American Journal of Computational and Applied Mathematics 2015, 5(5): 140-146
DOI: 10.5923/j.ajcam.20150505.02

Probabilistic Models for MTSF Comparison between Systems Connected to Supporting Device for Operation

Ibrahim Yusuf[1], U. A. Ali[2,*], Mansur Babagana[3], Bello Gimba[2]

[1]Department of Mathematical Sciences, Bayero University, Kano, Nigeria
[2]Department of Mathematics, Federal University, Dutse, Nigeria
[3]Department of Computer Science, Bayero University, Kano, Nigeria

Abstract In this paper, we study the mean time to system failure of three dissimilar redundant systems consisting of two subsystems A and B in series each. System I consists of Subsystem A containing two units A_1 and A_2 in cold standby, subsystem B has two units B_1 and B_2 unit in cold standby and with one external supporting unit connected to subsystems A and B. System II consists of Subsystem A containing two units A_1 and A_2 in cold standby, subsystem B has two units B_1 and B_2 unit in cold standby and with two external supporting units, one connected each to subsystems A and B. System III consists of Subsystem A containing two units A_1 and A_2 in active parallel, subsystem B contains two units B_1 and B_2 unit in cold standby and with two external supporting units, one connected each to subsystems A and B. The systems are analyzed using Kolmogorov forward equation method. Explicit expressions for mean time to system failure are derived. Comparisons are made analytically and numerically to determine the optimal configuration.

Keywords Mean time to system failure, Supporting device, Operation

1. Introduction

With advancement of modern science and technology, complex systems have been manufactured to meet the demand of industries, economic growth and populace in general. Companies and organizations heavily rely on these systems to conduct their business. Due to their importance in promoting and sustaining industries and economy, reliability measures of such systems have become an area of interest. Among the reliability measures of interest are, the steady-state availability, busy period, profit function and mean time to system failure, (MTSF). Modelling the reliability and availability of the system is important because it will assist in diagnosing the best time to carry out a preventive maintenance which leads to increase in mean time to system failure and availability of a system and related profit. Reliability modeling and analysis of complex systems have drawn the attention different researchers. [3] Investigated multi-objective reliability redundancy allocation series-parallel problem using efficient epsilon-constraint. [4] Proposed multi-objective particle swarm optimization method for solving reliability redundancy allocation problems.

Many research results have been reported on the analysis and comparison of reliability measures mention above. [1] studied MTSF and cost effectiveness of 2-out-of-3 cold standby system with probability of repair and inspection, [2] have analyzed the Availability of k-out-of-n: G systems with non identical components subject to repair priorities, [5] performed computational comparisons of confidence intervals for the steady-state availability of a repairable system, [6] performed comparative analysis between two unit cold standby and warm standby outdoor electric power systems in changing weather. [7] performed comparative analysis of availability between three systems with general repair times, reboot delay and switching failures. [8] performed comparative analysis of availability between two systems with warm standby units and different imperfect coverage. [9] Performed comparative analysis of some reliability characteristics between redundant systems requiring supporting units for their operation. [10] Have analyzed the mean time to system failure of 2-out-of-4 warm standby system attended repair machines and repairmen while [11] performed comparative analysis of profit between three dissimilar repairable redundant systems using supporting external device for operation.

The present paper is devoted to deal with mean time to system failure (MTSF) comparison between three dissimilar redundant systems that worked with the help of an external supporting device. Explicit expressions for mean time to system failure have been developed. Analytical and numerical investigation on the optimal configuration has been performed.

* Corresponding author:
ubahamad@yahoo.co.uk (U. A. Ali)
Published online at http://journal.sapub.org/ajcam

2. Notations, Description and States of the Systems

α_1 : Repair rate of unit A_k for both systems, $k = 1,2$

α_2 : Repair rate of unit B_k for both systems, $k = 1,2$

β_1 : Failure rate of unit A_k for both systems, $k = 1,2$

β_2 : Failure rate of unit B_k for both systems, $k = 1,2$

α_3 : Repair rate of the supporting unit for both systems

β_3 : Failure rate of the supporting unit for both systems

$P^k(t),\ k = I, II, II$: Probability row vector

$P_y(t), y = 1,2,3$: Probability that the system is in state S_i

3. Mean Time to System Failure Models Formulation

Let $P_i(t)$ to be the probability that the systems at time $t \geq 0$ are in the states S_i, $i = 0,1,2,3,4,5,6$ for system I, $i = 0,1,2,3,4,5,6,7,8,9$ for System II and $i = 0,1,2,3,4,5,6,7,8$ for System III. Also let $P^j(t)$, $j = I, II, III$ be the probability row vector at time t, we have the following initial conditions for system I, II and III respectively:

3.1. Mean Time to System Failure of System I

Let $P_i(t)$ to be the probability that the systems at time $t \geq 0$ are in the states S_i, $i = 0,1,2,3,4,5,6$ and $P^I(t) = [1,0,0,0,0,0,0]$ be the probability row vector for system I with the following differential equations.

