# Blood Flow through a Sine-Shaped Atherosclerotic Aorta with a Source Wall Temperature and Magnetic Field

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**Abstract**  This research is focused on investigating the blood flow through a sine-shaped atherosclerotic aorta with a source wall temperature and magnetic field. In the study we formulate models for the problem, scaled the governing models to be dimensionless, and reduced the coupled system to ordinary differential equations using perturbation method with an oscillation. The resulting ODEs are solved directly for velocity and temperature profiles with some pertinent parameters obtained. Mathematica codes were developed to simulate each of the functions to investigate the influence of the pertinent parameters such as radiation parameter, Hartmann number, womersley number, the treatment parameter, the wall temperature and the height of stenosis on the blood flow through an aorta that has lost its elasticity assuming the radius of the aorta as $R_0 = 1\,\text{mm}$. Conclusively, we noticed that the blood velocity increases for the increase in Darcy number and treatment parameter but reversed for the increase of wall temperature, Hartmann number, womersley number, height of stenosis and oscillatory frequency parameter. The temperature profile decreases with the increase of radiation parameter, oscillatory parameter, height of stenosis and womersley number, while it increases with the increase of wall temperature and treatment parameters. Finally, the flow rate, shear stress and rate of heat transfers are also increased and decreased with the increase of radiation parameter, treatment parameter, womersley number, Hartmann number, height of stenosis and the oscillatory parameter, and this work is recommended for clinicians and other scientists/mathematicians alike who are interested in investigating blood flow through the circulation system.

**Keywords**  Atherosclerosis, Aorta, Blood, Hartmann number, Magnetic field, Plaque, Source wall

## 1. Background Introduction

Atherosclerosis of the aorta means that a material called plaque (fat and calcium) build up in the inside wall of a large blood vessel called the aorta. This plaque build-up is sometimes called "hardening (sclerosis) of the arteries."

The aorta is the main artery that sends oxygenated blood from the heart out to the body and to the brain through the carotid artery. It runs from the heart down through to the stomach. When plaque builds up, it can cause some serious problems such as: the plaque can weaken the wall of the aorta, causes stretch or tear, pieces of the plaque can break open, which causes a blood clot to form. A blood clot or a piece of plaque can travel to other parts of your body and block or impair blood flow. The disease is sometimes symptomless and likely cause serious problems such as stroke, aortic aneurysm, aortic dissection and limb ischemia.

Atherosclerosis of the aorta can be treated with lifestyle changes and medicines that help lower the risk of serious complications. These medicines include the blood pressure medicines such as ACE (angiotensin-converting enzyme) inhibitors, ARBs (angiotensin II receptor blockers), and beta-blockers, cholesterol medicines, such as statin and other medicines to lower cholesterol and medicine that prevents blood clots, such as aspirin. These medicines prevent blood clots that can cause a heart attack. One may also get a CT (computed tomography) or MRI (magnetic resonance imaging) scans to check the health of your aorta. There are some few literatures and researchers who have worked on atherosclerosis and blood flow through arterial channels using different mathematical and experimental techniques are as follows:

Tang and Yang [1] studied and reported that stenosed segment of the artery may collapse because of insufficient pressure around the region of stenosis. As such it is very important to investigate the blood flow behaviour in stenosed arteries. There is a report that the shape and size of stenosis

are the major parameters that significantly affect the circulation of blood. That once atherosclerosis is noticed in an arterial segment, some issues such as increase in blood flow resistance and its associated reduction in blood flow to the tissues and organs and thus could lead to serious circulation disorder [2]. The investigation of some non-Newtonian fluid flow models in addition with power law and Carreau fluid models was carried out by Cho and Kensey [3], and the results gotten were compared with the existing experimental data. Bunonyo *et al* [4] elucidated on lipid profile effect on blood flow in an artery channel, they formulated the flow models and solved it, performed numerical simulation using Mathematica and found in their results that the length and onset of stenosis influenced the blood flow profile. Bunonyo and Amos [5] also investigated treatment and radiation effects on oscillatory blood flow through a stensoed artery. In their model the region of stenosis is infused into the flow equation with treatment and found that the treatment parameters can actually influence heat transfer level in the blood in an arterial channel. Laminar pulsatile flow of blood under the influence of magnetic field was investigated by Mandal *et al* [6]. Chang et al [7] elucidated fluid-solid structure interaction between blood and solid wall modelling of blood as power law and Carreau fluids using FDM to solve the resulted differential equations. Their investigated revealed that the stress concentration is maxima at the throat of stenosis. Akbar [8] carried out a study to investigate heat and mass transfer effects in blood flow in a tapered stenotic artery, they considered blood as Carreau fluid. Vaidya *et al* [9] investigated heat and mass transfer analysis of MHD peristaltic flow through a compliant porous channel with variable thermal conductivity, in this model examined the influence of long wavelength and small Reynolds number and impact of thermal variable conductions and the numerical simulation were done using MATLAB. Shit and Amal [10] formulated a mathematical model to verify the role of magnetic field on blood flow and its impact on thermal behaviour of biological tissue for tumour treatment. In their study, numerical computation was performed to study the effect of static magnetic field on thermal behaviour of the tumour surrounded by living biological tissues and blood vessels and they found that the application of magnetic field increases the heat transfer rate within the tumour tissues which in turn attribute to an enhancement of temperature of about 316K or above for hyperthermia treatment of cancer therapy. Shrivastava *et al* [11] also examined thermal equilibrium of blood and its surrounding tissue by considering different blood vessels diameter. Pennes [12] proposed that the exchange of energy from blood tissue is proportional to the temperature difference between the blood vessel and tissue. Jimoh *et al* [13] studied the haematocrit and slip velocity influence on third grade blood flow and heat transfer through a stenosed artery theoretically. They considered geometry of the stenosis and put into account Galerkin weighted residual and Newton Raphson methods to

solve the resulted equations that govern the flow and found that increase in magnetic field leads to decrease in velocity profile.

In this present research, the investigation of blood flow through a sine-shaped atherosclerotic plaque with a source wall temperature and magnetic field. The research focuses on mathematical formulation of the problem subject to some realistic boundary condition, consideration of the treatment of cholesterol in the aorta in aiding blood flow in the system. The geometry of the plaqued wall is follows Raju and Devanathan [14], but their work did not consider the treatments, which this investigated is focusing on, in ensuring that atherosclerosis can be treated. The research aim is to solve the models to obtain the governing functions using oscillatory perturbation technique and numerically simulate the obtained functions for velocity and temperature profiles, as well as volumetric flow rate, shear stress and the rate of heat transfer by varying the important parameters and study their impact on blood circulation.

# 2. Mathematical Formulation

Under this section, we consider blood to be Newtonian, viscous, incompressible and electrically conducting fluid, flowing through an axially symmetric direction with a velocity $w^*\left(x^*, y^*\right)$, where $x^*, y^*$ are dimensionless coordinates in axial and perpendicular directions respectively. The channel cross sectional area under investigation is assumed to have lost its elasticity due to atherosclerosis as a result of plaque, and the plaque is assumed to be in sine-shaped form detected using lumenographic technique. The flow is also considered to be parallel due to arterial valves, and in view of the aforementioned, the Navie-Stoke equation governing the flow, energy equation and geometry of atherosclerosis are presented as follows:

### 2.1. The Continuity Equation for Blood Flow

$$\frac{\partial w^*}{\partial x^*} = 0 \qquad (1)$$

### 2.2. The Momentum Equation for Blood Flow

$$\rho_b \frac{\partial w^*}{\partial t^*} = \frac{\partial P^*}{\partial x^*} + \mu_b \frac{\partial^2 w^*}{\partial y^{*2}} - \frac{\mu_b w^*}{k^*} - \sigma B_0^2 w^* + \rho_b g \beta_T \left(T_b^* - T_a\right) (2)$$

### 2.3. The Energy Equation

$$\rho_b c_b \frac{\partial T_b^*}{\partial t^*} = k_b \frac{\partial^2 T_b^*}{\partial y^{*2}} + Q_b^* \left(T_b^* - T_a\right) \qquad (3)$$

Atherosclerotic geometry:

$$y^* = R_0 + \delta^* Sin\left(\frac{2\pi}{\lambda}\left(x^* - ct^*\right)\right) \qquad (4)$$

where $x^* = \left(d_0 + \dfrac{\lambda}{2}\right)$

The above equations are subject to the following boundary conditions:

$$\left.\begin{aligned} w^* = 0, T_b^* = T_a \quad \text{at } y^* = 0 \\ w^* = 0, T_b^* = T_b \quad \text{at } y^* = R \end{aligned}\right\} \quad (5)$$

We introduce the following scaling parameters:

$$x = \frac{x^*}{\lambda}; y = \frac{y^*}{R_0}; c = f_p \lambda; t^* = \frac{1}{f_p}; w = \frac{w^*}{w_\infty}; t = t^* \omega;$$

$$Gr = \frac{g\beta_T T_a R_0^2}{w_\infty \upsilon_b}; Rd = \frac{Q_b^* R_0^2}{k_b}; Sin(x - 2\pi) = Sinx, \chi = \frac{y}{h}$$

$$Ha = \frac{\sigma B_0^2 R_0^2}{\mu_b}; Da = \frac{R_0^2}{k^*}; \theta_b = \frac{T_b^* - T_a}{T_a}; P = \frac{P^* R_0^2}{\mu_b w_\infty};$$

$$\delta^* = \frac{\delta}{R_T R_0}; \alpha^2 = \frac{\omega R_0^2}{\upsilon_b}; \upsilon_b = \frac{\mu_b}{\rho_b}; Pr = \frac{\mu_b c_b}{k_b}; \theta_{wb} = \frac{T_b}{T_a}$$

Using the scaling parameters in reducing equations (1) – (5) as follows:

$$\alpha^2 \frac{\partial w}{\partial t} = \frac{\partial P}{\partial x} + \frac{\partial^2 w}{\partial y^2} - \frac{w}{Da} - Ha^2 w + Gr\theta_b \quad (6)$$

$$\alpha^2 Pr \frac{\partial \theta_b}{\partial t} = \frac{\partial^2 \theta_b}{\partial y^2} + RdPr\theta_b \quad (7)$$

