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The Revised New Iterative Method for Solving the Model Describing Biological Species Living Together

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Abstract In this paper, a system of two nonlinear delay integro-differential equations derived from considering biological species living together and the revised new iterative method proposed by Bhalekar and Daftardar-Gejji (2012) is implemented for finding the solution of this system. Also, to demonstrate the validity and applicability of the method, examples are presented and results are compared with Adomian decomposition method, Variational iteration method, Pseudospectral method, and the Taylor collocation method. The method yields a series with accelerated convergence.

Keywords New iterative method, Nonlinear integro-differential equation, Revised new iterative method, Variational iteration method, Adomian decomposition method, Pseudospectral Lengendre method, Taylor collocation method

1. Introduction

System of Volterra Integro-differential equation (IDEs) arise in scientific fields such as biology [1], Medicine [2], Ecology [4], Population growth [3], physics such as electromagnetism theory [5], one dimensional visco elasticity and reactor dynamics [6]. This class of equations plays an important role in modelling of various problems of engineering and natural science and hence, attracted much attention in numerical computation and analysis.

This paper is concerned with the dynamic of two interacting species which was first modelled by [7]. It is considered two separated two species with number $u_1(t)$ and $u_2(t)$ at time t, where first species increases and second decreases. If they put together that the second species will feed on the first, there will be increase in the rate of the second species $\frac{du_2}{dt}$, which depends not only on the present population $u_1(t)$ but also on all previous values of the first species. When a steady-state or equilibrium is reached between the two species, it is described by the following system of nonlinear delay Volterra Integro-differential equations:

$$\frac{du_1}{dt} = u_1(t)\left[h_1 - \gamma_1 u_2(t) - \int_{t-T_0}^{t} f_1(t-\tau)u_2(\tau)d\tau\right] + g_1(t) \tag{1}$$

$$\frac{du_1}{dt} = u_1(t)\left[h_1 - \gamma_1 u_2(t) - \int_{t-T_0}^{t} f_1(t-\tau)u_2(\tau)d\tau\right] + g_1(t) \tag{2}$$

where $h_1, \gamma_1, h_2, \gamma_2 > 0,\ 0 \le t \le b$ with initial conditions

$$u_1(0) = \alpha_1, \quad u_2(0) = \alpha_2, \tag{3}$$

where h_1 and h_1 are coefficients of increase and decrease of the first and the second species respectively. The parameters f_1,f_2,g_1 and g_2 are given functions while u_1 and u_2 are unknown functions and $T_0 \epsilon R$ is assumed to be the finite heredity duration of both species.

Several numerical methods of approximating the solution of this model are known; Adomian decomposition method (ADM) [9], Variational Iteration Method (VIM) [10], Legendre Multiwavelent Method (LMM) [11], Differential Transform Method (DTM) [12] and Taylor Collocation Method (TCM) [13]. Recently, [14] have introduced a new Iterative Method (NIM) to solve general functional equation: $y = f + N(y)$, where f is specified function and N, a given nonlinear function of y. [15] obtained the solution of $nth-$ order linear and nonlinear Integro-differential equation using NIM. [16] applied NIM to system of Volterra Integro-differential equations

NIM is simple in its principles and easy to implement on computer packages such as Mathematica and Maple. This method is better than numerical methods as it is free from rounding off errors and does not require large computer power. NIM has proven successful over other methods in many cases [17], [18] [19], [20] [21], [22], [16].

Bhaleker, S. and Datterder-Gejj, V. [18] presented a modification of the NIM called the Revised NIM (RNIM), to solve the following system of functional equations with improved convergence:

$$U_i = f_i + N_i(U_1, U_2, \dots., U_n), \quad i = 1, 2, \dots, n \tag{4}$$

The main purpose of this paper is to solve equations (1) – (3) using the revised new Iterative algorithm [18]. The

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RNIM has been applied to solve various examples, some of which have already been solved by other methods. A comparison with other solutions reveals the usefulness and rapid convergence of this method.