$$\frac{d}{dt}(P(t)) = T_1 P(t) \tag{1}$$

where

$$T_1 = \begin{bmatrix} -q_0 & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 \\ \beta_1 & -q_1 & 0 & \alpha_2 & 0 & 0 & 0 \\ \beta_2 & 0 & -q_2 & \alpha_1 & 0 & 0 & 0 \\ 0 & \beta_2 & \beta_1 & -q_3 & \alpha_3 & \alpha_1 & \alpha_2 \\ 0 & 0 & 0 & \beta_3 & -\alpha_3 & 0 & 0 \\ 0 & 0 & 0 & \beta_1 & 0 & -\alpha_1 & 0 \\ 0 & 0 & 0 & \beta_2 & 0 & 0 & -\alpha_2 \end{bmatrix}$$

$q_0 = \beta_1 + \beta_2$, $q_1 = \alpha_1 + \beta_2$, $q_2 = \alpha_2 + \beta_1$, $q_3 = \alpha_1 + \alpha_2 + \beta_1 + \beta_2 + \beta_3$

It is difficult to evaluate the transient solutions, the procedure to develop the explicit expression for $MTSF_1$ is to delete the fifth, sixth and seventh rows and fifth, sixth and seventh column of matrix T_1 and take the transpose to produce a new matrix, say M_1. The expected time to reach an absorbing state is obtained from

$$E\left[T_{P(0)\to P(absorbing)}\right] = MTSF_1 = P(0)(-M_1^{-1})\begin{bmatrix}1\\1\\1\\1\end{bmatrix} \tag{2}$$

$$= \frac{\delta_1 \sum_{n=1}^{3} \beta_n + \delta_2\delta_3\beta_1 \sum_{n=1}^{3} \beta_n + \delta_3\delta_4\beta_2 \sum_{n=1}^{3} \beta_n + \delta_3\beta_1\beta_2}{\delta_3\beta_1\beta_2 \sum_{n=1}^{3} \beta_n}$$

where

$$M_1 = \begin{bmatrix} -q_0 & \beta_1 & \beta_2 & 0 \\ \alpha_1 & -q_1 & 0 & \beta_2 \\ \alpha_2 & 0 & -q_2 & \beta_1 \\ 0 & \alpha_2 & \alpha_1 & -q_3 \end{bmatrix}$$

$$\delta_1 = 2\alpha_1\alpha_2\beta_1 + 2\alpha_1\alpha_2\beta_2 + \alpha_1\alpha_2\beta_3 + \alpha_1^2\alpha_2 + \alpha_1\alpha_2^2 + \alpha_1\beta_1^2 + \alpha_1\beta_1\beta_2 + \alpha_1\beta_1\beta_3 + \alpha_2\beta_1\beta_2 + \alpha_2\beta_2^2 + \alpha_2\beta_2\beta_3 + \beta_1^2\beta_2 + \beta_1\beta_2^2 + \beta_1\beta_2\beta_3$$

$$\delta_2 = 2\alpha_2\beta_1 + 2\alpha_2\beta_2 + \alpha_2\beta_3 + \alpha_1\alpha_2 + \alpha_2^2 + \beta_1^2 + \beta_1\beta_2 + \beta_1\beta_3$$

$$\delta_3 = \beta_1\beta_3 + \alpha_2\beta_3 + \alpha_1\beta_3 + \alpha_2\beta_2 + \alpha_1\beta_1 + \alpha_2\beta_1 + \alpha_1\beta_2 + \beta_2\beta_3 + \beta_2^2 + 2\beta_1\beta_2 + \beta_1^2$$

$$\delta_4 = 2\alpha_1\beta_1 + 2\alpha_1\beta_2 + \alpha_1\beta_3 + \alpha_1^2 + \alpha_1\alpha_2 + \beta_1\beta_2 + \beta_2^2 + \beta_2\beta_3$$

3.2. Mean Time to System Failure Analysis of System II

Let $P_2(t)$ to be the probability that the systems at time $t \geq 0$ are in the states $S_i, i = 0,1,2,3,4,5,6,7,8,9$ and $P^{II}(t) = [1,0,0,0,0,0,0,0,0]$ be the probability row vector for system II with the following differential equations.

$$\frac{d}{dt}(P(t)) = T_2 P(t) \tag{3}$$

where

$$T_2 = \begin{bmatrix} -h_0 & \alpha_1 & \alpha_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \beta_1 & -h_1 & 0 & \alpha_2 & \alpha_1 & \alpha_3 & 0 & 0 & 0 & 0 \\ \beta_2 & 0 & -h_2 & \alpha_1 & 0 & 0 & \alpha_2 & \alpha_3 & 0 & 0 \\ 0 & \beta_2 & \beta_1 & -h_3 & 0 & 0 & 0 & 0 & \alpha_1 & \alpha_2 \\ 0 & \beta_1 & 0 & 0 & -\alpha_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \beta_3 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_2 & 0 & 0 & 0 & -\alpha_2 & 0 & 0 & 0 \\ 0 & 0 & \beta_3 & 0 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 \\ 0 & 0 & 0 & \beta_1 & 0 & 0 & 0 & 0 & -\alpha_1 & 0 \\ 0 & 0 & 0 & \beta_2 & 0 & 0 & 0 & 0 & 0 & -\alpha_2 \end{bmatrix}$$