Modified Atherosclerotic geometry:

$$y(x) = 1 + \frac{\delta}{R_0 R_T} Sin 2\pi x \quad (8)$$

where $x = \dfrac{1}{\lambda}\left(d_0 + \dfrac{\lambda}{2}\right)$

The above equations are subject to the following boundary conditions:

$$\left.\begin{aligned} w = 0, \theta_b = 0 \quad \text{at } y = 0 \\ w = 0, \theta_b = \theta_w - 1 \quad \text{at } y = h \end{aligned}\right\} \quad (9)$$

## 3. Method of Solution

In order to solve the dimensionless coupled partial differential equations, we consider the oscillatory perturbation technique to reduce the equations to ODE by assuming the solution in the form:

$$\left.\begin{aligned} w(\chi,t) = w_0(\chi)e^{i\omega t} \\ \theta_b(\chi,t) = \theta_{b0}(\chi)e^{i\omega t} \end{aligned}\right\} \quad (10)$$

Substituting equation (10) into equations (6), (7) & (9), they are reduced to the following:

$$\frac{\partial^2 w_0}{\partial y^2} - \left(\frac{1}{Da} + Ha^2 + \alpha^2 i\omega\right)w_0 = -Gr\theta_{b0} \quad (11)$$

$$\frac{\partial^2 \theta_{b0}}{\partial y^2} + \left(Rd - \alpha^2 i\omega\right)Pr\theta_{b0} = 0 \quad (12)$$

Applying the transformation $\chi = \dfrac{y}{h}$ in equation (11) & (12) we have:

$$\frac{\partial^2 w_0}{\partial \chi^2} - \beta_1 w_0 = -hGr\theta_{b0} \quad (13)$$

$$\frac{\partial^2 \theta_{b0}}{\partial \chi^2} + \beta_2 \theta_{b0} = 0 \quad (14)$$

where $\beta_1 = h\left(\dfrac{1}{Da} + Ha^2 + \alpha^2 i\omega\right)$ & $\beta_2 = h\left(Rd - \alpha^2 i\omega\right)Pr$

The above equations are subject to the following boundary conditions:

$$\left.\begin{aligned} w_0 = 0, \theta_{0b} = 0 \quad \text{at } \chi = 0 \\ w_0 = 0, \theta_{0b} = (\theta_w - 1)e^{-i\omega t} \quad \text{at } \chi = 1 \end{aligned}\right\} \quad (15)$$

The general solution of equation (14) is given as:

$$\theta_{b0}(\chi) = Asin\left(\sqrt{\beta_2}\chi\right) + Bcos\left(\sqrt{\beta_2}\chi\right) \quad (16)$$

Solving for the function coefficient in equation (16) using equation (15), it is simplified as:

$$\theta_{b0}(\chi) = \left(\frac{(\theta_w - 1)e^{-i\omega t}}{sin\left(\sqrt{\beta_2}\right)}\right)sin\left(\sqrt{\beta_2}\chi\right) \quad (17)$$

Substitute equation (17) into equation (13), we have the following:

$$\frac{\partial^2 w_0}{\partial \chi^2} - \beta_1 w_0 = \beta_3 sin\left(\sqrt{\beta_2}\chi\right) \quad (18)$$

where $\beta_3 = \left(\dfrac{(1-\theta_w)hGre^{-i\omega t}}{sin\left(\sqrt{\beta_2}\right)}\right)$

The homogenous solution of equation (18) is given as:

$$w_{0h} = A_1 sinh\left(\sqrt{\beta_1}\chi\right) + B_1 cosh\left(\sqrt{\beta_1}\chi\right) \quad (19)$$

Let us assume the particular solution of equation (18) takes the form:

$$w_{0p} = A_2 sin\left(\sqrt{\beta_2}\chi\right) + B_2 cos\left(\sqrt{\beta_2}\chi\right) \quad (20)$$

Differentiate equation (20) to second order and substitute into equation (18), we have:

$$w_{0p} = \left(-\frac{\beta_3}{(\beta_2 + \beta_1)}\right)sin\left(\sqrt{\beta_2}\chi\right) \quad (21)$$

Adding equations (19) and (21), we obtained the general solution of equation (18) as:

$$w_0 = A_1 sinh\left(\sqrt{\beta_1}\chi\right) + B_1 cosh\left(\sqrt{\beta_1}\chi\right) - \frac{\beta_3}{(\beta_2 + \beta_1)} sin\left(\sqrt{\beta_2}\chi\right) \qquad (22)$$

Solve for the function coefficients of equation (22) using equation (15) as follows:

$$w_0 = \left(\frac{\beta_3}{(\beta_2 + \beta_1)} \frac{sin\left(\sqrt{\beta_2}\right)}{sinh\left(\sqrt{\beta_1}\right)}\right) sinh\left(\sqrt{\beta_1}\chi\right) - \frac{\beta_3}{(\beta_2 + \beta_1)} sin\left(\sqrt{\beta_2}\chi\right) \qquad (23)$$

We can now substitute equation (23) & (17) into equation (10), we obtained the velocity and temperature profiles as following:

$$\theta_b(\chi) = \left(\frac{(\theta_w - 1)}{sin\left(\sqrt{\beta_2}\right)}\right) sin\left(\sqrt{\beta_2}\chi\right) \qquad (24)$$

$$w(\chi,t) = \left[\left(\frac{\beta_3}{(\beta_2 + \beta_1)} \frac{sin\left(\sqrt{\beta_2}\right)}{sinh\left(\sqrt{\beta_1}\right)}\right) sinh\left(\sqrt{\beta_1}\chi\right) - \frac{\beta_3}{(\beta_2 + \beta_1)} sin\left(\sqrt{\beta_2}\chi\right)\right] e^{i\omega t} \qquad (25)$$

The volumetric blood flow rate is mathematically stated as:

$$Q(x,t) = h\left[\begin{array}{l} \frac{1}{\sqrt{\beta_1}}\left(\frac{\beta_3}{(\beta_2 + \beta_1)} \frac{sin\left(\sqrt{\beta_2}\right)}{sinh\left(\sqrt{\beta_1}\right)}\right) cosh\left(\sqrt{\beta_1}\right) + \frac{1}{\sqrt{\beta_2}} \frac{\beta_3}{(\beta_2 + \beta_1)} cos\left(\sqrt{\beta_2}\right) \\ -\frac{1}{\sqrt{\beta_1}}\left(\frac{\beta_3}{(\beta_2 + \beta_1)} \frac{sin\left(\sqrt{\beta_2}\right)}{sinh\left(\sqrt{\beta_1}\right)}\right) - \frac{1}{\sqrt{\beta_2}} \frac{\beta_3}{(\beta_2 + \beta_1)} \end{array}\right] e^{i\omega t} \qquad (26)$$

The wall shear stress is stated mathematically as:

$$\tau_w(x,t) = \frac{1}{h}\left[\left(\frac{\beta_3\sqrt{\beta_1}}{(\beta_2 + \beta_1)} \frac{sin\left(\sqrt{\beta_2}\right)}{sinh\left(\sqrt{\beta_1}\right)}\right) cosh\left(\sqrt{\beta_1}\right) - \frac{\beta_3\sqrt{\beta_2}}{(\beta_2 + \beta_1)} cos\left(\sqrt{\beta_2}\chi\right)\right] e^{i\omega t} \qquad (27)$$

The rate of heat transfer is mathematically stated as:

$$Nu(x) = \frac{1}{h}\left(\frac{(\theta_w - 1)\sqrt{\beta_2}}{sin\left(\sqrt{\beta_2}\right)}\right) cos\left(\sqrt{\beta_2}\right) \qquad (28)$$

# 4. Simulated Graphical Results

This section presents the simulated results graphical to show the influence of the various pertinent parameters on the velocity, $w(\chi,t)$ and temperature, $\theta_b(\chi)$, as well as the volumetric flow rate, $Q$, shear stress, $\tau_w$ and rate of heat transfer, $Nu$ of the blood through the channel in the axial direction to validate the efficacy of the models are: $0.05 \le Da \le 1$ , $1 \le Ha \le 7$ , $0.1 \le \theta_w \le 5$ , $0.3 \le Rd \le 0.5$ , $0.3 \le R_T \le 1, 1.2 \le \omega \le 2$ , $\alpha = 0.2, Gr = 15, R_0 = 1, \sigma = 0.5$, $\mu_b = 0.004, Pr = 21$   Shit and Amal [10], $\rho_b = 1050$ , $c_b = 14.65$ and $k_b = 0.0022$ Chato [15].

