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Exact Solitary Wave Solutions of the (3+1) Modified B-type Kadomtsev-Petviashvili Family Equations

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Abstract We derive the new equations of B-type Kadomtsev-Petviashili equations family defined in (3 + 1) dimensions admitting solitary wave solutions whose analytical sequences are chosen at prior. The principle consists in defining the general B-type Kadomtsev-Petviashili equation by assigning arbitrary coefficients to its various terms, whose resolution of the constraint equations linking them makes it possible to define exactly the equations of this family as well as the corresponding solutions. The introduction of the new implicit function of Bogning in the method used facilitates the calculation management as well as the obtaining of the new equations.

Keywords Soliton, Solitary waves, Nonlinear phenomena, Kadomtsev-Petviashvili equations, Bogning functions, Bogning-Djeumen Tchaho-Kofane method

1. Introduction

In physics sciences, and mathematics, a nonlinear phenomenon is phenomenon in which the change of output is not proportional to the change of input. Typically, the behaviour of a nonlinear system is mathematically describe by a system of nonlinear equations, which is a set of simultaneous equations in which the unknowns functions (in the case of differential equations) appear as variables of a polynomial of degree higher than one or argument of a function which is not polynomial of degree one. When the terms of equations are partial derivatives, the equation is said partial nonlinear. Nonlinear equations are of interest to engineers, mathematicians, physicists and other scientists because most systems are inherently nonlinear in the nature [1-8]. So, we can quote: chaos, singularities and solitons, dynamic of population, the organization of the nature [9, 10].

Solitary wave solutions of nonlinear evolution equations have begun playing important roles in nonlinear science field, especially in nonlinear physical science. The solitary wave solution provides physical informations and more insight into the physical aspects of the problem thus leading to further applications [11]. It is well known that there are many methods for finding special solutions of nonlinear partial differential equations (NPDEs), such as the inverse scattering method [12], the homogeneous balance method [13], the Darboux transformation method [14, 15], the Hirota bilinear method [16, 17], the extended tanh method, the sine-cosine method [18], the Painlevé analysis [19, 20], the Bogning-Djeumen Tchaho-Kofané method (BDKm) [21-35] and so on. In this paper, we will focus our attention on the studies of NPDEs named the modified B-type (3+1)-dimensional Kadomtsev Petviashvili equation [(3+1)-BKPE].

The Kadomtsev-Petviashvili equation (KP) [36]

$$(-4\phi_t + \phi_{xxx} + 6\phi\phi_x)_x + 3\lambda^2\phi_{yy} = 0, \qquad (1)$$

were $\lambda^2 = \pm 1$, originate from a 1970 paper by two soviet physicists, Boris Kadomtsev (1928-1998) and Vladimir Petviashvili (1936-1993). The two researchers derived the equation that now bears their name as a model to study the evolution of long ion-acoustic waves of small amplitude propagating in plasma under the long transverse perturbations. In the absence of transverse dynamics, this problem is described by the Korteweg-de Vries (KdV) equation. The KP equation was soon widely accepted as a natural extension of classic KdV equation in two spatial dimensions, and was later derived as model for surface and internal water waves by Ablowitz and Segur (1979), and in nonlinear optics by Pelinovsky, Stepanyants and Kivshar (1995), as well as in other physical settings.

The focus of the 1970 paper was on a particular problem, the stability of solitons of KdV equation with respect to transverse perturbations. The authors showed that the KdV solitons are stable to such perturbations in the case of media characterized by negative dispersion (that when the phase speed of infinitesimal perturbation decreases with the wave number), in opposite case of positive dispersion media (where the phases speed increases with the wave number).