$h_0 = \beta_1 + \beta_2$,

$h_1 = \alpha_1 + \beta_1 + \beta_2 + \beta_3$,

$h_2 = \alpha_2 + \beta_1 + \beta_2 + \beta_3$,

$h_3 = \alpha_1 + \alpha_2 + \beta_1 + \beta_2$

Following the procedure used in computing $MTSF_1$

$$E\left[T_{P(0)\to P(absorbing)}\right] = MTSF_2 = P(0)(-M_2^{-1})\begin{bmatrix}1\\1\\1\\1\end{bmatrix} = \frac{N_1}{D_1} \quad (4)$$

$$M_2 = \begin{bmatrix} -h_0 & \beta_1 & \beta_2 & 0 \\ \alpha_1 & -h_1 & 0 & \beta_2 \\ \alpha_2 & 0 & -h_2 & \beta_1 \\ 0 & \alpha_2 & \alpha_1 & -h_3 \end{bmatrix}$$

$$\begin{aligned} N_1 = {} & \beta_3\alpha_1^2 + \alpha_1\beta_1^2 + \alpha_1\alpha_2^2 + \alpha_2\alpha_1^2 + \beta_1^3 + \beta_2^3 \\ & + \beta_2\alpha_1^2 + \alpha_2\beta_2^2 + \beta_3\alpha_2^2 + \beta_3^2\alpha_1 + \beta_3^2\alpha_2 + \beta_3^2\beta_1 \\ & + \beta_3^2\beta_2 + \beta_1\alpha_2^2 + 3\alpha_1\beta_1\beta_2 + 3\alpha_1\alpha_2\beta_2 + 2\alpha_1\beta_3\beta_1 \\ & + 3\alpha_1\beta_3\beta_2 + 3\beta_1\alpha_2\beta_2 + 2\alpha_1\beta_3\alpha_2 + 2\beta_2\beta_3\alpha_2 + 2\alpha_1\beta_2^2 \\ & + 2\alpha_2\beta_1^2 + 3\beta_1^2\beta_2 + 3\beta_1\beta_2^2 + 2\beta_3\beta_1^2 + 2\beta_3\beta_2^2 \\ & + 3\beta_1\beta_3\alpha_2 + 3\alpha_1\alpha_2\beta_1 + 4\beta_1\beta_3\beta_2 + \alpha_2\beta_1\beta_2 + 2\beta_1^2\beta_2 \\ & + 2\beta_1\beta_2^2 + 2\beta_1\beta_2\beta_3 + \alpha_1\beta_1\beta_2 + 3\alpha_1\beta_2 + \alpha_1^2\beta_2 + \alpha_1\alpha_2\beta_2 \\ & + 2\alpha_1\beta_2^2 + \alpha_2\beta_1\beta_2 + \beta_1^2\beta_2 + 2\beta_1\beta_2^2 + \beta_2^3 + \alpha_1\beta_2\beta_3 \\ & + \alpha_2\beta_2\beta_3 + \beta_1\beta_2\beta_3 + \beta_2^2\beta_3 + \alpha_1\alpha_2\beta_1 + \alpha_2^2\beta_1 + 2\alpha_2\beta_1^2 \\ & + 3\alpha_2\beta_1\beta_2 + \beta_1^3 + 2\beta_1^2\beta_2 + \alpha_1\beta_1\beta_2 + \beta_1\beta_2^2 + \alpha_1\beta_1\beta_3 \\ & + \alpha_2\beta_1\beta_3 + \beta_1^2\beta_3 + \beta_1\beta_2\beta_3 \end{aligned}$$

$$\begin{aligned} D_1 = {} & \beta_1^4 + \beta_2^4 + 6\beta_1^2\beta_3\beta_2 + 2\beta_1\alpha_2\beta_2^2 + 4\beta_1^2\alpha_2\beta_2 \\ & + 2\beta_1\beta_3^2\beta_2 + 6\beta_1\beta_3\beta_2^2 + 2\alpha_1\beta_1^2\beta_2 + 3\alpha_1\beta_3\beta_2^2 \\ & + 3\beta_1^2\beta_3\alpha_2 + \beta_1^2\alpha_2^2 + \beta_3^2\beta_1^2 + \beta_2^2\alpha_1^2 + \beta_3^2\beta_2^2 \\ & + \beta_2\beta_3^2\alpha_2 + \beta_1\beta_3^2\alpha_1 + \beta_2\beta_3^2\alpha_1 + 4\beta_1\alpha_1\beta_2^2 \\ & + \alpha_1\alpha_2\beta_1^2 + \alpha_1\beta_3\beta_1^2 + \alpha_1\alpha_2\beta_2^2 + \beta_2^2\beta_3\alpha_2 \\ & + \beta_1\beta_3\alpha_2^2 + \beta_1\beta_3^2\alpha_2 + \beta_2\beta_3\alpha_1^2 + \beta_1\alpha_1\beta_3\alpha_2 \\ & + \beta_2\alpha_1\beta_3\alpha_2 + 2\alpha_2\beta_1^3 + 4\beta_1^3\beta_2 + 6\beta_1^2\beta_2^2 \end{aligned}$$

