

Combined Effect of Viscous Dissipation and Radiation on Unsteady Free Convective Non-Newtonian Fluid Along a Continuously Moving Vertically Stretched Surface with No-Slip Phenomena

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Abstract Day by day research of non-Newtonian fluids is increasing because of its huge existence in the universe. Applications of non-Newtonian fluids in many industrial implementations have been an interesting topic to many researchers. Atmospheric elements, bio fluids such as blood, multiphase mixers, pharmaceutical formulation, cosmetics and toiletries, paints, and beverage items are examples of non-Newtonian fluids. Through this study, an analysis has been carried out to study the combined effect of unsteady free convection boundary layer flow of non-Newtonian fluid along a vertically stretched surface with viscous dissipation and thermal radiation in the presence of magnetic field. The governing nonlinear partial differential equations have been transformed to dimensionless equations with the help of similarity consideration to determine similarity solutions. Transformed equations have been discretized by implicit finite difference approximation to get solutions. The result of non-dimensional velocity and temperature profiles are depicted and discussed for different parameter such as Prandtl number Pr , Eckert number Ec , magnetic parameter M , and radiation parameter N . Moreover, for scientific interest the effects of skin friction coefficient (C_f) and Nusselt number (Nu) are presented in tables.

Keywords Unsteady non-Newtonian fluid, Variable thermal conductivity, Thermal radiation, Heat transfer, Natural convection, Viscous dissipation

1. Introduction

Since the study of non-Newtonian fluid flow has become an important matter for the applicants. Schowalter [1] and [2] was the first who observed the formula of boundary layer flow of a Non-Newtonian fluid as well as the similarity solution which is a significant process for solving most of the fluid mechanics problem in recent time. Elbashbeshy [3] found heat transfer over a stretching surface with variable and uniform surface heat flux subject to injection and suction. Sakiadis [4] and [5] was the first who analyze the boundary layer flow on a continuous moving solid surface in a Newtonian fluid. Later, it was studied by Crane [6], who had gained an exact solution of the boundary-layer flow of the Newtonian fluid caused by the stretching of an elastic sheet moving in its own plane linearly. For various applications in industry, the study of non-Newtonian fluid flow and heat transfer over a stretched surface gets significant attention. These kinds of fluid exhibit non linier relationship between

shear stress and rate of strain such as polymer solutions, molten plastics, paints and foods. Chamkha [7] also studied radiation effects on free convection flow past a semi-infinite vertical plate. Raptis and Perdakis [8] analyzed radiation and free convection flow past a moving plate. Nuclear power plants, satellites, space vehicles, and submarines are interest of such engineering areas. In case of high temperature the interaction of radiation with hydro-magnetic flow has become industrially more noticeable. Kishan and Sashidha Reddy [9] were studied the momentum and heat transfer in laminar boundary layer flow of non- Newtonian fluids past a semi-infinite flat plates with the thermal dispersion in the presence of a uniform magnetic field for both the cases of static plate and continuous moving plate. At present time, the radiation effect on MHD flow and heat transfer problems has become more significant industrially. Effects of radiation on non-Newtonian fluids have been studied by many researchers Mansour and Gorla [10]. Moreover, the study of viscous dissipation has been interestingly increased for decreasing the waste of energy. The effect of viscous dissipation and pressure stress work on free convection flow along a vertical flat plate has been investigated by Alam, Alim and Chowdhury [11]. B.K. Jha and A.O. Ajibade [12] studied the Effect of viscous dissipation on natural

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convection flow between vertical plates with time period boundary conditions. Recently, viscous dissipation Effect on Natural Convection Flow along a vertically wavy surface has been studied by Parveen et al. [13]. A. Shahzad, R Ali [14] & [15] investigated the power law fluid over a vertical stretching sheet with the convective boundary condition for MHD flows. Furthermore, approximate analytic solution has been studied. M. Khan et al. [16] analysed on Falken-Skan flow with mixed convection. J Ahmed et al. [17] studied on boundary layer flow of Sisko fluid over a radially stretching sheet where Lorentz forces are effective. A Shahzad et al. [18] investigated the unsteady case for axisymmetric flow and heat transfer over time-dependent radially stretching sheet. J Ahmed et al. [19] also studied Axis symmetrical flow and heat transfer over an unsteady stretching sheet for power law fluid. Saha et al. [20] investigated the effect of viscous dissipation on MHD free convection flow heat and mass transfer of non-Newtonian fluids along a continuously moving stretching sheet. In this present investigation, combined effect of viscous dissipation and radiation on unsteady non-Newtonian fluid along a vertically stretched surface has been investigated. In this work, flow has been discussed for free convection and no-slip condition. The similarity transformation is used to transform the governing equations into dimensionless equations and Finite Difference Method has been used to obtain approximate solutions. Finally the study has been compared with Chen, C.H. [21] to obtain standard verifications.

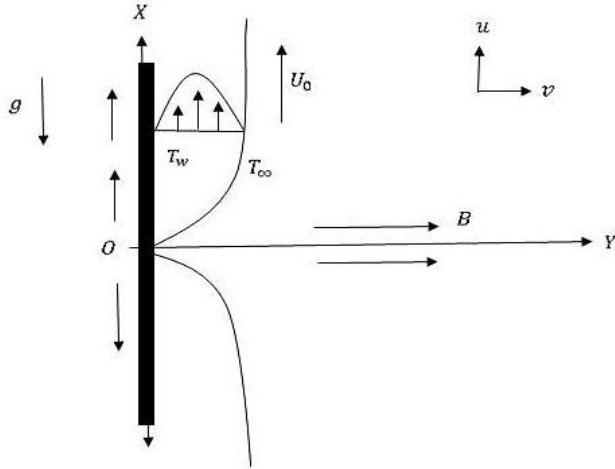


Figure 2.1. Physical model of plate

2. Formulation

We consider the unsteady free convection laminar boundary layer flow of a non-Newtonian fluid along a stretching surface in the vertical direction which is moving continuously. We also assume the combined effect of viscous dissipation and radiation for this particular study. Boundary layer development has been generalized by the existence of zero velocity at the surface of the plate. For this particular study, the velocity component u of X axis pertains along the surface and the v of Y axis normal to

the surface of the sheet. Two equal and opposite forces are performed to keep the origin fixed for X - axis so that the sheet is stretched. The power law is used to describe fluid behavior, permit mathematical predictions, and correlate computerized experimental data in this case.