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The B-type (3+1)-dimensional Kadomtsev Petviashvili equations are given by [37]

$$\phi_{xxxy}+\lambda(\phi_x\phi_y)_x+(\phi_x+\phi_y+\phi_z)_t-(\phi_{xx}+\phi_{yy}+\phi_{zz})=0, \tag{2}$$

and

$$\phi_{yt}-\phi_{xxxy}-3(\phi_x\phi_y)_x+3\phi_{xx}+3\phi_{zz}=0, \tag{3}$$

These equations describe the wave propagation in three spatial and one temporal coordinates such that λ is a real constant. The main objective in this work is to establish the conditions which verified the coefficients $n_i(i=0,1,2,3)$ and $\lambda_i(i=0,1,...,9)$ such that the following (3+1) B-type Kadomtsev Petviashvili family equations defined as

$$\begin{aligned} &n_0\phi_{xxxy}+n_1(\phi_x\phi_y)_x+n_2(\phi_x+\phi_y+\phi_z)_t \\ &+n_3(\phi_{xx}+\phi_{yy}+\phi_{zz})=0, \end{aligned} \tag{4}$$

and

$$\begin{aligned} &(\lambda_0+i\lambda_1)\phi_{yt}+(\lambda_2+i\lambda_3)\phi_{xxxy}+(\lambda_4+i\lambda_5)(\phi_x\phi_y)_x \\ &+(\lambda_6+i\lambda_7)\phi_{xx}+(\lambda_8+i\lambda_9)\phi_{zz}=0, \end{aligned} \tag{5}$$

admit solitary wave solutions. Thus, the paper is organized as follows: section 2 presents the BDKm and where the Bogning implicit function (BIf) is associated; section 3 gives a brief presentation of Bogning's function; Section 4, solves equation (4); section (5) solves equation (5) and to end the work, we propose a conclusion.

2. Presentation of BDKm

J. R. Bogning et al. have established an analytical method for obtaining solutions of shape $a\sinh^m\zeta/\cosh^n\zeta$ in certain classes of NPDEs, where a is a real or complex and n,m the real constants. This method is focused on the construction of solitary wave solution. It has been adopted to facilitate the resolution or the construction of certain types of nonlinear partial differential equations where the nonlinear terms and dispersive terms coexist. It is also devoted to the construction of the solitary wave solutions of certain categories of NPDEs of the form

$$\begin{aligned} &a_i\sum_i\left(\frac{\partial u}{\partial x_i}\right)_i+b_i\sum_i\left(\frac{\partial^2 u}{\partial x_i^2}\right)+...+c_i\sum_i\left(\frac{\partial^l u}{\partial x_i^l}\right) \\ &+d_i\sum_{m,n}\left(\frac{\partial^n u\partial^m u}{\partial x_i^n\partial x_i^m}\right)+f(u,|u|^2)=0, \end{aligned} \tag{6}$$

where a_i,b_i,c_i and d_i are constants that characterize partial differential equations, i,l,m and n positive natural integers, f a linear arbitrary function of u and $|u|^2$, and u the unknown function to determine and $|u|$ the magnitude of u. We look for the solution of equation (6) under the form of linear combination of the hyperbolic function as follows

$$u=\sum_{n,m}a_{nm}\sinh^m(\alpha x)/\cosh^n(\alpha x), \tag{7}$$

where α is constant which depends on the parameter of the system which models NPDE and a_{nm} the constants to be determined. Introducing the Bogning function defined by

$$J_{n,m}(\alpha x)=\sinh^m(\alpha x)/\cosh^n(\alpha x), \tag{8}$$

The form of the solution becomes

$$u=\sum_{n,m}a_{nm}J_{n,m}(\alpha x), \tag{9}$$

Then, the consideration of equation (9) in equation (6) permits to obtain the range equation in the form

$$\begin{aligned} &\sum_{i,j,n}F(a_{ij})J_{n,0}(\alpha x)+\sum_{i,j,m}G(a_{ij})J_{n,1}(\alpha x) \\ &+\sum_{i,j,k}H(a_{ij})J_{-k,0}(\alpha x) \\ &+\sum_{i,j,l}T(a_{ij})J_{l,1}(\alpha x)+\sum_{i,j}W(a_{ij})=0, \end{aligned} \tag{10}$$

where m,n,k and l are the positive whole integers.

Setting the coefficients $F(a_{ij})$, $G(a_{ij})$, $H(a_{ij})$, $T(a_{ij})$ and $W(a_{ij})$ to zero we obtain the coefficient equations then, the resolution permits to determine the coefficients a_{ij}.