3.3. Mean Time to System Failure Analysis of System III

Let $P_3(t)$ to be the probability that the systems at time $t \geq 0$ are in the states $S_i, i = 0,1,2,3,4,5,6,7,8,9$ and $P^{III}(t) = [1,0,0,0,0,0,0,0,0,0]$ be the probability row vector for system II with the following differential equations. $P^{III}(t) = [1,0,0,0,0,0,0,0,0,0]$ be the probability row vector for system II with the following differential equations.

$$\frac{d}{dt}(P(t)) = T_3 P(t) \quad (5)$$

$$T_3 = \begin{bmatrix} -y_0 & \alpha_1 & \alpha_2 & 0 & \alpha_3 & 0 & 0 & 0 & 0 \\ \beta_1 & -y_1 & 0 & \alpha_2 & 0 & 0 & 0 & \alpha_1 & 0 \\ \beta_2 & 0 & -y_2 & \alpha_1 & 0 & 0 & 0 & 0 & \alpha_2 \\ 0 & \beta_2 & \beta_1 & -y_3 & 0 & \alpha_1 & \alpha_2 & 0 & 0 \\ \beta_3 & 0 & 0 & 0 & -\alpha_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_1 & 0 & -\alpha_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \beta_2 & 0 & 0 & -\alpha_2 & 0 & 0 \\ 0 & \beta_1 & 0 & 0 & 0 & 0 & 0 & -\alpha_1 & 0 \\ 0 & 0 & \beta_2 & 0 & 0 & 0 & 0 & 0 & -\alpha_2 \end{bmatrix}$$

$y_0 = \beta_1 + \beta_2 + \beta_3$, $y_1 = \alpha_1 + \beta_1 + \beta_2$, $y_2 = \alpha_2 + \beta_1 + \beta_2$, $y_3 = \alpha_1 + \alpha_2 + \beta_1 + \beta_2$

Following the procedure used in computing $MTSF_1$, the expected time to reach an absorbing state is obtained from

$$E\left[T_{P(0)\to P(absorbing)}\right] = MTSF_3 = P(0)(-M_3^{-1})\begin{bmatrix}1\\1\\1\\1\end{bmatrix} = \frac{N_2}{D_2} \quad (6)$$

$$M_3 = \begin{bmatrix} -y_0 & \beta_1 & \beta_2 & 0 \\ \alpha_1 & -y_1 & 0 & \beta_2 \\ \alpha_2 & 0 & -y_2 & \beta_1 \\ 0 & \alpha_2 & \alpha_1 & -y_3 \end{bmatrix}$$

$$N_2 = 3\alpha_1\alpha_2\beta_1 + 3\alpha_1\beta_1\beta_2 + 3\beta_1\alpha_2\beta_2 + 3\alpha_1\alpha_2\beta_2 + 3\beta_1^2\beta_2 + \alpha_1\beta_1^2 + 2\alpha_2\beta_1^2 + 2\alpha_1\beta_2^2 + \alpha_2\beta_2^2 + 3\beta_1\beta_2^2 + \beta_1\alpha_2^2 + \beta_2\alpha_1^2 + \beta_1^3 + \beta_2^3 + \alpha_2\alpha_1^2 + \alpha_1\alpha_2^2 + \beta_1\left(\alpha_2\alpha_1 + \alpha_2^2 + 2\alpha_2\beta_1 + 3\alpha_2\beta_2 + \beta_1^2 + 2\beta_1\beta_2 + \beta_2\alpha_1 + \beta_2^2\right) + \beta_2\left(3\beta_1\alpha_1 + \alpha_1^2 + \alpha_2\alpha_1 + 2\beta_2\alpha_1 + \alpha_2\beta_1 + \beta_1^2 + 2\beta_1\beta_2 + \beta_2^2\right) + \beta_1\beta_2\left(2\beta_1 + 2\beta_2 + \alpha_1 + \alpha_2\right)$$

$$D_2 = \beta_1^4 + \beta_2^4 + 3\beta_1^2\beta_3\beta_2 + 2\beta_1\alpha_2\beta_2^2 + 4\beta_1^2\alpha_2\beta_2 + 3\beta_1\beta_3\beta_2^2 + 2\alpha_1\beta_1^2\beta_2 + 2\alpha_1\beta_3\beta_2^2 + 2\beta_1^2\beta_3\alpha_2 + \beta_1^2\alpha_2^2 + \beta_2^2\alpha_1^2 + 4\beta_1\alpha_1\beta_2^2 + \alpha_1\alpha_2\beta_1^2 + \alpha_1\beta_3\beta_1^2 + \alpha_1\alpha_2\beta_2^2 + \beta_2^2\beta_3\alpha_2 + \beta_1\beta_3\alpha_2^2 + \beta_2\beta_3\alpha_1^2 + 3\beta_1\alpha_1\beta_3\alpha_2 + 3\beta_2\alpha_1\beta_3\alpha_2 + 2\alpha_2\beta_1^3 + 4\beta_1^3\beta_2 + 6\beta_1^2\beta_2^2 + \beta_3\beta_1^3 + 4\beta_1\beta_2^3 + 2\alpha_1\beta_2^3 + \beta_3\beta_2^3 + 3\beta_1\alpha_1\beta_3\beta_2 + 3\beta_1\beta_2\beta_3\alpha_2 + \beta_3\alpha_1^2\alpha_2$$

4. Comparative Analysis

The purpose of this section is to present analytical and numerical comparisons for the mean time to system failure between the systems using MAPLE and MSTLAB software. Five different cases are presented below: Cases I – IV are analytical comparison between the systems while case V is the numerical comparison.

