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# Assessment of Human Skills Using Trapezoidal Fuzzy Numbers (Part II)

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**Abstract** In an earlier work, recently published in this journal, we have used the Trapezoidal Fuzzy Numbers (TpFNs) as an assessment tool of human skills. This approach led to an approximate linguistic characterization of the corresponding group's overall performance, but it was not proved to be always sufficient for comparing the performance of two different human groups, since two TpFNs are not always comparable. In this paper we complete the above fuzzy assessment approach by presenting a defuzzification method of TpFNS based on the Center of Gravity (COG) technique, which enables the required comparison of the performance of two (or more) groups.

**Keywords** Human Assessment, Fuzzy Logic, Fuzzy Numbers (FNs), Triangular (TFNs) and Trapezoidal (TpFNs) Fuzzy Numbers, Center of Gravity (COG) Defuzzification Technique

## 1. Introduction

The social demand of classifying humans according to their qualifications makes the assessment of human skills a very important task. *Fuzzy logic*, due to its nature of characterizing an ambiguous case with multiple values, offers rich resources for the assessment purposes. This gave us several times in past the impulse to apply principles of fuzzy logic for assessing human skills using as tools the *corresponding system's uncertainty* (e.g. see [8] and its relevant references, Section 3 of [9], etc), the *COG defuzzification technique* (e.g. see [5], Section 3 of [9], etc), as well as two recently developed variations of the COG technique, i.e. the *Triangular (TFAM) and Trapezoidal (TpFAM) Fuzzy Assessment Models* (e.g. see [6] and [11] respectively, etc). It is of worth to notice that the TFAM and TpFAM, which are equivalent to each other since they obtain exactly the same results, treat better than the COG technique the ambiguous assessment cases being at the boundaries between two successive assessment grades. The use of the COG technique for assessment purposes, as well as the above mentioned two variations of it were initiated by Igor Subbotin, Professor of Mathematics at State University in Los Angeles and Voskoglou's coauthor in many publications (e.g. [6], [11], etc).

In a recently published paper [12] we have extend our above researches by using the *Trapezoidal Fuzzy Numbers* (*TpFNs*) as an assessment tool of human skills. This approach, while it is better than our older fuzzy methods for the *individual assessment* [10], in case of *group assessment* led (in [12]) to an approximate characterization of the group's overall performance and *it was not proved to be always sufficient for comparing the performance of two different groups* (for more details see below Example 1 of Section 4).

In this paper we *complete* the above fuzzy assessment approach by presenting *a defuzzification method of TpFNS* based on the Center of Gravity (COG) technique, which enables the required comparison of the performance of two (or more) groups. The rest of the paper is organized as follows: In Section 2 we recall in brief some definitions presented in [12], which are necessary for the understanding of the present paper. In Section 3 we present the defuzzification method for TpFNs based on the COG technique. In Section 4 we reconsider and we extend an example originally presented in [12] and we apply our defuzzification method on the outputs (TpFNs) of the extended example for comparing the performance of two basket-ball teams. Finally, Section 5 is devoted to our conclusion and a brief discussion on the perspectives of future research on the subject.

## 2. Introductory Definitions

In this Section we recall in brief some definitions presented in [12], which are necessary for the understanding of the present paper. For general facts on *fuzzy sets* we refer to the book of Klir and Folger [4].

We start with the definition of a fuzzy number:

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***Definition 1:*** A *Fuzzy Number* is a normal (i.e. there exists x in ***R***, such that m(x) = 1) and convex (i.e. its x-cuts[1] $A^x$ are ordinary closed real intervals, for all x in [0, 1]) fuzzy set A on the set ***R*** of real numbers with a piecewise continuous membership function *y = m(x).*

The following statement defines a *partial order* on the set of all FNs:

***Definition 2**:* Given the FNs A and B we write A ≤ B (or ≥ ) if, and only if, $A_l^x \le B_l^x$ and $A_r^x \le B_r^x$ (or ≥ ) for all x in [0, 1]. Two FNs for which the above relation holds are called *comparable*, otherwise they are called *non comparable*.

FNs play a fundamental role in fuzzy mathematics, analogous to the role played by the ordinary numbers in classical mathematics. For general facts on FNs we refer to Chapter 3 of the book [7], which is written in Greek language, and also to the classical on the subject book [3].

