

Accuracy of Predictions of Ice Slurry Properties and Technical Characteristics of Machines Producing It

Boris Menin

Mechanical & Refrigeration Consultant Expert, Beer-Sheba, Israel

Abstract The heat transfer properties of ice slurries and technical characteristics of ice-slurry makers remain unclear and need to be better understood. We aim to introduce a specific metric called the comparative error by which the discrepancy between the chosen model and the observed object is verified. Information theory methods are applied in order to calculate this error caused only by the finite number of recorded variables after they have been chosen by the conscious observer and before defining the model structure. The comparative error can be realized by field tests or computer simulation. We show that the quantity of information contained in the model is calculated and defines the required number of variables that should be taken into account. The use of the suggested approach can help researchers to find the value of the experimental error for the confirmation of the eligibility of the chosen model for different ice slurry applications. Examples of practical application of the considered concept for the measurement of ice slurry behaviors and the ice-slurry maker parameters are discussed.

Keywords Error Analysis, Ice Slurry, Information Theory, Heat Transfer, Modeling, Theory of Similarity

1. Introduction

Pumpable ice [1] technology is known from the 70th years of the twentieth century. It is based on the production of fluids or secondary refrigerants, also called coolants, with the viscosity of water or jelly and the cooling capacity of ice. Pumpable Ice is typically a slurry of ice crystals or particles ranging from 5 to 10,000 μm (1cm) in diameter and transported in brine, seawater, food liquid, or gas bubbles of air, ozone, or carbon dioxide [2].

Besides generic terms such as “pumpable”, “jelly”, and “slurry ice”, there are many trademark names [2] for such coolants, like “Deepchill”, “Beluga”, “optim”, “flow”, “fluid”, “jel”, “binary”, “liquid”, “maxim”, “whipped”, and “bubble slurry” ice. These brands are authorized by more than 40 industrial ice-maker production companies [3]. Following the recommendations of the Working Party on Ice Slurries, we will use the term “ice slurry” (IS).

Compared to crystal ice (crushed, plate, tube, shell, flake ice), IS is the most attractive cooling medium [4]. Many applications of it are possible and have already been realized [5, 6]. IS covers a full surface of a product protecting it from damage. IS removes heat from the product at significantly higher rates than other types of ice. The increased heat flux from IS causes a long-shelf life of products. The overall heat transfer coefficient between the cooled medium and boiled

refrigerant is higher than during the production of other kinds of ice. IS is now well recognized not only as an incomparable cooling technology but also as an excellent preservation medium. These advantages have resulted in a huge interest in the development and manufacture of IS machines.

In spite of many experimental and numerical investigations regarding the thermal-hydraulic characteristics of IS generator and IS itself (see Chapter 2), the industrial technical parameters of IS generators and IS properties are still unsatisfactory and there is a need to support everyday engineering problems and to develop a single simplified method that allows the use of engineering judgment based on experience. This will enhance our ability to understand the relative impact of the main technical and technological parameters of IS machines.

At the same time, there is no universally accepted recommendation on how to calculate a pure ice concentration in IS. Although there is a standard for the calculation of the performance of generators of crystalline ice [7], the “Working Party on Ice Slurries” of the International Institute of Refrigeration has not yet put forward any relevant proposals. That is why the vast majority of industrial IS generator manufacturers currently do not provide sufficient technical information for understanding the real advantages and disadvantages of IS machines and establishing an objective comparison of specific indicators of ice machines. The companies use different technologies to produce their panels, as well as different methods of testing and verifying their claims.

In this note, we prepared a comparison table of specific

* Corresponding author:

meninbm@gmail.com (Boris Menin)

Published online at <http://journal.sapub.org/ajcam>

Copyright © 2016 Scientific & Academic Publishing. All Rights Reserved

parameters of IS makers (see Table 1 at the end of the paper) made by several companies.

Some of the companies mentioned in Table 1 claim that their systems allow the refrigerant charge to be reduced but, among all the manufacturers analyzed by [8], only Crytec® provides sufficient technical information to establish a comparison between direct and indirect cooling.

Considerable variation of the values of specific parameters of ice machines, especially the coefficient of performance, according to the author's opinion, confirms the need to develop a uniform methodology for calculating the pure ice concentration in IS. In addition, it creates a situation in the ice machine markets that does not allow potential buyers to obtain true technical information to help them choose a preferred model.

It is obvious that many business people and engineers cannot agree with the presented results introduced in Table 1. This situation will persist as long as the "Working Party on Ice Slurries" of the International Institute of Refrigeration does not publish its advice and recommendations.

