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Compromise Solutions for Fuzzy Multi-Level Multiple Objective Decision Making Problems

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Abstract This paper extended the concept of the technique for order preference by similarity to ideal solution (TOPSIS) to develop a methodology to find compromise solutions for the Multi-Level Multiple Objective Decision Making (MLMODM) Problems with fuzzy parameters in the objective functions and the right hand side of the constraints (FMLMODM) of mixed (Maximize/Minimize)-type. Anew interactive algorithm is presented for the proposed TOPSIS approach for solving these types of mathematical programming problems. Also, an illustrative numerical example is solved and compared the solution of proposed algorithm with the solution of Global Criterion (GC) method.

Keywords Compromise Programming, Fuzzy Programming, TOPSIS method, Global Criterion method, Interactive Decision Making, Multiple Objective Programming, Multi-level Programming, and Fuzzy Parameters

1. Introduction

Compromise programming (CP) was initially proposed by Zeleny (1973) and subsequently used by many researchers. [24]. Yu (1973) and Zeleny (1974) define the ideal solution (Yu describes this solution as the "utopia point") as any solution that would simultaneously optimize each individual objective. CP assumes that any DM seeks a solution as close as possible to the ideal point, [21, 25].

The non-centralized planning has been recognized as an important decision making problem. It searches for a simultaneous compromise among the various objectives of the different departments. Multi-Level programming, a tool for modeling non-centralized decisions, consists of the objective(s) of the Manager at its first level and that is of the followers at the other levels. The decision-maker at each level seeks to optimize his individual objective Functions, which depends in part on the variables controlled by the decision makers at the other levels and their final decisions are executed sequentially where the upper-level decision-maker makes his decision firstly, [5, 7, 8, 11, 12, 16, 20].

An extended TOPSIS method for solving interactive large scale multiple objective optimization problems involving fuzzy parameters is introduced in [4].

Several algorithms for solving different kinds of large scale multiple objective optimization problems using TOPSIS approach are presented in [1].

A review on theory, applications and softwares of bi-level, multi-level multiple criteria decision making and TOPSIS approach is presented in [2].

Interactive TOPSIS algorithms for solving multi-level non-linear multi-objective decision-making problems are given in [6].

A modified TOPSIS method for solving large scale two-level linear multiple objective optimization problems with fuzzy parameters in the right-hand side of the independent constraints is introduced in [3].

We extend the TOPSIS method [4, 15] to find compromise solutions [14, 22, 25, 26, 27] for the Multi-Level Multiple Objective Decision Making (MLMODM) Problems with fuzzy parameters [18, 19, 23, 27] in the objective functions and the right hand side of the constraints (FMLMODM).

In the following sections, the formulation of FMLMODM problems is given in section (2).By use of TOPSIS method, a new interactive algorithm for solving MLMODM problems is proposed in section (3). For the sake of illustration, we present an example for the extended TOPSIS method and compared the solution of proposed algorithm with the solution of traditional Global Criterion (GC) method in section (4).

2. Formulation of the Problem

Consider the following FMLMODM problem:

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$$[\mathrm{DM}_{L_1}]$$
$$\underset{X_1}{Maximize/Minimize}\left(f_{11}(X_1,X_2,\ldots,X_h,\tilde{V}_{11}),..,f_{1k_1}(X_1,X_2,\ldots,X_h,\tilde{V}_{1k_1})\right) \quad (1\text{-}1)$$

where X_2 *solves the* 2^{nd} *level*

$$[\mathrm{DM}_{L_2}]$$
$$\underset{X_2}{Maximize/Minimize}\left(f_{21}(X_1,X_2,,X_h,\tilde{V}_{21}),\ldots.,f_{2k_2}(X_1,X_2,\ldots,X_h,\tilde{V}_{2k_2})\right) \quad (1\text{-}2)$$

where X_3 *solves the* 3^{rd} *level*

$$[\mathrm{DM}_{L_3}]$$
$$\underset{X_3}{Maximize/Minimize}\left(f_{31}(X_1,X_2,\ldots,X_h,\tilde{V}_{31}),..,f_{3k_3}(X_1,X_2,\ldots,X_h,\tilde{V}_{3k_3})\right) \quad (1\text{-}3)$$

where X_{I_h} *solves the* h^{th} *level*

$$[\mathrm{DM}_{L_h}]$$
$$\underset{X_h}{Maximize/Minimize}\left(f_{h1}(X_1,X_2,\ldots,X_h,\tilde{V}_{h1}),..,f_{hk_h}(X_1,X_2,\ldots,X_h,\tilde{V}_{hk_h})\right) \quad (1\text{-}4)$$

Subject to

$$X\in M=\{X\in R^n: DX\le \widetilde{U}\} \quad (1\text{-}5)$$

where
m: the number of constraints,
n: the number of variables,
h: the number of levels
k: the number of objective functions,
k_j: The number of objective functions of the DM_{L_j}, j=1,2,….,h,
n_j: The number of variables of the DM_{L_j}, j=1,2,….,h,
$\tilde{V}_{jk_j}$: an n-dimensional row vector of fuzzy parameters
for the f_{jk_j} objective functions=1,2,…,h.
$\tilde{U}$: An *m*-dimensional column vector of right-hand sides of constraints whose elements are fuzzy Parameters
D: an $(m\times n)$ coefficient matrix,
R: the set of all real numbers,
X: an *n*-dimensional column vector of variables,
X_j: an n_j -dimensional column vector of variables for the j^{th} level, j=1,2,…,h,
$K_j=\{1,2,\ldots,k_j\}, j=1,2,\ldots,h,$
$K=K_1\cup K_2\cup\ldots.\cup K_j=\{1,2,\ldots..,k\},$
$N=\{1,2,\ldots..,n\},$
$R^n=\{X=(x_1,x_2,\ldots,x_n)^T: x_i\in R,\ i\in N\}.$

Throughout this paper, we assume that the column vectors of fuzzy parameters $\tilde{U}$ and $\tilde{V}_{jk_j}$, *j= =1,2,…,h,* the row vectors of fuzzy parameters are characterized as the column vectors of fuzzy numbers and row vectors of fuzzy numbers respectively [18, 19, 23, 29].

It is appropriate to recall that a real fuzzy number $\widetilde{\lambda}$ whose membership function $\mu_{\tilde{\lambda}}(\lambda)$ is defined as, [18, 19, 23, 29]:

(1) A continuous mapping from R^1 to the closed interval *[0,1]*,
(2) $\mu_{\tilde{\lambda}}(V)=0$ for all $\lambda\in(-\infty,\lambda_1]$,
(3) Strictly increasing on $[\lambda_1,\lambda_2]$,
(4) $\mu_{\tilde{\lambda}}(\lambda)=1$ for all $\lambda\in[\lambda_2,\lambda_3]$,
(5) Strictly decreasing on $[\lambda_3,\lambda_4]$,
(6) $\mu_{\tilde{\lambda}}(\lambda)=0$ for all $\lambda\in[\lambda_4,+\infty)$.

A possible shape of fuzzy number $\widetilde{\lambda}$ is illustrated in figure (1). The concept of α-cut of the vectors parameters $\widetilde{U}$ and $\tilde{V}_{jk_j}$, *j= =1,2,…,h,* whose elements are fuzzy numbers is introduced as follows:

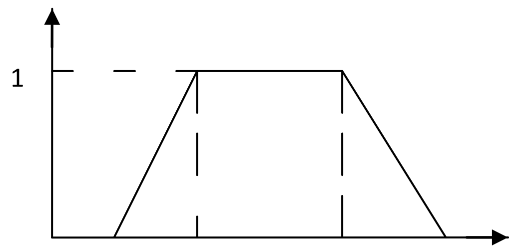

Figure (1). Membership function of fuzzy number $\tilde{\lambda}$

Definition 1. (α-cut).

The α-cut of $(\tilde{V}_{jk_j}, \tilde{U})$, *j= 0,1,2,…,h,* is defined as the ordinary set $(\tilde{V}_{jk_j}, \tilde{U})_\alpha$ for which the degree of their membership function exceeds the level α ϵ [0,1]:

$$(\tilde{V}_{jk_j}, \tilde{U})_\alpha = \{\left(V_{jk_j}, U\right): \mu_{\tilde{V}_{j,t,i}}(v_{j,t,i}) \geq \alpha, \mu_{\tilde{U}_s}(u_s) \geq \alpha,\ j=1,...,h,\ t=1,\dots,k_j,\ i=1,...,n, s=1,\dots,m\} \tag{2}$$

For a certain degree α, the FMLMODM problem (1) can be understood as the following nonfuzzy α-Multi-Level Multiple Objective Decision Making (α- MLMODM) problem:

$$[\alpha - \mathrm{DM}_{L_1}]$$

$$\underset{X_1}{Maximize/Minimize} \left(f_{11}(X_1, X_2, \dots, X_h, V_{11}), \dots .., f_{1k_1}\left(X_1, X_2, \dots, X_h, V_{1k_1}\right)\right) \tag{3-1}$$

where X_2 *solves the* 2^{nd} *level*

$$[\alpha - \mathrm{DM}_{L_2}]$$

$$\underset{X_2}{Maximize/Minimize} \left(f_{21}(X_1, X_2, , X_h, V_{21}), \dots ., f_{2k_2}\left(X_1, X_2, \dots, X_h, V_{2k_2}\right)\right) \tag{3-2}$$

where X_3 *solves the* 3^{rd} *level*

$$[\alpha - \mathrm{DM}_{L_3}]$$

$$\underset{X_3}{Maximize/Minimize} \left(f_{31}(X_1, X_2, \dots, X_h, V_{31}), \dots ., f_{3k_3}\left(X_1, X_2, , X_h, V_{3k_3}\right)\right) \tag{3-3}$$

where X_{l_3} *solves the* h^{th} *level*

$$[\alpha - \mathrm{DM}_{L_h}]$$

$$\underset{X_h}{Maximize/Minimize} \left(f_{h1}(X_1, X_2, , X_h, V_{h1}), \dots ., f_{hk_h}\left(X_1, X_2, \dots, X_h, V_{hk_h}\right)\right) \tag{3-4}$$

Subject to

$$X \in M^{/} = \{X \in R^n: DX \leq U, \tag{3-5}$$

$$(V_{jk_j}, U) \in (\tilde{V}_{jk_j}, \tilde{U})_\alpha, j = 0,1,2, \dots, h,\ \} \tag{3-6}$$

In the (α - MLMODM) problem (3), the parameters $V_{jk_j}, j = 0,1,2, \dots, h \; and \; U$ are treated as decision variables rather than constants.