$$\tau = K \left[\left| \frac{\partial u}{\partial y} \right|^{n-1} \frac{\partial u}{\partial y} \right] \quad (1)$$

If the surface and flow temperature differs, there will be a region where the fluid temperature varies from, $T = T_w$, at $y = 0$ to $T = T_\infty$, at $y \rightarrow \infty$, in the flow of outer region. This region is known as thermal boundary layer. Suppose, no change of temperature profiles of the surface T_w as well as the free stream temperature profile T_∞ are taken respectively. For natural convection the flow is induced by a force known as buoyancy forces, which arise from density ρ differences caused by the variation of temperature in the region. If we go through molecular level of the fluid, it shows us the changes of temperature profiles such as $T - T_\infty$ is arisen by the different values of temperature of the fluid particle. The relative change is calculated as $\beta(T - T_\infty)$, where β is designated as the volumetric coefficient of thermal expansion. Gravitational force is involved because of buoyancy force. In this case, $g\beta(T - T_\infty)$ is the lift force per unit volume where gravitational acceleration g is working through vertical axis. Hence, $g\beta(T - T_\infty)$ is active at X direction. B is used as applied magnetic field which is reliable on fluid's characters. But it causes the growth of magnetic force f .

$$f = (\sigma B^2 u)/\rho$$

To describe this discrete mathematical problem let us formulate the governing equations as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

$$\frac{\partial u}{\partial t} + v \frac{\partial u}{\partial y} = \frac{k}{\rho} \frac{\partial}{\partial x} \left(\left| \frac{\partial u}{\partial y} \right|^{n-1} \frac{\partial u}{\partial y} \right) + g\beta(T - T_\infty) - \frac{\sigma B^2 u}{\rho} \quad (3)$$

$$\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{1}{\rho C_p} \frac{4\sigma_1}{k_1} \frac{\partial^2 T^4}{\partial y^2} + \frac{k}{\rho C_p} \left(\left| \frac{\partial u}{\partial y} \right|^{n-1} \frac{\partial u}{\partial y} \right)^2 \quad (4)$$

Initial conditions: $u = 0, T = 0$ at $t = 0$ and

Boundary conditions: $u = U_0, T = T_w$, at $y = 0$ and $u = 0, T = T_\infty$, at $y \rightarrow \infty$; for $t > 0$.

u	Velocity components in the x direction
v	Velocity components in the y direction
t	Time
U	Dimensionless velocity components along x-axis
V	Dimensionless velocity components along y-axis
\bar{t}	Dimensionless time
M	Magnetic parameter
B	Magnetic field
T	Temperature of the field
\bar{T}	Dimensionless temperature
T_w	Temperature at wall
Ec	Eckert number
K	Flow consistency coefficient

N	Radiation parameter
Re	Local Reynolds number
Pr	Generalized Prandtl number
Gr	Grashof number
ρ	Density of fluid
μ	Dynamic viscosity
τ	Shear stress
δ	Thermal boundary layer thickness
Φ	Dissipation function
∇	Del operator
k	Thermal conductivity of the fluid
ν	Kinematic viscosity
α	Thermal diffusivity
β	Volumetric coefficient expansion
σ	Electric conductivity
σ_1	Stephan-Boltzmann constant

3. Similarity Consideration

Introducing $\bar{t} = \frac{tu_0}{L}$, $X = \frac{x}{L}$, $Y = \frac{y}{L}$, $U = \frac{u}{U_0}$, $v = \frac{v}{U_0}$, $\bar{T} = \frac{T-T_\infty}{T_w-T_\infty}$, Where L is the characteristic length and U_0 is an arbitrary reference velocity which is related to this problem. Since free stream conditions are quiescent in free convection, there is no logical external reference velocity (U_0), as in forced convection. The radiative heat flux q_1 is described by the Rosseland approximation such that, $q_1 = -\frac{4\sigma_1}{3k_1} \frac{\partial T^4}{\partial y}$, Note that Rosseland approximation is valid for optically thick fluids. σ_1 is the Stefan-Boltzmann constant and k_1 is the Rosseland mean absorption coefficient. It is surmised that the temperature difference within the flow is sufficiently small such that T^4 can be expressed in the Taylor series, where free stream temperature T_∞ and neglecting the higher order terms from Taylor series expansion $T^4 \approx 4T_\infty^3 T - 3T_\infty^4$.

Therefore, $\frac{\partial q_1}{\partial y} = -\frac{16\sigma_1 T_\infty^3}{3k_1} \frac{\partial^2 T}{\partial y^2}$. So, the dimensionless momentum equation and ultimate dimensionless energy equation are:

$$\frac{\partial U}{\partial \bar{t}} + V \frac{\partial U}{\partial Y} = \frac{1}{Re} \frac{\partial}{\partial Y} \left[\left| \frac{\partial U}{\partial Y} \right|^{n-1} \frac{\partial U}{\partial Y} \right] + Gr \bar{T} - MU \quad (5)$$

$$\frac{\partial \bar{T}}{\partial \bar{t}} + V \frac{\partial \bar{T}}{\partial Y} = \frac{1}{Pr Re} \left(1 + \frac{4}{3N} \right) \frac{\partial^2 \bar{T}}{\partial Y^2} + \frac{Ec}{Re} \left(\left| \frac{\partial U}{\partial Y} \right|^{n-1} \frac{\partial U}{\partial Y} \right)^2 \quad (6)$$