The aims of this paper are to formulate a novel approach for the development of physical-mathematical models (PMMs) and a reliable universal metric for use in the validation process in order to calculate the discrepancy between the PMM and the observed and researched material object (MO), in our case, the IS properties and IS generator characteristics that obtained from measurements. The presented metric is based on the calculation of the comparative error which corresponds to the error inherent in the model and caused only by its finiteness.

Table 1. Comparison of IS maker characteristics

Kind of Ice	Pumpable ice												
	Stream Ice	Liquid Ice			Slurry Ice			Slurry Ice			Bubble Slurry Ice		
Company	Ziegra [31]	Optimar [32]			Recom Ice systems [33]			Zhejiang Kingfit Environment Co., Ltd. [34]			Crytec Ltd. [35]		
Country	Germany	Iceland			Netherlands			China			Israel		
Type of Ice Maker	SI 7500	B-105	B-110	B-120	SL 100	SL 150	SL 200	QSI-5T	QSI-10T	QSI-15T	CR-004	CR-010	CR-020
Ice Capacity (IC) * : Metric tons/24h kg/h	4.5 188	2.1 90	4.8 200	12.2 510	5.0 208	7.5 313	10.0 417	1.3 52	2.5 104	3.8 156	3.5 146	8.5 354	15.5 646
Power (P), kW	20	7	17	24	10	15	20	12	22	30	8.9	25.4	44.4
Ice concentration, %	15-50	up to 40			up to 70			up to 50			up to 50		
Application	marine												
Refrigerant type	R 404A	R 404A			R 404A			R 404A			R507		
Dimensions, m													
Length	2.2	1.4	1.3	1.7	1.9	2.2	3.0	1.2	1.5	1.7	1.8	1.8	2.0
Width	2.2	0.6	1.2	1.4	1.7	1.8	1.8	0.9	1.0	1.2	0.6	1.2	1.4
Height	1.4	0.9	1.7	1.7	1.2	1.8	2.1	2.0	1.9	1.9	1.1	1.4	1.4
Shipping weight, kg	1,400	336	620	1,040	1,300	1,700	2,200	1,200	1,500	1,700	400	820	1,350
SPECIFIC PARAMETERS													
IC per footprint, (kg/h) / m²	39	104	139	226	64	79	76	48	69	79	144	169	237
IC per volume, (kg/h) / m³	28	112	80	131	55	44	36	31	37	42	129	125	168
IC per weight, (kg/h) / kg	0.1	0.26	0.32	0.49	0.16	0.18	0.19	0.04	0.07	0.09	0.36	0.43	0.48
**COP of ice maker	0.87	1.16	1.09	1.98	1.94	1.94	1.94	0.40	0.44	0.48	1.52	1.30	1.35

*Capacities are based on 15°C seawater

** COP = [IC (kg/h) X 0.093] / P

■ Pure ice capacity is 8-12 metric tons per 24h

■ Pure ice capacity is 3-8 metric tons per 24h

■ Pure ice capacity is up to 3.0 metric tons per 24h

The results show that the proposed validation metric provides an error value that is able to quantify the agreement between the developed PMM and the researched MO and is easily interpretable.

2. Background

It is obvious that in order to use the unique properties of IS and high energy efficiency IS makers in industry, agriculture, medicine and the fishing industry, we must have exact knowledge of the properties of IS and IS machine specifications.

For these purposes, in recent years, there has been a dramatic increase in the use of mathematical modeling and numerical simulation as a tool of evaluation of multivariable or complex physical processes. Scientists, no doubt, are interested too in testing the veracity of models used in their investigations. However, comprehensive testing is currently impossible [9]. In practice, model validation aims to increase confidence in the accuracy of the model. Estimation of the model can be made with different levels of detail, but there is no generally accepted or standard procedure that would establish the minimum quantitative requirements for the making of a test model [10]. At the same time, the general strategies of matching models and MOs that have been particularly popular from both a theoretical and an applied perspective are *verification* and *validation* (V&V) techniques [11].

Verification is the process of the conforming that a computational model accurately represents the basic mathematical model and its solution; *validation* is the process of determining the degree to which a model guarantees an accurate representation of the real world from the perspective of the intended use of the PMM [12].