Case I: Comparison $MTSF_i \quad i = 1,2,3$ when $\alpha_1 \neq \alpha_2$ and $\beta_1 \neq \beta_2$

$$MTSF_1 - MTSF_2 > 0 \tag{7}$$

$$MTSF_1 - MTSF_3 > 0 \tag{8}$$

$$MTSF_2 - MTSF_3 > 0 \tag{9}$$

$$\forall \alpha_1 > 0, \alpha_2 > 0, \beta_1 > 0, \beta_2 > 0, \beta_3 > 0$$

Equations (7), (8) and (9) are too spacious to be shown here. From (7), (8) and (9)

$$MTSF_1 > MTSF_2 > MTSF_3$$

Case II: Comparison $MTSF_i \quad i = 1,2,3$ when $\alpha_1 = \alpha_2$

$$MTSF_1 - MTSF_2 = \frac{N_3}{D_3} > 0 \tag{10}$$

$$N_3 = 2\alpha_1^3\beta_1 + 2\alpha_1^3\beta_2 + 8\alpha_1^2\beta_1^2 + 8\alpha_1^2\beta_1\beta_2 + \alpha_1^2\beta_2^2 + 12\alpha_1\beta_1^3 + 13\alpha_1\beta_1^2\beta_2 + 3\alpha_1\beta_1\beta_2^2 + 6\beta_1^4 + 7\beta_1^3\beta_2 + 2\beta_1^2\beta_2^2$$

$$D_3 = 2\beta_1^2\left(2\beta_1 + \beta_2\right)\left(\alpha_1\beta_1 + \alpha_1\beta_2 + 2\beta_1^2 + \beta_1\beta_2\right)$$

$$MTSF_1 - MTSF_3 = \frac{N_4}{D_4} > 0 \tag{11}$$

$$N_4 = 2\alpha_1^4\beta_2 + 4\alpha_1^3\beta_1^2 + 10\alpha_1^3\beta_1\beta_2 + \alpha_1^3\beta_2^2 + 16\alpha_1^2\beta_1^3 \quad 24\alpha_1^2\beta_1^2\beta_2 + 5\alpha_1^2\beta_1\beta_2^2 + 24\alpha_1\beta_1^4 + 30\alpha_1\beta_1^3\beta_2 + 12\beta_1^5 + 8\alpha_1\beta_1^2\beta_2^2 + 18\beta_1^4\beta_2 + 6\beta_1^3\beta_2^2$$

$$D_4 = 2\beta_1^2(2\beta_1 + \beta_2)(\alpha_1^2\beta_2 + 2\alpha_1\beta_1^2 + 2\alpha_1\beta_1\beta_2 + 4\beta_1^3 + 2\beta_1^2\beta_2)$$

$$MTSF_2 - MTSF_3 = \frac{N_5}{D_5} > 0 \tag{12}$$

$$N_5 = \beta_2(\alpha_1^4 + 4\alpha_1^3\beta_1 + \alpha_1^3\beta_2 + 7\alpha_1^2\beta_1^2 + 3\alpha_1^2\beta_1\beta_2 + 6\alpha_1\beta_1^3 + 4\alpha_1\beta_1^2\beta_2 + 4\beta_1^4 + 2\beta_1^3\beta_2)$$

$$D_5 = 2\beta_1(\alpha_1\beta_1 + \alpha_1\beta_2 + 2\beta_1^2 + \beta_1\beta_2)(\alpha_1^2\beta_2 + 4\beta_1^3 \quad 2\alpha_1\beta_1^2 + 2\alpha_1\beta_1\beta_2 + 2\beta_1^2\beta_2)$$

From (10), (11) and (12)