2. Description of the Methods

2.1. New Iterative Method

Consider the system of nonlinear functional equations in equation (4) above, where f_i are known functions and N_i are nonlinear operators. Let $U_i = (u_1, u_2, \dots, u_n)$ be a solution of system (4), where u_i having the series form:

$$U_i = \sum_{j=0}^{\infty} u_{i,j} \qquad i = 1, 2, \dots, n \qquad (5)$$

The nonlinear operator N_i can be decompose as

$$N_i(\overline{\boldsymbol{u}}_{\boldsymbol{i}}) = \boldsymbol{N_i}\left(\sum_{j=0}^{\infty} \boldsymbol{u_{1,j}}_{,\dots,} \sum_{j=0}^{\infty} \boldsymbol{u_{n,j}}\right)$$
$$= N_i(U_{1,0}, \dots, U_{n,0}) + \sum_{k=1}^{\infty}\left\{N_i\left(\sum_{j=0}^{k} U_{1,j}, \dots, \sum_{j=0}^{k} U_{n,j}\right)\right\} - N_i\left(\sum_{j=0}^{k-1} U_{1,j} \dots \sum_{j=0}^{k-1} U_{n,j}\right) \qquad (6)$$

By virtue of equations (5) and (6), system (4) is equivalent to

$$\sum_{j=0}^{\infty} u_{i,j} = f_i + N_i(u_{1,0}, \dots, u_{n,0}) + \sum_{k=1}^{\infty}\left\{N_i\left(\sum_{j=0}^{k} u_{1,j} \dots \sum_{j=0}^{k} u_{n,j}\right) - N_i\left(\sum_{j=0}^{k-1} u_{1,j} \dots \sum_{j=0}^{k-1} u_{n,j}\right)\right\}$$
$$(i = 1, 2, \dots, n)$$

for $i = 1, 2, \dots, n$, we define the recurrence relation:

$$u_{i,0} = f_i$$
$$u_{i,1} = N_i(u_{1,0}, \dots, u_{n,0})$$
$$u_{i,m+1} = N_i\left(\sum_{j=0}^{m} u_{1,j}, \dots, \sum_{j=0}^{m} u_{n,j}\right) - N_i\left(\sum_{j=0}^{m-1} u_{1,j}, \dots, \sum_{j=0}^{m-1} u_{n,j}\right)$$

then

$$u_i = \sum_{j=0}^{\infty} u_{i,j}$$

The $k-th$ order approximation of u_i is given by

$$u_i = \sum_{j=0}^{k-1} u_{i,j}$$

2.2. Revised New Iterative Method

In this section we present the algorithm of the RNIM suggested by [18] to illustrate the technique. We consider the system of equation (4).

Initial step $(n = 0)$:

$$u_{i,0} = f_i, \qquad i = 1, 2, \dots, n$$

First Iteration $(n = 1)$:

$$u_{1,1} = N_1(u_{1,0,} u_{2,0}, \dots, u_{n,0})$$
$$u_{2,1} = N_2(u_{1,0} + u_{1,1}, u_{2,0}, \dots, u_{n,0})$$
$$u_{3,1} = N_3(u_{1,0} + u_{1,1}, u_{2,0} + u_{2,1}, u_{3,0} \dots, u_{n,0})$$
$$\vdots$$
$$\vdots$$
$$\vdots$$
$$u_{n,1} = N_n(u_{1,0} + u_{1,1}, u_{2,0} + u_{2,1}, u_{3,0} + u_{3,1} \dots, u_{n-1,0} + u_{n-1,1, u_{n,0}})$$

kth Iteration $(n = k = 2, 3, \dots)$

$$u_{1,k} = N_1\left(\sum_{i=0}^{k-1} u_{1,i}, \dots, \sum_{i=0}^{k-1} u_{n,i}\right) - N_1\left(\sum_{i=0}^{k-2} u_{1,i}, \dots, \sum_{i=0}^{k-2} u_{n,i}\right)$$

$$u_{2,k} = N_2\left(\sum_{i=0}^{k} u_{1,i}, \sum_{i=0}^{k-1} u_{2,i}, \dots, \sum_{i=0}^{k-1} u_{n,i}\right) - N_2\left(\sum_{i=0}^{k-1} u_{1,i}, \sum_{i=0}^{k-2} u_{2,i} \dots, \sum_{i=0}^{k-2} u_{n,i}\right)$$