We can say that quality validation may be useful in certain scenarios, especially in the process of identification of possible causes of errors in the model. However, at the moment, the validation is not able to provide a quantitative measure of the agreement or difference between the experimental and computer data. A more detailed discussion of the existing developments in V&V is introduced in [13, 14]. Claims about V&V of PMM results, in general, are now routinely found in published literature.

However, some scholars suggest that the V&V of numerical models of natural systems is impossible [15]. The authors argue that the model can never fully simulate reality in all conditions and, therefore, cannot be confirmed.

Taking into account the above mentioned statements we reviewed several technical information sources (in English) about the activities related to IS and usage of V&V including publications of IJR (International Journal of Refrigeration) and Proceedings of IIR (International congresses on refrigeration). The review covers a total amount of publications of 1,006 papers and the period of 2000-2015 [17]. The number of articles specifically devoted to the study of the properties of IS is 78, while the numbers of articles

focused on the calculation of the IS machine characteristics is 18. Most of the authors declare satisfactory results such as “*model predictions are confirmed by experiments*”, “*model captures the trends found in experimental investigations*”, “*comparison showed a very good agreement between these data*”, “*it can supply plentiful parametric information in the interested region*” and so on.

As mentioned above, numerical models can be verified or validated. At the same time, the above-stated assertion regarding the research region in which we are interested is, unfortunately, far from the reality.

Factually, *no one has calculated a generalized absolute error (AE) of a specific goal function (similarity criterion/consolidated criteria) that describes the "life-activity" of the researched MO, characterizes the interaction of the MO with the environment, and is compiled from the observed physical variables measured at certain accuracy. Nobody has calculated the difference between the numerical predictions and experimental results with a total absolute error achieved during field tests.*

Thus, there is a sizable body of literature on the methods of development of PMMs that describe MOs with the lowest possible discrepancy. At the same time, nobody has tried to quantify the conceptual PMM error caused by choosing a finite number of variables to take into account.

Using the proposed method, it will be possible to assess the proximity of the PMM to the real characteristics of IS and IS machine settings and to calculate the error observed in experimental studies or computer modeling. Besides that, by applying the suggested approach, one can significantly reduce the duration of the research and design phase and thus reduce the cost of the project.

3. Applied Techniques

First of all, let us specify the concept of "comparative error." We define the "error" as the minimum discrepancy between the tested MO and PMM that was caused only by the fact that only a finite number of variables were taken into account.

The comparative error ε [18] is the ratio between ΔU and S^* :

$$\varepsilon = \Delta U / S^* \quad (1)$$

where ΔU is the alleged error of the measured dimensional (DL) variable U ; S^* is the considered DL range of changes of the measured DL variable U .

If S^* is not known, the information obtained during the measurement cannot be defined. S^* depends on the previous knowledge that the conscious observer had before performing the research. If nothing is known about the studied system, then S^* is defined by the limits of the measuring devices used. For this reason, it would be useful to express the closeness of the PMM to the studied MO with the help of the comparative error.

We take into account that comparative errors of U and the

dimensionless (DS) researched variable u are equal:

$$(\Delta u/S) = (\Delta U/t^*)/(S^*/t^*) = (\Delta U/S^*) \quad (2)$$

Before using the methods of the information theory of thermodynamic processes in order to formulate the approach of calculation of the model's error in the description of the IS properties and IS makers, we introduce the term "System of Primary Variables (SPV)", which is like the International System of Units (SI). It is a set of primary DL variables and secondary DL variables, designed on the basis of the primary ones [19], which are necessary and sufficient to describe and characterize the observed MO quantitatively and with regard to its physical content. The number of DL variables is finite.

In addition, it is worth underling that the limits of description of the studied MO are caused by the choice of the class of phenomenon (**CoP**) and the number of secondary parameters taken into account in the PMM. **CoP** is a set of physical phenomena and processes described by a finite number of primary and secondary variables that characterize certain specific features of the MO with qualitative and quantitative aspects [20]. For example, in thermodynamics, the base set of dimensions often includes the length L (m), weight M (kg), time T (s), and temperature Θ (K).

The dimension of any q in a frame of SI can only express a unique combination of the dimensions of seven main primary variables in different degrees [19]:

$$q \ni L^l \cdot M^m \cdot T^t \cdot \Theta^\theta \cdot J^j \cdot F^f \quad (3)$$

where I is the strength of electric current (A), J is the luminous intensity (cd), and F is the amount of substance (mol).