Initial condition at $\bar{t} = 0$, $U = 0$, $\bar{T} = 0$ and

Boundary condition at $\bar{t} > 0$, $U = 1$, $\bar{T} = 1$ when $Y = 0$, $U = 0$, $\bar{T} = 0$ when $Y \rightarrow \infty$. (7) Where, the radiation number $N = \frac{k_1 k}{4\sigma_1 T_\infty^3}$, Reynolds number $Re = \frac{U_0^2 L^n}{\frac{k}{\rho}}$,

Prandtl Number $Pr = \frac{k/\rho}{\alpha} \left(\frac{U_0}{L} \right)^{n-1}$, Eckert number $Ec = \frac{U_0^{n+1}}{C_p \Delta T L^n}$. Here the modified version of dimensionless numbers are used. Note that, those numbers are valid for this fluid flow only. It may appear different in different flows.

4. Numerical Computation

Although several procedures are available for solving such equations, the solutions typically involve complicated mathematical series and functions and may be obtained for only a set of simple geometries and boundary condition. We will use implicit finite difference technique to approximate the solution of governing equations. Finally we will discuss its convergences. Rectangular region of the flow field is chosen, the region is divided into a grid of lines parallel to X and Y axes, where X is chosen along the stretching surface and Y is normal to the plate.

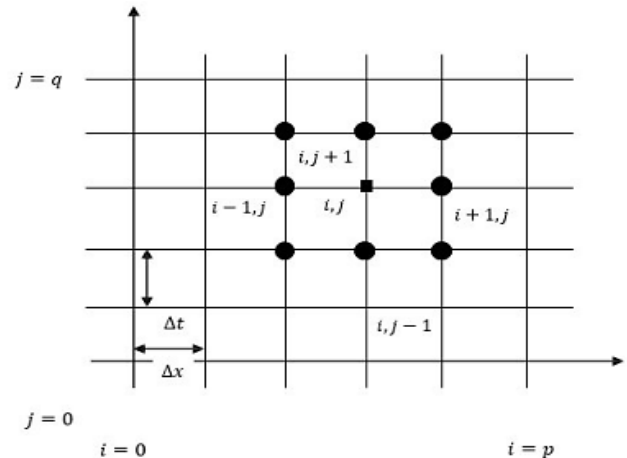


Figure 4.1. Numerical model

Let assume the length of the sheet is $X_{max} = 50$ which indicates that x varies from 0 to 50 where y varies for 0 to $Y_{max} = 20$. So the step size is calculated below.

Here $h = \frac{Y_{max}}{n+1}$, $\Delta t = k = 0.000001$ and for grid spacing p (space = 200) and q (time = 1000000) in the X and Y directions respectively.

Two point forward difference formula for determining time derivatives, two point central difference method has chosen to approximate the space involving derivatives and three point central difference approximation to find double derivative.

Now the equations (5) to (7) becomes:

$$U_{i,j+1} = U_i + k \left[-V \frac{U_{i+1,j} - U_{i-1,j}}{2h} + \frac{n}{Re} \left| \frac{U_{i+1,j} - U_{i-1,j}}{2h} \right|^{n-1} \frac{U_{i+1,j} - U_{i-1,j}}{2h} \right. \\ \left. \frac{U_{i-1,j} - 2U_{i,j} + U_{i+1,j}}{h^2} + Gr \bar{T}_{i,j} - MU_{i,j} \right] \quad (7)$$

$$\frac{\bar{T}_{i,j+1} - \bar{T}_{i,j}}{k} + V \frac{\bar{T}_{i+1,j} - \bar{T}_{i-1,j}}{2h} = \\ \frac{1}{Pr Re} \frac{\bar{T}_{i-1,j} - 2\bar{T}_{i,j} + \bar{T}_{i+1,j}}{h^2} + \\ \frac{Ec}{Re} \left(\left| \frac{U_{i+1,j} - U_{i-1,j}}{2h} \right|^{n-1} \frac{U_{i+1,j} - U_{i-1,j}}{2h} \right)^2 \quad (8)$$

for $i = 1, 2, \dots, p$ and $j = 1, 2, \dots, q$

Initial condition at $\bar{t} = 0$, $U_{i,0} = 0$, $\bar{T}_{i,0} = 0$;

Boundary condition at $\bar{t} > 0$, $U_{0,j} = 1$, $\bar{T}_{i,0} = 1$

where $Y = 0, U_{p+1,j} = 0, \bar{T}_{p+1,j} = 0$

where $Y \rightarrow \infty, j = 1, 2, \dots, q$. (9)

The skin friction coefficient is, $C_f = \frac{\tau_w}{\frac{1}{2}\rho U_0^2}$ and the

Nusselt number is, $N_u = \frac{hL}{k}$. Using FDM the discretize equations are $C_f = \frac{2}{Re} \begin{cases} \left(\frac{U_{1,q} - U_{0,q}}{h}\right)^n, & \text{if } U_{1,q} \geq U_{0,q} \\ -\left(\frac{U_{0,q} - U_{1,q}}{h}\right)^n, & \text{if } U_{1,q} < U_{0,q} \end{cases}$ and $N_u = \frac{\bar{T}_{0,q} - \bar{T}_{1,q}}{h}$ (10)

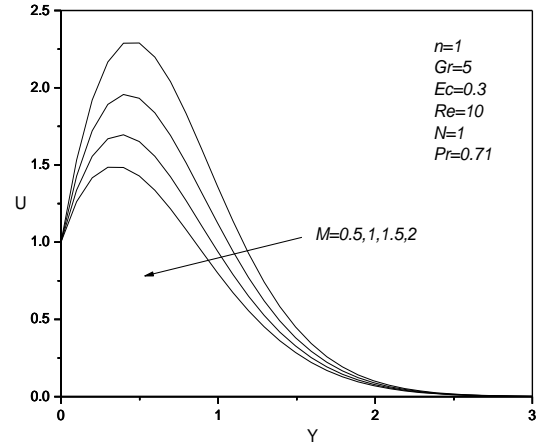
5. Results and Explanations

The updated velocity profile U and temperature profile T are obtained at all internal node points by successive application of the equations (7) to (9). The whole process will be done again and again and used time step will be sufficiently small. Effects of dimensionless numbers on velocity and temperature has been depicted through graphs. Using (10) skin friction coefficients and Nusselt numbers are obtained then solutions are shown in the table.