$$u_{j,k} = N_j\left(\sum_{i=0}^{k} u_{1,j}, \dots, \sum_{i=0}^{k-1} u_{j-1,i}, \sum_{i=0}^{k-1} u_{j,i}, \dots, \sum_{i=0}^{k-1} u_{n,i}\right) - N_j\left(\sum_{i=0}^{k-1} u_{1,\pi} \dots \sum_{i=0}^{k-1} u_{j-1,i}, \sum_{i=0}^{k-2} u_{j,i} \dots \sum_{i=0}^{k-1} u_{n,i}\right)$$

$$\vdots$$

$$\vdots$$

$$\vdots$$

$$u_{n,k} = N_n\left(\sum_{i=0}^{k} u_{1,i} \dots \sum_{i=0}^{k} u_{n-1,i}, \sum_{i=0}^{k-1} u_{n,i}\right) - N_n\left(\sum_{i=0}^{k-1} u_{1,i} \dots \sum_{i=0}^{k-1} u_{n-1,i}, \sum_{i=0}^{k-2} u_{n,i}\right)$$

Thus $N_i(u_i) = N_i\left(\sum_{j=0}^{\infty} u_{1,j} \dots \sum_{j=0}^{\infty} u_{n,j}\right) = \sum_{j=1}^{\infty} u_{i,j}$

Hence,

$$u_i = \sum_{j=1}^{\infty} u_{i,j}$$

3. Revised NIM for the System (1) - (3)

In view of the RNIM, the system (1) - (3) is equivalent to the system of Integral equation:

$$u_1(t) = \propto_1 + I_t\big(g_1(t)\big) + I_t\left[u_1\big(h_1 - \gamma_1 u_1(t)\big) - \int_{t-T_0}^{t} f_1(t-\tau)u_2(\tau)d\tau\right]$$

$$u_2(t) = \propto_2 + I_t\big(g_2(t)\big) + I_t\left[u_2\big(-h_2 - \gamma_2 u_1(t)\big) - \int_{t-T_0}^{t} f_2(t-\tau)u_1(\tau)d\tau\right]$$

where I_t is an integral operator with respect to t.

We set

$$u_{1,0} = \propto_1 + I_t\big(g_1(t)\big)$$

$$u_{2,0} = \propto_2 + I_t\big(g_2(t)\big)$$

Therefore,

$$N_1(u_i) = I_t\left[u_1\big(h_1 - \gamma_1 u_2(t)\big) - \int_{t-T_0}^{t} f_1(t-\tau)u_2(\tau)d\tau\right]$$

and

$$N_2(u_i) = I_t\left[u_2\big(-h_2 - \gamma_2 u_1(t)\big) - \int_{t-T_0}^{t} f_2(t-\tau)u_1(\tau)d\tau\right]$$

Now for n=1, we get

$$u_{1,1} = I_t\left[u_{1,0}\left(h_1 - \gamma_1 u_{2,0}(t)\right) - \int_{t-T_0}^{t} f_1(t-\tau)u_{2,0}(\tau)d\tau\right]$$

$$u_{2,1} = I_t\left[u_{2,0}\left(-h_2 - \gamma_2\big(u_{1,0}(t) + u_{1,1}\big)\right) + \int_{t-T_0}^{t} f_2(t-\tau).\big(u_{1,0} + u_{1,1}\big)(\tau)d\tau\right]$$

Second iteration: n=2 gives;

$$u_{1,2} = I_t\left[\left(u_{1,0}+u_{1,1}\right)\left(h_1 - \gamma_1\left(u_{2,0}+u_{2,1}\right)\right) - \int_{t-T_0}^{t} f_1(t-\tau).\left(u_{2,0}+u_{2,1}\right)d\tau\right] - I_t\left[u_{1,0}\left(h_1 - \gamma_1 u_{2,0}\right) - \int_{t-T_0}^{t} f_1(t-\tau).u_{2,0}(\tau)d\tau\right]$$

$$u_{2,2} = I_t\left[\left(u_{2,0}+u_{2,1}\right)\left(-h_2 + \gamma_2\left(u_{1,0}+u_{1,1}+u_{1,2}\right)\right) + \int_{t-T_0}^{t} f_2(t-\tau).\left(u_{1,0}+u_{1,1}+u_{1,2}\right)d\tau\right] - I_t\left[u_{2,0}\left(-h_2 + \gamma_2\left(u_{1,0}+u_{1,1}\right)\right) + \int_{t-T_0}^{t} f_2(t-\tau).\left(u_{1,0}+u_{1,1}\right)d\tau\right]$$