# 5. Discussion of the Graphical Results

We shall discuss the simulated graphical results from the previous section to show the importance of the various pertinent parameters on the velocity and temperature, as well as the volumetric flow rate, shear stress and rate of heat transfer of the blood through the channel in the axial direction. The discussion follows a sequence in the form of velocity results, Figure 1 – Figure 8, the blood temperature results, Figure 9 – Figure 14, volumetric flow rate, Figure 15 – Figure 20, wall shear stress, Figure 21 – Figure 27, and rate of heat transfer, Figure 28 – Figure 33.
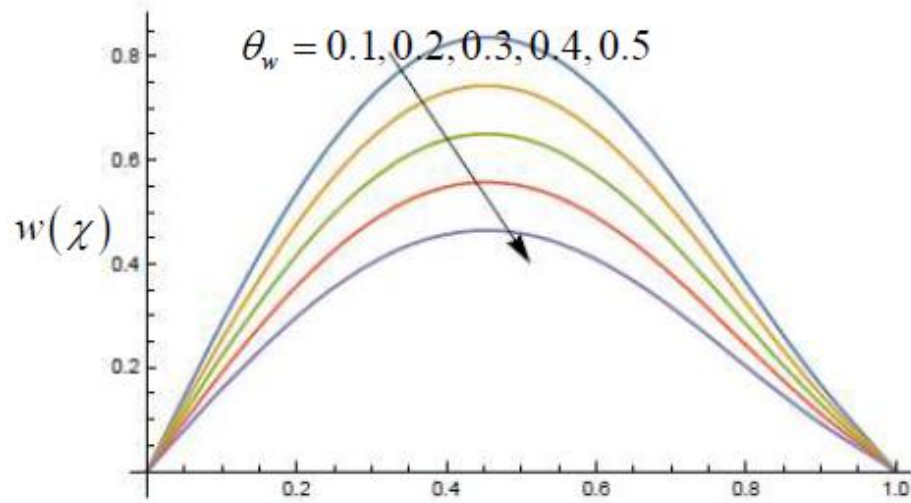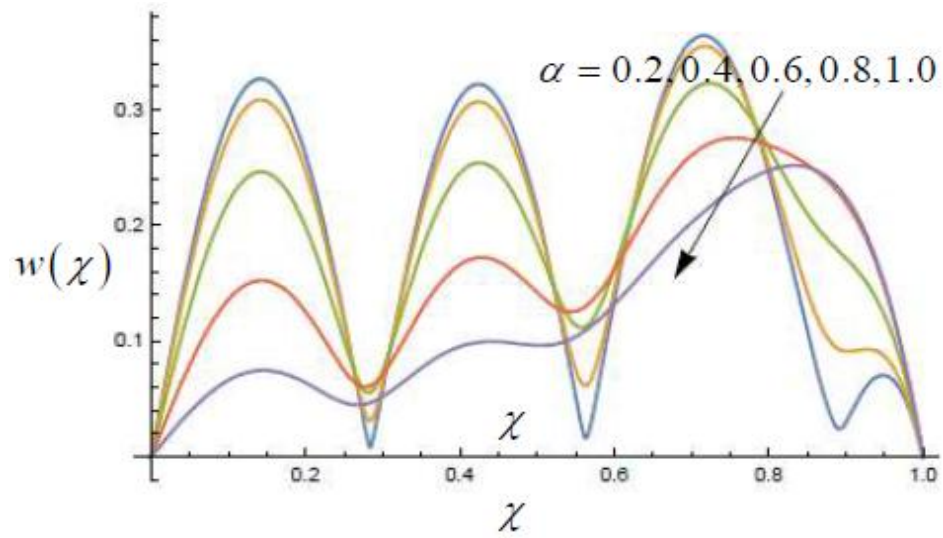
**Figure 1.** Influence of $\theta_w$ values on velocity profile
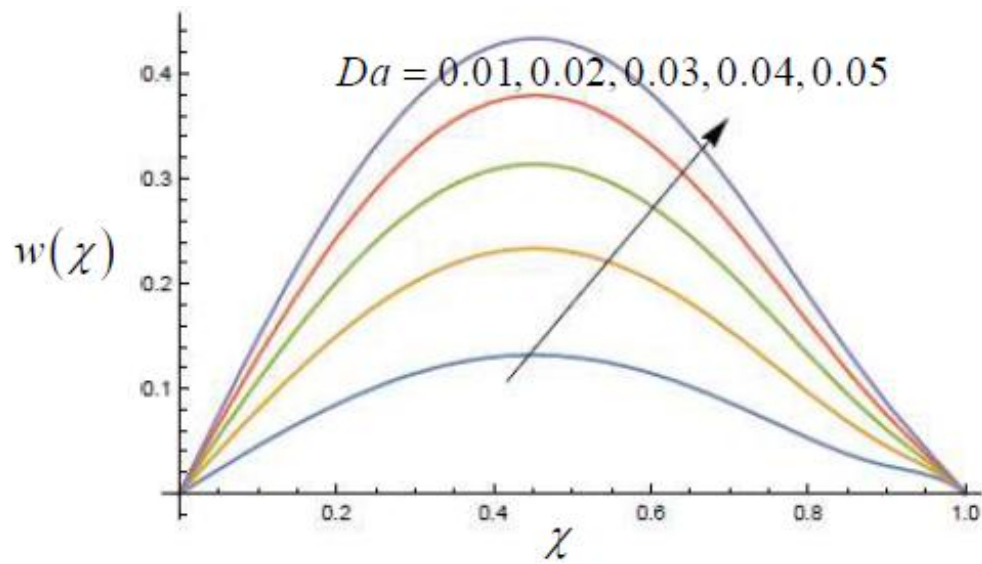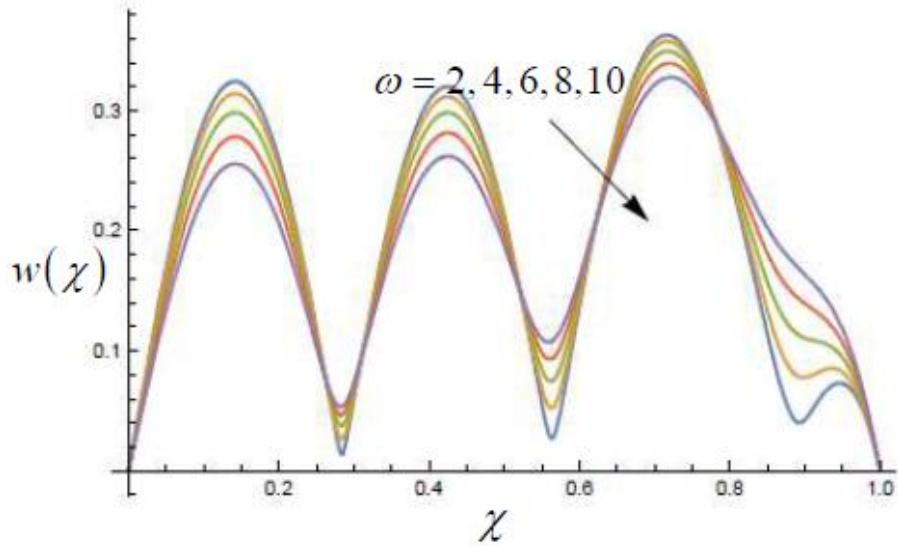


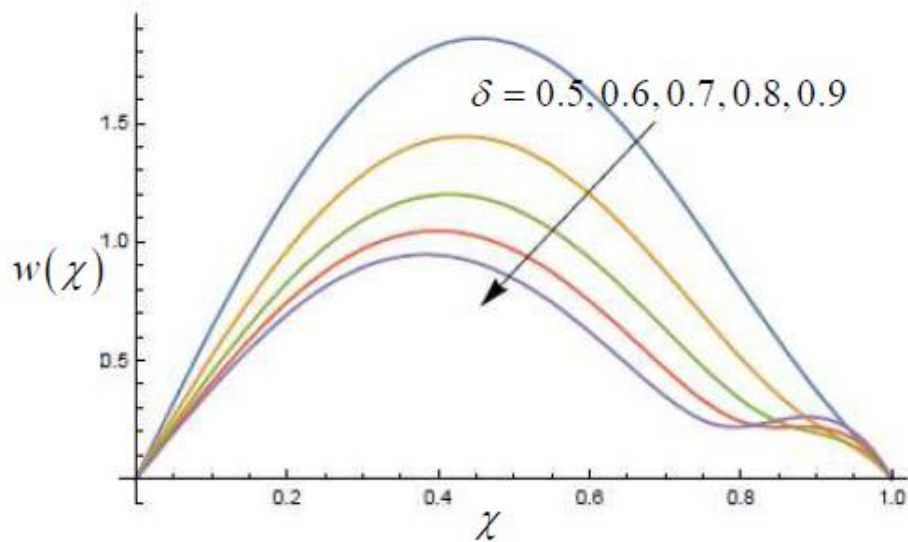**Figure 2.** Influence of $Rd$ values on velocity profile
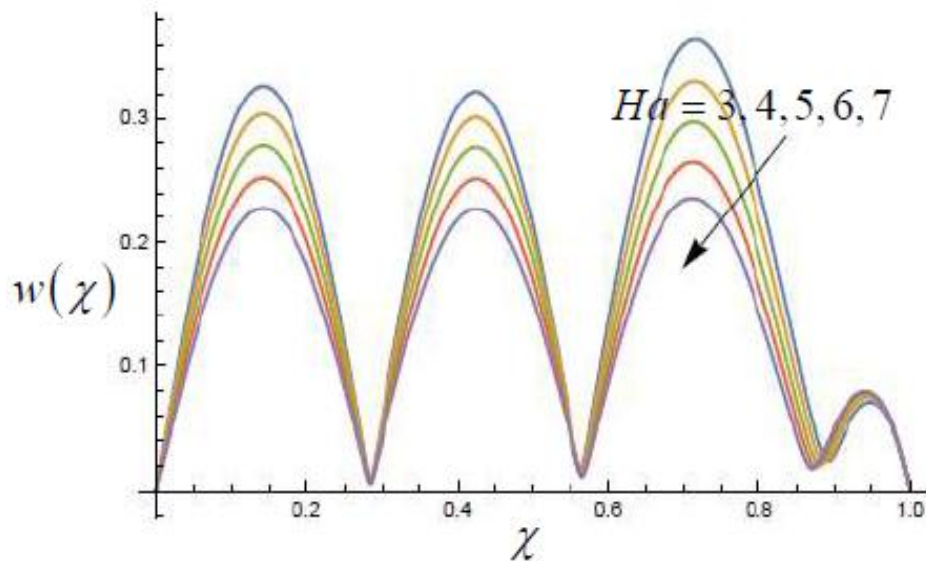


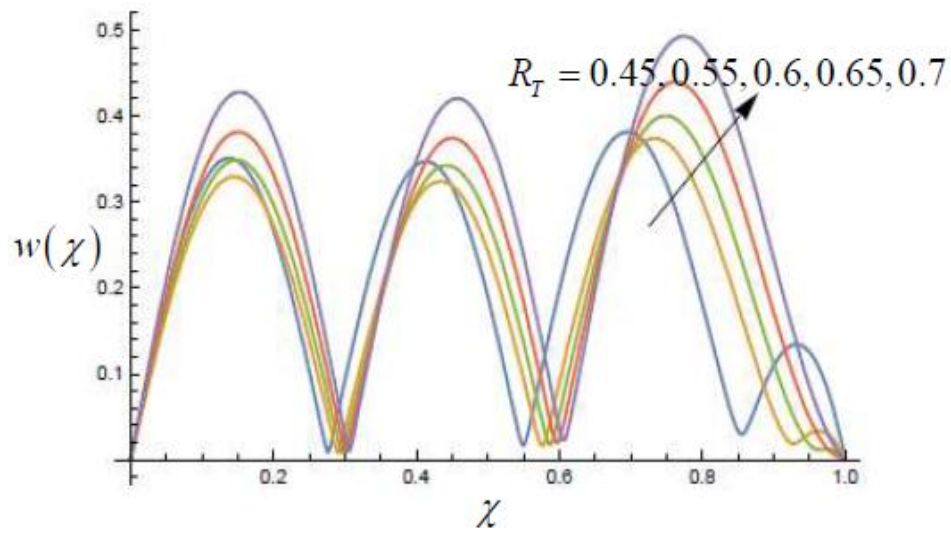**Figure 3.** Influence of $Ha$ values on velocity profile