Based on the definition of α-cut of the fuzzy numbers, we characterize α–efficient solution of α- MLMODM problem (3):

Definition 2: (α- efficient solution).

A solution $X^* \in M^{/}$ is said to be an α- efficient solution to the α- MLMODM problem (4), if and only if there does not exist another $X, (V_{jk_j}, U) \in (\tilde{V}_{jk_j}, \tilde{U})_\alpha$, *j=0,1,2,…h,* such that $f_{jk_j}\left(X_1, X_2, \dots, X_h, V_{jk_j}\right) \geq f_{jk_j}\left(X_1^*, X_2^*, \dots, X_h^*, V_{jk_j}\right)$, $j = 1,2, \dots, h,$ for maximizati $\left(f_{jk_j}\left(X_1, X_2, \dots, X_h, V_{jk_j}\right) \leq f_{jk_j}\left(X_1^*, X_2^*, \dots, X_h^*, V_{jk_j}\right), j = 1,2, \dots ., h, \text{for minimization}\right)$ and with strictly inequality holding for at least one jk_j where the corresponding value of the parameter $(V_{jk_j}^*, U^*)$, *j= 0,1,2,…,h* is called α-cut optimal parameters.

Thus, the α-MLMODM problem (3) can be written as follows:

$$[\alpha - \mathrm{DM}_{L_1}]$$

$$\underset{X_1}{Maximize/Minimize} \left(f_{11}(X_1, X_2, \dots, X_h, V_{11}), \dots .., f_{1k_1}\left(X_1, X_2, \dots, X_h, V_{1k_1}\right)\right) \tag{4-1}$$

where X_2 *solves the* 2^{nd} *level*

$$[\alpha - \mathrm{DM}_{L_2}]$$

$$\underset{X_2}{Maximize/Minimize} \left(f_{21}(X_1, X_2, X_h, V_{21}), \dots., f_{2k_2}(X_1, X_2,, X_h, V_{2k_2})\right) \tag{4-2}$$

where X_3 *solves the* 3^{rd} *level*

$$[\alpha - \mathrm{DM}_{L_3}]$$

$$\underset{X_3}{Maximize/Minimize} \left(f_{31}(X_1, X_2,, X_h, V_{31}),., f_{3k_3}(X_1, X_2,, X_h, V_{3k_3})\right) \tag{4-3}$$

where X_{I_3} *solves the* h^{th} *level*

$$[\alpha - \mathrm{DM}_{L_h}]$$

$$\underset{X_h}{Maximize/Minimize} \left(f_{h1}(X_1, X_2,, X_h, V_{h1}),, f_{hk_h}(X_1, X_2, \dots, X_h, V_{hk_h})\right) \tag{4-4}$$

Subject to

$$X \in M^{//} = \{X \in R^n: DX \le U, \tag{4-5}$$

$$L \le U \le Q, \tag{4-6}$$

$$\eta^T_{j\,k_j} \le V^T_{j\,k_j} \le \gamma^T_{j\,k_j}, j = 1,2, \dots., h\}. \tag{4-7}$$

It should be noted that the constraint (3-6) is replaced by the equivalent constraints (4-6) and (4-7), where $L, \eta_{j\,k_j}$ and $Q, \gamma_{j\,k_j}$ are lower and upper bound on U *and* $V_{j\,k_j}$ respectively.

3. TOPSIS for FMLMODEMs

A modified version of TOPSIS method is introduced to find compromise solutions for the FMLMODM problems. Modified equations for the distance function [17] from the positive ideal solution (PIS) and the distance function from the negative ideal solution (NIS) are introduced to include all the objective functions of multi-level of the problem. An interactive decision making algorithm to find a compromise solution through TOPSIS approach is provided in (3-1) where the first level decision maker (DM_{L_1}) is asked to specify the membership function for each fuzzy parameter, the maximum negative and positive tolerance values, the power p of the distance functions, the degree α and the relative importance of the objectives. Then, the jth Level decision maker (DM_{L_j}) is asked to specify the maximum negative and positive tolerance values, and the relative importance of the objectives. An illustrative numerical example for the extended TOPSIS method is given in section (4).

In order to obtain a compromise solutions to the FMLMODM problems using the modified TOPSIS approach, a generalized formulas for the distance function from the PIS and the distance function from the NIS are proposed and modeled to include all the objective functions of all the levels. Thus, we propose an interactive decision making algorithm to find a compromise solutions through TOPSIS approach where the DM_{L_1} is asked to specify the membership function for each fuzzy parameter, the maximum negative and positive tolerance values, the power p of the distance functions, the degree α and the relative importance of the objectives. Then, the DM_{L_j}, j=2,3,….,h, is asked to specify the maximum negative and positive tolerance values, and the relative importance of the objectives.

Algorithm (I):

Phase (1):

Step 1:

(!-1): Let h = the number of the levels of the FMLMODM problem (1). Set j=1, "The 1st level".

(1-2): Ask the DM_{L_1} to specify a membership function for each fuzzy numbers $\widetilde{U}$ and $\widetilde{V}_{j\,k_j}$ *j= 1,2,…,h,* in the FMLMODM problem (1).

For example, the fuzzy numbers $\tilde{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)$ can have a membership function of the following form [13]:

$$\mu_{\tilde{\lambda}}(\lambda) = \begin{cases} 0, \lambda \le \lambda_1, \\ 1 - \left[\frac{(\lambda-\lambda_2)}{(\lambda_1-\lambda_2)}\right]^2, \lambda_1 \le \lambda \le \lambda_2, \\ 1, \lambda_2 \le \lambda \le \lambda_3, \\ 1 - \left[\frac{(\lambda-\lambda_3)}{(\lambda_4-\lambda_3)}\right]^2, \lambda_3 \le \lambda \le \lambda_4 \\ 0, \lambda \ge \lambda_4 \end{cases} \tag{5}$$

(1-3): Ask the DM_{L_1} to select *α= α*∈[0,1].*

(1-4): Transform the FMLMODM problem (1) to the form of α-MLMODM problem (4) by using steps (1-2) and (1-3).

Step 2:

Construct the PIS payoff table of the following problem:

$$[\alpha - \mathrm{DM}_{L_1}]$$

$$\underset{X_1}{Maximize/Minimize} \left(f_{11}(X_1, X_2, \ldots, X_h, V_{11}), \ldots\ldots, f_{1k_1}(X_1, X_2, \ldots, X_h, V_{1k_1})\right)$$

Subject to (6)

$$X \in M^{//}$$

and obtain $\left(f_{11}^{*^{\alpha - \mathrm{DM}\,L_1}}, f_{12}^{*^{\alpha - \mathrm{DM}\,L_1}}, \ldots, f_{1k_1}^{*^{\alpha - \mathrm{DM}\,L_1}}\right)$, the individual positive ideal solutions.

where $K_1 = \theta_{11} \cup \theta_{12}$,

$f_{1i_{max}}$, Objective Functions for Maximization, $i_{max} \in \theta_{11} \sqsubset K_1$, and

$f_{1i_{min}}$, Objective Functions for Minimization, $i_{min} \in \theta_{12} \sqsubset K_1$.

Step 3:

Construct the NIS payoff table of problem (6) and obtain $\left(f_{11}^{-^{\alpha - \mathrm{DM}\,L_1}}, f_{12}^{-^{\alpha - \mathrm{DM}\,L_1}}, \ldots, f_{1k_1}^{-^{\alpha - \mathrm{DM}\,L_1}}\right)$, the individual negative ideal solutions.