3. Brief Presentation of Bogning Function

The search for solitary wave solutions by means of the Bogning-Djeumen Tchaho-Kofané method enabled us to detect a function with very interesting properties, whose thorough study obliged us to baptized " Bogning’s function" and denoted [38]

$$J_{n,m}\left(\sum_{i=0}^p\alpha_i x_i\right)=\sinh^m\left(\sum_{i=0}^p\alpha_i x_i\right)/\cosh^n\left(\sum_{i=0}^p\alpha_i x_i\right), \tag{11}$$

where $J_{n,m}\left(\sum_{i=0}^p\alpha_i x_i\right)$ represents the implicit form of the function, $\sinh^m\left(\sum_{i=0}^p\alpha_i x_i\right)/\cosh^n\left(\sum_{i=0}^p\alpha_i x_i\right)$ the explicit form of the function, α_i ($i=0,1,2,...,p$) represent the parameters associated to the independent variables x_i ($i=0,1,2,...,p$), the couple $(n,m)\in R^2$ indicate the power of the function. In a more precise way, n is the power of $\cosh\left(\sum_{i=0}^p\alpha_i x_i\right)$ and m the power of

$\sinh\left(\sum_{i=0}^{p}\alpha_i x_i\right)$. In dimension one; it is defined and according to the choice of the constant of anybody by

$$J_{n,m}\left(\alpha x\right)=\sinh^m\left(\alpha x\right)/\cosh^n\left(\alpha x\right), \tag{12}$$

where α, represents the parameter associated with the independent variable x, and the couple (n,m) indicates the power of the function. n is the power of $\cosh\left(\alpha x\right)$ and m the power of $\sinh\left(\alpha x\right)$. Some of the main properties of this function we are going to use to solve the equations are given by:

$$\frac{d^p J_{n,m}}{dx^p}=m\alpha\frac{d^{p-1}J_{n-1,m-1}}{dx^{p-1}}-n\alpha\frac{d^{p-1}J_{n+1,m+1}}{dx^{p-1}}, \tag{13}$$

$$J_{n,m}^{p}=J_{np,mp}. \tag{14}$$

The trigonometric form of the Bogning's function is also defined as

$$J_{n,m}\left(ix\right)=\left(i\right)^m\frac{\sin^m\left(x\right)}{\cos^n\left(x\right)}. \tag{15}$$

But taking into account the type of problem to be solved in this paper, the trigonometric form of the Bogning function will not be used. The great developments concerning this function as its spectacular properties are consigned in the book such as the reference [38] indicates it.

4. Resolution of Equation (4)

In optics to use the function $J_{n,m}$ of parameter $\alpha=1$, in order to simplify calculations of the various terms of equation (4), we pose the change of variable $\zeta=\alpha x+\beta y+\gamma z+\alpha_0 t$ and equation (4) becomes

$$\begin{aligned}&n_0\alpha^3\beta\phi_{\zeta\zeta\zeta\zeta}+n_1\alpha^2\beta(\phi_\zeta{}^2)_\zeta\\&+\left[n_2\alpha_0(\alpha+\beta+\gamma)+n_3(\alpha^2+\beta^2+\gamma^2)\right]\phi_{\zeta\zeta}=0,\end{aligned} \tag{16}$$

Integrating first time equation (9) with respect to the variable ζ, gives

$$\begin{aligned}&n_0\alpha^3\beta\phi_{\zeta\zeta\zeta}+n_1\alpha^2\beta\phi_\zeta{}^2\\&+\left[n_2\alpha_0(\alpha+\beta+\gamma)+n_3(\alpha^2+\beta^2+\gamma^2)\right]\phi_\zeta=cst,\end{aligned} \tag{17}$$

where cst, is an arbitrary real constant. Substituting

$$\phi=aJ_{n,m}\left(\zeta\right), \tag{18}$$

in equation (10) in the case where $cst=0$, yields to the range equation which is the main equation in the centre of all analysis