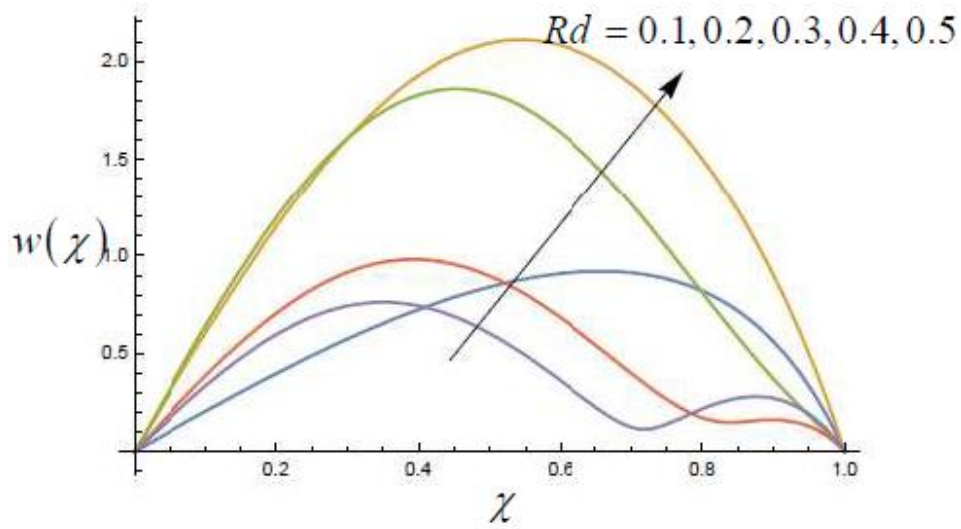**Figure 4.** Influence of $\alpha$ values on velocity profile



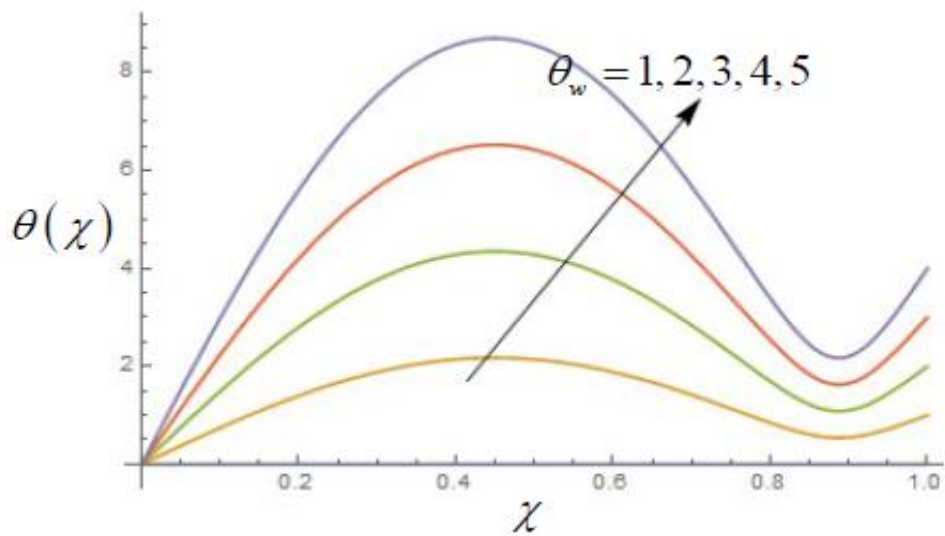**Figure 5.** Influence of $\omega$ values on velocity profile



**Figure 6.** Influence of $\delta$ values on velocity profile

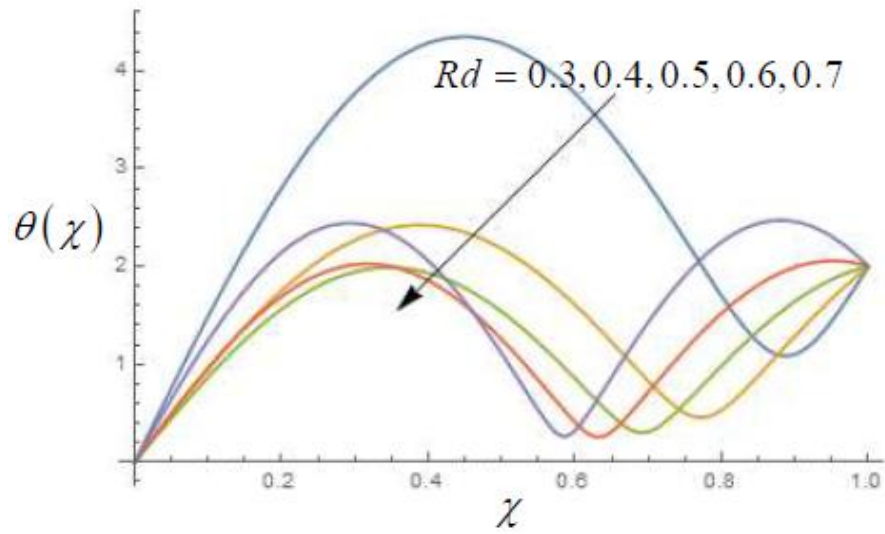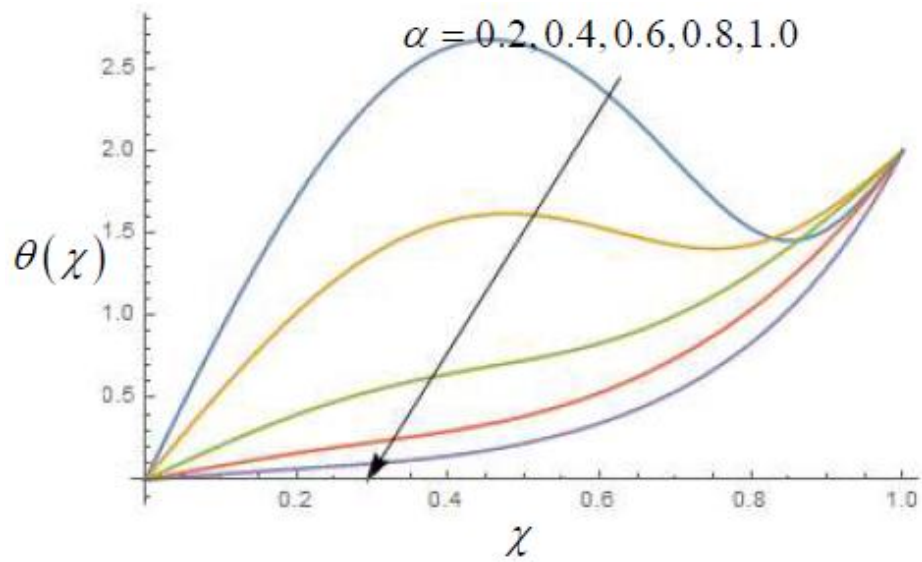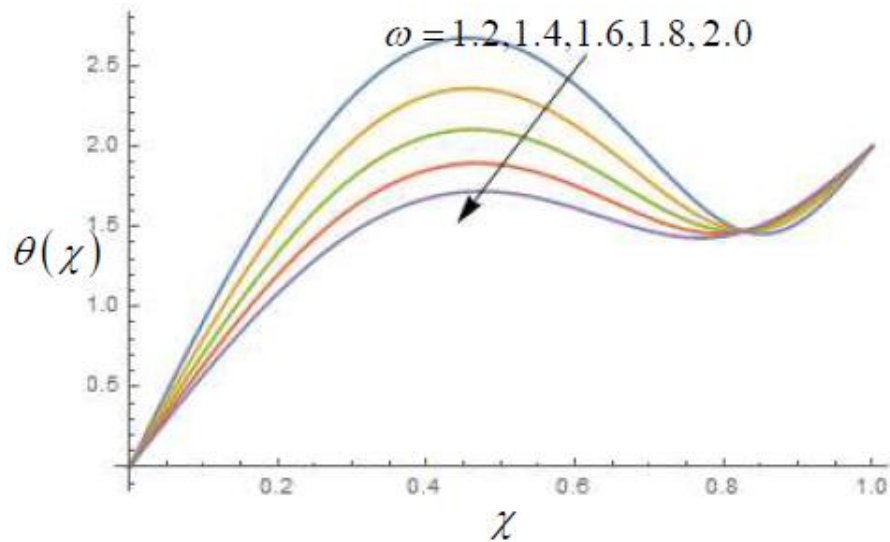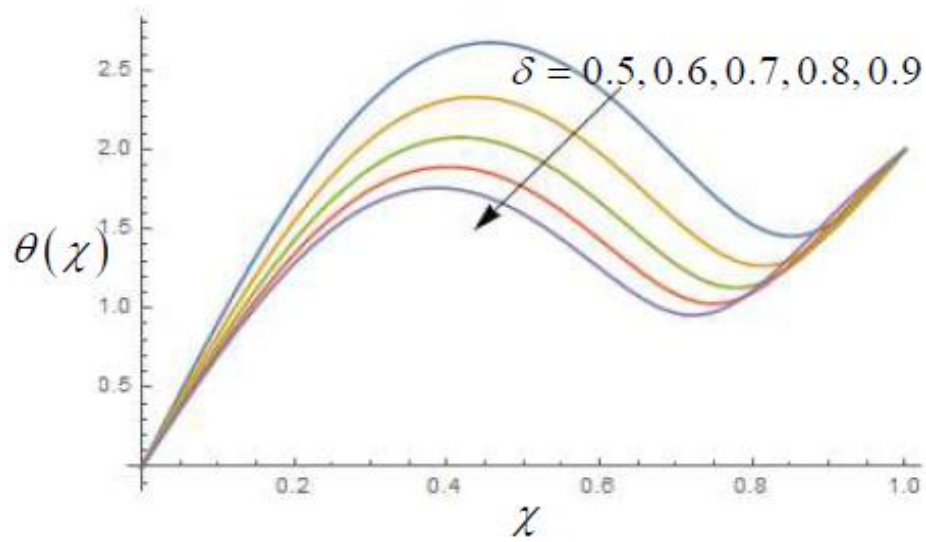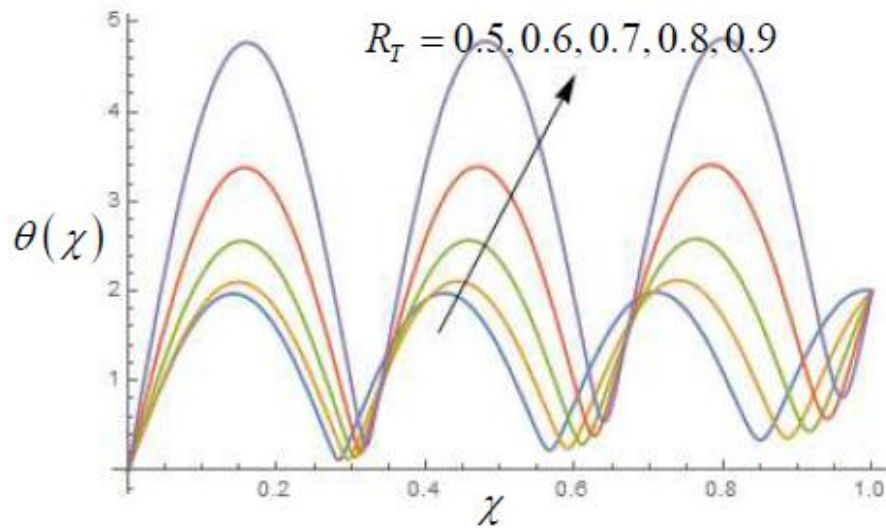**Figure 7.**   Influence of  $R_T$   values on velocity profile