According to [21], the exponents of variables $l, m, \dots, f$ take only integer values and change in the following ranges:

$$\begin{aligned} -3 \leq l \leq +3, -1 \leq m \leq +1, -4 \leq t \leq +4, -2 \leq \theta \leq +2, \\ -4 \leq \theta \leq +4, -1 \leq j \leq +1, -1 \leq f \leq +1 \end{aligned} \quad (4)$$

The number of choices of dimensions for each variable according to (4) is the following:

$$e_l = 7; e_m = 3; e_t = 9; e_\theta = 5; e_j = 3; e_f = 3 \quad (5)$$

It can be shown [22, 23] that a set of DS variables constructed in a frame of SI with seven basic DS $\mathbf{N}_{SI}$ may be considered as a subgroup of the infinite Abelian group representing all physical DL variables. This is a fictitious system, since it does not exist in physical reality. At the same time, the actually existing MO may be expressed by this set. Then, taking into account (5), $\mathbf{N}_{SI}$ equals [23]

$$\begin{aligned} \mathbf{N}_{SI} &= (e_l \cdot e_m \cdot e_t \cdot e_\theta \cdot e_j \cdot e_f - 1)/2 - 7 \\ &= (7 \cdot 3 \cdot 9 \cdot 5 \cdot 3 \cdot 3 - 1)/2 - 7 = 38,265 \end{aligned} \quad (6)$$

where "-1" corresponds to the occasion when all integers of primary variables in the formula (3) are treated as having zero dimensions; dividing by 2 means that there are both required and reverse variables, for example, the length L^1 and the running length L^{-1} ; and 7 is the number of primary variables in SI.

Let us consider the situation where all DS variable of SI have equal probabilities. Then, there is an uncertainty

directly related to $\mathbf{N}_{SI}$. This approach completely ignores the human evaluation of information. In other words, the set of 100 notes played by chimpanzees and the melody of Mozart's 100 notes in Piano Concerto No.21 Andante, have exactly the same amount of information.

It is useful to be able to calculate information theory quantities for the chosen mathematical model as a part of an investigation into effective V&V methods or as a tool for quantifying the discrepancy between the developed PMM and MO.

Based on these assumptions, the DS error of the PMM caused by the finite number of recorded variables Δu_{pmm} can be obtained [24]

$$\varepsilon = \Delta u_{pmm}/S \leq [(z' - \beta')/\mathbf{N}_{SI} + (z'' - \beta'')/(z' - \beta')] \quad (7)$$

where S is the DS interval of observation/supervision of DS variable u ; z' is the number of physical DL variables in selected **CoP**; β' is the number of primary physical DL variables in selected **CoP**; z'' is the number of physical DL variables recorded in PMM; β'' is the number of primary physical DL variables recorded in the PMM.

Following the above formulated approach, it can be argued that when comparing different PMMs (according to a value of $\Delta u_{pmm}/S$) describing the same MO, preference should be given to the PMM for which the ratio τ is closer to 1, that is,

$$\tau = \Delta u_{pmm}/\Delta u_{exp} \rightarrow 1 \quad (8)$$

4. Examples of Approach Applications

It should be mentioned that a style of the examples below is instructive and didactic that can be explained by a necessity to show features of the $\mathbf{N}_{SI}$ -hypothesis application.

Besides that, the sizable body of publications does not provide enough input data for calculation and verification of the results obtained by (7). Here, an attempt will be made to analyze the published results and compare them with the data obtained by the approach introduced using Equation (7).

4.1. IS Properties

4.1.1. In [25], a similarity theory with non-dimensional numbers and criteria was applied in order to formulate the PMM describing the influence of the mass flow, pipe diameter or ice contents on the pressure drop characteristics and heat transfer process of flowing IS. The researchers measured the thermal and hydraulic characteristics of IS using original testing equipment. The experimental procedure and treatment of results were carefully explained.

Due to analysis of the recorded variables dimensions, the PMM was classified with **CoP** $\equiv LMT\Theta$. Let us calculate the conditions of realizing the minimum comparative error of the PMM $(\varepsilon_{min})_{LMT\Theta}$:

$$\begin{aligned} (\Delta u_{pmm}/S)'_{z'-\beta'} &= [(z' - \beta')/\mathbf{N}_{SI} + (z'' - \beta'')/(z' - \beta')] \\ &= [1/\mathbf{N}_{SI} - (z'' - \beta'')/(z' - \beta')^2] \end{aligned} \quad (9)$$

$$[1/\mathbf{N}_{SI} - (z'' - \beta'')/(z' - \beta')^2] = 0 \quad (10)$$

$$(z' - \beta')^2 / \kappa_{SI} = (z'' - \beta'') \quad (11)$$

where ' - denotes the partial derivative with respect to $z' - \beta'$.