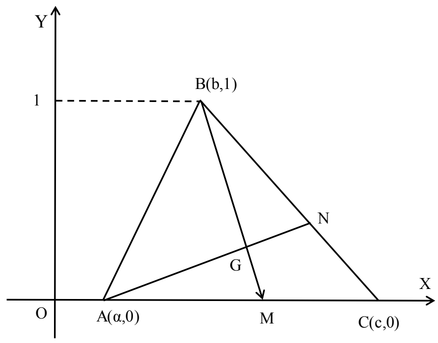


**Figure 1.** Graph and COG of the TFN (a, b, c)

The simplest form of FNs is probably the *Triangular FNs (TFNs)*. Roughly speaking a TFN (*a, b, c*), with *a, b* and *c* real numbers means "*approximately equal to b*" or, if you prefer, that "*b lies in the interval [a, c]*". The graph of the TFN (*a, b, c*) in the interval [*a, c*] is the union of two straight line segments forming a triangle with the X-axis, while it is zero outside [*a, c*] (see Figure 1). Therefore the analytic definition of a TFN is given as follows:

***Definition 3:*** Let *a, b* and *c* be real numbers with *a* < *b* < *c*. Then the *Triangular Fuzzy Number (TFN)* A = (*a*, *b*, *c*) is the FN with membership function:

$$y = m(x) = \begin{cases} \dfrac{x-a}{b-a} &, \ x \in [a,b] \\ \dfrac{c-x}{c-b}, & x \in [b,c] \\ 0, & x < a \ and \ x > c \end{cases}$$

Obviously we have that *m(b)=1*, while *b* need not be in the "middle" of *a* and *c*.

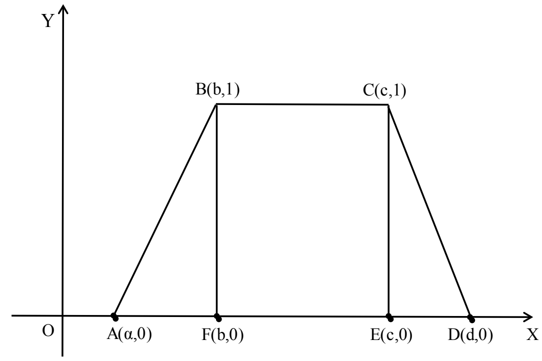


**Figure 2.** Graph of the TpFN (a, b, c, d)

A TpFN (*a, b, c, d*) with *a, b, c, d* in ***R*** actually means "*approximately in the interval [b, c]"*. Its membership function *y=m(x)* is constantly 0 outside the interval [*a, d*], while its graph in this interval [*a, d*] is the union of three straight line segments forming a trapezoid with the X-axis (see Figure 2), Therefore, its analytic definition is given as follows:

***Definition 4:*** Let *a* < *b* < *c*< *d* be given real numbers. Then the TpFN (*a, b, c, d*) is the FN with membership function:

$$y = m(x) = \begin{cases} \dfrac{x-a}{b-a} &, \ x \in [a,b] \\ x = 1, \ , & x \in [b,c] \\ \dfrac{d-x}{d-c}, & x \in [c,d] \\ 0, & x < a \ and \ x > d \end{cases}$$

Obviously the *TpFNs are generalizations of TFNs*. In fact, the TFN (*a, b, d*) can be considered as a special case of the TpFN *(a, b, c, d)* with *b=c.*

It can be shown that the two well known general methods for performing operations between FNs (e.g. see Section 3 of [12]) lead to the following simple rules for the *addition* and *subtraction* of TpFNs:

***Definition 5:*** Let A = ($a_1$, $a_2$, $a_3$, $a_4$) and B = ($b_1$, $b_2$, $b_3$, $b_4$) be two TFNs. Then

- The sum A + B = ($a_1$+$b_1$, $a_2$+$b_2$, $a_3$+$b_3$, $a_4$+$b_4$).
- The difference A - B = A + (-B) = ($a_1$-$b_4$, $a_2$-$b_3$, $a_3$-$b_2$, $a_4$-$b_1$), where

–B = (-$b_4$, -$b_3$, -$b_2$, -$b_1$) is defined to be the *opposite* of B.