Step 4:

Use steps (2 and 3) to construct the distance function from the PIS and the distance function from the NIS:

$$d_P^{PIS^{\alpha - \mathrm{DM}\,L_1}} = \left(\sum_{i_{max} \in \theta_{11}} w_{i_{max}}^p \left(\frac{f_{1i_{max}}^{*^{\alpha - \mathrm{DM}\,L_1}} - f_{1i_{max}}^{\alpha - \mathrm{DM}\,L_1}(X)}{f_{1i_{max}}^{*^{\alpha - \mathrm{DM}\,L_1}} - f_{1i_{max}}^{-^{\alpha - \mathrm{DM}\,L_1}}}\right)^p + \sum_{i_{min} \in \theta_{12}} w_{i_{min}}^p \left(\frac{f_{1i_{min}}^{\alpha - \mathrm{DM}\,L_1}(X) - f_{1i_{min}}^{*^{\alpha - \mathrm{DM}\,L_1}}}{f_{1i_{min}}^{-^{\alpha - \mathrm{DM}\,L_1}} - f_{1i_{min}}^{*^{\alpha - \mathrm{DM}\,L_1}}}\right)^p\right)^{1/p} \quad (7\text{-}1)$$

and

$$d_P^{NIS^{\alpha - \mathrm{DM}\,L_1}} = \left(\sum_{i_{max} \in \theta_{11}} w_{i_{max}}^p \left(\frac{f_{1i_{max}}^{\alpha - \mathrm{DM}\,L_1}(X) - f_{1i_{max}}^{-^{\alpha - \mathrm{DM}\,L_1}}}{f_{1i_{max}}^{*^{\alpha - \mathrm{DM}\,L_1}} - f_{1i_{max}}^{-^{\alpha - \mathrm{DM}\,L_1}}}\right)^p + \sum_{i_{min} \in \theta_{12}} w_{i_{min}}^p \left(\frac{f_{1i_{min}}^{-^{\alpha - \mathrm{DM}\,L_1}} - f_{1i_{min}}^{\alpha - \mathrm{DM}\,L_1}(X)}{f_{1i_{min}}^{-^{\alpha - \mathrm{DM}\,L_1}} - f_{1i_{min}}^{*^{\alpha - \mathrm{DM}\,L_1}}}\right)^p\right)^{1/p} \quad (7\text{-}2)$$

Where $w_i, i = 1,2, \ldots, k_1$, are the relative importance (weighs) of objectives, and $p = 1,2, \ldots\ldots, \infty$.

Step 5:

Transfer the $\alpha - \mathrm{DM}_{L_1}$ problem (6) into the following bi-objective problem with two commensurable (but often conflicting) objectives:

$$Minimize\ d_p^{PIS^{\alpha - \mathrm{DM}\,L_1}}(X)$$

$$Maximize\ d_p^{NIS^{\alpha - \mathrm{DM}\,L_1}}(X)$$

Subject to (8)

$$X \in M^{//}$$

Where $p = 1,2, \ldots\ldots, \infty$.

Step 6:

(6-1): Ask the DM_{L_1} to select p= p* ∈ {1, 2, …, ∞ },

(6-2): Ask the DM_{L_1} to select $w_i = w_i{}^*, i = 1,2, \ldots, k_1$, where $\sum_{i=1}^{k_1} w_i = 1$,

Step 7: Use step (6) and equation (7) to compute $d_p^{PIS^{\alpha - \mathrm{DM}\,L_1}}$ and $d_p^{NIS^{\alpha - \mathrm{DM}\,L_1}}$.

Step 8: Construct the payoff table of problem (8) and obtain:

$$d_p^{-^{\alpha - \mathrm{DM}\,L_1}} = \left(\left(d_p^{PIS^{\alpha - \mathrm{DM}\,L_1}}\right)^-, \left(d_p^{NIS^{\alpha - \mathrm{DM}\,L_1}}\right)^-\right),$$

$$d_p^{*^{\alpha - \mathrm{DM}\,L_1}} = \left(\left(d_p^{PIS^{\alpha - \mathrm{DM}\,L_1}}\right)^*, \left(d_p^{NIS^{\alpha - \mathrm{DM}\,L_1}}\right)^*\right).$$

Step 9:

(9-1): Construct the following membership functions $\mu_1(X)$ and $\mu_2(X)$, (figure (2)):

$$\mu_1(X)=\mu_{d_p^{PIS^{\alpha-\mathrm{DM}\,L_1}}}(X)==\begin{cases}1,\ if\ d_p^{PIS^{\alpha-\mathrm{DM}\,L_1}}(X)<\left(d_p^{PIS^{\alpha-\mathrm{DM}\,L_1}}\right)^*,\\ 1-\dfrac{d_p^{PIS^{\alpha-\mathrm{DM}\,L_1}}(X)-\left(d_p^{PIS^{\alpha-\mathrm{DM}\,L_1}}\right)^*}{\left(d_p^{PIS^{\alpha-\mathrm{DM}\,L_1}}\right)^{/}-\left(d_p^{PIS^{\alpha-\mathrm{DM}\,L_1}}\right)^*},\ if\ \left(d_p^{PIS^{\alpha-\mathrm{DM}\,L_1}}\right)^-\ge d_p^{PIS^{\alpha-\mathrm{DM}\,L_1}}(X)\ge\left(d_p^{PIS^{\alpha-\mathrm{DM}\,L_1}}\right)^*,\\ 0,\ if\ d_p^{PIS^{\alpha-\mathrm{DM}\,L_1}}(X)>\left(d_p^{PIS^{\alpha-\mathrm{DM}\,L_1}}\right)^-,\end{cases}\quad (9\text{-}1)$$

$$\mu_2(X)=\mu_{d_p^{NIS^{\alpha-\mathrm{DM}\,L_1}}}(X)==\begin{cases}1,\ if d_p^{NIS^{\alpha-\mathrm{DM}_{L_1}}}(X)>\left(d_p^{NIS^{\alpha-\mathrm{DM}_{L_1}}}\right)^*,\\ 1-\dfrac{\left(d_p^{NIS^{\alpha-\mathrm{DM}_{L_1}}}\right)^*-d_p^{NIS^{\alpha-\mathrm{DM}_{L_1}}}(X)}{\left(d_p^{NIS^{\alpha-\mathrm{DM}_{L_1}}}\right)^*-\left(d_p^{NIS^{\alpha-\mathrm{DM}_{L_1}}}\right)^-},\ if\left(d_p^{NIS^{\alpha-\mathrm{DM}_{L_1}}}\right)^-\le d_p^{NIS^{\alpha-\mathrm{DM}_{L_1}}}(X)\le\left(d_p^{NIS^{\alpha-\mathrm{DM}_{L_1}}}\right)^*,\\ 0,\ if d_p^{NIS^{\alpha-\mathrm{DM}_{L_1}}}(X)<\left(d_p^{NIS^{\alpha-\mathrm{DM}_{L_1}}}\right)^-,\end{cases}\quad (9\text{-}2)$$

where $\left(d_p^{PIS^{\alpha-\mathrm{DM}_{L_1}}}\right)^*=\underset{X\in M^{//}}{Minimize}\ d_P^{PIS^{\alpha-\mathrm{DM}_{L_1}}}(X)$ *and the solution is* $X^{PIS^{\alpha-\mathrm{DM}_{L_1}}}$,

$\left(d_p^{NIS^{\alpha-\mathrm{DM}_{L_1}}}\right)^*=\underset{X\in M^{//}}{Maximize}\ d_P^{NIS^{\alpha-\mathrm{DM}_{L_1}}}(X)$ *and the solution is* $X^{NIS^{\alpha-\mathrm{DM}_{L_1}}}$,

$\left(d_p^{PIS^{\alpha-\mathrm{DM}_{L_1}}}\right)^-=d_p^{PIS^{\alpha-\mathrm{DM}_{L_1}}}\left(X^{NIS^{\alpha-\mathrm{DM}_{L_1}}}\right)$ *and* $\left(d_p^{NIS^{\alpha-\mathrm{DM}_{L_1}}}\right)^-=d_p^{NIS^{\alpha-\mathrm{DM}_{L_1}}}\left(X^{PIS^{\alpha-\mathrm{DM}_{L_1}}}\right)$.

(9-2): By using the max-min decision model, [9], the Tchebycheff model, [10], and the membership functions (9), construct the following satisfactory level model:

$$Maximize\ \delta^{\alpha-\mathrm{DM}_{L_1}},\quad (10\text{-}1)$$

subject to

$$\mu_1(X)\ge\delta^{\alpha-\mathrm{DM}\,L_1},\quad (10\text{-}2)$$

$$\mu_2(X)\ge\delta^{\alpha-\mathrm{DM}\,L_1},\quad (10\text{-}3)$$

$$X\in M^{//},\delta^{\alpha-\mathrm{DM}\,L_1}\in[0,1],\quad (10\text{-}4)$$

where $\delta^{\alpha-\mathrm{DM}\,L_1}$ is the satisfactory level for both criteria of the shortest distance from the PIS and the farthest distance from the NIS.

(9-3): Solve problem (10) to obtain the Pareto optimal solution $\left(\delta^{*^{\alpha-\mathrm{DM}\,L_1}},X^{*^{\alpha-\mathrm{DM}\,L_1}}\right)$, then $X^{*^{\alpha-\mathrm{DM}\,L_1}}$ is a nondominated solution of (8) and a compromise solution of the $\alpha-\mathrm{DM}_{L_1}$ problem (6).

(9-4): If the DM_{L_1} is satisfied with $X^{*^{\alpha-\mathrm{DM}\,L_1}}$, then go to step (10). Otherwise, go to step (6-2).