$$MTSF_1 > MTSF_2 > MTSF_3$$

$$\forall \alpha_1 > 0,\ \beta_1 > 0, \beta_2 > 0, \beta_3 > 0$$

Case III: Comparison $MTSF_i \quad i = 1,2,3$ when $\beta_1 = \beta_2$

$$MTSF_1 - MTSF_2 = \frac{N_6}{D_6} > 0 \tag{13}$$

$$N_6 = \alpha_1^4\alpha_2\beta_1 + \alpha_1^4\alpha_2\beta_3 + \alpha_1^4\beta_1^2 + \alpha_1^4\beta_1\beta_3 + 4\alpha_1^3\alpha_2^2\beta_1 + 3\alpha_1^3\alpha_2^2\beta_3 + 15\alpha_1^3\alpha_2\beta_1^2 + 16\alpha_1^3\alpha_2\beta_1\beta_3 + 3\alpha_1^3\alpha_2\beta_3^2 + 11\alpha_1^3\beta_1^3 + 13\alpha_1^3\beta_1^2\beta_3 + 3\alpha_1^3\beta_1\beta_3^2 + 5\alpha_1^2\alpha_2^3\beta_1 + 3\alpha_1^2\alpha_2^3\beta_3 + 37\alpha_1^2\alpha_2^2\beta_1^2 + 33\alpha_1^2\alpha_2^2\beta_1\beta_3 + 6\alpha_1^2\alpha_2^2\beta_3^2 + 80\alpha_1^2\alpha_2\beta_1^3 + 93\alpha_1^2\alpha_2\beta_1^2\beta_3 + 29\alpha_1^2\alpha_2\beta_1\beta_3^2 + 2\alpha_1^2\alpha_2\beta_3^3 + 46\alpha_1^2\beta_1^4 + 64\alpha_1^2\beta_1^3\beta_3 + 64\alpha_1^2\beta_1^3\beta_3 + 26\alpha_1^2\beta_1^2\beta_3^2 + 3\alpha_1^2\beta_1\beta_3^3 + 2\alpha_1\alpha_2^4\beta_1 + \alpha_1\alpha_2^2\beta_3 + 25\alpha_1\alpha_2^3\beta_1^2 + 19\alpha_1\alpha_2^3\beta_1\beta_3 + 3\alpha_1\alpha_2^3\beta_3^2 + 2\alpha_1\alpha_2^2\beta_3^3 + 105\alpha_1\alpha_2^2\beta_1^3 + 108\alpha_1\alpha_2^2\beta_1^2\beta_3 + 31\alpha_1\alpha_2^2\beta_1\beta_3^2 + 48\beta_1^6 + 172\alpha_1\alpha_2\beta_1^4 + 221\alpha_1\alpha_2\beta_1^3\beta_3 + 87\alpha_1\alpha_2\beta_1^2\beta_3^2 + 10\alpha_1\alpha_2\beta_1\beta_3^3 + 84\alpha_1\beta_1^5 + 130\alpha_1\beta_1^4\beta_3 + 64\alpha_1\beta_1^3\beta_3^2 + 10\alpha_1\beta_1^2\beta_3^3 + 2\alpha_2^4\beta_1^2 + \alpha_2^4\beta_1\beta_3 + 20\alpha_2^3\beta_1^3 + 108\alpha_2\beta_1^5 + 16\alpha_2^3\beta_1^2\beta_3 + 3\alpha_2^3\beta_1\beta_3^2 + 74\alpha_2^2\beta_1^4 + 83\alpha_2^2\beta_1^3\beta_3 + 29\alpha_2^2\beta_1^2\beta_3^2 + 3\alpha_2^2\beta_1\beta_3^3 + 150\alpha_2\beta_1^4\beta_3 + 44\beta_1^4\beta_3^2 + 68\alpha_2\beta_1^3\beta_3^2 + 10\alpha_2\beta_1^2\beta_3^3 + 80\beta_1^5\beta_3 + 8\beta_1^3\beta_3^3$$

$$D_6 = \beta_1^2(2\beta_1 + \beta_3)(\alpha_1 + \alpha_2 + 2\beta_1)(\alpha_1 + \alpha_2 + 4\beta_1 + 2\beta_3)(\alpha_1\beta_1 + \alpha_1\beta_3 + 2\alpha_2\beta_1 + \alpha_2\beta_3 + 4\beta_1^2 + 2\beta_1\beta_3)$$

$$MTSF_1 - MTSF_3 = \frac{N_7}{D_7} > 0 \tag{14}$$

$$N_7 = \alpha_1^3\alpha_2^2\beta_3 + \alpha_1^3\alpha_2\beta_1^2 + 2\alpha_1^3\alpha_2\beta_1\beta_3 + \alpha_1^3\beta_1^3 + \alpha_1^3\beta_1^2\beta_3 + \alpha_1^2\alpha_2^3\beta_3 + 2\alpha_1^2\alpha_2^2\beta_1^2 + 8\alpha_1^2\alpha_2^2\beta_1\beta_3 + \alpha_1^2\alpha_2^2\beta_3^2 + 9\alpha_1^2\alpha_2\beta_1^3 + 16\alpha_1^2\alpha_2\beta_1^2\beta_3 + 7\alpha_1^2\beta_1^4 + 3\alpha_1^2\alpha_2\beta_1\beta_3^2 + 9\alpha_1^2\beta_1^3\beta_3 + 2\alpha_1^2\beta_1^2\beta_3^2 + \alpha_1\alpha_2^3\beta_1^2 + 2\alpha_1\alpha_2^3\beta_1\beta_3 + 9\alpha_1\alpha_2^2\beta_1^3 + 16\alpha_1\alpha_2^2\beta_1^2\beta_3 + 18\alpha_1\beta_1^5 + 3\alpha_1\alpha_2^2\beta_1\beta_3^2 + 26\alpha_1\alpha_2\beta_1^4 + 36\alpha_1\alpha_2\beta_1^2\beta_3 + \alpha_2^3\beta_1^3 + 9\alpha_1\alpha_2\beta_1^2\beta_3^2 + 24\alpha_1\beta_1^4\beta_3 + 7\alpha_1\beta_1^3\beta_3^2 + \alpha_2^3\beta_1^2\beta_3 + 7\alpha_2^2\beta_1^4 + 9\alpha_2^2\beta_1^3\beta_3 + 2\alpha_2^2\beta_1^2\beta_3^2 + 18\alpha_2\beta_1^5 + 12\beta_1^6 + 24\alpha_2\beta_1^4\beta_3 + 7\alpha_2\beta_1^3\beta_3^2 + 18\beta_1^5\beta_3 + 6\beta_1^4\beta_3^2$$