In other words, the opposite of a TpFN, as well as the sum and the difference of two TpFNs are also TpFNs. On the contrary, the product and the quotient of two TFNs, although they are FNs, *they are not always TpFNs*, apart from some special cases, or in terms of suitable approximating formulas (for more details see [2]).

***Definition 6***: We define the following two scalar operations:

[1] Let x be a real number of the interval [0, 1]. We recall then that the x-*cut* of a fuzzy set A on U, denoted by $A^x$, is defined to be the crisp set $A^x$ = {y ∈ U: m(y) ≥ x}.

- $k + A = (k+a_1, k+a_2, k+a_3, k+a_4)$, $k \in \boldsymbol{R}$
- $kA = (ka_1, ka_2, ka_3, ka_4)$, if $k>0$ and $kA = (ka_4, ka_3, ka_2, ka_1)$, if $k<0$.

We close this section with the following definition, which will be used Section 4 for assessing the overall performance of a human group with the help of TpFNs:

***Definition 7:*** Let $A_i = (a_{1i}, a_{2i}, a_{3i}, a_{4i})$, i = 1, 2,…, *n* be TpFNs, where *n* is a non negative integer, $n \geq 2$. Then we define the *mean value* of the above TpFNs to be the TpFN:

$$A = \frac{1}{n}(A_1 + A_2 + \ldots + A_n).$$

## 3. Defuzzification of TFNs/TpFNs

In this section we shall use the COG technique for defuzzifying a given TFN/TpFN. We start with the case of TFNs:

***Proposition 1:*** The coordinates (*X, Y*) of the COG of the graph of the TFN (*a*, *b*, *c)* are calculated by the formulas $X = \frac{a+b+c}{3}$, $Y = \frac{1}{3}$.

*Proof:* The graph of the TFN (*a*, *b*, *c*) is the triangle ABC of Figure 1, where A (*a*, 0),

B (*b*, 1) and C (*c*, 0). Then, the COG, say G, of ABC is the intersection point of its medians AN and BM, where N $(\frac{b+c}{2}, \frac{1}{2})$ and M $(\frac{a+c}{2}, 0)$. Therefore the equation of the straight line on which AN lies is $\frac{x-a}{\frac{b+c}{2}-a} = \frac{y}{\frac{1}{2}}$, or

$$x + (2a - b - c)y = a \quad (1)$$

In the same way one finds that the equation of the straight line on which BM lies is

$$2x + (a + c - 2b)y = a + c \quad (2)$$

Since $D = \begin{vmatrix} 2 & a+c-2b \\ 1 & 2a-b-c \end{vmatrix} = 3(a-c) \neq 0$, the linear system of (1) and (2) has a unique solution with the respect to the variables *x* and *y* determining the coordinates of the triangle's COG.

The proof of the Proposition is completed by observing that

$$D_x = \begin{vmatrix} a+c & a+c-2b \\ a & 2a-b-c \end{vmatrix} = a^2 - c^2 + ba - bc$$

$$=(a+c)(a-c)+b(a-c)$$

$= (a - c)(a + c + b)$ and $D_y = \begin{vmatrix} 2 & a+c \\ 1 & a \end{vmatrix} = a - c$.

Next, Proposition 1 will be used as a *Lemma* for the defuzzification of TpFNs. The corresponding result is the following:

***Proposition 2:*** The coordinates (*X, Y*) of the COG of the graph of the TpFN (*a, b, c, d*) are calculated by the formulas $X = \frac{c^2+d^2-a^2-b^2+dc-ba}{3(c+d-a-b)}$, $Y = \frac{2c+d-a-2b}{3(c+d-a-b)}$.

*Proof:* We divide the trapezoid forming the graph of the TpFN (*a, b, c, d*) in three parts, two triangles and one rectangle (Figure 2). The coordinates of the three vertices of the triangle ABE are (*a,* 0), (*b,* 1) and (*b,* 0) respectively, therefore by Proposition 1 the COG of this triangle is the point $C_1$ $(\frac{a+2b}{3}, \frac{1}{3})$. Similarly one finds that the COG of the triangle FCD is the point $C_2$ $(\frac{d+2c}{3}, \frac{1}{3})$. Also, it is easy to check that the COG of the rectangle BCFE is the point $C_3$ $(\frac{b+c}{2}, \frac{1}{2})$. Further, the areas of the two triangles are equal to $S_1 = \frac{b-a}{2}$ and $S_2 = \frac{d-c}{2}$ respectively, while the area of the rectangle is equal to $S_3 = c - b$.