Step 10:

(10-1): Ask the DM_{L_1} to select the maximum acceptable negative and positive tolerance (relaxation) values, [18, 20], τ_i^L and $\tau_i^R, i=1,2,\dots,n_1$ on the decision vector, $X_1^{*^{\alpha-\mathrm{DM}\,L_1}}=\left(x_{11}^{*^{\alpha-\mathrm{DM}\,L_1}},x_{12}^{*^{\alpha-\mathrm{DM}\,L_1}},\dots.,x_{1n_1}^{*^{\alpha-\mathrm{DM}\,L_1}}\right)$.

(10-2): Construct the linear membership functions (Figure 3) for each of the n_1 components of the decision vector $\left(x_{11}^{*^{\alpha-\mathrm{DM}\,L_1}},x_{12}^{*^{\alpha-\mathrm{DM}\,L_1}},\dots.,x_{1n_1}^{*^{\alpha-\mathrm{DM}\,L_1}}\right)$ controlled by the DM_{L_1} can be formulated as:

$$\mu_{1i}(x_{1i})=\begin{cases}\dfrac{x_{1i}-\left(X_{1i}^{*^{\alpha-\mathrm{DM}\,L_1}}-\tau_i^L\right)}{\tau_i^L},\ if\ x_{1i}^{*^{\alpha-\mathrm{DM}\,L_1}}-\tau_i^L\le x_{1i}\le x_{1i}^{*^{\alpha-\mathrm{DM}\,L_1}}\\ \dfrac{\left(X_{1i}^{*^{\alpha-\mathrm{DM}\,L_1}}+\tau_i^R\right)-x_{1i}}{\tau_i^R},\ if\ x_{1i}^{*^{\alpha-\mathrm{DM}\,L_1}}\le x_{1i}\le x_{1i}^{*^{\alpha-\mathrm{DM}\,L_1}}+\tau_i^R, i=1,2,\dots,n_1,\\ 0, otherwise.\end{cases}\quad (11)$$

(10-3): Set j = j+1, go to the next phase.

$$\mu_{d_p^{PIS^{\alpha-\mathrm{DM}\,L_1}}}(X),\ \mu_{d_p^{NIS^{\alpha-\mathrm{DM}\,L_1}}}(X)$$

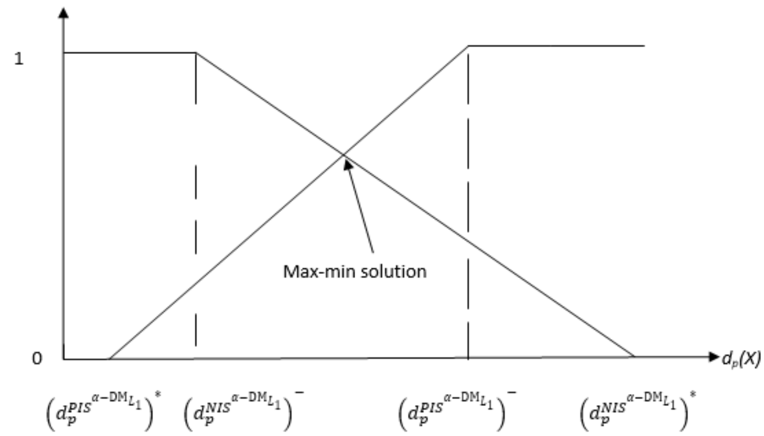

Figure (2). The membership functions of $\mu_{d_p^{PIS^{\alpha-\mathrm{DM}_{L_1}}}}(X)$ and $\mu_{d_p^{NIS^{\alpha-\mathrm{DM}_{L_1}}}}(X)$

Phase (2):

Step 11:

Set j = 2, "The 2nd level".

Construct the PIS payoff table of the following problem:

$$[\alpha - \mathrm{DM}_{L_2}]$$

$$\underset{X_2}{Maximize/Minimize}\left(f_{21}(X_1, X_2, \dots, X_h, V_{21}), \dots., f_{2k_2}(X_1, X_2, \dots, X_h, V_{2k_2})\right)$$

Subject to (12)

$$X \in M^{//}$$

and obtain $\left(f_{21}^{*^{\alpha-\mathrm{DM}_{L_2}}}, f_{22}^{*^{\alpha-\mathrm{DM}_{L_2}}}, \dots., f_{2k_2}^{*^{\alpha-\mathrm{DM}_{L_2}}}\right)$, the individual positive ideal solutions.

where $K_2 = \theta_{21} \cup \theta_{22}$,

$f_{2i_{max}}$, Objective Functions for Maximization, $i_{max} \in \theta_{21} \sqsubset K_2$, and

$f_{2i_{min}}$, Objective Functions for Minimization, $i_{min} \in \theta_{22} \sqsubset K_2$.

Step 12:

Construct the NIS payoff table of problem (12) and obtain $\left(f_{11}^{-^{\alpha-\mathrm{DM}_{L_2}}}, f_{12}^{-^{\alpha-\mathrm{DM}_{L_2}}}, \dots., f_{1k_i}^{-^{\alpha-\mathrm{DM}_{L_2}}}\right)$, theindividual negative ideal solutions.

Step 13:

Use steps (11 and 12) to construct the distance function from the PIS and the distance function from the NIS:

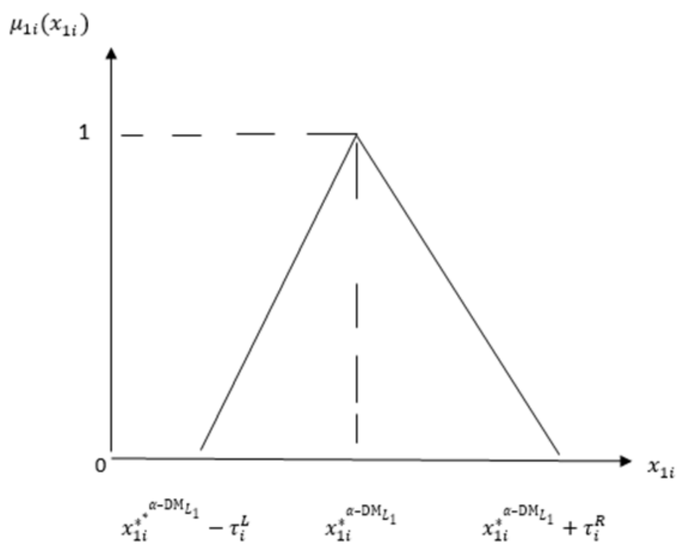

Figure (3). The membership function of the decision variable x_{1i}

$$d_P^{PIS^{\alpha-\mathrm{DM}\,L_2}} = \left(\sum_{i_{max}\in\theta_{11}} w_{i_{max}}^p \left(\frac{f_{1i_{max}}^{*\,\alpha-\mathrm{DM}\,L_1} - f_{1i_{max}}^{\alpha-\mathrm{DM}\,L_1}(X)}{f_{1i_{max}}^{*\,\alpha-\mathrm{DM}\,L_1} - f_{1i_{max}}^{-\,\alpha-\mathrm{DM}\,L_1}} \right)^p + \sum_{i_{min}\in\theta_{12}} w_{i_{min}}^p \left(\frac{f_{1i_{min}}^{\alpha-\mathrm{DM}\,L_1}(X) - f_{1i_{min}}^{*\,\alpha-\mathrm{DM}\,L_1}}{f_{1i_{min}}^{-\,\alpha-\mathrm{DM}\,L_1} - f_{1i_{min}}^{*\,\alpha-\mathrm{DM}\,L_1}} \right)^p + \sum_{i_{max}\in\theta_{21}} w_{i_{max}}^p \left(\frac{f_{2i_{max}}^{*\,\alpha-\mathrm{DM}L_2} - f_{2i_{max}}^{\alpha-\mathrm{DM}L_2}(X)}{f_{2i_{max}}^{*\,\alpha-\mathrm{DM}L_2} - f_{2i_{max}}^{-\,\alpha-\mathrm{DM}L_2}} \right)^p + \sum_{i_{min}\in\theta_{22}} w_{i_{min}}^p \left(\frac{f_{2i_{min}}^{\alpha-\mathrm{DM}L_2}(X) - f_{2i_{min}}^{*\,\alpha-\mathrm{DM}L_2}}{f_{2i_{min}}^{-\,\alpha-\mathrm{DM}L_2} - f_{2i_{min}}^{*\,\alpha-\mathrm{DM}L_2}} \right)^p \right)^{1/p} \quad (13\text{-}1)$$

and

$$d_P^{NIS^{\alpha-\mathrm{DM}\,L_1}} = \left(\sum_{i_{max}\in\theta_{11}} w_{i_{max}}^p \left(\frac{f_{1i_{max}}^{\alpha-\mathrm{DM}\,L_1}(X) - f_{1i_{max}}^{-\,\alpha-\mathrm{DM}\,L_1}}{f_{1i_{max}}^{*\,\alpha-\mathrm{DM}\,L_1} - f_{1i_{max}}^{-\,\alpha-\mathrm{DM}\,L_1}} \right)^p + \sum_{i_{min}\in\theta_{12}} w_{i_{min}}^p \left(\frac{f_{1i_{min}}^{-\,\alpha-\mathrm{DM}\,L_1} - f_{1i_{min}}^{\alpha-\mathrm{DM}\,L_1}(X)}{f_{1i_{min}}^{-\,\alpha-\mathrm{DM}\,L_1} - f_{1i_{min}}^{*\,\alpha-\mathrm{DM}\,L_1}} \right)^p + \sum_{i_{max}\in\theta_{21}} w_{i_{max}}^p \left(\frac{f_{2i_{max}}^{\alpha-\mathrm{DM}L_2}(X) - f_{2i_{max}}^{-\,\alpha-\mathrm{DM}L_2}}{f_{2i_{max}}^{*\,\alpha-\mathrm{DM}L_1} - f_{2i_{max}}^{-\,\alpha-\mathrm{DM}L_1}} \right)^p + \sum_{i_{min}\in\theta_{22}} w_{i_{min}}^p \left(\frac{f_{2i_{min}}^{-\,\alpha-\mathrm{DM}L_2} - f_{2i_{min}}^{\alpha-\mathrm{DM}L_2}(X)}{f_{2i_{min}}^{-\,\alpha-\mathrm{DM}L_2} - f_{2i_{min}}^{*\,\alpha-\mathrm{DM}L_2}} \right)^p \right)^{1/p} \quad (13\text{-}2)$$

where $w_i, i = 1,2,\ldots.,k_1 + k_2$, are the relative importance (weighs) of objectives, and $p = 1,2,\ldots\ldots,\infty$.