$$\begin{aligned}&\alpha^3\beta a n_0 m(m-1)(m-2)J_{n-3,m-3}\left(\zeta\right)\\&-\alpha^3\beta a n_0 n(n+1)(n+2)J_{n+3,m+3}\left(\zeta\right)\\&-ma\left\{\begin{array}{l}\alpha^3\beta n_0(3mn-3m+2)\\-\left[\begin{array}{l}n_2(\alpha+\beta+\gamma)\alpha_0\\+n_3(\alpha^2+\beta^2+\gamma^2)\end{array}\right]\end{array}\right\}J_{n-1,m-1}\left(\zeta\right)\\&+na\left\{\begin{array}{l}\alpha^3\beta n_0 a(3mn+3n+2)\\-\left[\begin{array}{l}n_2(\alpha+\beta+\gamma)\alpha_0\\+n_3(\alpha^2+\beta^2+\gamma^2)\end{array}\right]\end{array}\right\}J_{n+1,m+1}\left(\zeta\right)\\&+a^2\alpha^2\beta n_1 m^2 J_{2n-2,2m-2}\left(\zeta\right)\\&+a^2\alpha^2\beta n_1 n^2 J_{2n-2,2m-2}\left(\zeta\right)\\&-2a^2\alpha^2\beta n_1 mnJ_{2n,2m}\left(\zeta\right)=0.\end{aligned} \tag{19}$$

The values of n and m for which certain terms of equation (19) gather are given by $n,m=\{-5,-3,-1,1,3,5\}$.

- When $n_0=n_1=0$, equation (4) reduces to

$$n_2(\phi_x+\phi_y+\phi_z)_t-n_3(\phi_{xx}+\phi_{yy}+\phi_{zz})=0, \tag{20}$$

and admits for solution

$$\begin{aligned}\phi(x,y,z,t)&=aJ_{n,m}\left(\zeta\right)\\&=aJ_{n,m}\left(\alpha x+\beta y+\gamma z+\alpha_0 t\right),\end{aligned} \tag{21}$$

with

$$\alpha_0=-\frac{(\alpha^2+\beta^2+\gamma^2)n_3}{(\alpha+\beta+\gamma)n_2}. \tag{22}$$

The values of m and n for which certain terms of equation (12) gather are given by $m\in\{0,1,2\}$ and $n\in\{-1,-2,0\}$. Thus, we observe these cases as follows:

- When $(m,n)=\left(0,-1\right)$ and $(m,n)=\left(1,0\right)$ we obtain respectively from equation (19), the following equations

$$\begin{aligned}&-a^2\alpha^2\beta n_1+a^2\alpha^2\beta n_1 J_{-2,0}\left(\zeta\right)\\&+a\left\{\alpha^3\beta n_0+\left[\begin{array}{l}n_2(\alpha+\beta+\gamma)\alpha_0\\+n_3(\alpha^2+\beta^2+\gamma^2)\end{array}\right]\right\}J_{0,1}\left(\zeta\right)=0,\end{aligned} \tag{20}$$

and

$$\begin{aligned}&\left\{(\alpha^3\beta n_0+\left[\begin{array}{l}n_2(\alpha+\beta+\gamma)\alpha_0\\+n_3(\alpha^2+\beta^2+\gamma^2)\end{array}\right]\right\}J_{-1,0}\left(\zeta\right)\\&+a^2\alpha^2\beta n_1 J_{-2,0}(\zeta)=0.\end{aligned} \tag{21}$$

The resolution of the algebraic system deduced from equation (20) and (21) for $n_1=0$ gives

$$\alpha_0 = \frac{n_0\alpha^3\beta + n_3(\alpha^2+\beta^2+\gamma^2)}{n_2(\alpha+\beta+\gamma)}. \tag{22}$$

The constraint relation $n_1 = 0$, reduces equation (4) as follows

$$\begin{aligned} &n_0\phi_{xxxy} - n_2\alpha^3\beta(\phi_x+\phi_y+\phi_z)_t \\ &-n_3\alpha^3\beta(\phi_{xx}+\phi_{yy}+\phi_{zz}) = 0, \end{aligned} \tag{23}$$

Such that its solutions are given by

$$\begin{aligned} \phi(x,y,z,t) &= aJ_{-1,0}(\zeta) \\ &= aJ_{-1,0}(\alpha x+\beta y+\gamma z+\alpha_0 t), \end{aligned} \tag{24}$$

and

$$\begin{aligned} \phi(x,y,z,t) &= aJ_{0,1}(\zeta) \\ &= aJ_{0,1}(\alpha x+\beta y+\gamma z+\alpha_0 t), \end{aligned} \tag{25}$$

where α_0 is defined by equation (25).