$$D_7 = \beta_1^2(2\beta_1 + \beta_3)(\alpha_1 + \alpha_2 + 2\beta_1)(\alpha_1\alpha_2\beta_3 + 4\beta_1^3 + \alpha_1\beta_1^2 + \alpha_1\beta_1\beta_3 + \alpha_2\beta_1^2 + \alpha_2\beta_1\beta_3 + 2\beta_1^2\beta_3)$$

$$MTSF_3 - MTSF_2 = \frac{N_8}{D_8} \tag{15}$$

$$N_8 = -(\alpha_1^3\alpha_2^2\beta_3 + 2\alpha_1^3\alpha_2\beta_1\beta_3 + \alpha_1^3\alpha_2\beta_3^2 + \alpha_1^3\beta_1^2\beta_3 + \alpha_1^3\beta_1\beta_3^2 + \alpha_1^2\alpha_2^3\beta_3 - \alpha_1^2\alpha_2^2\beta_1^2 + 8\alpha_1^2\alpha_2^2\beta_1\beta_3 - \alpha_2^3\beta_1^3 + 2\alpha_1^2\alpha_2^2\beta_3^2 - 2\alpha_1^2\alpha_2\beta_1^3 + 14\alpha_1^2\alpha_2\beta_1^2\beta_3 + 8\alpha_1^2\alpha_2\beta_1\beta_3^2 + \alpha_1^2\alpha_2\beta_3^3 + 7\alpha_1^2\beta_1^3\beta_3 + 6\alpha_1^2\beta_1^2\beta_3^2 + \alpha_1^2\beta_1\beta_3^3 + 4\beta_1^3\beta_3^3 - \alpha_1\alpha_2^3\beta_1^2 + 3\alpha_1\alpha_2^3\beta_1\beta_3 + \alpha_1\alpha_2^3\beta_3^2 - 7\alpha_1\alpha_2^2\beta_1^3 - 12\alpha_2\beta_1^5 + 15\alpha_1\alpha_2^2\beta_1^2\beta_3 + 9\alpha_1\alpha_2^2\beta_1\beta_3^2 + \alpha_1\alpha_2^2\beta_3^3 - 11\alpha_1\alpha_2\beta_1^4 + 25\alpha_1\alpha_2\beta_1^3\beta_3 + 23\alpha_1\alpha_2\beta_1^2\beta_3^2 + 4\alpha_1\alpha_2\beta_1\beta_3^2 - 7\alpha_2^2\beta_1^4 + 16\alpha_1\beta_1^3\beta_3^2 + 4\alpha_1\beta_1^2\beta_3^3 + 2\alpha_2^3\beta_1^2\beta_3 + \alpha_2^3\beta_1\beta_3^2 + 16\beta_1^5\beta_3 + 8\alpha_2^2\beta_1^3\beta_3 + 7\alpha_2^2\beta_1^2\beta_3^2 + \alpha_2^2\beta_1\beta_3^3 + 14\alpha_2\beta_1^4\beta_3 + 16\beta_1^4\beta_3^2 + 18\alpha_2\beta_1^3\beta_3^2 + 4\alpha_2\beta_1^2\beta_3^3$$

$$D_8 = \beta_1(\alpha_1\beta_1 + \alpha_1\beta_3 + 2\alpha_2\beta_1 + \alpha_2\beta_3 + 4\beta_1^2 + 2\beta_1\beta_3)(\alpha_1 + \alpha_2 + 4\beta_1 + 2\beta_3)(\alpha_1\alpha_2\beta_3 + \alpha_1\beta_1^2 + \alpha_1\beta_1\beta_3 + 4\beta_1^3\alpha_2\beta_1^2 + \alpha_2\beta_1\beta_3 + 2\beta_1^2\beta_3)$$

From (13), (14) and (15)

$$MTSF_1 > MTSF_2,$$

$$MTSF_1 > MTSF_3$$

$$\forall \alpha_1 > 0,\ \beta_1 > 0, \beta_2 > 0, \beta_3 > 0$$

while the sign of $MTSF_3 - MTSF_2$ depend on N_8

Case IV: Comparison $MTSF_i \quad i = 1,2,3$ when $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$

$$MTSF_1 - MTSF_2 = \frac{N_9}{D_9} > 0 \tag{16}$$

$$N_9 = 6\alpha_1^3\beta_1 + 4\alpha_1^3\beta_3 + 22\alpha_1^2\beta_1^2 + 17\alpha_1^2\beta_1\beta_3 + 12\beta_1^4 \; 2\alpha_1^2\beta_3^2 + 30\alpha_1\beta_1^3 + 28\alpha_1\beta_1^2\beta_3 + 6\alpha_1\beta_1\beta_3^2 + 14\beta_1^3\beta_3 + 4\beta_1^2\beta_3^2$$