Third Iteration: n=3 results to;

$$u_{1,3} = I_t\left[\left(u_{1,0}+u_{1,1}+u_{1,2}\right)\left(h_1 - \gamma_2\left(u_{2,0}+u_{2,1}+u_{2,2}\right)\right) - \int_{t-T_0}^{t} f_1(t-\tau).\left(u_{2,0}+u_{2,1}+u_{2,2}\right)(\tau)d\tau\right] - I_t\left[u_{1,0}+u_{1,1}\left(h_1 - \gamma_2(u_{2,0}+u_{2,1})\right) - \int_{t-T_0}^{t} f_1(t-\tau)(u_{2,0}+u_{2,1})(\tau)d\tau\right]$$

$$u_{2,3} = I_t\left[\left(u_{2,0}+u_{2,1}+u_{2,2}\right)\left(h_1 - \gamma_2\left(u_{1,0}+u_{1,1}+u_{1,2}+u_{1,3}\right)\right) - \int_{t-T_0}^{t} f_2(t-\tau).\left(u_{1,0}+u_{1,1}+u_{1,2}+u_{1,3}\right)(\tau)d\tau\right] - I_t\left[u_{2,0}+u_{2,1}\left(h_1 - \gamma_2(u_{1,0}+u_{1,1}+u_{1,2}+u_{1,3})\right) - \int_{t-T_0}^{t} f_2(t-\tau)(u_{2,0}+u_{2,1})(\tau)d\tau\right]$$

$$\vdots$$

and so on. The rest iterations can be obtained.

The solution is of the form

$$u_1 = u_{1,0} + u_{1,1} + u_{1,2} + u_{1,3} + \cdots = \sum_{i=0}^{n} u_{1,i}$$

$$u_2 = u_{2,0} + u_{2,1} + \cdots += \sum_{i=0}^{n} u_{2,i} \qquad i = 0, 1, 2,$$

4. Illustrative Examples

To give a clear over view of this method, we present the following examples. We apply the revised NIM and compare the results with the other numerical methods. Results are shown with tables and figures below. All of them were performed on the computer using a program written in Maple 15.

Example 4.1. [13]: Consider the system of Integro-differential equation (1) and (2) with $f_1(t) = f_1(t) = 1$, $h_1 = h_2 = 1$, $\gamma_1 = \gamma_2 = 1$, $T_0 = 1$, $\alpha_1 = 0$, $\alpha_2 = 0$, $g_1(t) = 1 + t$, and $g_2(t) = \frac{3}{2} - 3t$ with the exact solution as $u_1(t) = t$, $u_2(t) = 1$.

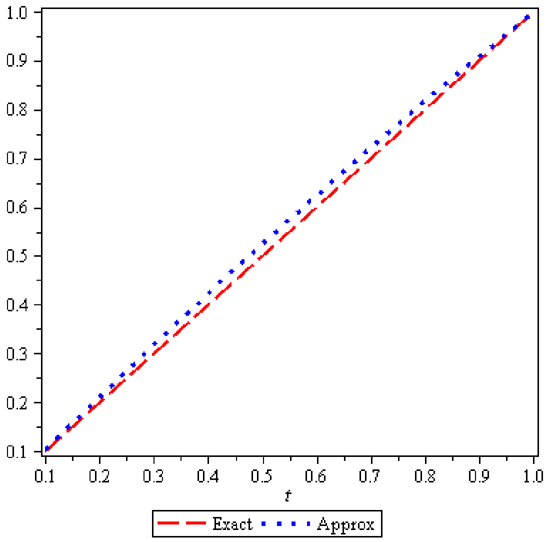
By solving the system (1) – (2) using the RNIM algorithm with this data, we obtain approximate solution as