- When $(n,m) = (-2,0)$ or $(n,m) = (0,2)$ we obtain respectively from equation (19)

$$\begin{aligned} &4a^2\alpha^2\beta n_1 J_{-4,0}(\zeta) - 4a^2\alpha^2\beta n_1 J_{-2,0}(\zeta) \\ &+\left\{8\alpha^3\beta n_0 + 2\begin{bmatrix} n_2(\alpha+\beta+\gamma)\alpha_0 \\ +n_3(\alpha^2+\beta^2+\gamma^2)\end{bmatrix}\right\} J_{-1,1}(\zeta) = 0, \end{aligned} \tag{26}$$

and

$$\left\{8\alpha^3\beta n_0 + 2\begin{bmatrix} n_2(\alpha+\beta+\gamma)\alpha_0 \\ +n_3(\alpha^2+\beta^2+\gamma^2)\end{bmatrix}\right\} J_{-1,1}(\zeta) = 0. \tag{27}$$

The resolution of equations (26) and (27) after setting $n_1 = 0$ gives

$$\alpha_0 = -\frac{4n_0\alpha^3\beta + n_3(\alpha^2+\beta^2+\gamma^2)}{n_2(\alpha+\beta+\gamma)}, \tag{28}$$

and solutions respectively read

$$\begin{aligned} \phi(x,y,z,t) &= aJ_{-2,0}(\zeta) \\ &= aJ_{-2,0}(\alpha x+\beta y+\gamma z+\alpha_0 t), \end{aligned} \tag{29}$$

and

$$\begin{aligned} \phi(x,y,z,t) &= aJ_{0,2}(\zeta) \\ &= aJ_{-2,0}(\alpha x+\beta y+\gamma z+\alpha_0 t), \end{aligned} \tag{30}$$

where α_0 is defined by equation (28).

- When $m = n = 1$ and $m = n = -1$ we obtain in that order from equation (19), equations

$$\begin{aligned} &(-6\alpha^3\beta n_0 + a\alpha^2\beta n_1)J_{4,0}(\zeta) \\ &+\left\{4\alpha^3\beta n_0 + \begin{bmatrix} n_2(\alpha+\beta+\gamma)\alpha_0 \\ +n_3(\alpha^2+\beta^2+\gamma^2)\end{bmatrix}\right\} J_{2,0}(\zeta) = 0, \end{aligned} \tag{31}$$

and

$$\begin{aligned} &\left\{-4\alpha^3\beta n_0 - \begin{bmatrix} n_2(\alpha+\beta+\gamma)\alpha_0 \\ +n_3(\alpha^2+\beta^2+\gamma^2)\end{bmatrix}\right\} J_{0,1}(\zeta) \\ &+\left\{-2\alpha^3\beta n_0 + \begin{bmatrix} n_3(\alpha+\beta+\gamma)\alpha_0 \\ +n_3(\alpha^2+\beta^2+\gamma^2)\end{bmatrix} + a\alpha^2\beta n_1\right\} J_{3,0}(\zeta) = 0. \end{aligned} \tag{32}$$

From the resolution of equations (31) and (32), we obtain the following constants

$$a = \frac{6\alpha n_0}{n_1}, \tag{33}$$

and

$$\alpha_0 = -\frac{4n_0\alpha^3\beta + n_3(\alpha^2+\beta^2+\gamma^2)}{n_2(\alpha+\beta+\gamma)}, \tag{34}$$

such that equation (4) admits for solutions

$$\begin{aligned} \phi(x,y,z,t) &= \frac{6\alpha n_0}{n_1} J_{1,1}(\zeta) \\ &= \frac{6\alpha n_0}{n_1} J_{1,1}(\alpha x+\beta y+\gamma z+\alpha_0 t), \end{aligned} \tag{35}$$

and

$$\begin{aligned} \phi(x,y,z,t) &= \frac{6\alpha n_0}{n_1} J_{-1,-1}(\zeta) \\ &= \frac{6\alpha n_0}{n_1} J_{-1,-1}(\alpha x+\beta y+\gamma z+\alpha_0 t), \end{aligned} \tag{36}$$

where α_0 is defined by equation (34).