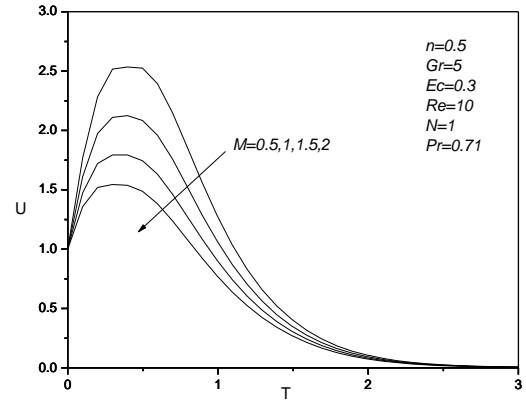
From the Figure 5.1 the velocity profile decreases with the increase of magnetic parameter M in each type of fluid. It has been obtained that, introducing magnetic field in the flow causes higher restriction to the fluid. For Newtonian and pseudo plastic fluid the velocity profile reduces in same rate but in case of dilatant fluid rate of reduction is little bit slower. Thus, velocity profile decreases as M increases in all types of fluids which reduces faster for Newtonian fluid compared to non-Newtonian fluids. But in case of temperature profile there is no significant effect found.

It is distinct from Figure 5.2 that velocity profile U decreases with the increase of radiation parameter N . However the rate of decreasing is almost same at the beginning of the surface but at the point 0.3 of Y scale. It is also clear that the momentum boundary layer thickness reduces rapidly for decreasing N . For small changes of N such as 0.5 to 1 the rate of decreasing velocity is about 0.94% for $n=1$, 0.77% for $n=1.5$ and 0.25% for $n=0.5$ respectively. But if the changes of radiation parameter is from 1 to 2, then reducing rate decreases because of free convection. However in case of free convection we cannot think huge radiation from the fluid.

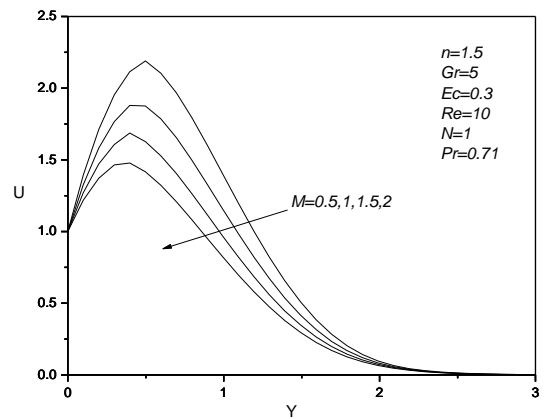
According to the above Figure 5.3 the temperature profile decreases with in positive increment of the magnitude of radiation parameter N . Form above result, we observe that the small increment cause a lot to the thermal boundary layer. Temperature profile reduces to zero slowly. So from the above illustration it is evident that numerical outcomes support the physical experiment (Newton's law of cooling). The rate of heat transfer is thus increased. So we can use the effect of radiation to control the velocity and temperature of boundary layer.



5.1(a)

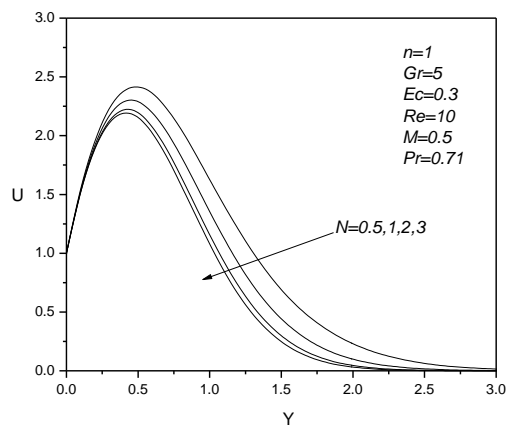


5.1(b)

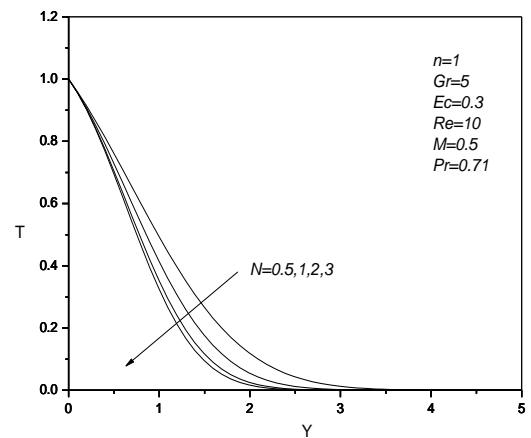


5.1(c)

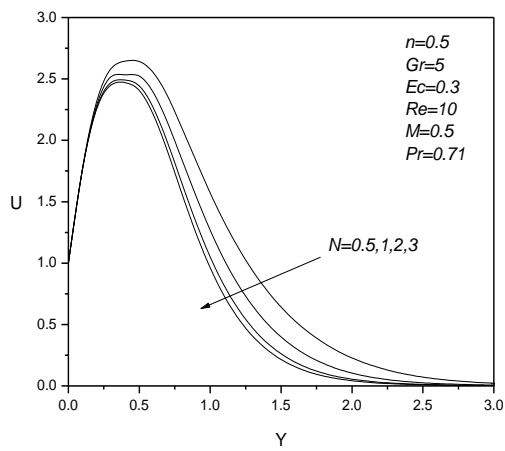
Figure 5.1. (a, b, c): effect of Magnetic parameter M on velocity profile for different values of n