**Figure 8.**   Influence of  $Rd$   values on velocity profile



**Figure 9.**   Influence of  $\theta_w$  values on temperature profile

**Figure 10.**   Influence of  $Rd$  values on temperature profile



**Figure 11.**   Influence of  $\alpha$  values on temperature profile



**Figure 12.**   Influence of  $\omega$  values on temperature profile

**Figure 13.**  Influence of $\delta$ values on temperature profile



**Figure 14.**  Influence of $R_T$ values on temperature profile
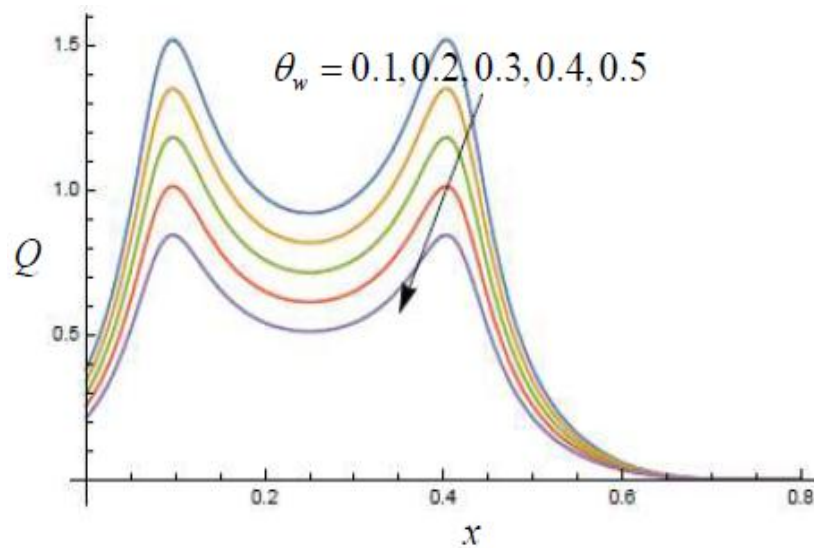


**Figure 15.**  Influence of $\theta_w$ values on flow rate along axial direction
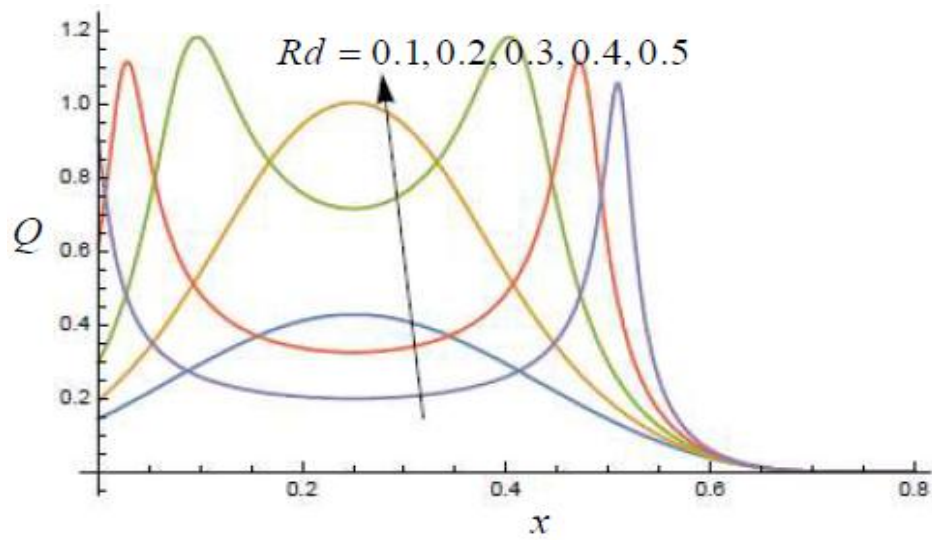
**Figure 16.**   Influence of *Rd* values on flow rate along axial direction
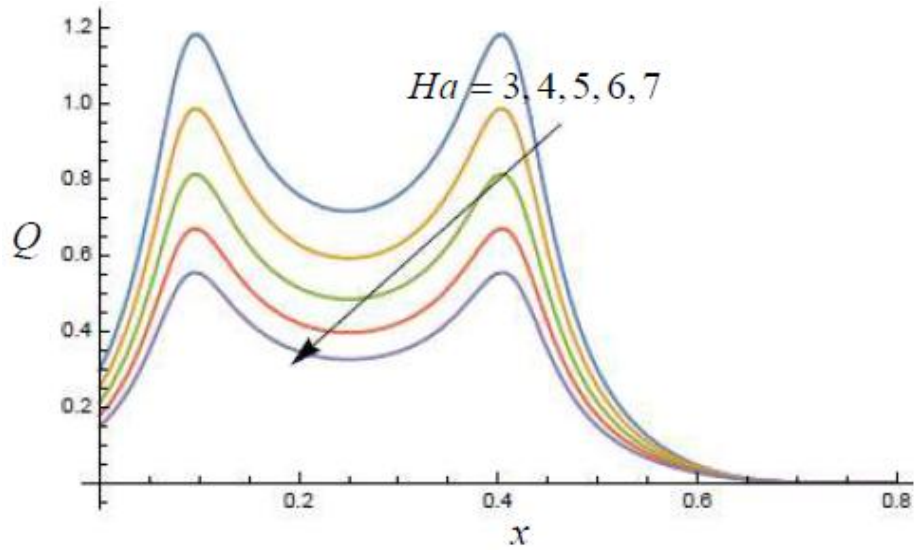


**Figure 17.**   Influence of  *Ha*  values on flow rate along axial direction
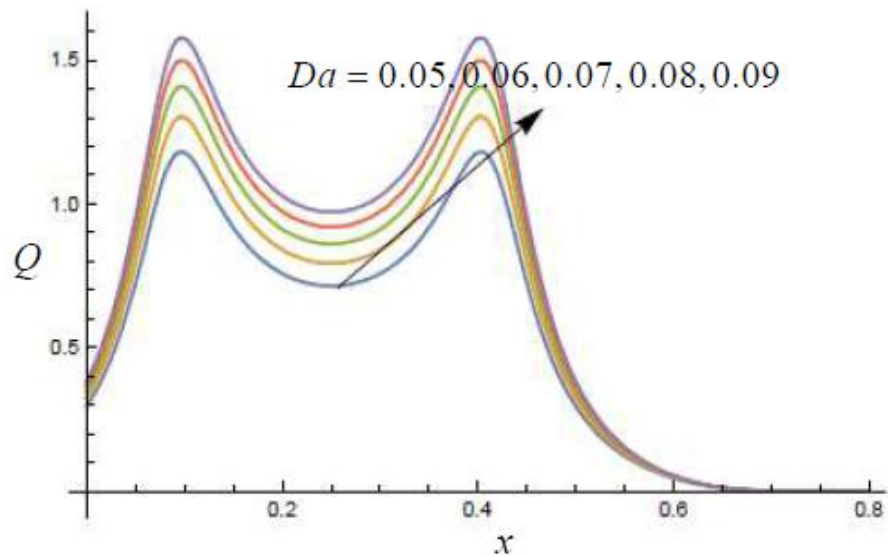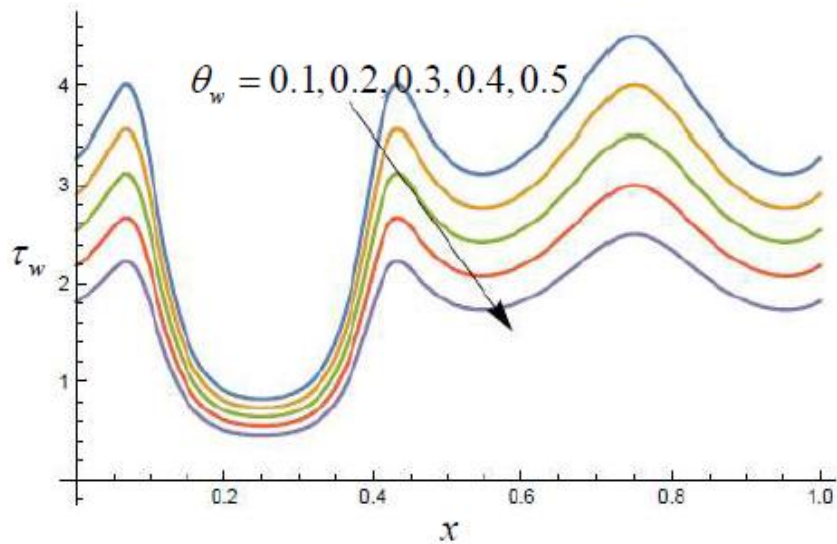