$$u_1(t) = t + 0.3962 \times 10^{-19}t^2 - 0.8406 \times 10^{20}t^3$$

$$u_2(t) = 1 + 1.852835 \times 10^{-22}t^{81}$$

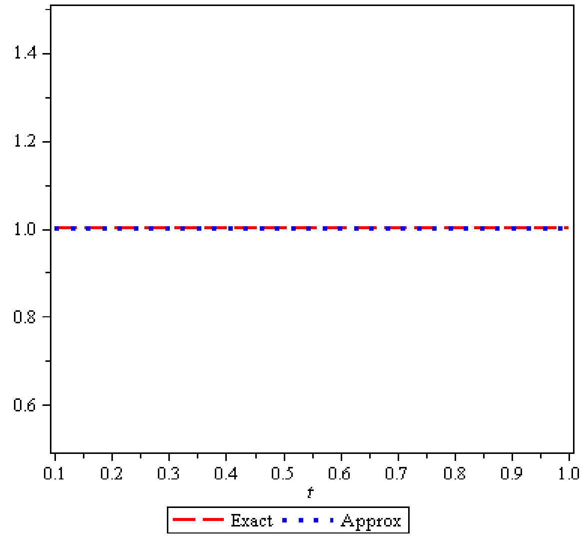
This is indeed the exact solution of the problem.

In figure (1) and (2), approximate and exact solutions are plotted.



(a) Result for u1(t) in Example 1

Figure 1.



(b) Result for u2(t) in Example 1

Figure 2.

Example 4.2. [10], [12], [13] and [11]: Now we consider the system (1) – (2) with

$f_1(t) = 1, f_2(t) = t - 1,\ h_1 = 1,\ h_2 = 2,\ \gamma_1 = {}^1/_3,\ T_0 = {}^1/_2,\ \alpha_1 = 1,\ \alpha_2 = 0,$

$g_1(t) = -{}^5/_2\, t^3 + {}^{49}/_{12}\, t^2 + {}^{17}/_2\, t - {}^{23}/_6$ and $g_2(t) = {}^5/_2\, t^3 - {}^1/_4\, t^2 + {}^3/_8\, t - 1.$

The exact solutions of this system are in the form

$$u_1(t) = -3t + 1, \qquad u_2(t) = t^2 - t$$

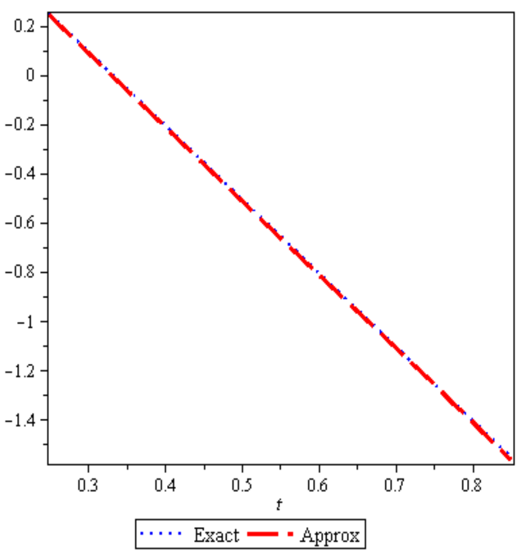
We obtain the approximate solutions by RNIM and get

$$u_1(t) = 1 - (3.01671549)t - \cdots$$

$$u_2(t) = -t + (1.00113807)t^2 + \cdots$$

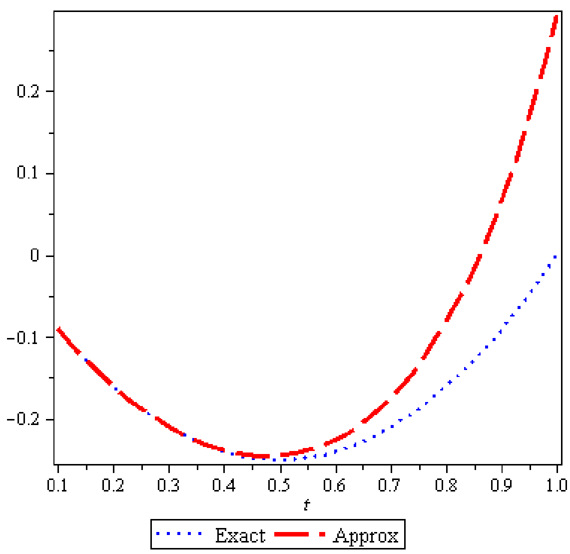
These results show the high accuracy of the technique only by two Iterations.