So, $(\epsilon_{\min})_{LMT\theta}$ can be reached with the following data:

$$\begin{aligned} (z' - \beta') &= (e_l \cdot e_m \cdot e_t \cdot e_\theta - 1) / 2 - 4 \\ &= (7 \cdot 3 \cdot 9 \cdot 9 - 1) / 2 - 4 = 846 \end{aligned} \quad (12)$$

$$(z'' - \beta'') = (z' - \beta')^2 / \kappa_{SI} = 846^2 / 38,265 = 19 \quad (13)$$

where 4 is the number of primary variables in $CoP \equiv LMT\theta$, and is equal to

$$(\epsilon_{\min})_{LMT\theta} = 846 / 38,265 + 19 / 846 = 0.0445 \quad (14)$$

The authors [25] did not provide data on the limits of observation (range of the size of change) and the claimed error of measurement for each parameter, and did not calculate the total relative or comparative error of key variables of the investigated process including the pressure drop and heat transfer.

Taking into account all of the above mentioned factors, we use only known data in addition to the following assumption.

To describe the pressure drop behavior, eight ($z' - \beta' = 8$) non-dimensional parameters have been used. To study the heat transfer, at the beginning, 14 DS criteria and numbers were applied but after simplification the experimental results were given by only four ($z' - \beta' = 4$). The comparative errors of measurement of the pressure drop ϵ_{1exp} and heat transfer ϵ_{2exp} are the following:

$$\epsilon_{1exp} = 850 / 38,265 + 8 / 850 = 0.0316 \quad (15)$$

$$\epsilon_{2exp} = 850 / 38,265 + 4 / 850 = 0.0269 \quad (16)$$

Therefore, these values are not closed to the comparative error calculated by Equation (15). So, the methods and apparatus used need to verify the IS properties' values with the right accuracy.

4.1.2. The demonstration of the influence of the glycol solution and the ice fraction on the rheological behavior of IS was presented in [26]. IS was considered as a single-phase fluid with space dependent physical properties. PMM was explained with clear assumptions, equations, boundary conditions and computerized algorithm including 27 DL variables. A calibrated experimental apparatus including information about the technical data of testing equipment and the realized measurement accuracy of the flow rate, temperature, and ice and glycol solution concentration was presented. Experimental, numerical and analytical results were given and illustrated in the graphs of pressure drop (Pa) versus flow velocity (m/s) for different process conditions. The authors [26] declared that the PMM can be considered to be very reliable and its results are very close to obtained experimental data.

However, as in the previous study, the authors [26] did not provide information about the overall error of differential pressure reached in the experiment. Therefore, we have to make the following assumptions.

The considered DL range of changes of the measured pressure drop is $S_{pd} = 3,800 - 2,000 = 1,800$ Pa and the overall DL error of measurement of pressure drop is $\Delta_{pd} = 150$ Pa [26]. Then the possible comparative error of the pressure drop of

the experimental study is

$$\epsilon_{pd} = \Delta_{pd} / S_{pd} = 150 / 1,800 = 0.0833 \quad (17)$$

Thus, this value is larger than the lowest comparative error calculated according to Equation (14). That is the reason why the research team knew, a priori, the limit value of the achievable accuracy and were able to try to find a more promising method for obtaining better results.

In justification of the findings we can mention that the practice of comparing numerical and analytical solutions is best referred to as bench-marking because the congruence between them does not indicate entails anything about the correspondence to material reality [15].

The results of the above mentioned papers show, without a doubt, significant progress in determining the IS properties. However, they are not sufficient for use in the calculation of the technical characteristics of working modes of air coolers or ice machines, which will be discussed below.

In general, when analyzing the results of the above-mentioned researches, the neglect of V&V methods and uncertainty analysis, which ultimately only help modelers to move closer to a more detailed understanding of the nature and properties of IS, is strange and difficult to explain.

4.2. IS Generators

4.2.1. Similarity theory was applied for a novel system using direct contact heat transfer between air and water solution and generating IS [27]. The empirical formula connects the volumetric heat transfer coefficient with the main essential parameters including geometric, fluid, and thermal material properties. This formula of the volumetric heat transfer coefficient agreed with the experimental values with a relative error of less than 15%. Based on this formula, a laboratory-scale direct-contact IS generator was constructed and tested.