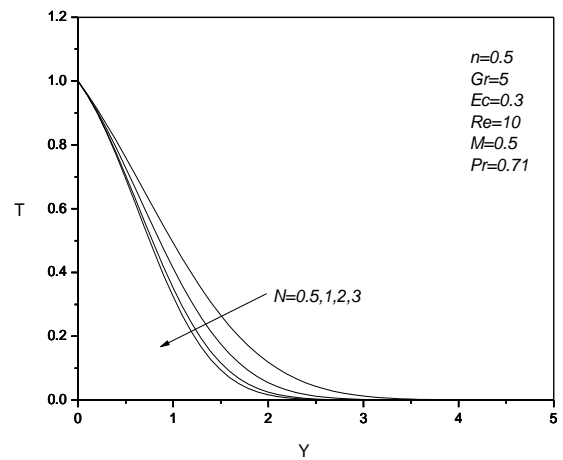
5.2(a)



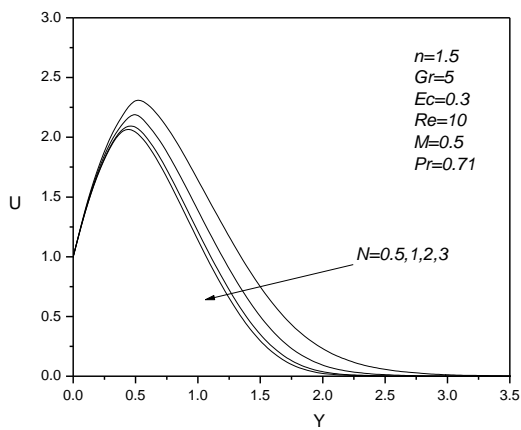
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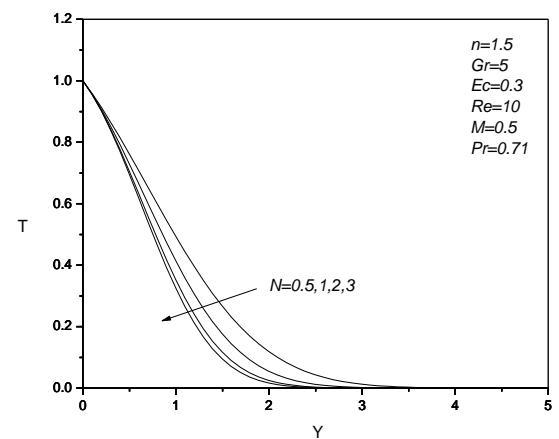
5.2(b)



5.3(b)



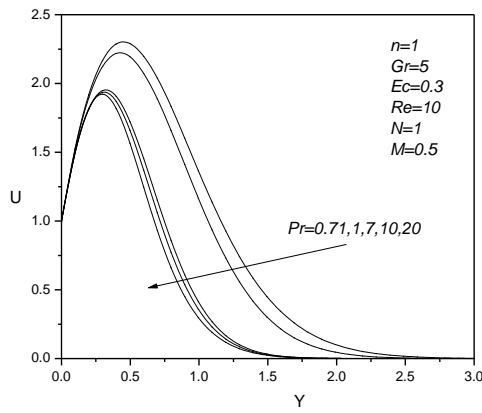
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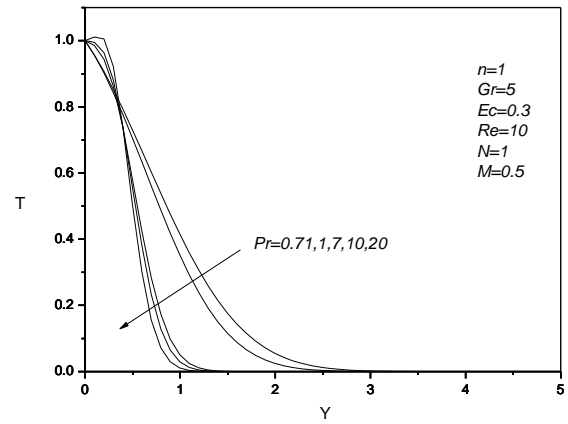
5.3(c)

Figure 5.2. (a, b, c): effect of Radiation parameter N on velocity profile for different values of n

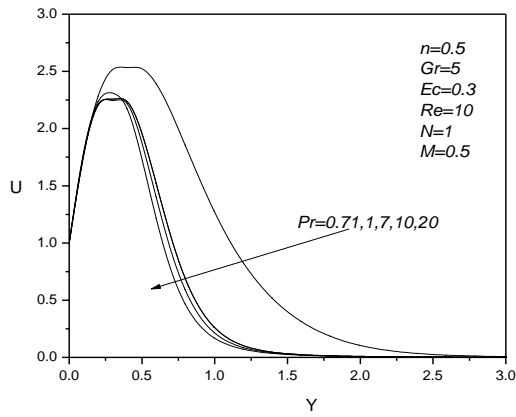
Figure 5.3. (a, b, c): effect of Radiation parameter N on temperature profile for different values of n



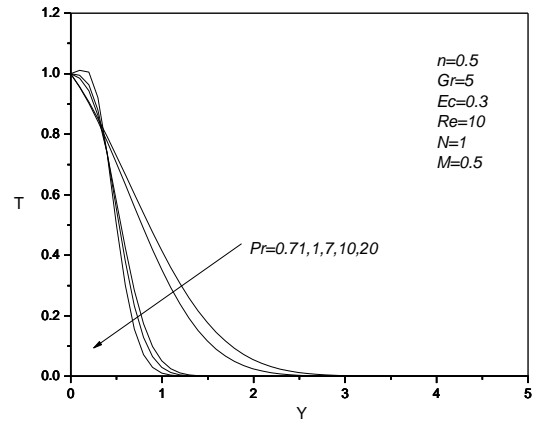
5.4(a)