We see that from the different relations linking the constants $\mathrm{n_i}(i = 0,1,2,3)$, we can define many equations derived from equation (4) as well as their solutions.

5. Resolution of Generalized (3+1) B-type Kadomtsev-Petviashvili Equation Family (5)

Using the same wave transformation as early, equation (5) becomes after a first integration

$$\begin{aligned} &\lambda_2\alpha^3\beta\phi_{\zeta\zeta\zeta} + \lambda_4\alpha^2\beta\phi_\zeta{}^2 \\ &+(\lambda_0\beta\alpha_0 + \lambda_6\alpha^2 + \lambda_8\gamma^2)\phi_\zeta \\ &+i\begin{bmatrix} \lambda_3\alpha^3\beta\phi_{\zeta\zeta\zeta} + \lambda_5\alpha^2\beta\phi_\zeta{}^2 \\ +(\lambda_1\beta\alpha_0 + \lambda_7\alpha^2 + \lambda_9\gamma^2)\phi_\zeta \end{bmatrix} = 0. \end{aligned} \tag{37}$$

We assume that the integration constant of equation (37) is zero. The real and imaginary part of equation (37) lead respectively to the following equations

$$\begin{aligned}&\left\{\begin{array}{l}-\lambda_2\alpha^3\beta a\left[m(m-1)(n-2)+(2mn-m+n)m\right]\\+am\left(\lambda_0\beta\alpha_0+\lambda_6\alpha^2+\lambda_8\gamma^2\right)\end{array}\right\}J_{n-1,m-1}\\&+\left\{\begin{array}{l}a\lambda_2\alpha^3\beta\left[(2mn-m+n)+n(n+1)(m+2)\right]\\-an\left(\lambda_0\beta\alpha_0+\lambda_6\alpha^2+\lambda_8\gamma^2\right)\end{array}\right\}J_{n+1,m+1}(\zeta)\\&-a\lambda_2\alpha^3\beta n(n+1)(n+2)J_{n+3,m+3}(\zeta)\\&+\lambda_4\alpha^2\beta a^2m^2J_{2n-2,2m-2}(\zeta)\\&-2\lambda_4\alpha^2\beta mna^2J_{2n,2m}(\zeta)\\&+\lambda_2\alpha^3\beta am(m-1)(m-2)J_{n-3,m-3}\\&+\lambda_4\alpha^2\beta a^2n^2J_{2n+2,2m+2}(\zeta)=0,\end{aligned} \tag{38}$$

and

$$\begin{aligned}&\left\{\begin{array}{l}-\lambda_3\alpha^3\beta a\left[m(m-1)(n-2)+(2mn-m+n)m\right]\\+am\left(\lambda_1\beta\alpha_0+\lambda_7\alpha^2+\lambda_9\gamma^2\right)\end{array}\right\}J_{n-1,m-1}\\&+\left\{\begin{array}{l}a\lambda_3\alpha^3\beta\left[(2mn-m+n)n+n(n+1)(m+2)\right]\\-an\left(\lambda_1\beta\alpha_0+\lambda_7\alpha^2+\lambda_9\gamma^2\right)\end{array}\right\}J_{n+1,m+1}(\zeta)\\&-a\lambda_3\alpha^3\beta n(n+1)(n+2)J_{n+3,m+3}(\zeta)\\&+\lambda_5\alpha^2\beta a^2m^2J_{2n-2,2m-2}(\zeta)\\&-2\lambda_5\alpha^2\beta mna^2J_{2n,2m}(\zeta)\\&+\lambda_3\alpha^3\beta am(m-1)(m-2)J_{n-3,m-3}\\&+\lambda_5\alpha^2\beta a^2n^2J_{2n+2,2m+2}(\zeta)=0,\end{aligned} \tag{39}$$

The values of n and m for which certain terms of equations (39) and (40) gather are given by $n,m=\{-5,-3,-1,1,3,5\}$.

The analysis of the equations (38) and (39) above enables to identify all the possibilities of solutions.