According to the modeler's claims, the charts built can help engineers to design and calibrate the size and operational conditions of IS generators for practical application in planned/envisaged practical applications. At the same time, V&V procedures are absent.

The chosen PMM relates to $CoP \equiv LMT\theta$ and the minimum comparative error that could be reached at $(z'' - \beta'') \approx 19$ (14). In turn, during the theoretical research and experimental tests eleven DL variables and five DS criteria and complexes were recorded, which is far from the recommended 19. In other words, in the proposed approach, to achieve the lower error of the applied PMM, the use of a distinctive additional amount of DS complexes is required. Currently, engineers cannot conveniently look up the charts to design and calibrate the size of IS makers and operational conditions without carrying out complicated individual modeling and additional computations.

4.2.2. Reference [28] presents a study in which an attempt was made to analyze the working mode of the serial production IS maker. Its ice capacity appears in the form depending on the selected essential parameters.

The object of the research is a generator of pumpable (slurry) ice called the Wiped Surface Crystallizer and includes several horizontal double-walled crystallizers-evaporators, with a revolving rotor (shaft and plastic wipers) in each, compressor-condenser unit (with R507A refrigerant) and all required refrigeration and liquid accessories.

As the goal function, the DS pure ice capacity $X = G/P$, (kg/s) of the IS generator was selected and presented as a product correlation function of one-parameter functions of the recorded essential parameters and a normalizing constant. G is the pure ice capacity (kg/s) and P is the flow rate of liquid (kg/s).

Twelve (z') input DL parameters were taken into account in this study including the geometric characteristics of evaporators, fluid parameters, as well as the operational conditions. The total number of DS criteria and numbers that were used for analyzing the working modes was $z' - \beta' = 4$ where β' is number of primary variables: $\beta' = 8$.

In order to find the explicit form of one-parameter functions and the constant factor, each parameter was varied within the range of its admissible values. Then a procedure for obtaining experimental results was realized by the method of least squares, keeping all the other parameters at the main level.

The achieved discrepancy between the experimental and computational data in the range of admissible values of the similarity criteria and dimensionless conversion factors did not exceed 11%.

The PMM is classified with $CoP \equiv LMT\theta$. Based on (12), we find $z' - \beta' = 846$. This corresponds to the lowest comparative error at ($z'' - \beta''$) ≈ 19 (13). This value is higher in comparison to the number of DS criteria used in the reviewed article.

The direct measurement errors are much smaller than the measured values, accounting for a few percent of those values or less. Then the error dX can be formally considered as a small increment ΔX of a measured variable: $\Delta X \approx dX$. In order to find the experimental value of the claimed error of ice capacity ΔX_{exp} , the mathematical apparatus of differential calculus is applied [29]:

$$\Delta X_{exp} = \left| \frac{\partial X}{\partial G} \Delta G \right| + \left| \frac{\partial X}{\partial P} \Delta P \right| \quad (18)$$

where $\frac{\partial X}{\partial G}$ and $\frac{\partial X}{\partial P}$ are partial derivatives with respect to the corresponding values; ΔG is the DL error of measurement of pure ice capacity, 0.018 kg/s, ΔP is the DL error of measurement of liquid flow rate, 0.03 kg/s.

So, the value of ΔX_{exp} reached in the experiment equals

$$\begin{aligned} \Delta X_{exp} &= \left| \frac{1}{G} \Delta G \right| + \left| \frac{G}{P^2} \Delta P \right| \\ &= \left| \frac{1}{0.25} 0.018 \right| + \left| -\frac{0.25}{1.0^2} 0.03 \right| = 0.08 \end{aligned} \quad (19)$$

where $P = 1.0$ kg/s; $G = 0.25$ kg/s.

From (7), using calculated values of κ_{SI} и $z' - \beta'$, one gets the DS ice concentration error of the chosen PMM ΔX_{pmm} :

$$\begin{aligned} (\Delta X)_{pmm} &\leq \Delta X_{max} \cdot ((z' - \beta')/\kappa_{SI} + (z'' - \beta'')/(z' - \beta')) \\ &= 0.5 \cdot [846/38,265 + 4/846] = 0.013 \end{aligned} \quad (20)$$

where ΔX_{max} is the given range of changes of DS ice concentration X allowed by the chosen PMM, 0.5 [28].