In Table 1 – 2, the absolute errors obtained by the RNIM are compared with the results obtained by variational Iteration method [10], Taylor collocation method [13], Adomian decomposition method [9] and Pseudospectral Legendre method [11]. In figure 3 – 4, the exact and approximate value for $u_1(t)$ and $u_2(t)$ is plotted. It is seen from this figure and tables that the present method is closer to exact solution than other method.



(a) Result for u1(t) in Example 2

Figure 3.



(b) Result for u2(t) in Example 2

Figure 4.

Table 1. Comparison of the absolute errors obtained by the ADM, PLM, TCM, VIM and the RNIM for $u_1(t)$ in Example 2

	Exact value	**VIM**	**ADM**	**RNIM**	**TCM**	**RNIM**
$\boldsymbol{t_i}$	$y_1(t_i)$					
0.10	0.7000	3.1519e-4	1.0918e-4	1.9863e-13	0.54621e-12	0
0.20	0.4000	4.2729e-4	1.7889-4	3.2312e-13	0.103513e-11	0.9760e-12
0.30	0.1000	4.7231e-4	1.0836e-4	3.6159e-13	0.130287e-11	1.1076e-14
0.40	-0.2000	4.8554e-4	3.0948e-4	3.2132e-13	0.15896e-11	0
0.50	-0.5000	4.7436e-4	0.0014	2.0962e-13	0.19494e-11	0
0.60	-0.8000	4.4598e-4	0.0014	3.3786e-14	0.23863e-11	0.8032e-12
0.70	-1.1000	4.3682e-4	0.0064	1.9889e-13	0.29375e-11	0.0125e-12
0.80	-1.4000	5.3581e-4	0.0105	4.8112e-13	0.35725e-11	0.7287e-12
0.90	-1.7000	9.1000e-4	0.0152	8.0559e-13	0.45035e-11	0.0053e-14
1.00	-2	0.0018	0.0199	1.1650e-12	0.8311e-11	0.6610e-14

Table 2. Comparison of the absolute errors obtained by the ADM, PLM, TCM, VIM and the RNIM for $u_2(t)$ in Example 2

	Exact value	**VIM**	**ADM**	**PLM**	**TCM for $N = 5$**	**RNIM**
$\boldsymbol{t_i}$	$y_1(t_i) = t_i^2 - t_i$					
0.10	-0.09	3.3414e-5	7.5832e-6	4.4685e—14	0.5948e-11	0.3887e-11
0.20	-0.16	8.5453e-5	1.3659e-8	3.7454e-14	0.3821e-11	0.6163e-12
0.30	-0.21	1.3335e-4	7.6562e-4	1.1098e-14	0.3404e-11	0.7011e-10
0.40	-0.24	1.7910e-4	0.0028	9.0378e-14	0.2611e-11	0.1584e-12
0.50	-0.25	2.2278e-4	0.0076	1.8979e-13	0.2127e-11	0.2341e-11
0.60	-0.24	2.3712e-4	0.0177	2.9875e-13	0.1652e-11	0.2586e-12
0.70	-0.21	1.6269e-4	0.0362	4.0665e-13	0.1141e-11	0.0012e-12
0.80	-0.16	1.0708e-4	0.0667	5.0291e-13	0.1501e-11	0.1223e-12
0.90	-0.09	7.3102e-4	0.1139	5.7693e-13	0.4403e-11	0.0301e-11
1.00	0	0.0019	0.1831	6.1812e-13	0.2519e-9	0.4246e-11

Example 4.3: [10] and [13]

In this example, we solve the system (1) – (2) with $f_1(t) = 2t - 3, f_2(t) = t,\ h_1 = h_2 = 2,\ \gamma_1 = \gamma_2 = 1,\ T_0 = {}^1\!/_3$, $\alpha_1 = \alpha_2 = 0$, $g_1(t) = t^2\left(2 - 3te^{-t} - {}^7\!/_2\, e^{-t} + {}^{13}\!/_6\, te^{1/3-t} + {}^{22}\!/_9\, te^{1/3-t}\right) - 2t$ and $g_2(t) = {}^1\!/_{648}\, e^{-t}(324t^3 -$ *8t2+325t+324*.