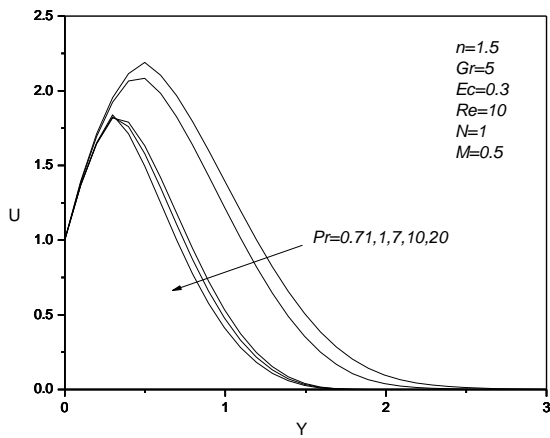
5.5(a)



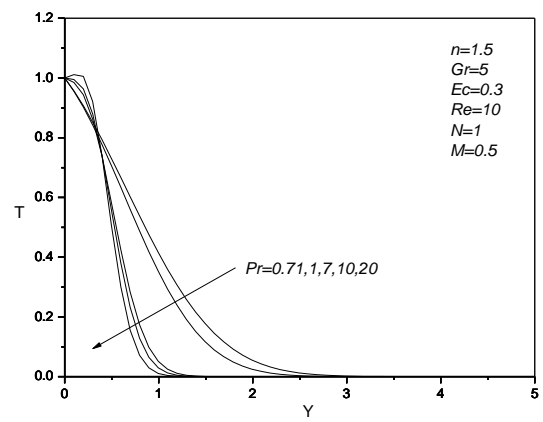
5.4(b)



5.5(b)

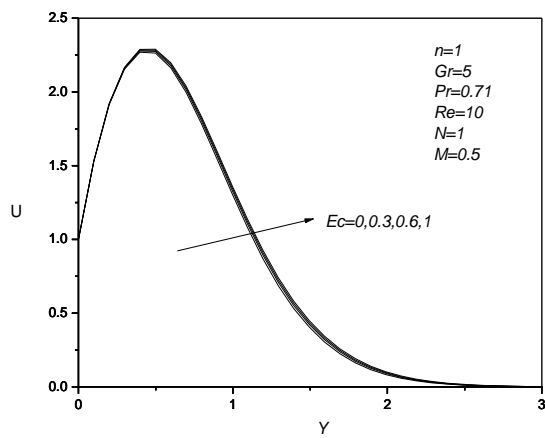


5.4(c)

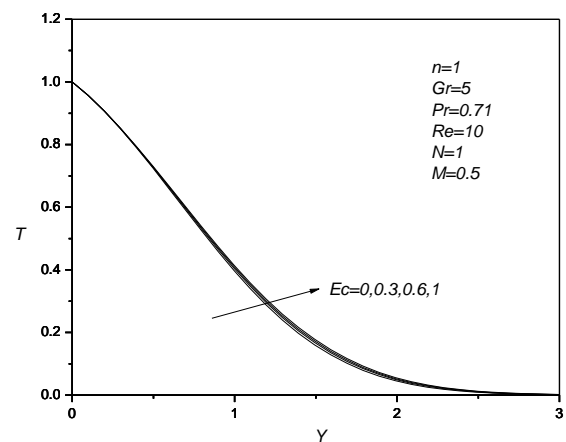


5.5(c)

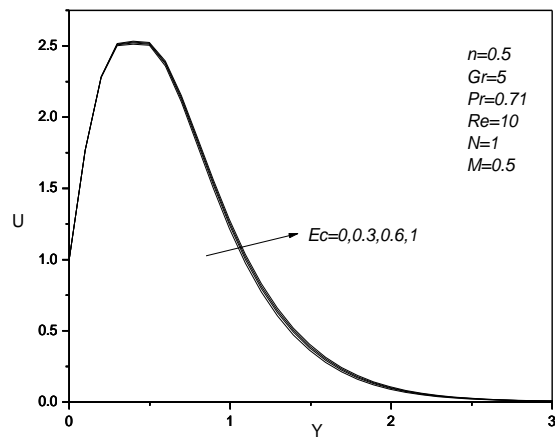
Figure 5.4. (a, b, c): effect of Prandtl number Pr on velocity profile for different values of n **Figure 5.5.** (a, b, c): effect of Prandtl number Pr on temperature profile for different values of n



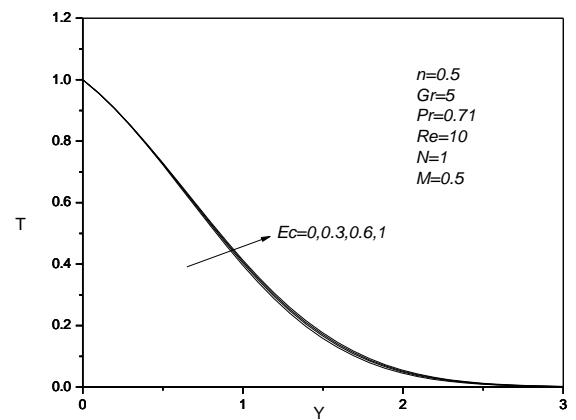
5.6(a)