From (19) and (20), we get $\Delta X_{exp} > \Delta X_{pmm}$, that is, the actual error in the experiment is 6.2 times (0.08/0.013) greater than the minimum. This means that at the chosen number of DS criteria the existing accuracy of measurement of DL variables is insufficient. In addition, the number of recorded DS variables $z' - \beta' = 4$ is less than the recommended number ≈ 19 (13), which corresponds to the lowest comparative error.

So, the declared discrepancy of 11% between the experimental and computational data does not guarantee that the choice of PMM structure is sufficiently complete at the number of variables taken into account. The PMM with formulated relationships between the main characteristics of the process under investigation may be wrong.

For further experimental work it is necessary to use devices of a higher class of accuracy that is sufficient to confirm and clarify a new PMM designed with many DS variables.

Hence, the use of the suggested approach can help researchers to find, in a few minutes, the minimum value of the *required* experimental error for the confirmation of the eligibility of the chosen PMM. This error will correspond to the error inherent in the model and caused only by its finiteness.

5. Conclusions

In line with the aim of this study this work has formulated the approach and the mathematically calculated metric in order to choose the PMM's number of recorded variables describing the researched phenomenon or process and corresponding to the measurement accuracy in the field experiments. For the chosen PMM, we proposed a formula for calculating the minimum absolute comparative error of the desired goal parameter with which it is possible to compare the experimental data and actual error of field tests.

The formula suggested is very simple and is derived with reference to the number of variables recorded in PMM.

The distance or similarity measure between the formulated PMM and the researched MO only requires calculation of the comparative error. Although IS properties and data on the characteristics of IS machines are used as a paradigmatic example, the formula can be applied to any physical phenomenon in different areas of science and technology.

The author has supposed that the estimate of the error of the mathematical model caused by the model's finiteness (number of chosen variables), can be a special metric for the adequacy of accuracy of the physical experiment measurements.

Author intends that the suggested tool be used by researchers and designers interested in learning how efficiently the comparative error is used in the development of modern refrigeration equipment compared to other metrics. This tool allows users to input the qualitative and

quantitative data needed to score results for any model.

The physical meaning of Δu_{pmm} lies in the fact that when any event or process is schematized, there is a discrepancy between the PMM and the MO called the threshold mismatch [30]. The value Δu_{pmm} , due to the threshold mismatch, should never be more than the permissible error of measurement. Otherwise, it is necessary to redefine the model before carrying out the experiment. In the above approach, Δu_{pmm} represents a sort of "model noise" (similar to "thermal noise").

