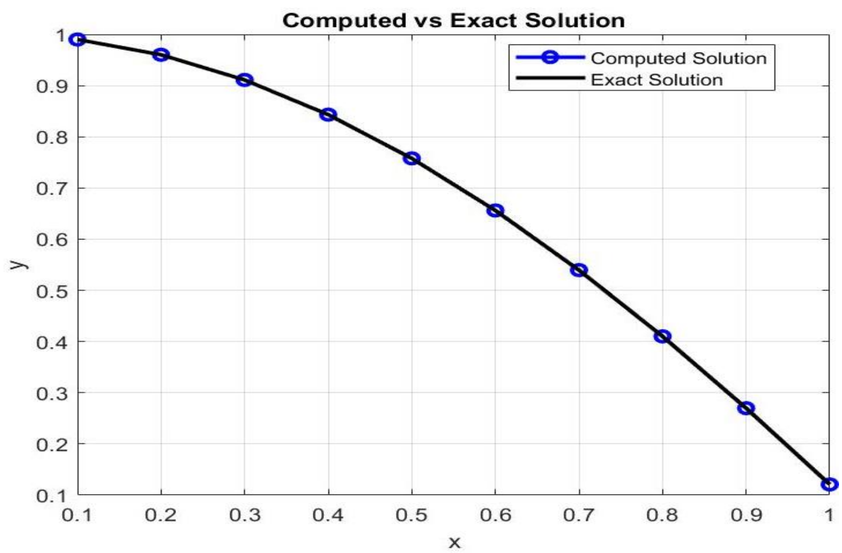

Figure 3. Graph of Numerical result and Exact solution for Problem 3

4. Discussion of Results

Analysis of the basic properties showed that the derived method has accuracy of order 6 with error constant of $\frac{-16}{1575}$. The method was verified to be consistent, zero – stable and convergent. The results obtained in Tables 1 – 3 and the graphical representation in Figures 1 – 3 confirmed that the two – step third derivative method has high accuracy when compared with exact solution. The table of results also showed that the new method performed better in terms of accuracy and convergence than the methods in [7], [13] and [14] which are of order 6, 6 and 7 respectively.

5. Conclusions

This study has produced an accurate, consistent, zero – stable and convergent implicit two – step third derivative method which is capable of direct solution of IVPs of third order ODEs. Taylors series expansion was used as the basis function for the approximate solution to the given problems. The method was implemented on some sampled non – linear test problems whose numerical results when compared with exact solution and numerical results from some existing block methods revealed that the developed method performed more favorably in terms of accuracy, stability and convergence hence, this method is recommended for direct solution of IVPs of third order ODEs.

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