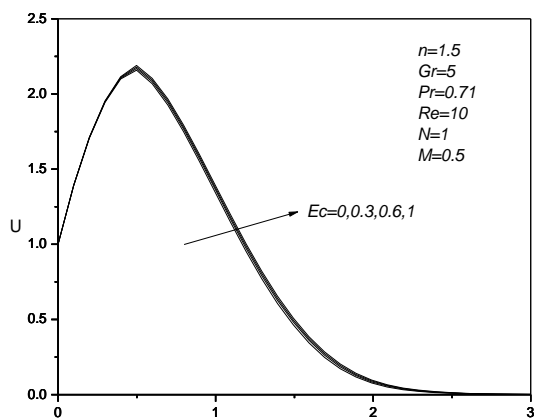
5.7(a)



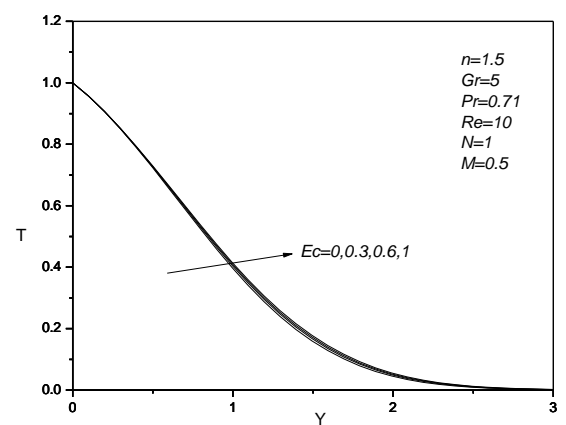
5.6(b)



5.7(b)



5.6(c)



5.7(c)

Figure 5.6. (a, b, c): effect of Eckert number Ec on velocity profile for different values of n

Figure 5.7. (a, b, c): effect of Eckert number Ec on temperature profile for different values of n

Figure 5.4 shows the effect of Prandtl number Pr on velocity profile shows significant changes for the increase of its magnitude. From the figure it can be told the rate of reduction is little for three types of fluid for large Prandtl number but for small Prandtl number such as $Pr = 0.71$ or 1 the reduction rate is larger. However for $Pr = 0.71$ there is a sharp rise in the velocity boundary layers near stretching surface. Physically $Pr = 0.71$ corresponds to air and $Pr = 1, 7, 10$ is for different values of water. For pseudo plastic fluid the velocity profile rise higher and slowly decreases to zero at the wall.

It is clear from Figure 5.5 that for small Prandtl number the temperature profile reduces to zero at the wall with the increase of Y steps. But when $Pr = 7, 10$ or 20 then profiles raises a little at the beginning and then slowly reduces to zero at the wall. The rate of reduction is almost same for these three types of fluid. Moreover, cross flows have been found for each case.

Figure 5.6: According to Eckert number it implies the ratio of advective transport and heat dissipation potential. It provides the connection between a flow's kinetic energy and the boundary layer enthalpy difference, and is used to characterize heat dissipation. Ec number shows small effect on velocity and temperature profile for its small changes. For Newtonian fluid the Figure 4.9, peaks of the velocity profile are just over 2.25 of velocity scale where in case pseudo plastic fluid it is almost 2.5 but for dilatant fluid it is just

under 2.25 of velocity scale. All these velocity profile increases at the beginning and slowly fall to zero with the increment of Y steps. The rate of increment of velocity and temperature profile with the increase of Eckert number Ec are almost same. For each type of fluid momentum boundary layer thickness and thermal boundary layer thickness have changed over the increment of Ec .

From Figure 5.7 it reveals there are not any significant changes of temperature profiles T at the beginning and at the end for different values of Ec . But at the mid-point of each profile it shows its variation. Although the rate of change is very small and profile slowly rises for the increment of Ec .

From the table 5.7 it indicates C_f reduces with the increment of M but N_u remain unchanged. C_f and N_u decreased significantly for the increase of Pr . In this case cross flow has been found. C_f and N_u shows opposite scenario. Where skin friction increases slowly and Nusselt number decreases, although the effect is very small. Thermal radiation can be used to control the boundary layer. Skin friction coefficient C_f has decreased slowly but Nusselt number N_u effect shows diversity for the increase of N in case of different power law index n .

To show the validity of the model described above, skin friction coefficient as well as heat transfer rate have been calculated and compared to the Chen (2008) model for specific parameter.

Table 5.1. Skin friction coefficient (C_f) and Nusselt number (N_u) for different n, M, N, Ec

n	M	C_f	N_u	N	C_f	N_u	Ec	C_f	N_u
0.5	0.5	0.5556170156	0.4375853918	0.5	0.5572589081	0.4183440638	0	0.5556174461	0.4375853918
	1.0	0.4951622412	0.4375853918	1	0.5556170156	0.4457585392	0.3	0.5555449049	0.4376441039
	1.5	0.436506391	0.4375853918	2	0.5542651235	0.4472651266	0.6	0.5554660597	0.437488349
	2.0	0.3790941134	0.4375853918	3	0.5536617753	0.4480264272	1.0	0.555343968	0.4385414382
1	0.5	1.068160893	0.4375853918	0.5	1.097097235	0.4183428075	0	1.068160893	0.4375853918
	1.0	0.601023964	0.4375853918	1	1.068160893	0.4472638734	0.3	1.066456534	0.4376441039
	1.5	0.6787112529	0.4375853918	2	1.048716957	0.4472638734	0.6	1.064808736	0.4374877560
	2.0	0.5271331731	0.4375853918	3	1.041236803	0.4480258343	1.0	1.062432411	0.4385408453
1.5	0.5	1.56877755	0.4375836029	0.5	1.633654363	0.4163322648	0	1.568775453	0.4375853918
	1.0	1.187896283	0.4375836029	1	1.56877755	0.43756029	0.3	1.564971397	0.4376428979
	1.5	0.8966871722	0.4375836029	2	1.525523567	0.4472626674	0.6	1.561205349	0.4374843541
	2.0	0.6505575034	0.4375836029	3	1.50737908	0.4480246383	1.0	1.55617659	0.4385354484