- When $\lambda_2=\lambda_3=0$, $\lambda_4=\lambda_5=0$, $m\neq 0$ and $n\neq 0$, we obtain respectively from equations (38) and (39) the following equations

$$\lambda_0\alpha_0\beta+\lambda_6\alpha^2+\lambda_8\gamma^2=0, \tag{40}$$

and

$$\lambda_1\alpha_0\beta+\lambda_7\alpha^2+\lambda_9\gamma^2=0. \tag{41}$$

The resolution of equations (40) and (41) permits to find

$$\alpha_0=-\frac{\lambda_6\alpha^2+\lambda_8\gamma^2}{\beta\lambda_0}=-\frac{\lambda_7\alpha^2+\lambda_9\gamma^2}{\beta\lambda_1}, \tag{42}$$

And

$$\alpha=\pm\gamma\sqrt{\frac{\lambda_9\lambda_0-\lambda_8\lambda_1}{\lambda_1\lambda_6-\lambda_0\lambda_7}}. \tag{43}$$

Under those conditions, equation (5) reduces to

$$(\lambda_0+i\lambda_1)\phi_{yt}+(\lambda_6+i\lambda_7)\phi_{xx}+(\lambda_8+i\lambda_9)\phi_{zz}=0, \tag{44}$$

such that according to the values obtained in equations (42) and (43) its solution for any value of *m* and *n* is given by

$$\begin{aligned}\phi(x,y,z,t)&=aJ_{n,m}(\zeta)\\&=aJ_{n,m}(\alpha x+\beta y+\gamma z+\alpha_0 t),\end{aligned} \tag{45}$$

- When $n=-1,m=0$ or $n=0,m=1$, the resolution of equations (38) and (39) impose to set $\lambda_4=\lambda_5=0$ and obtain the constants given by

$$\begin{aligned}\alpha_0&=\frac{\lambda_2\alpha^3\beta+\lambda_6\alpha^2+\lambda_8\gamma^2}{\lambda_0\beta}\\&=\frac{\lambda_3\alpha^3\beta+\lambda_7\alpha^2+\lambda_9\gamma^2}{\lambda_1\beta},\end{aligned} \tag{46}$$

and

$$\beta=-\frac{(\lambda_0\lambda_7-\lambda_1\lambda_6)\alpha^2+(\lambda_0\lambda_9-\lambda_1\lambda_8)\gamma^2}{(\lambda_0\lambda_3-\lambda_1\lambda_2)\alpha^3}. \tag{47}$$

Those constraint relations reduce equation (5) in the form

$$\begin{aligned}&(\lambda_0+i\lambda_1)\phi_{yt}+(\lambda_2+i\lambda_3)\phi_{xxxy}\\&+(\lambda_6+i\lambda_7)\phi_{xx}+(\lambda_8+i\lambda_9)\phi_{zz}=0.\end{aligned} \tag{48}$$

Thus, the solutions of equation (48) in both cases are given respectively according to equations (46) and (47) the following solutions

$$\begin{aligned}\phi(x,y,t)&=aJ_{-1,0}(\zeta)\\&=aJ_{-1,0}(\alpha x+\beta y+\gamma z+\alpha_0 t),\end{aligned} \tag{49}$$

and

$$\begin{aligned}\phi(x,y,t)&=aJ_{0,1}(\zeta)\\&=aJ_{0,1}(\alpha x+\beta y+\gamma z+\alpha_0 t).\end{aligned} \tag{50}$$

- When $(n,m)=(-2,0)$ or $(n,m)=(0,2)$, we obtain from equations (38) and (39) the constraint $\lambda_4=\lambda_5=0$, such that

$$\begin{aligned}\alpha_0&=\frac{4\lambda_2\alpha^3\beta+\lambda_6\alpha^2+\lambda_8\gamma^2}{\lambda_0\beta}\\&=\frac{4\lambda_3\alpha^3\beta+\lambda_7\alpha^2+\lambda_9\gamma^2}{\lambda_1\beta},\end{aligned} \tag{51}$$

and

$$\beta=-\frac{(\lambda_0\lambda_7-\lambda_1\lambda_6)\alpha^2+(\lambda_0\lambda_9-\lambda_1\lambda_8)\gamma^2}{4(\lambda_0\lambda_3-\lambda_1\lambda_2)\alpha^3}. \tag{52}$$