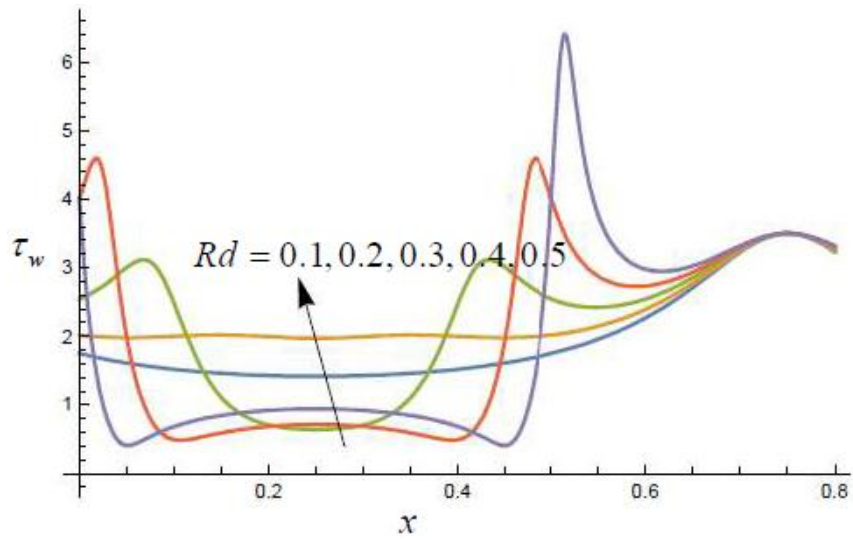**Figure 18.**   Influence of  *Da*  values on flow rate along axial direction
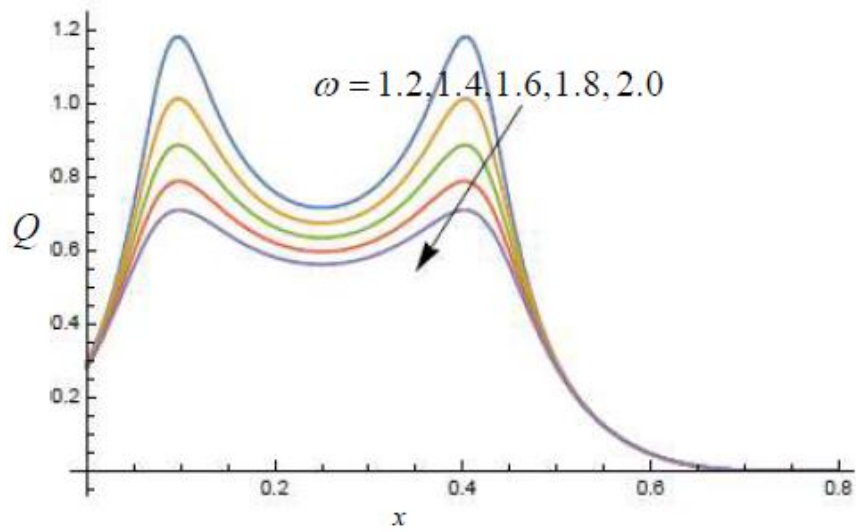
**Figure 19.** Influence of $\omega$ values on flow rate along axial direction
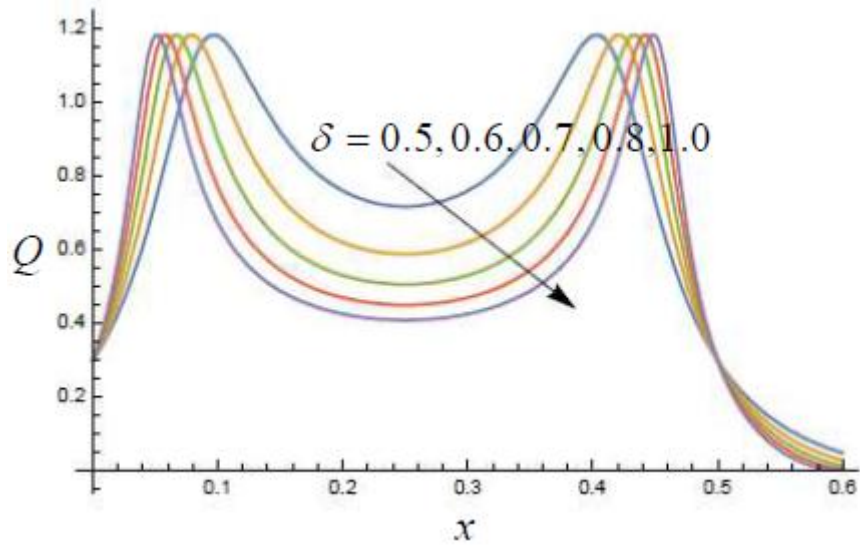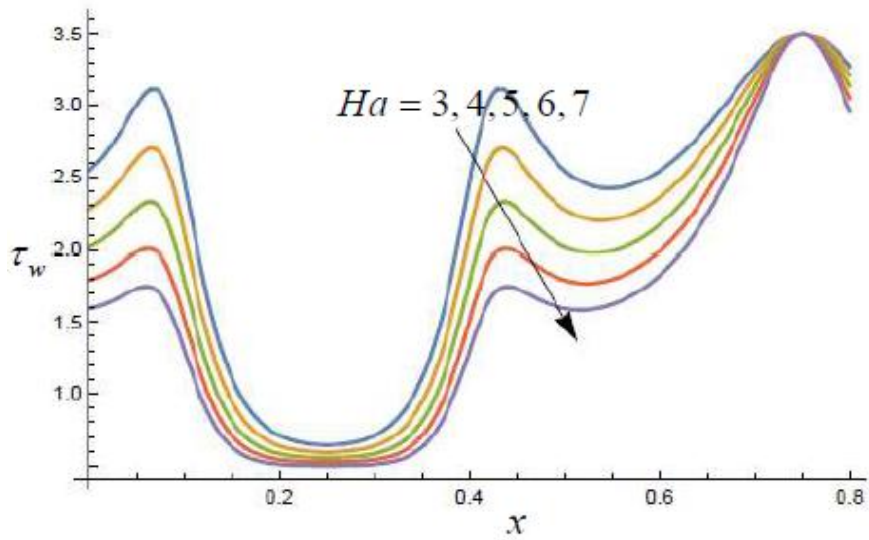


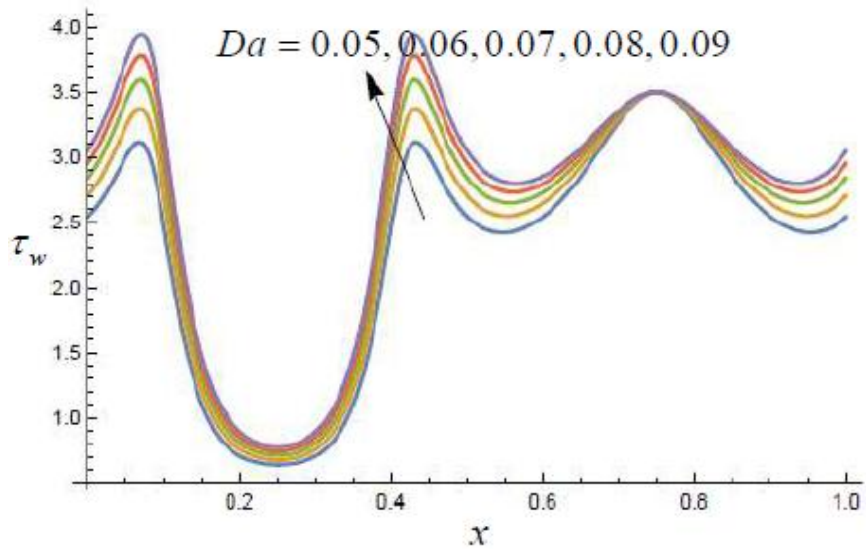**Figure 20.** Influence of $\delta$ values on flow rate along axial direction



**Figure 21.** Influence of $\theta_w$ values on shear stress along axial direction

**Figure 22.**  Influence of  $Rd$   values on shear stress along axial direction
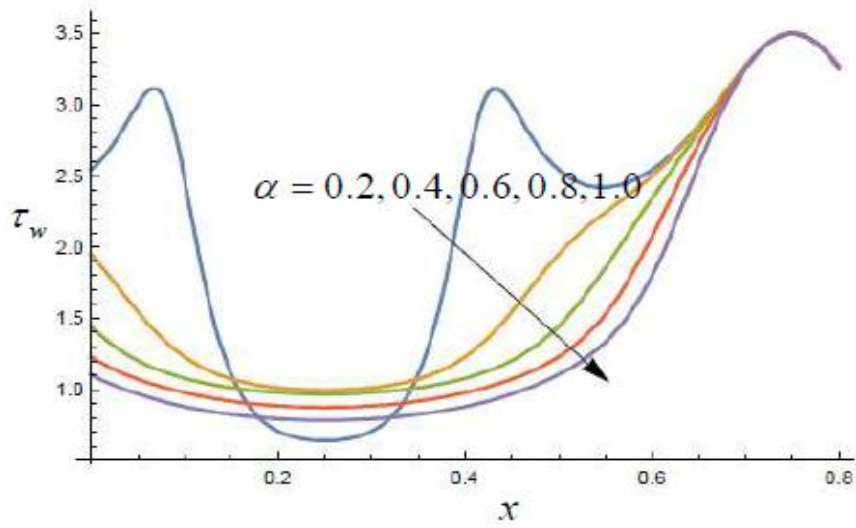