Table 5.2. Comparison table for Skin friction coefficient C_f and Nusselt number N_u excluding unsteady term, viscous dissipation and radiation term for free convection

n	M	Chen 2008: C_f	Present C_f	Chen 2008: N_u	Present N_u
1.5	1.0	-2.412287	-2.4122119	1.689593	1.6881962
	5.0	-5.768535	-5.7688105	3.874307	3.8729347
1.0	1.0	-2.519363	-2.5193228	1.578424	1.5785991
	5.0	-5.366930	-5.3669556	3.754876	3.7549825
0.5	1.0	-2.633241	-2.6333226	1.346116	1.3461124
	5.0	-4.608094	-4.6079521	3.528467	3.5286989

6. Conclusions

- To control the velocity and thermal boundary layer, thermal radiation can be used significantly.
- Wall friction has been increased due to buoyancy force which has thickened the boundary layer.
- The heat transfer rate has been increased by absorbing thermal energy from the surface.
- Drag of pseudo-plastic fluid along a vertically stretched surface has been decreased as velocity profile increased.
- A strong magnetic field can be applied to increase the wall temperature of the pseudo-plastic fluids.
- Cross flow has been found for variation of Prandtl number.
- Eckert number shows small effect. C_f decreased but N_u has increased with the growth of Ec .

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REFERENCES

- [1] Schowalter, W.R.: The application of boundary layer theory to power-law pseudo plastic fluid Similar solutions, *AICHE J.* 6, 24-28 (1960).
- [2] Schowalter W.R.: *Mechanics of Non-Newtonian Fluids*, Pergamon Press, Oxford, (1978).
- [3] Elbashbeshy, E.M.A.: Heat transfer over a stretching surface with variable surface heat flux, *J. Phys. D Appl. Phys.* 31, 1951-1954. (1998).
- [4] Sakiadis. B.C.: Boundary-layer behavior on continuous solid surface on a continuous flat surface-I, The boundary layer on a continuous flat surface, *AICHE J.* 7, 26-28(1961a).
- [5] Sakiadis. B.C.: Boundary-layer behavior on continuous solid surface on a continuous flat surface-II, The boundary layer on a continuous flat surface, *AICHE J.* 7, 221-225 (1961b).
- [6] Crane, L. J.: Flow Past a Stretching Plate, *ZAMP.* 21, 645-647(1970).
- [7] Chamkha, A.J., Takhar H.S. and Soundalgekar V.M.: Radiation effect on free convection flow past a semi-Infinite vertical plate with mass transfer. *Chem. Eng. J.* 84, 335-342 (2001).
- [8] Raptis, A. and Perdikis C.: Radiation and free convection flow past a moving plate. *App. Mech. Eng.* 4, 817-821(1999).
- [9] Kishan Naikoti, Shashidar Reddy Borra.: Quasi-linearization Approach to MHD Effects on Boundary Layer Flow of Power-Law Fluids Past A Semi Infinite Flat Plate with Thermal Dispersion", *Int. J. of Non-Linear Science.* 11, 301-311 (2011).
- [10] Mansour M. A. and Gorla R. S. R.: Mixed convection-radiation interaction in power-law fluids along a non-isothermal wedge embedded in a porous medium. *Transport in Porous Media.* 30, 113-124 (1998).
- [11] M. M. Alim, M.A. Alim and M.K. Chowdhury.: Effect of pressure stress work and viscous dissipation in natural convection flow along a vertical plate with heat conduction. *J. Naval and Marine Eng.* 3, 69-76 (2006).
- [12] Jha B.K. and Ajibade A.O.: Effect of viscous dissipation on natural convection flow between vertical plates with time period boundary conditions, *Thermophys. Aero tech.* 17, 1576-1587 (2011).
- [13] Nazma Parveen, Sujon Nath, Md. Abdul Alim.: Viscous dissipation Effect on Natural Convection Flow along a vertically wavy surface. 90, 294-300 (2014).
- [14] A. Shahzad, R Ali, MHD flow of a non-Newtonian Power law fluid over a vertical stretching sheet with the convective boundary condition, *Walailak Journal of Science and Technology (WJST)* 10 (1), 43-56 (2013).
- [15] A. Shahzad, R. Ali, Approximate analytic solution for magneto-hydrodynamic flow of a non-Newtonian fluid over a vertical stretching sheet. *Can J Appl Sci* 2, 202-215 (2012).
- [16] M. Khan, R. Ali, A. Shahzad MHD Falkner-Skan flow with mixed convection and convective boundary conditions. *Walailak Journal of Science and Technology (WJST)* 10 (5), 517-529 (2013).
- [17] J Ahmed, A Shahzad, A Begum, R Ali, N Siddiqui, Effects of inclined Lorentz forces on boundary layer flow of Sisko fluid over a radially stretching sheet with radiative heat transfer, *Journal of the Brazilian Society of Mechanical Sciences and Engineering*, 1-12 (2017).
- [18] A Shahzad, R Ali, M Hussain, M Kamran, Unsteady axisymmetric flow and heat transfer over time-dependent radially stretching sheet, *Alexandria Engineering Journal* 56, 35-41 (2017).
- [19] J Ahmed, T Mahmood, Z Iqbal, A Shahzad, R Ali: Axisymmetric flow and heat transfer over an unsteady stretching sheet in power law fluid, *Journal of Molecular Liquids* 221, 386-393 (2016).
- [20] K.C. Saha, M.A. Samad and M.R. Hossain: Effect of viscous dissipation on MHD Free Convection flow Heat and Mass Transfer of Non-Newtonian Fluids along a Continuously moving Stretching Sheet, *Research J. of App. Sciences, Eng. And Tech.*, pp. 1058-1073 (2015).
- [21] Chen, C.H.: Effects of magnetic field and suction/injection on the convective heat transfer on the non-Newtonian power law fluids past a power law stretched sheet with surface heat flux. *Int.J. Therm. Sci* 47, 954-961(2008).