Step 14:

Transfer the $\alpha - \mathrm{DM}_{L_2}$ problem (12) into the following bi-objective problem with two commensurable (but often conflicting) objectives:

$$Minimize\ d_p^{PIS^{\alpha-\mathrm{DM}\,L_2}}(X),\ Maximize\ d_p^{NIS^{\alpha-\mathrm{DM}\,L_2}}(X)$$

Subject to (14)

$$X \in M^{//}$$

where $p = 1,2,\ldots\ldots,\infty$.

Step 15:

(15-1): Ask the DM_{L_2} to select p= $p^* \in \{1, 2, \ldots, \infty\}$,

(15-2): Ask the DM_{L_2} to select $w_i = w_i{}^*, i = 1,2,\ldots,k_1 + k_2$, where $\sum_{i=1}^{k_1+k_2} w_i = 1$,

Step 16: Use step (12) and equation (13) to compute $d_p^{PIS^{\alpha-\mathrm{DM}\,L_2}}$ and $d_p^{NIS^{\alpha-\mathrm{DM}\,L_2}}$.

Step 17: Construct the payoff table of problem (14) and obtain:

$$d_p^{-\,\alpha-\mathrm{DM}\,L_2} = \left(\left(d_p^{PIS^{\alpha-\mathrm{DM}\,L_2}}\right)^-, \left(d_p^{NIS^{\alpha-\mathrm{DM}\,L_2}}\right)^-\right), d_p^{*\,\alpha-\mathrm{DM}\,L_2} = \left(\left(d_p^{PIS^{\alpha-\mathrm{DM}\,L_2}}\right)^*, \left(d_p^{NIS^{\alpha-\mathrm{DM}\,L_2}}\right)^*\right).$$

Step 18:

(18-1): Construct the following membership functions $\mu_3(X)$ and $\mu_4(X)$:

$$\mu_3(X) = \mu_{d_p^{PIS^{\alpha-\mathrm{DM}\,L_2}}}(X) = \begin{cases} 1, if\ d_p^{PIS^{\alpha-\mathrm{DM}\,L_2}}(X) < \left(d_p^{PIS^{\alpha-\mathrm{DM}\,L_2}}\right)^*, \\ 1 - \dfrac{d_p^{PIS^{\alpha-\mathrm{DM}\,L_2}}(X) - \left(d_p^{PIS^{\alpha-\mathrm{DM}\,L_2}}\right)^*}{\left(d_p^{PIS^{\alpha-\mathrm{DM}\,L_2}}\right)^/ - \left(d_p^{PIS^{\alpha-\mathrm{DM}\,L_2}}\right)^*}, if\ \left(d_p^{PIS^{\alpha-\mathrm{DM}\,L_2}}\right)^- \geq d_p^{PIS^{\alpha-\mathrm{DM}\,L_2}}(X) \geq \left(d_p^{PIS^{\alpha-\mathrm{DM}\,L_2}}\right)^*, \\ 0, if\ d_p^{PIS^{\alpha-\mathrm{DM}\,L_2}}(X) > \left(d_p^{PIS^{\alpha-\mathrm{DM}\,L_2}}\right)^-, \end{cases} \quad (15\text{-}1)$$

$$\mu_4(X) = \mu_{d_p^{NIS^{\alpha-\mathrm{DM}\,L_2}}}(X) = \begin{cases} 1, if d_p^{NIS^{\alpha-\mathrm{DM}\,L_2}}(X) > \left(d_p^{NIS^{\alpha-\mathrm{DM}\,L_2}}\right)^*, \\ 1 - \dfrac{\left(d_p^{NIS^{\alpha-\mathrm{DM}\,L_2}}\right)^* - d_p^{NIS^{\alpha-\mathrm{DM}\,L_2}}(X)}{\left(d_p^{NIS^{\alpha-\mathrm{DM}\,L_2}}\right)^* - \left(d_p^{NIS^{\alpha-\mathrm{DM}\,L_2}}\right)^-}, if\ \left(d_p^{NIS^{\alpha-\mathrm{DM}\,L_2}}\right)^- \leq d_p^{NIS^{\alpha-\mathrm{DM}\,L_2}}(X) \leq \left(d_p^{NIS^{\alpha-\mathrm{DM}\,L_2}}\right)^*, \\ 0, if d_p^{NIS^{\alpha-\mathrm{DM}\,L_2}}(X) < \left(d_p^{NIS^{\alpha-\mathrm{DM}\,L_2}}\right)^-, \end{cases} \quad (15\text{-}2)$$

Where

$$\left(d_p^{PIS^{\alpha-\mathrm{DM}\,L_2}}\right)^* = \underset{X\in M^{//}}{Minimize}\ d_P^{PIS^{\alpha-\mathrm{DM}\,L_2}}(X)\ and\ the\ solution\ is\ X^{PIS^{\alpha-\mathrm{DM}\,L_2}},$$

$$\left(d_p^{NIS^{\alpha-\mathrm{DM}\,L_2}}\right)^* = \underset{X\in M^{//}}{Maximize}\, d_P^{NIS^{\alpha-\mathrm{DM}\,L_2}}(X) \text{ and the solution is } X^{NIS^{\alpha-\mathrm{DM}\,L_2}},$$

$$\left(d_p^{PIS^{\alpha-\mathrm{DM}\,L_2}}\right)^- = d_p^{PIS^{\alpha-\mathrm{DM}\,L_2}}\left(X^{NIS^{\alpha-\mathrm{DM}\,L_2}}\right) \text{ and } \left(d_p^{NIS^{\alpha-\mathrm{DM}\,L_2}}\right)^- = d_p^{NIS^{\alpha-\mathrm{DM}\,L_2}}\left(X^{PIS^{\alpha-\mathrm{DM}\,L_2}}\right)$$

(18-2): By using the max-min decision model [9], the Tchebycheff model, [10], and the membership functions (15), construct the following satisfactory level model:

$$Maximize\ \delta^{\alpha-\mathrm{DM}\,L_2}, \tag{16-1}$$

Subject to

$$\mu_3(X) \geq \delta^{\alpha-\mathrm{DM}\,L_2}, \tag{16-2}$$

$$\mu_4(X) \geq \delta^{\alpha-\mathrm{DM}\,L_2}, \tag{16-3}$$

$$X \in M^{//}, \delta^{\alpha-\mathrm{DM}\,L_2} \in [0,1], \tag{16-4}$$

$$\frac{x_{1i}-\left(X_{1i}^{*\,\alpha-\mathrm{DM}\,L_1}-\tau_i^L\right)}{\tau_i^L} \geq \delta^{\alpha-\mathrm{DM}\,L_2}, i = 1,2,\dots,n_1 \tag{16-5}$$

$$\frac{\left(X_{1i}^{**\,\alpha-\mathrm{DM}\,L_1}+\tau_i^R\right)-x_{1i}}{\tau_i^R} \geq \delta^{\alpha-\mathrm{DM}\,L_2}, i = 1,2,\dots,n_1 \tag{16-6}$$

Where $\delta^{\alpha-\mathrm{DM}\,L_2}$ is the satisfactory level for both criteria of the shortest distance from the PIS and the farthest distance from the NIS.

(18-3): Solve problem (16) to obtain the Pareto optimal solution $\left(\delta^{*^{\alpha-\mathrm{DM}\,L_2}}, X^{*^{\alpha-\mathrm{DM}\,L_2}}\right)$, then $X^{*^{\alpha-\mathrm{DM}\,L_2}}$ is a non-dominated solution of (14) and a compromise solution of the $\alpha-\mathrm{DM}_{L_2}$ problem (12).

(18-4): If the DM_{L_2} is satisfied with $X^{*^{\alpha-\mathrm{DM}\,L_2}}$, then go to step (19). Otherwise, go to step (15-2).

Step 19: If h =j, stop. Otherwise, go to step (20).

Step 20:

(20-1): Ask the DM_{L_1} to select the maximum acceptable negative and positive tolerance (relaxation) values τ_i^L and $\tau_i^R, i = 1,2,\dots,n_2$ on the decision vector, $X_2^{*^{\alpha-\mathrm{DM}\,L_2}} = \left(x_{21}^{*^{\alpha-\mathrm{DM}\,L_2}}, x_{22}^{*^{\alpha-\mathrm{DM}\,L_2}}, \dots, x_{2n_2}^{*^{\alpha-\mathrm{DM}\,L_2}}\right)$.