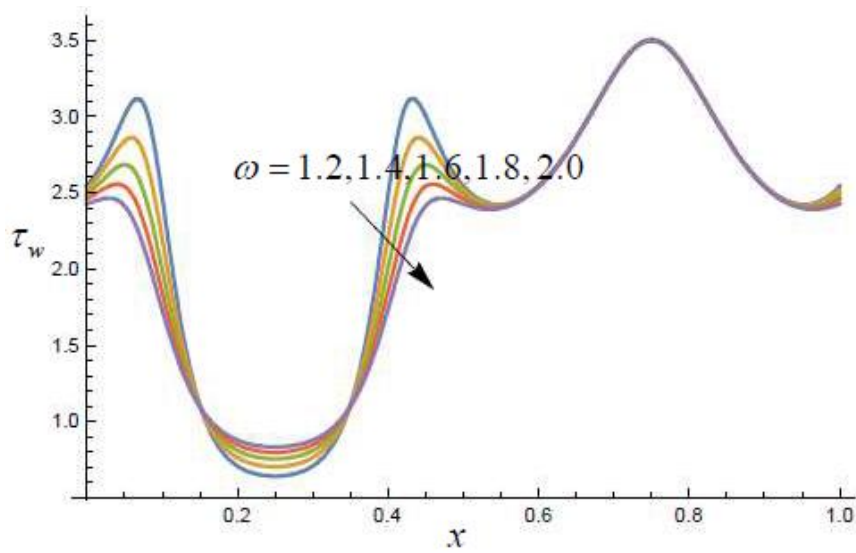**Figure 23.**  Influence of  $Ha$   values on shear stress along axial direction



**Figure 24.**  Influence of  $Da$   values on shear stress along axial direction
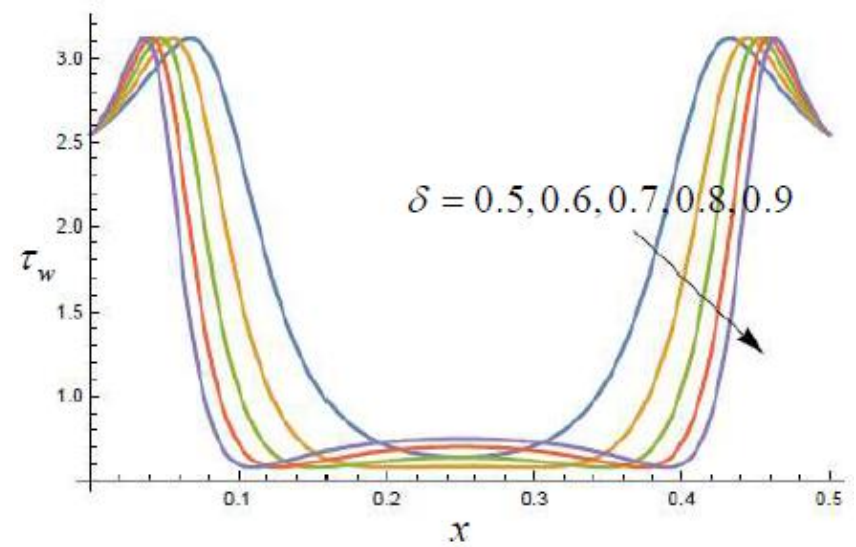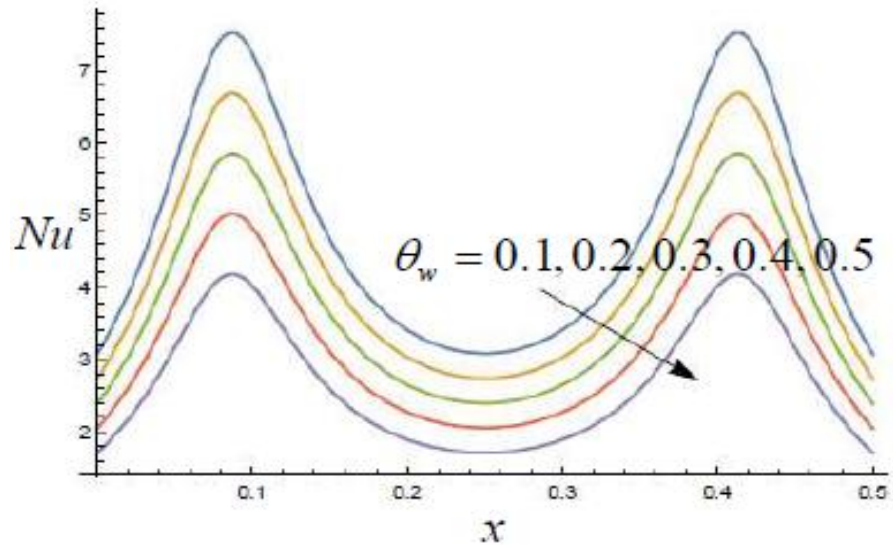
**Figure 25.** Influence of $\alpha$ values on shear stress along axial direction

**Figure 26.** Influence of $\omega$ values on shear stress along axial direction

**Figure 27.** Influence of $\delta$ values on shear stress along axial direction

**Figure 28.**   Influence of $\theta_w$ values on rate of heat transfer along axial direction



**Figure 29.**   Influence of $\alpha$ values on rate of heat transfer along axial direction



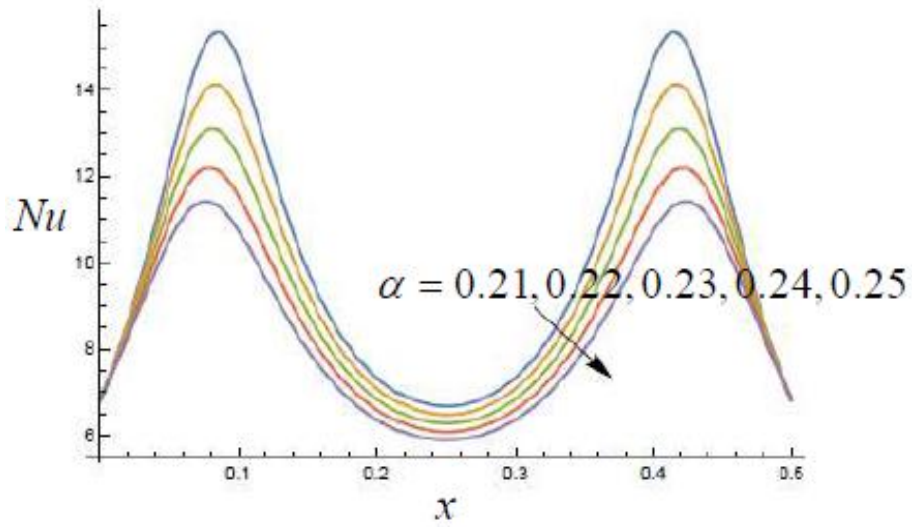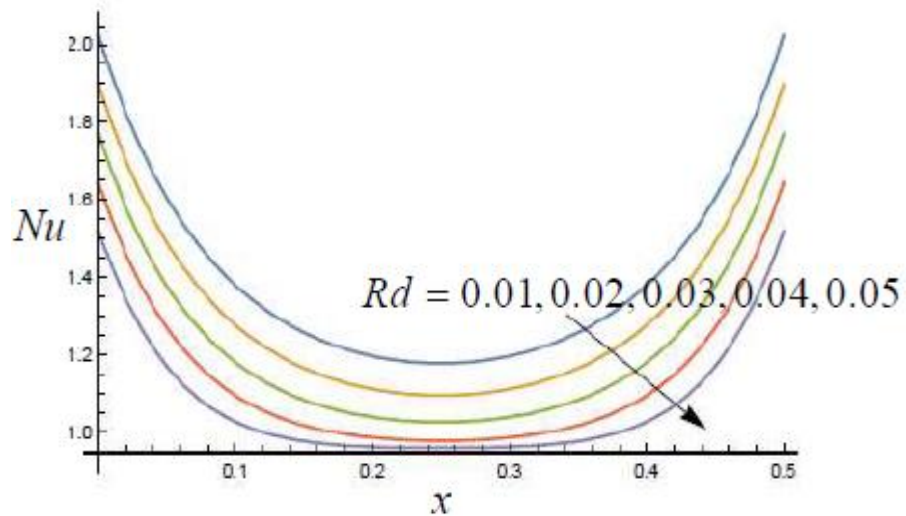**Figure 30.**   Influence of $Rd$ values on rate of heat transfer along axial direction