The exact solution of this system is in the form

$$u_1(t) = t^2, \qquad u_2(t) = {}^1\!/_2\, te^{-t}$$

Using the RNIM algorithm, we calculate the approximate solutions $u_1(t)$ and $u_2(t)$ and get

$$u_1(t) = t^2 + (0.001348 \times 10^{-8})e^{-t}t^4 + \cdots$$

$$u_2(t) = {}^1\!/_2\, te^{-t} - (0.013481 \times 10^{8})e^{-t}t^2 + \cdots$$

The exact solutions, absolute errors obtained by other methods and the present method are given in table 3 – 4.

Table 3. Comparison of the absolute errors obtained by the ADM, PLM, TCM, VIM and the RNIM for $u_2(t)$ in Example 3

	Exact value	VIM	ADM	PLM	TCM	RNIM
t_i						
0.10	-0.01	4.5022e-10	1.6853e-6	1.0230e-4	0.5461e-12	0.0001e-10
0.20	-0.04	4.0722e-9	2.5611e-6	1.7615e-4	0.1035e-11	0.0201e-12
0.30	-0.09	4.7234e-8	3.7002e-5	2.2932e-4	0.1303e-11	0.2216e-11
0.40	-0.16	3.6410e-7	1.8821e-4	2.6958e-4	0.1590e-11	0.1002e-10
0.50	-0.25	2.0360e-6	6.9224e-4	3.0468e-4	0.1949e-11	0.1234e-10
0.60	-0.36	8.8060e-6	0.0021	3.4240e-4	0.2386e-11	0.2867e-10
0.70	-0.49	3.1212e-5	0.0053	3.9051e-4	0.2938e-11	0.4321e-10
0.80	-0.64	9.4447e-5	0.0118	4.5676e-4	0.3573e-11	0.3772e-8
0.90	-0.81	2.5162e-4	0.0240	5.4894e-4	0.4504e-11	0.0213e-9
1.00	-1	6.0400e-4	0.0452	6.7480e-4	0.8311e-11	0.4239e-8

Table 4. Comparison of the absolute errors obtained by the ADM, PLM, TCM, VIM and the RNIM for $u_2(t)$ in Example 3

	Exact value	VIM	ADM	PLM	TCM	RNIM
t_i						
0.10	0.0452	9.8098e-8	2.5313e-6	0.0018	0.5948e-11	0.6624e-12
0.20	0.0819	6.9337e-8	2.1284e-5	0.0022	0.3821e-11	0.4231e-12
0.30	0.1111	2.6971e-7	1.5029e-4	0.0019	0.3404e-11	0.2341e-11
0.40	0.1341	3.5541e-7	6.2907e-4	0.0013	0.2611e-11	0.3554e-9
0.50	0.1516	2.4947e-6	0.0019	7.3996e-4	0.2127e-11	0.6223e-11
0.60	0.1646	1.0872e-5	0.0047	3.1787e-4	0.1652e-11	0.2631e-10
0.70	0.1738	3.8523e-5	0.0101	1.3068e-4	0.1141e-11	0.1032e-10
0.80	0.1797	1.1488e-4	0.0195	1.5143e-4	0.1501e-11	0.2000e-9
0.90	0.1830	3.0093e-4	0.0349	2.7099e-4	0.4403e-11	0.0234e-9
1.00	0.1839	7.1128e-4	0.0585	3.0815e-4	0.2519e-9	0.3201e-9

5. Conclusions

In this present paper, we employed the modification of NIM; termed as “revised NIM” for solving a system of nonlinear delay Integro-differential equations which arises in a model describing biological species living together. The method yields a series solution which converges faster than numerical methods. It is then observed from figures and tables that the method is simple and powerful tool to obtain the approximate solution of this system.

Maple 15 was used to carried out the computations.

6. Recommendations

This method can also be extended to other models in future.

REFERENCES

[1] Abdou, M.A. (2002). Fredholm-Voltera Integral Equations of the First Kind and Contact Problems. *Appl. Math.. and comput.* 125: 177-193.