(20-2): Construct the linear membership functions for each of the n_2 components of the decision vector $\left(x_{21}^{*^{\alpha-\mathrm{DM}\,L_2}}, x_{22}^{*^{\alpha-\mathrm{DM}\,L_2}}, \dots, x_{2n_2}^{*^{\alpha-\mathrm{DM}\,L_2}}\right)$ controlled by the DM_{L_1} can be formulated as:

$$\mu_{2i}(x_{2i}) = \begin{cases} \dfrac{x_{2i}-\left(X_{2i}^{*^{\alpha-\mathrm{DM}\,L_2}}-\tau_i^L\right)}{\tau_i^L}, if\ x_{2i}^{*^{\alpha-\mathrm{DM}\,L_2}}-\tau_i^L \leq x_{2i} \leq x_{2i}^{*^{\alpha-\mathrm{DM}\,L_2}} \\ \dfrac{\left(X_{2i}^{*^{\alpha-\mathrm{DM}\,L_2}}+\tau_i^R\right)-x_{2i}}{\tau_i^R}, if\ x_{2i}^{*^{\alpha-\mathrm{DM}\,L_2}} \leq x_{2i} \leq x_{2i}^{*^{\alpha-\mathrm{DM}\,L_2}}+\tau_i^R, i = 1,2,\dots,n_2, \\ 0, otherwise. \end{cases} \tag{17}$$

(20-3): Set j = j+1, go to the next phase.

4. Illustrative Numerical Example for Algorithm (I)

Consider the following FMLMODM problem:

$$[\mathrm{DM}_{L_1}]$$

$$\underset{x_1,x_2}{Maximize} f_{11}(X) = 6\tilde{u}_{11}x_1 + 7x_2 + 3x_3 + 5x_4 + x_5 + x_6$$

$$\underset{x_1,x_2}{Minimize} f_{12}(X) = 3\tilde{u}_{12}x_1 + 4x_2 + 2x_3 + 3x_4 + 2x_5 + x_6$$

where x_1 *and* x_2 *solves the second level*

$$[\mathrm{DM}_{L_2}]$$

$$\underset{x_3,x_4}{Maximize} f_{21}(X) = 13x_1 + 3\tilde{u}_{21}x_2 + 5x_3 + 2x_4 + x_5 + 2x_6$$

$$\underset{x_3,x_4}{Minimize} f_{22}(X) = 10x_1 + 7\tilde{u}_{22}x_2 + 4x_3 + 6x_4 + 2x_5 + 3x_6$$

where x_3 *and* x_4 *solves the third level*

$[\mathrm{DM}_{L_3}]$

$$\underset{x_5,x_6}{\mathit{Maximize}} f_{31}(X) = 12x_1 + 5x_2 + 6\tilde{u}_{31}x_3 + 5x_4 + x_5 + x_6$$

$$\underset{x_5,x_6}{\mathit{Minimize}} f_{32}(X) = 9x_1 + 4x_2 + 5\tilde{u}_{32}x_3 + 4x_4 + 3x_5 + 2x_6$$

Subject to

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \le 50, x_1 + x_2 \le 10, x_2 \le 8\tilde{y}, 5x_3 + x_4 \le 12,$$

$$x_5 + x_6 \ge 5, x_5 + 5x_6 \le 50, x_1, x_2, x_3, x_4, x_5, x_6 \ge 0$$

where

$$\tilde{u}_{11} = (0,1,3,5), \tilde{u}_{12} = (1,6,7,8), \tilde{u}_{21} = (0,5,7,10), \tilde{u}_{22} = (0,2,4,6),$$

$\tilde{u}_{31}$= (1,6,7,9), $\tilde{u}_{32}$= (2,7,8,10), ỹ=(1,6,7,8) and let α =0.59.

Use the introduced algorithm in subsection (3-1) to solve the above problem.

Solution:

- h= 3, j=1,
- Use the membership function (5) to convert the above problem to the following α-MLMODM problem,

$[\propto -\mathrm{DM}_{L_1}]$

$$\underset{x_1,x_2}{\mathit{Maximize}} f_{11}(X) = 6u_{11}x_1 + 7x_2 + 3x_3 + 5x_4 + x_5 + x_6$$

$$\underset{x_1,x_2}{\mathit{Minimize}} f_{12}(X) = 3u_{12}x_1 + 4x_2 + 2x_3 + 3x_4 + 2x_5 + x_6$$

where x_1 and x_2 solves the second level

$[\propto -\mathrm{DM}_{L_2}]$

$$\underset{x_3,x_4}{\mathit{Maximize}} f_{21}(X) = 13x_1 + 3u_{21}x_2 + 5x_3 + 2x_4 + x_5 + 2x_6$$

$$\underset{x_3,x_4}{\mathit{Minimize}} f_{22}(X) = 10x_1 + 7u_{22}x_2 + 4x_3 + 6x_4 + 2x_5 + 3x_6$$

where x_3 and x_4 solves the third level

$[\propto -\mathrm{DM}_{L_3}]$

$$\underset{x_5,x_6}{\mathit{Maximize}} f_{31}(X) = 12x_1 + 5x_2 + 6u_{31}x_3 + 5x_4 + x_5 + x_6$$

$$\underset{x_5,x_6}{\mathit{Minimize}} f_{32}(X) = 9x_1 + 4x_2 + 5u_{32}x_3 + 4x_4 + 3x_5 + 2x_6$$

Subject to

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \le 50,\ x_1 + x_2 \le 10, x_2 \le 8\mathrm{y}, 5x_3 + x_4 \le 12, x_5 + x_6 \ge 5, x_5 + 5x_6 \le 50,$
$x_1, x_2, x_3, x_4, x_5, x_6 \ge 0$, 0.36 $\le u_{11} \le$ 4.282.8 $\le u_{12} \le$ 4.64, 1.8≤ $u_{21} \le$ 8.92, 0.72≤ u_{22} ≤5.28,
2.8≤ u_{31} ≤8.28, 3.8≤ u_{32} ≤9.28, 2.8≤y≤ 8.28.

- Obtain *PIS* and *NIS* payoff tables for the $[\propto -\mathrm{DM}_{L_1}]$ of the Problem:

Table (1). PIS payoff table for $[\propto -\mathrm{DM}_{L_1}]$ the problem

	$f_{11}(X)$	$f_{12}(X)$	x_1	x_2	x_3	x_4	x_5	x_6	u_{11}	u_{12}	u_{21}	u_{22}	u_{31}	u_{32}	y
$\underset{x_1,x_2}{Max} f_{11}(X)$	344.8	174.765	10	0	0	12	26.765	1.235	4.28	2.8	1.8	0.72	2.8	3.8	8.28
$\underset{x_1,x_2}{Min} f_{12}(X)$	5	5	0	0	0	0	0	5	0.36	2.8	1.8	0.72	2.8	3.8	8.28

PIS: $f^{*^{\propto-\mathrm{DM}_{L_1}}}$ = (344.8, 5)

Table (2). NIS payoff table for $[\propto -\mathrm{DM}_{L_1}]$ the problem

	$f_{11}(X)$	$f_{12}(X)$	x_1	x_2	x_3	x_4	x_5	x_6	u_{11}	u_{12}	u_{21}	u_{22}	u_{31}	u_{32}	y
$\underset{x_1,x_2}{Min} f_{11}(X)$	5	8.765	0	0	0	0	3.765	1.235	0.36	2.8	1.8	0.72	2.8	3.8	8.28
$\underset{x_1,x_2}{Max} f_{12}(X)$	137.6	321.2	10	0	0	12	28	0	0.36	7.64	1.8	0.72	2.8	3.8	8.28

NIS: $f^{-^{\propto-DM_{L_1}}}$ = (5, 321.2)

- Next, construct equation and obtain the following equations:

$$d_P^{PIS^{\propto-\mathrm{DM}\,L_1}} = \left[w_1^p\left(\frac{344.8 - f_{11}(X)}{344.8-5}\right)^p + w_2^p\left(\frac{f_{12}(x) - (5)}{321.2-(5)}\right)^p\right]^{1/p}$$

$$d_P^{NIS^{\propto-\mathrm{DM}\,L_1}} = \left[w_1^p\left(\frac{f_{11}(X) - 5}{344.8-5}\right)^p + w_2^p\left(\frac{321.2 - f_{12}(x)}{321.2-(5)}\right)^p\right]^{1/p}$$

- Thus, problem is obtained. In order to get numerical solutions, assume that $w_1^p = w_2^p = 0.5$ and $p=2$,