REFERENCES

- [1] P. Prout, and T. Misson, 2004, Trials of the Pumpable Icing of Fish, Seafish Technology and Training. Available: <http://goo.gl/JkYPJv>.
- [2] Pumpable ice technology, 2009, Wikipedia. Available: <https://goo.gl/OExA43>.
- [3] FIS Israel, Companies Directory: Ice machines > Liquid / Fluid / Slurry. Available: <http://goo.gl/3oEc9x>.
- [4] M. Kauffeld, M.J. Wang, V. Goldstein, K.E. Kasza, 2010, Ice Slurry Applications, Int. J. Refrigeration, 33(8), 1491–1505. Available: <http://goo.gl/EZljqY>.
- [5] P. Rivet, 2008, Ice slurries – state of the art, Ecolibrium, 28-33. Available: <http://goo.gl/npNsWs>.
- [6] M. Kauffeld, M. Kawaji, and P.W. Egolf, 2005, Handbook on Ice Slurries – Fundamentals and Engineering. Published by the International Institute of Refrigeration (IIR/IIR). Available: <http://goo.gl/X0uxoZ>.
- [7] ASHRAE Standard 29-1988R (RA 2005), Method of Testing Automatic Ice Makers. Available: <https://goo.gl/ND66Ju>.
- [8] F. Illan, A. Viedma, 2010, Using ice slurry secondary refrigerant for charge reduction in industrial facilities. IIR Second Workshop on Refrigerant Charge Reduction, KTH, Stockholm, Sweden, 1-8. Available: <http://goo.gl/hUZdOV>.
- [9] B. Osman, 1997, Principles of Simulation Model Validation, Verification, and Testing, Trans. of Society for Computer Simulation International, 14(1), 3-12.
- [10] G. Bellocchi, R. Confalonieri, and M. Donatelli, 2006. Crop modelling and validation: integration of Irene DLL in the warm environment, Italian Journal of Agrometeorology, 3, 35-39. Available: <http://goo.gl/9MO8Ie>.
- [11] C.-J. Kat, and P.S. Els, 2012, Validation metric based on relative error. Mathematical and Computer Modelling of Dynamical Systems: Methods, Tools and Applications in Engineering and Related Sciences 18(5) (2012) 487-520. Available: <http://goo.gl/sFk9UR>.
- [12] ASME V&V 10-2006, 2006, Guide for Verification and Validation in Computational Solid Mechanics, The American Society of Mechanical Engineers standards. Available: <http://goo.gl/9gjVdA>.
- [13] P.G. Maropoulos, and D. Ceglarek, 2010, Design verification and validation in product lifecycle, CIRP Annals – Manufacturing Technology, 59, 740–759. Available: <http://goo.gl/sJc02G>.
- [14] A.T. Bahill, and S.J. Henderson, 2005, Requirements development, verification and validation exhibited in famous failures systems engineering, Systems Engineering, 8(1), 1-14. Available: <http://goo.gl/srriyy>.
- [15] N. Oreskes, K. Shrader-Frechette, and K. Belitz, 1994, Verification, validation and confirmation of numerical models in the Earth sciences, Science, 263, 641-646. Available: <http://goo.gl/H2temb>.
- [16] O. Tedeschi, 2004, Assessment of the mathematical models. Workshop on Mathematical Model analysis and Evaluation, 1-28. Available: <http://goo.gl/CIWWdO>.
- [17] Publications of IJR, Proceedings of IIR, 2015. Selection of Publications are by keyword "slurry ice". Available: <http://goo.gl/58UrAl>.
- [18] L. Brillouin, Science and Information Theory. USA: Dover, New York, 2004.
- [19] A.A. Guhman, Introduction in the Theory of Similarity, Russia: High School, Moscow, 253-292, 1973, in Russian.
- [20] L.I. Sedov, Similarity and Dimensional Methods in Mechanics, USA: CRC Press, Florida, Chapter 1, 1993.
- [21] B.M. Yavorskij, and A.A. Detlaf, Handbook of Physics, Russia: Nauka, Moscow, Chapter 9, 1985, in Russian.
- [22] A. Laszlo, 1964, Systematization of dimensionless quantities by group theory, International Journal of Heat and Mass Transfer, 7(4), 423–430.
- [23] B.M. Menin, 2015, Estimation of required complexity of physical-mathematical model: universal method, American Journal of Computational and Applied Mathematics, 5(6), 159-163. Available: <http://goo.gl/OHyri8>.
- [24] B.M. Menin, 2014, Comparative error of the phenomena Model, International Referred Journal of Engineering and Science, 3(11), 68-76. Available: <http://goo.gl/zMtMAU>.
- [25] F. Illan, A. Viedma, and P. Martnez, 2005, Dimensional analysis and experimental study of pressure drop and heat transfer for Na-Cl ice slurry in pipes. Sixth Workshop on Ice Slurries of the International Institute of Refrigeration, Yverdon-les-Bains, Switzerland, 1-11.
- [26] S. Mellari, M. Boumaza, and P.W. Egolf, 2012, Physical modeling, numerical simulations and experimental investigations of Non-Newtonian ice slurry flows, Int. J. Refrigeration, 35, 1284-1291.
- [27] X. Zhang, et al., 2013, Analysis of ice slurry production by direct contact heat transfer of air and water solution, Journal of Zhejiang University-SCIENCE A (Applied Physics & Engineering), 14(8), 583-588. Available: <http://goo.gl/JcbNW9>.
- [28] B.M. Menin, 2010, Calculation of pumpable (slurry) ice machine capacity, Journal of Scientific Israel– Technological Advantages, Civil & Mechanical Engineering, 12(2), 178–181. Available: <http://goo.gl/YmZna2>.
- [29] J. Taylor, An Introduction to Error Analysis, USA: University Science Books, Mill Valley, California, Chapter 1, 1982. Available: <http://goo.gl/Pkk50s>.
- [30] S.G. Rabinovich, Measurement Errors and Uncertainties:

- Theory and Practice. USA: Springer, New York, Chapter 1.3, 2005.
- [31] ZIEGRA Eismaschinen, Germany, web-site: <http://goo.gl/C HL8uN>.
- [32] Optimar, Iceland, web-site: <http://goo.gl/8brdJG>.
- [33] Recom Ice systems, Netherlands, web-site: <http://goo.gl/Zm 57AH>.
- [34] Zhejiang Kingfit Environment Co., Ltd., China, web-site: <http://goo.gl/zxYLA9>.
- [35] Crytec Ltd., Israel, web-site: <http://goo.gl/77yQDY>.