It is well known then (e.g. see [35]) that the coordinates of the COG of the trapezoid, being the resultant of the COGs $C_i$ $(x_i, y_i)$, for i=1, 2, 3, are calculated by the formulas $X = \frac{1}{S}\sum_{i=1}^{3} S_i x_i$, $Y = \frac{1}{S}\sum_{i=1}^{3} S_i y_i$ (3), where $S = S_1 + S_2 + S_3 = \frac{c+d-b-a}{2}$ is the area of the trapezoid.

The proof of the Proposition is completed by replacing the above found values of $S$, $S_i$, $x_i$ and $y_i$, i = 1, 2, 3, in formulas (3) and by performing the corresponding operations.

## 4. Use of the TpFNs for Assessing Human Skills

We reconsider first the following example originally presented in [12]:

***Example 1:*** The performance of the five players of a basket-ball team who started a game was individually assessed by six different athletic journalists using a scale from 0 to 100 as follows: $\mathbf{P_1}$ (player 1): 43, 48, 49, 49, 50, 52, $\mathbf{P_2}$**:** 81, 83. 85, 88, 91, 95, $\mathbf{P_3}$**:** 76, 82, 89, 95, 95, 98, $\mathbf{P_4}$**:** 86, 86, 87, 87, 87, 88 and $\mathbf{P_5}$**:** 35, 40, 44, 52, 59, 62.

The players' performance is characterized by the *fuzzy linguistic labels (grades)* A, B, C, D and F corresponding to the above scores as follows: A (85-100) = excellent, B (84-75) = very good, C (74-60) = good, D (59-50) = fair and F (<50) = unsatisfactory. How one can assess their individual and overall performances in this game?

For this, in [12] we have assigned to each basket-ball player $\mathbf{P_i}$ a TpFN (denoted, for simplicity, by the same letter) as follows: $\mathbf{P_1}$ = (0, 43, 52, 59), $\mathbf{P_2}$ = (75, 81, 95, 100), $\mathbf{P_3}$ = (75, 76, 98, 100), $\mathbf{P_4}$ = (85, 86, 88, 100) and $\mathbf{P_5}$ **=** (0, 35, 62, 74). Each of the above TpFNs characterizes the *individual performance* of the corresponding player in the form (*a, b, c,* *d*), where *a* and *d* are the lower and upper bounds respectively of his performance with respect to the scores assigned to the linguistic grades A, B, C, D and F, while *b* and *c* are the lower and higher scores respectively assigned to the corresponding player by the athletic journalists.

Further, for assessing the overall performance of the above five players, we have calculated in [12] the mean value of the TpFNs $\mathbf{P_i}$ , i = 1, 2, 3, 4, 5 (see Definition 7), which is equal to $\mathbf{P} = \frac{1}{5}\sum_{i=1}^{5} P_i$ = (47, 64.2, 79, 86.6). The value of **P** gives the information that *the five players' performance fluctuates from unsatisfactory (a = 47) to excellent (d = 86.6), while their overall performance lies in the interval [b, c] = [64.2, 79], i.e. it can be characterized from good (C) to very good (B).*

Further, in [12] we have noticed that this approach is not always appropriate to be used alone when one wants to compare the overall performance of two or more groups of players, because two (or more) TpFNs are not always comparable (see Definition 2). However, under the light of Proposition 2, we are now in position to overcome this difficulty by defuzzifying the corresponding TpFNs. In order to illustrate this in practice we extend Example 1 as follows:

***Example 2:*** Reconsider Example 1 and assume that the same six journalits assessed also the performance of the five players of the opponent team who started the same basket-ball game. Assume further that the overall performance of the first five players of the second team was assessed as in Example 1 using TpFNs and that the mean value of the corresponding TpFNs was found to be equal to **P′** = (47.8, 65.3, 78.1, 85.9). How one can compare the overall performance of the two teams?