Table (3). PIS payoff table of problem [$\propto -\mathrm{DM}_{L_1}$] when p=2

	$d_2^{PIS^{\propto-\mathrm{DM}_{L_1}}}$	$d_2^{NIS^{\propto-\mathrm{DM}_{L_1}}}$	$f_{11}(X)$	$f_{12}(X)$	x_1	x_2	x_3	x_4	x_5	x_6	u_{11}	u_{12}	u_{21}	u_{22}	u_{31}
$d_2^{PIS^{\propto-\mathrm{DM}_{L_1}}}$	0.251	0.756	279.314	99.509	10	0	0	3.503	0	5	4.28	2.8	1.8	1.235	2.8
$d_2^{NIS^{\propto-\mathrm{DM}_{L_1}}}$	0.707	0.707	5	5	0	0	0	0	0	5	1.235	2.8	1.8	1.235	2.8

$$d_2^{*^{\propto-\mathrm{DM}\,L_1}} = (0.2514704,\ 0.7560625088),\quad d_2^{-^{\propto-\mathrm{DM}\,L_1}} = (0.7071067812,\ 0.7071068).$$

- Now, it is easy to compute problem (15):

$$Maximize\ \delta^{\propto-\mathrm{DM}\,L_1}$$

Subject to

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \le 50, x_1 + x_2 \le 10, x_2 \le 8\tilde{y}, 5x_3 + x_4 \le 12,\ x_5 + x_6 \ge 5, x_5 + 5x_6 \le 50, x_1, x_2, x_3,$
$x_4, x_5, x_6 \ge 0$, $0.36 \le u_{11} \le 4.28$, $2.8 \le u_{12} \le 4.64$, $1.8 \le u_{21} \le 8.92$, $0.72 \le u_{22} \le 5.28$,
$2.8 \le u_{31} \le 8.28$, $3.8 \le u_{32} \le 9.28$, $2.8 \le y \le 8.28$

$$\left(\frac{d_2^{PIS^{\propto-\mathrm{DM}\,L_1}}(X) - 0.2514704}{0.7071067812 - 0.2514704}\right) \ge \delta^{\mathrm{FLDM}},\ \left(\frac{0.7560625088 - d_2^{NIS^{\propto-\mathrm{DM}\,L_1}}(X)}{0.7560625088 - 0.7071068}\right) \ge \delta^{\mathrm{FLDM}},\ \ \delta^{\mathrm{FLDM}} \in [0,1].$$

- The maximum "satisfactory level" ($\delta^{\propto-\mathrm{DM}\,L_1}=1$) is achieved for the solution $X_1^{*^{\propto-\mathrm{DM}\,L_1}}=0.119803$, $X_2^{*^{\propto-\mathrm{DM}\,L_1}}=$zero, $X_3^{*^{\propto-\mathrm{DM}\,L_1}}=$zero, $X_4^{*^{\propto-\mathrm{DM}\,L_1}}=$zero, $X_5^{*^{\propto-\mathrm{DM}\,L_1}}=3.780026$, $X_6^{*^{F\propto-\mathrm{DM}\,L_1}}=1.219974$

- Let the DM_{L_1} decide $X_1^{*^{\propto-\mathrm{DM}\,L_1}}=0.119803$ with positive tolerance $\tau^R = 0.001$ and $\tau^l = 0.001$ and $X_2^{*^{\propto-\mathrm{DM}\,L_1}}=$zero with positive tolerance $\tau^R = 0.3$ and $\tau^l = 0.3$.

- j=2. Obtain *PIS* and *NIS* payoff tables for the $\propto -\mathrm{DM}_{L_2}$ Problem.

Table (4). *PIS*payoff table for the [$\propto -\mathrm{DM}_{L_2}$] problem

	$f_{21}(X)$	$f_{22}(X)$	x_1	x_2	x_3	x_4	x_5	x_6	u_{11}	u_{12}	u_{21}	u_{22}	u_{31}	u_{32}	y
$\underset{x_1,x_2}{Max} f_{21}(X)$	325.1	182.9	0	10	0	12	22	5.5	0.36	2.8	8.92	0.72	2.8	3.8	8.28
$\underset{x_1,x_2}{Min} f_{22}(X)$	5	10	0	0	0	0	5	0	0.36	2.8	1.8	0.72	2.8	3.8	8.28

PIS: $f^{*^{\propto-\mathrm{DM}\,L_2}}$ = (325.1, 10)

Table (5). *NIS* payoff table for the [$\propto -\mathrm{DM}_{L_2}$] problem

	$f_{21}(X)$	$f_{22}(X)$	x_1	x_2	x_3	x_4	x_5	x_6	u_{11}	u_{12}	u_{21}	u_{22}	u_{31}	u_{32}	y
$\underset{x_1,x_2}{Min} f_{21}(X)$	5	5	0	0	0	0	5	0	0.36	2.8	1.8	0.72	2.8	3.8	8.28
$\underset{x_1,x_2}{Max} f_{22}(X)$	111	503.1	0	10	0	12	22	5.5	0.36	7.64	1.8	5.28	2.8	3.8	8.28

NIS: $f^{-^{\propto-\mathrm{DM}\,L_2}}$ = (5, 503.1)

- Next, compute and obtain the following equations:

$$d_P^{PIS^{\propto-\mathrm{DM}\,L_2}} = \left[w_1^p\left(\frac{344.8-f_{11}(X)}{344.8-5}\right)^p + w_2^p\left(\frac{f_{12}(x)-\ (5)}{321.2-(5)}\right)^p + w_3^p\left(\frac{325-f_{21}(X)}{325-5}\right)^p + w_4^p\left(\frac{f_{22}(X)-\ (10)}{503.1-(10)}\right)^p\right]^{1/p}$$

$$d_P^{NIS^{\propto-\mathrm{DM}\,L_2}} = \left[w_1^p\left(\frac{f_{11}(X)-5}{344.8-5}\right)^p + w_2^p\left(\frac{321.2-\ f_{12}(x)}{321.2-(5)}\right)^p + w_3^p\left(\frac{f_{21}(X)-5}{325-5}\right)^p + w_4^p\left(\frac{503.1-\ f_{22}(X)}{503.1-(10)}\right)^p\right]^{1/p}$$

- Thus, problem is obtained. In order to get numerical solutions, assume that $w_1^p = w_2^p = w_3^p = w_4^p = 0.25$ and *p=2*,

Table (6). PIS payoff table of problem [$\propto-\mathrm{DM}_{L_2}$] when p=2

	$d_2^{PIS^{\propto-\mathrm{DM}_{L_2}}}$	$d_2^{NIS^{\propto-\mathrm{DM}_{L_2}}}$	$f_{21}(X)$	$f_{22}(X)$	x_1	x_2	x_3	x_4	x_5	x_6	u_{11}	u_{12}	u_{21}	u_{22}	u_{31}
$Min.\, d_2^{PIS^{\propto-\mathrm{DM}_{L_2}}}$	0.311	0.695	288.945	139.715	5.649	4.351	0.984	7.079	0	10	4.28	2.8	8.92	0.72	2.8
$Max.\, d_2^{NIS^{\propto-\mathrm{DM}_{L_2}}}$	0.492	0.704	10	15	0	0	0	0	0	5	4.28	2.8	8.92	0.72	2.8

$d_2^{*\,\propto-\mathrm{DM}\,L_2}$=(0.311, 0.7035743), $d_2^{-\,\propto-\mathrm{DM}\,L_2}$=(0.4922742398, 0.6947074141).

- Now, it is easy to compute:

$$Maximize\ \delta^{\propto-\mathrm{DM}\,L_2}$$

subject to

$x_1+x_2+x_3+x_4+x_5+x_6 \le 50, x_1+x_2 \le 10,\ x_2 \le 8y, 5x_3+x_4 \le 12, x_5+x_6 \ge 5, x_5+5x_6 \le 50,$

$x_1, x_2, x_3, x_4, x_5, x_6 \ge 0$, 0.36 $\le u_{11} \le$ 4.28, 2.8$\le u_{12} \le$ 4.64, 1.8$\le u_{21} \le$ 8.92, 0.72$\le u_{22} \le$5.28,

2.8$\le u_{31} \le$8.28, 3.8$\le u_{32} \le$9.28, 2.8$\le$y$\le$ 8.28

$$\left(\frac{d_2^{PIS^{\propto-\mathrm{DM}\,L_2}}(X)-0.311}{0.4922742398-0.311}\right) \ge \delta^{\propto-\mathrm{DM}\,L_2}, \left(\frac{0.7035743-d_2^{NIS^{\propto-\mathrm{DM}\,L_2}}(X)}{0.7035743-0.6974074141}\right) \ge \delta^{\propto-\mathrm{DM}\,L_2},$$

$$\left(\frac{(0.119803+0.001)-x_1}{0.001}\right) \ge \delta^{\propto-\mathrm{DM}\,L_2},\ \left(\frac{x_1-(0.119803-0.001)}{0.001}\right) \ge \delta^{\propto-\mathrm{DM}\,L_2}$$

$$\left(\frac{(0+0.3)-x_1}{0.3}\right) \ge \delta^{\propto-\mathrm{DM}\,L_2},\ \left(\frac{x_1-(0-0.3)}{0.3}\right) \ge \delta^{\propto-\mathrm{DM}\,L_2},\ \delta^{\propto-\mathrm{DM}\,L_2} \in [0,1]\,.$$