**Figure 31.** Influence of $\omega$ values on rate of heat transfer along axial direction
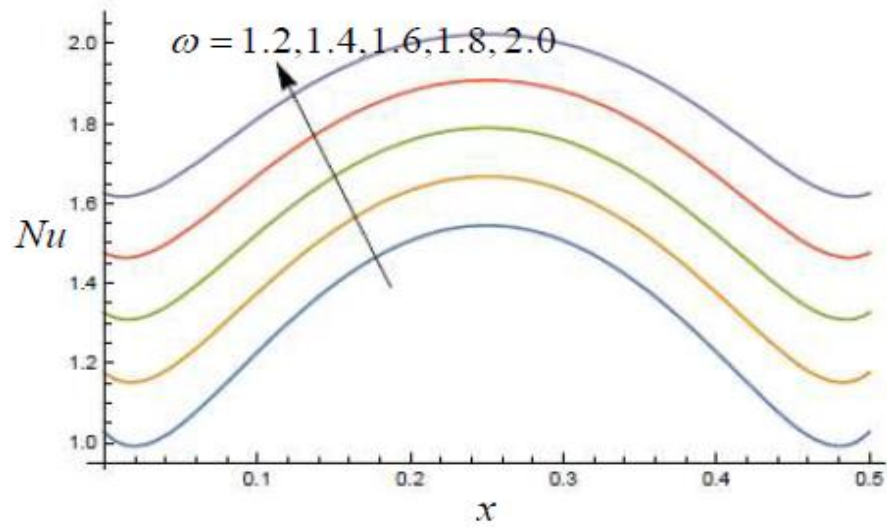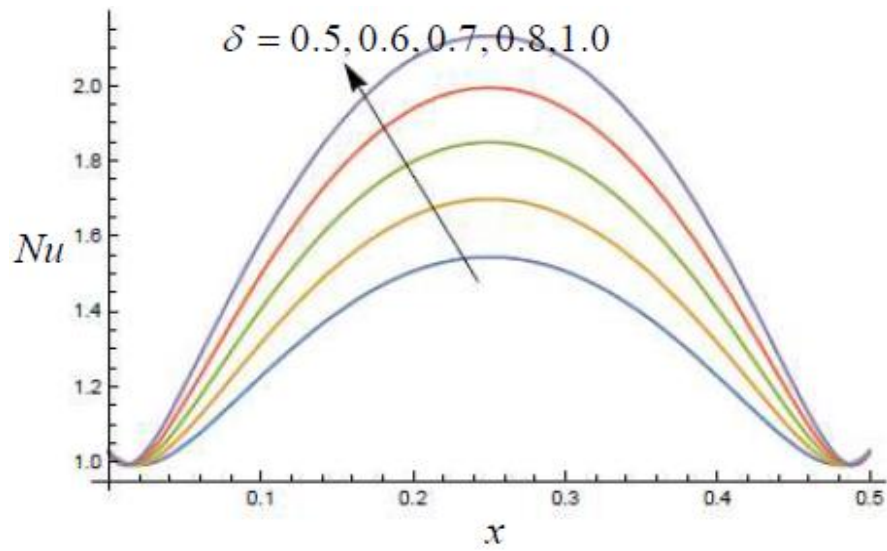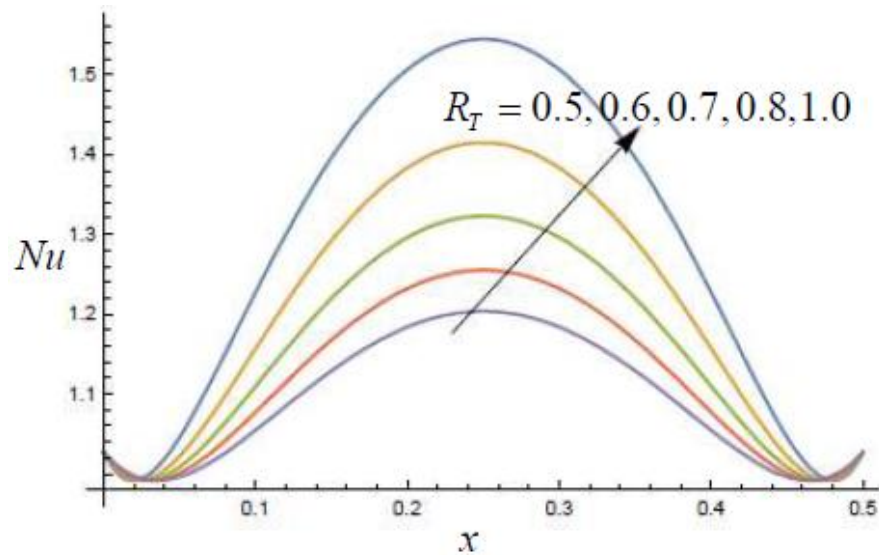


**Figure 32.** Influence of $\delta$ values on rate of heat transfer along axial direction



**Figure 33.** Influence of $R_T$ values on rate of heat transfer along axial direction

It is observed in Figure 1 that the velocity profile decreases along the cross section as the wall temperature increase. This is so because the wall temperature increases does not make much difference in much stiffer atherosclerosis and it rather require surgical procedure to care take of such scenario. However, the velocity attained peaks between $0.3 \le x \le 0.6$ for different values of the wall temperature before decreasing to zero as the boundary gets larger. In Figure 2 it is noticed that blood velocity profile increases as the value of Darcy number increase. This result is realistic in the sense that, the porosity increase aided the velocity increase before decreasing as the boundary layer grows. This result clearly showed different peak for different level of porosity along artery within $0.3 \le x \le 0.6$ with the other pertinent parameters values. Figure 3 depicts that the velocity profile decreases by increasing the values of the Hartmann number. This is because when a magnetic field strength of between $1 \le B_0 \le 3$ applied to a flowing electrically conducting fluid such blood it generate a force called the Lorentz force which opposes the flow. However, the oscillatory behaviour of this result is as a result of pumping rate of the heart.

Figure 4 illustrates that increase in womersley number $\alpha$, decreases the blood velocity profile which gives upper influence to the inertia force over the viscous force as the fluid flow in the arterial segment. The oscillatory behaviour of the result is the due to the pumping rate of the heart with the other pertinent parameters stated in section 4. The influence of the oscillatory parameter was investigated with outcome shown in Figure 5, and the figure show that increase the oscillatory frequency parameter, resulted to the decrease in velocity profile.

Figure 6 clearly showed that as the height of stenosis increases as a result of the increase in the risk factor of atherosclerosis, the more the velocity reduces owing to the factor that increase thickens the stenosed region but velocity attains maximum between $0.3 \le x \le 0.6$ with the other pertinent parameters as stated in section 4. It is seen in Figures 7 & 8 that the increase in the treatment and radiation parameters values $R_T$ & $Rd$ increases the velocity of the fluid.

It is observed in Figure 10 – Figure 13 that increase in radiation, womersley, oscillatory frequency and height of stenosis parameters influenced the temperature profile to decrease, while Figure 9 and Figure 14 depict an increase in velocity profile holding the other variables values constant. The volumetric flow was investigated by varying the values of $\theta_w, Ha, \omega$ and $\delta$ in Figure 15, Figure 17, Figure 19 and Figure 20, and we noticed that increase in the aforementioned parameters values, decreases the volumetric flow rate with other parameters values. Main while, the flow rate increases by increasing the values of the wall temperature and Darcy number respectively. The wall shear stress was investigated with graphical results shown in Figure 21 to Figure 27. In Figures 21, 23, 25, 26 and 27, we noticed a decrease in shear stress as a result of the increase in the values of $\theta_w, Ha, \alpha, \omega$ and $\delta$, while the stress increases

for the increase in Darcy number and radiation parameters, while we maintain the values of the other pertinent parameters as stated in section 4.

Figure 31 – Figure 33 show the results of pertinent parameters on the rate of heat transfer, it is seen that oscillatory frequency parameter, height of stenosis, and treatment parameters increases resulted to an increase in the rate of heat transfer, while Figure 28 – Figure 30 depicts a decrease in rate of heat transfer as a result of the increase in wall temperature, womersley number and radiation parameter.

# 6. Conclusions

The aim of this research is to study blood flow through a sine-shaped atherosclerotic plaque with a source wall temperature and magnetic field using mathematical models. The exact solutions were obtained for the velocity and temperature profiles after scaling the governing equations and adopting oscillatory perturbation. The resulted functions are simulated with some pertinent variable parameters using Mathematica, and we conclude the following observations:

(a) The velocity of blood decreases considerably with the increase in the wall temperature $\theta_w$, the Hartmann number $Ha$, oscillatory parameter $\omega$, the height of stenosis $\delta$ and the womersley number, $\alpha$, and it increases marginally with the increase of Darcy number $Da$ and the treatment parameter $R_T$.

(b) The temperature distribution increases with the increase of the wall temperature $\theta_w$ and the treatment parameter $R_T$ while it decreases with the increase of radiation parameter $Rd$, womersley parameter $\alpha$, oscillatory parameter $\omega$ and height of stenosis $\delta$.

(c) The volumetric blood flow rate decreases with the increase of $\theta_w$, Hartmann number $Ha$, oscillatory parameter $\omega$ and the height of the stenosis $\delta$ while it increases with the increase of the radiation parameter $Rd$, the Darcy parameter $Da$.

(d) The shear stress of blood at the vessel wall increases considerably with the increase of the radiation parameter $Rd$ and the Darcy number $Da$, while it decreases with the increase of the wall temperature $\theta_w$, the Hartmann number $Ha$, the womersley number $\alpha$, the oscillatory parameter $\omega$ and the stenostic height $\delta$.

(e) The rate of heat transfer increases with the increase of the oscillatory parameter $\omega$, the height of stenosis $\delta$ and the treatment parameter $R_T$, while it decreases with the increase of wall temperature parameter $\theta_w$, womersley number $\alpha$ and the radiation parameter $Rd$.

Given the above points, we can finally conclude that blood

flow situation can be accurately determined by knowing the flow profile of an individual. This investigation made known the importance of early treatment in ailment related to cardiovascular challenges such as atherosclerosis and the use of heat in the form of massaging in aiding blood flow in constricted area of an aorta in order to break the molecular bond within that area in supporting life. The study will be of interest to clinicians and researchers who are involved in the treatment of cancer and other tumour ill conditions.

## Nomenclature

| | |
|---|---|
| $w^*$ | Dimensional velocity profile |
| $w_0$ | Perturbed velocity profile |
| $w_\infty$ | Average fluid velocity |
| $x^*, y^*$ | Dimensional distances |
| $R$ | Radius of an abnormal tissue |
| $R_0$ | Radius of normal tissue |
| $R_T$ | Treatment parameter |
| $Ha$ | Hartmann number |
| $\delta$ | Height of stenosis |
| $d_0$ | Onset of stenosis |
| $\alpha$ | Womersley number |
| $Rd$ | Heat absorption constant |
| $k_b$ | Thermal conductivity of blood |
| $Da$ | Darcy number |
| $Gr$ | Thermal Grashof number |
| $C_b$ | The specific heat capacity of blood |
| $B_0$ | Strength of applied magnetic field |
| $Pr$ | Prandtl number for blood |
| $c$ | Wave speed |
| $f_p$ | Pulse frequency |
| $t^*$ | Dimensionless time |
| $T_a^*$ | Arterial temperature |
| $T_b^*$ | Blood temperature |

**Greek Symbols**

| | |
|---|---|
| $\upsilon$ | Kinematic viscosity of blood |
| $\mu_b$ | Dynamic viscosity of blood |
| $g$ | Acceleration due to gravity |
| $\lambda$ | Length of tumour |
| $\omega$ | Oscillatory frequency |
| $\theta \equiv \theta_b$ | Dimensionless temperature |
| $\theta_o$ | Dimensionless perturbed temperature |
| $\theta_w$ | Wall temperature of blood |
| $\rho_b$ | Density of the fluid |

**Subscripts**

| | |
|---|---|
| $w$ | Wall |
| $b$ | Blood |
| $a$ | Arterial |
| $t$ | Tissue |
| $T$ | Treatment |
| $p$ | Pulse |

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