For this, applying Proposition 2 one finds that the x-coordinate of the COC of the trapezoid constituting the graph of the TpFN **P** is equal to

$$X = \frac{79^2 + (86.6)^2 - (64.2)^2 - 47^2 + 79*86.6 - 47*(64.2)}{3(79 + 86.6 - 47 - 64.2)} \approx 68.84 .$$

In the same way one finds that the x-coordinate *X* 'of the graph of **P′** is approximately equal to 68.13.

Observe now that the GOGs of the graphs of **P** and **P′** lie in a rectangle with sides of length 100 units on the X-axis (player scores from 0 to 100) and one unit on the Y-axis (normal fuzzy sets). Therefore, *the nearer the x-coordinate of the COG to 100, the better the corresponding team's performance*, Thus, since $X > X'$, the first team demonstrates a better overall performance than the second one.

***Remark:*** In the same way as in Example 2 one can defuzzify the TpFNs $\mathbf{P_i}$, i=1, 2, 3, 4, 5 of Example 1 corresponding to the five players of the first basket-ball team. In this way it becomes possible to compare the individual performance *of any two players*, in contrast to our method presented in [10] and the equivalent to it method of A. Jones [1] that define a *partial order only* on the individual performances.

## 5. Conclusions

In the present paper we used the TpFNs as a tool for assessing human skills. The main advantage of this approach is that in case of *individual assessment* it is sufficient for comparing the performances of *all students*, in contrast to the alternative fuzzy assessment methods applied in earlier works, which define *a partial order* only on the individual performances. However, in case of *group assessment* the TpFNs approach *initially leads to an approximate characterization of the group's overall performance, which is not always sufficient for comparing the performances of two different groups*, as our fuzzy assessment methods applied in earlier works do. This is due to the fact that the inequality between TpFNs defines on them a relation of partial order only. Therefore, in cases where our fuzzy outputs are not comparable, *some extra calculations are needed* in order to obtain the required comparison by defuzzifying these outputs. This could be considered as a disadvantage of this approach, although the extra calculations needed are very simple.

Further, our new method of using TpFNs for the assessment of human skills is of general character, which means that it could be utilized for assessing a great variety of human (or machine; e.g. CBR systems [11]) activities. This is one of the main targets of our future research on the subject.


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## REFERENCES


[1] Jones, A., Kaufmman, A. & Zimmerman, H. J. (Eds.), Fuzzy Sets: Theory and Applications, NATO ASI Series, Series C: Mathematical and Physical Sciences, Vol. 177, Reidel Publishing Company, Dordrecht, Holland, 1986.

[2] Hanss, M., Applied Fuzzy Arithmetic, An Introduction with Engineering Applications, Springer-Verlag,Berlin-Heidelber g, 2005.

[3] Kaufmann, A. & Gupta, M., Introduction to Fuzzy Arithmetic, Van Nostrand Reinhold Company, New York, 1991.

[4] Klir, G. J., Folger, T. A., Fuzzy Sets, Uncertainty and Information, London, Prentice-Hall, London, 1988.

[5] Subbotin, I. Ya., Badkoobehi, H., Bilotskii, N. N., Application of fuzzy logic to learning assessment. Didactics of Mathematics: Problems and Investigations, 22, 38-41, 2004.

[6] Subbotin, I. Ya, & Voskoglou, M. Gr., A Triangular Fuzzy Model for Assessing Critical Thinking Skills, International

Journal of Applications of Fuzzy Sets and Artificial intelligence, 4, 173 -186, 2014.

[7] Theodorou, J., Introduction to Fuzzy Logic, Tzolas Publications, Thessaloniki, Greece, 2010 (in Greek language).

[8] Voskoglou, M. Gr., Stochastic and fuzzy models in Mathematics Education, Artificial Intelligence and Management, Saarbrucken - Germany, Lambert Academic Publishing, 2011.

[9] Voskoglou, M. Gr., A Study on Fuzzy Systems, American Journal of Computational and Applied Mathematics, 2(5), 232-240, 2012.

[10] Voskoglou, M. Gr., Assessing Students Individual Problem Solving Skills, International Journal of Applications of Fuzzy Sets and Artificial Intelligence, 3, 39-49, 2013.

[11] Voskoglou, M. Gr. & Subbotin, I. Ya., Evaluating the Effectiveness of a CBR System: A Fuzzy Logic Approach, American Journal of Computational and Applied Mathematics, 5(2), 27-32, 2015.