- The maximum "satisfactory level" ($\delta^{\propto-\mathrm{DM}\,L_2}$=1) is achieved for the solution $X_1^{*\,\propto-\mathrm{DM}\,L_2}$=0.119803, $X_2^{*\,\propto-\mathrm{DM}\,L_2}$=zero, $X_3^{*\,\propto-\mathrm{DM}\,L_2}$ =1.234568, $X_4^{*\,\propto-\mathrm{DM}\,L_2}$ =1.234568, $X_5^{*\,\propto-\mathrm{DM}\,L_2}$ =3.765432, $X_6^{*\,\propto-\mathrm{DM}\,L_2}$ =1.234568, Let the DM_{L_2} decide $X_3^{*\,\propto-\mathrm{DM}\,L_2}$=1.234568, $X_4^{*\,\propto-\mathrm{DM}\,L_2}$=1.234568 with positive tolerance $\tau^R = 0.3$ and $\tau^l = 0.3$.
- j=3. Obtain *PIS* and *NIS* payoff tables for the [$\propto-\mathrm{DM}_{L_3}$] Problem:

Table (7). PIS payoff table for the $\propto-\mathrm{DM}_{L_3}$ problem

	$f_{31}(X)$	$f_{32}(X)$	x_1	x_2	x_3	x_4	x_5	x_6	u_{11}	u_{12}	u_{21}	u_{22}	u_{31}	u_{32}	y
$\underset{x_1,x_2}{Max} f_{31}(X)$	276.832	239.965	10	0	2.4	0	36.365	1.235	0.36	2.8	1.8	0.72	8.28	3.8	8.28
$\underset{x_1,x_2}{Min} f_{32}(X)$	5	10	0	0	0	0	0	5	0.36	2.8	1.8	0.72	2.8	3.8	8.28

PIS: $f^{*\,\propto-\mathrm{DM}\,L_3}$= (276.832, 10)

Table (8). NIS payoff table for the [$\propto-\mathrm{DM}_{L_3}$] problem

	$f_{31}(X)$	$f_{32}(X)$	x_1	x_2	x_3	x_4	x_5	x_6	u_{11}	u_{12}	u_{21}	u_{22}	u_{31}	u_{32}	y
$\underset{x_1,x_2}{Min} f_{31}(X)$	5	13.765	0	0	0	0	3.765	1.235	0.36	2.8	1.8	0.72	2.8	3.8	8.28
$\underset{x_1,x_2}{Max} f_{32}(X)$	197.92	314.16	10	0	2.4	0	37.6	0	0.36	2.8	1.8	0.72	2.8	9.28	8.28

NIS: $f^{-\,\propto-\mathrm{DM}\,L_3}$= (5, 314.6)

- Next, compute and obtain the following equations:

$$d_P^{PIS^{\propto-\mathrm{DM}_{L3}}} = \left[w_1^p \left(\frac{344.8 - f_{11}(X)}{344.8-5}\right)^p + w_2^p \left(\frac{f_{12}(x) - (5)}{321.2-(5)}\right)^p + w_3^p \left(\frac{325 - f_{21}(X)}{325-5}\right)^p + w_4^p \left(\frac{f_{22}(X) - (10)}{503.1-(10)}\right)^p + w_5^p \left(\frac{276.832 - f_{31}(X)}{276.832-5}\right)^p + w_6^p \left(\frac{f_{32}(X) - (10)}{314.6-(10)}\right)^p \right]^{1/p}$$

$$d_P^{NIS^{\propto-\mathrm{DM}_{L3}}} = \left[w_1^p \left(\frac{f_{11}(X) - 5}{344.8-5}\right)^p + w_2^p \left(\frac{321.2 - f_{12}(x)}{321.2-(5)}\right)^p + w_3^p \left(\frac{f_{21}(X) - 5}{325-5}\right)^p + w_4^p \left(\frac{503.1 - f_{22}(X)}{503.1-(10)}\right)^p + w_5^p \left(\frac{f_{31}(X) - 5}{276.832-5}\right)^p + w_6^p \left(\frac{314.6 - f_{32}(X)}{314.6-(10)}\right)^p \right]^{1/p}$$

- Thus, problem is obtained. In order to get numerical solutions, assume that $w_1^p = w_2^p = w_3^p = w_4^p == w_5^p = w_6^p = 1/6$ and $p=2$

Table (9). PIS payoff table of problem [$\propto -\mathrm{DM}_{L_3}$], when p=2

	$d_2^{PIS^{\propto-\mathrm{DM}_{L3}}}$	$d_2^{NIS^{\propto-\mathrm{DM}_{L3}}}$	$f_{31}(X)$	$f_{32}(X)$	x_1	x_2	x_3	x_4	x_5	x_6	u_{11}	u_{12}	u_{21}	u_{22}	u_{31}	u_{32}	y
$Min.\, d_3^{PIS^{\propto-\mathrm{DM}_{L_3}}}$	0.319	0.735	219.591	127.999	6.479	3.520	2.4	0	0	5	4.28	2.8	8.92	0.72	2.8	3.8	2.8
$Max.\, d_3^{NIS^{\propto-\mathrm{DM}_{L_3}}}$	0.862	0.705	5	10	0	0	0	0	0	5	1.235	2.8	8.92	0.72	2.8	3.8	2.8

$$d_2^{*\,\propto-\mathrm{DM}_{L3}} = (0.3194993,\ 0.7365085114),\ d_2^{-\,\propto-\mathrm{DM}_{L3}} = (0.8615533876,\ 0.7047538).$$

- Now, it is easy to compute:

$$Maximize\ \delta^{\propto-\mathrm{DM}_{L3}}$$

Subject to

$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 \le 50, x_1 + x_2 \le 10, x_2 \le 8\text{y}, 5x_3 + x_4 \le 12, x_5 + x_6 \ge 5,$

$x_5 + 5x_6 \le 50, x_1, x_2, x_3, x_4, x_5, x_6 \ge 0$, $0.36 \le u_{11} \le 4.28$, $2.8 \le u_{12} \le 4.64$,

$1.8 \le u_{21} \le 8.92$, $0.72 \le u_{22} \le 5.28$, $2.8 \le u_{31} \le 8.28$, $3.8 \le u_{32} \le 9.28$, $2.8 \le \text{y} \le 8.28$

$$\left(\frac{d_2^{PIS^{\propto-\mathrm{DM}_{L3}}}(X) - 0.3194993}{0.8615533876 - 0.3194993}\right) \ge \delta^{\propto-\mathrm{DM}_{L3}},$$

$$\left(\frac{0.7365085114 - d_2^{NIS^{\propto-\mathrm{DM}_{L3}}}(X)}{0.7365085114 - 0.7047538}\right) \ge \delta^{\propto-\mathrm{DM}_{L3}}, \left(\frac{(0.119803+0.001)-x_1}{0.001}\right) \ge \delta^{\propto-\mathrm{DM}_{L3}},$$

$$\left(\frac{x_1-(0.119803-0.001)}{0.001}\right) \ge \delta^{\propto-\mathrm{DM}_{L3}},\ \left(\frac{(0+0.3)-x_2}{0.3}\right) \ge \delta^{\propto-\mathrm{DM}_{L3}},\ \left(\frac{x_2-(0-0.3)}{0.3}\right) \ge \delta^{\propto-\mathrm{DM}_{L3}}$$

$$\left(\frac{(1.234568+0.3)-x_3}{0.3}\right) \ge \delta^{\propto-\mathrm{DM}_{L3}},\ \left(\frac{x_3-(1.234568-0.3)}{0.3}\right) \ge \delta^{\propto-\mathrm{DM}_{L3}},$$

$$\left(\frac{(1.234568+0.3)-x_4}{0.3}\right) \ge \delta^{\propto-\mathrm{DM}_{L3}},\ \left(\frac{x_4-(1.234568-0.3)}{0.3}\right) \ge \delta^{\propto-\mathrm{DM}_{L3}},\ \delta^{\propto-\mathrm{DM}_{L3}} \in [0,1]$$

- The "satisfactory level" ($\delta^{\propto-\mathrm{DM}_{L3}}$ =0.76) is achieved for the solution $X_1^{*\,\propto-\mathrm{DM}_{L3}}$ =0.1194589, $X_2^{*\,\propto-\mathrm{DM}_{L3}}$ =zero, $X_3^{*\,\propto-\mathrm{DM}_{L3}}$=1.234224, $X_4^{*\,\propto-\mathrm{DM}_{L3}}$=1.234224, $X_5^{*\,\propto-\mathrm{DM}_{L3}}$=5, $X_6^{*\,\propto-\mathrm{DM}_{L3}}$=0,

The comparison between the proposed TOPSIS method and the traditional GC method is given in Table (10). In general, the results show that the proposed interactive modified TOPSIS method is introducing better results than (or closer results to) the traditional GC method.

Table (10)

Objective	Proposed TOPSIS method	GC method	Ideal Objective Vector	
			PIS	NIS
f_{11}	15.13182322	8.834487	344.8	5
f_{12}	18.90916599	7.3481148	5	321.2
f_{21}	15.1925657	14.43363892	325.1	5
f_{22}	23.536893	19.89672821	10	503.1
f_{31}	33.3399388	33.8571471	276.832	5
f_{32}	78.2800197	21.6883874	10	314.16

5. Conclusions

This paper extended TOPSIS approach to find compromise solutions for the FMLMODM of mixed (Maximize/Minimize)-type. Anew interactive algorithm is presented for the proposed TOPSIS approach for solving these type of mathematical programming problems. Also, an illustrative numerical example is solved and compared the solution of proposed algorithm with the solution of the traditional GC method. In general, the results show that the proposed TOPSIS method is introducing better results than (or closer results to) the traditional GC method.

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