So, equation (5) in this case is reduced to equation (48) such that the solutions are

$$\begin{aligned}\phi(x,y,z,t)&=aJ_{0,2}(\zeta)\\&=aJ_{0,2}(\alpha x+\beta y+\gamma z+\alpha_0 t),\end{aligned} \tag{53}$$

and

$$\begin{aligned}\phi(x,y,z,t) &= aJ_{-2,0}(\zeta) \\ &= aJ_{-2,0}(\alpha x+\beta y+\gamma z+\alpha_0 t).\end{aligned} \tag{54}$$

- When $m=n=1$ and $m=n=-1$ we find from equations (38) and (39) that

$$\frac{\lambda_4}{\lambda_2}=\frac{\lambda_5}{\lambda_3}, \tag{55}$$

$$\begin{aligned}\alpha_0 &= \frac{4\lambda_2\alpha^3\beta+\lambda_6\alpha^2+\lambda_8\gamma^2}{\lambda_0\beta} \\ &= \frac{4\lambda_3\alpha^3\beta+\lambda_7\alpha^2+\lambda_9\gamma^2}{\lambda_1\beta},\end{aligned} \tag{56}$$

$$\beta=-\frac{(\lambda_0\lambda_7-\lambda_1\lambda_6)\alpha^2+(\lambda_0\lambda_9-\lambda_1\lambda_8)\gamma^2}{(\lambda_0\lambda_3-\lambda_1\lambda_2)\alpha^3}, \tag{57}$$

and

$$a=\frac{6\alpha\lambda_2}{\lambda_4}. \tag{58}$$

Equations (55), (56), ..., (58) are verified if and only if the following conditions are satisfied: $\beta\lambda_0\neq 0$, $\beta\lambda_1\neq 0$, $(\lambda_9\lambda_0-\lambda_8\lambda_1)(\lambda_1\lambda_6-\lambda_0\lambda_7)\succ 0$, $(\lambda_0\lambda_3-\lambda_1\lambda_2)\alpha\neq 0$, $\lambda_2\neq 0$, $\lambda_3\neq 0$, $\lambda_4\neq 0$.

With the above conditions, equation (5) reduces to

$$\begin{aligned}&\lambda_2(\lambda_0+i\lambda_1)\phi_{yt}+\lambda_2(\lambda_2+i\lambda_3)\phi_{xxxy} \\ &+\lambda_4(\lambda_2+i\lambda_3)(\phi_x\phi_y)_x \\ &+\lambda_2(\lambda_6+i\lambda_7)\phi_{xx}+\lambda_2(\lambda_8+i\lambda_9)\phi_{zz}=0,\end{aligned} \tag{59}$$

and admits for solutions

$$\begin{aligned}\phi(x,y,z,t) &= \frac{6\alpha\lambda_2}{\lambda_4}J_{1,1}(\zeta) \\ &= \frac{6\alpha\lambda_2}{\lambda_4}J_{1,1}(\alpha x+\beta y+\gamma w+\alpha_0 t),\end{aligned} \tag{60}$$

and

$$\begin{aligned}\phi(x,y,z,t) &= \frac{6\alpha\lambda_2}{\lambda_4}J_{-1,-1}(\zeta) \\ &= \frac{6\alpha\lambda_2}{\lambda_4}J_{-1,-1}(\alpha x+\beta y+\gamma w+\alpha_0 t).\end{aligned} \tag{61}$$

6. Conclusions

The objective of this work was to construct the solitary wave solutions of KP equations whose analytic sequences are constituted by hyperbolic functions. At the end of the studies we believe that we have achieved this goal; because in the light of the results obtained by the use of the BDKm extended to the implicit functions of Bogning as well as the principle which consisted in modifying the coefficients of the different terms of the two forms of KP equations, we obtained different forms of solutions as well as the new equations they check. Among the solutions obtained, some are solitary wave solutions and others are not.

From a physical point of view, these coefficients, arbitrarily assigned to the different terms of the equations considered, are interpreted as quantities which make it possible to characterize the propagation medium or the moving physical system. These coefficients can also be considered as indicative factors of the properties of the physical system studied. In purely mathematical jargon, the use of the BDKm extended to the implicit functions greatly facilitated the calculations while allowing to obtain easily the expected solutions as well as the family of the nonlinear partial differential equations of the KP type which admit these solutions.

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