[2] Bloom, F. (1980). A symptotic bounds for solutions of damped Integro-differential of electromagnetic theory. *J. Math. Anal Appl.* 73: 524-542.

[3] Lokta, A.J. (1939). On an Integral equation in population analysis. *Anal. Math. Stat.* 10: 144-161.

[4] Kot, M. (2001). Elements of Mathematical Ecology. *Cambridge University press.*

[5] Pougaza, D.B. (2007). The lokta Integral equation as a stable population model, post-graduate essay. *African Institutes of Mathematical Society (AIMS).*

[6] Kopeikin, I.D. and Shishkin, V.P. (1984). Integral Form of the General Solution of Equations of Steady-state thermo

elasticity. *Journal of Appl. Math. Mech*. (PMMU.S.S.R). 48(1), 117-119.

[7] Volterra, V. (1927). Variazionifluttuazioni del numerod'Individui in specie animali convinenti, memorie del R. *comitato talessografico italiano.*

[8] Jerri, A.J. (1999). Introduction to Integral equation with applications. *John wiley and sons Inc.*

[9] Babolian, E. and Biazar, J. (2002). Solving the problem of biological species living together by Adomian decomposition method. *Appl. Malt and comput.* 129: 339-343.

[10] Shakeri, F. and Dehghen. (2008). Solution of model describing biological species living together using the variational Iteration method. *Mathematical and computer modeling*. 48: 659-69.

[11] Yousefi, S.A. (2011). Numerical solution of a model describing biological species living together by using Legendre multiwalet method. *International Journal of nonlinear science*. 11(1): 109-113.

[12] Tari, A. (2012). The differential transforms method for solving the model describing biological species living together. *Iranian Journal of Mathematical Science and informatics.* 7(2), 63-74.

[13] Gokmen, E., Sezer, M. (2005). Approximate solution of a model describing biological species living together by Taylor collocation method .*New Trends in Mathematical Sciences*. 3(2): 147-158.

[14] Datterder-Gejj, V. and Jafari H. (2006). An Iterative method for solving functional equations. *J. Math. and Appl*. 316: 753.

[15] Hemeda, A.A. (2012). New Iterative method; Application to nth-order Integro-differential equations. *International Mathematical Forum.* 7: 2317-2332.

[16] Ibrahim, H. and Ayoo, P.V. (2013). Approximation of systems of volteraIntegro-differential equations using the new Iterative method. *International Journal of Science and Research.* Index Copernicus value (2013); 6.14, ISSN (online), 2319-7064.

[17] Ambreem B., Abid U., Hayat K. and Moliyud-Hin S.T., (2013). New iterative method for Time-fractional Schrodiger equations. *World Journal of Modelling and Simulation*. 9: 89-95.

[18] Bhaleker S. and Daftardar-Gejji V. (2012). Solving fractional-order logistic equation using the new iterative method. *Hindawi publishing corporation, International Journal of Differential Equations*. 20: 12-16.

[19] Daftardar-Gejji, V. and Bhaleker, S. (2008). Solution using fractional Diffusion-wave equation using new Iterative method. *International Journal for Theory and Applications*. 11: 1311-1454.

[20] Rameden, J. and Al-luhaibi, M.S. (2014). New Iterative method for solving the formberg-wittam equation and comparision with the homotopy perturbation transform method. *British Journal of Math. and comput.* 4(9): 1231-1227.

[21] Yaseen, M., Samaiz, M. and Naheed, S. (2012). The DJ method for exact solutions of Laplace equation. *International Journal of Mathematical Physics.* 12: 1208-3350.

[22] Aboiyar, T. and Ibrahim, H. (2015) Approximation of systems of Volterra Integral equations of the second kind using the new Iterative method. *International Journal of Applied Science and Mathematical Theory.* 1(4).

[23] Ibrahim, H. Attah, F. and Gwegwe, G.T. (2016). On the solution of Voltera-Fredholm and mixed Voltera-Fredholm Integral equations using the new Iterative method (accepted for publication). *Journal of Applied Mathematics* (SAP).

[24] Bhaleker, S. and Daftardar-Gejji, V. (2012). Solving system of nonlinear functional equations using Revised new Iterative method. *World Academy of Science, Engineering and Technology*. 68.