$$D_9 = 2\beta_1^2(2\beta_1 + \beta_3)(3\alpha_1\beta_1 + 2\alpha_1\beta_3 + 4\beta_1^2 + 2\beta_1\beta_3)$$

$$MTSF_1 - MTSF_3 = \frac{N_{10}}{D_{10}} > 0 \tag{17}$$

$$N_{10} = 2\alpha_1^4\beta_3 + 4\alpha_1^3\beta_1^2 + 10\alpha_1^3\beta_1\beta_3 + \alpha_1^3\beta_3^2 + 12\beta_1^5 \; 16\alpha_1^2\beta_1^3 + 24\alpha_1^2\beta_1^2\beta_3 + 5\alpha_1^2\beta_1\beta_3^2 + 24\alpha_1\beta_1^4 + 30\alpha_1\beta_1^3\beta_3 + 8\alpha_1\beta_1^2\beta_3^2 + 18\beta_1^4\beta_3 + 6\beta_1^3\beta_3^2$$

$$D_{10} = 2\beta_1^2(2\beta_1 + \beta_3)(\alpha_1^2\beta_3 + 2\alpha_1\beta\beta_3 + 4\beta_1^3 + 2\beta_1^2\beta_3)$$

$$MTSF_2 - MTSF_3 = \frac{N_{11}}{D_{11}} > 0, \tag{18}$$

$$N_{11} = 2\alpha_1^4\beta_3 - 2\alpha_1^3\beta_1^2 + 9\alpha_1^3\beta_1\beta_3 + 2\alpha_1^3\beta_3^2 - 6\alpha_1^2\beta_1^3 \; 16\alpha_1^2\beta_1^2\beta_3 + 6\alpha_1^2\beta_3^2\beta_1 - 6\alpha_1\beta_1^4 + 14\alpha_1\beta_1^3\beta_3 + 8\beta_1^4\beta_3^2 + 4\beta_1^3\beta_3^2$$

$$D_{11} = 2\beta_1(\alpha_1^2\beta_3 + 2\alpha_1\beta_1^2 + 2\alpha_1\beta_1\beta_3 + 4\beta_1^3 + 2\beta_1^2\beta_3)(3\alpha_1\beta_1 + 2\alpha_1\beta_3 + 4\beta_1^2 + 2\beta_1\beta_3)$$

From (16), (17) and (18)

$$MTSF_1 > MTSF_2 > MTSF_3$$

$$\forall \alpha_1 > 0,\ \beta_1 > 0, \beta_3 > 0$$

Case V: The objective here is to present specific numerical comparisons for the mean time to system failure. In this part, we numerically compare the results for mean time to system failure for all the developed models. For each model the following set of parameters values are fixed throughout the simulations for consistency: $\alpha_1 = 0.1, \alpha_2 = 0.3, \quad \beta_1 = 0.2, \beta_2 = 0.1, \beta_3 = 0.4$

From Figures 1 the MTSF results for the three systems being studied against the repair rate α_1. It is clear from the figure that it is clear that system I has higher MTSF with respect to α_1 as compared with the other two systems. The differences between the MTSF of system I and the other two systems widens as α_1 increases. These tend to suggest that system I is better than the other systems. Figure 2 depicts the MTSF calculations for the three systems against β_1. The observations that can be made here are much similar to those made on Figure 1. Figure 3 shows the MTSF results for the three systems being studied against the repair rate α_2. It is clear from the figure that it is clear that system I has higher MTSF with respect to α_2 as compared with the other two systems. The differences between the MTSF of system I and the other two systems widens as α_2 increases. There is slight difference between the MTSF of system II and that of system III with respect to α_2. It is evident here that system I is has higher MTSF than systems II and III. From Figures 4, it can be seen that the MTSF of system I decreases much slower than those of the other two systems with increase in β_2. By comparing systems II and III, it can be observed that there is slight difference between the two. System III is decreasing a little faster than system II with respect to β_2. We can conclude as before that system I is better than the other two systems in all the three figures.

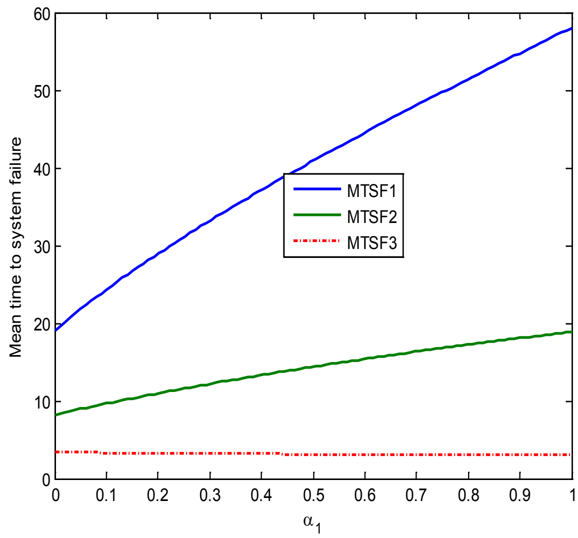

Figure 1. MTSF against α_1

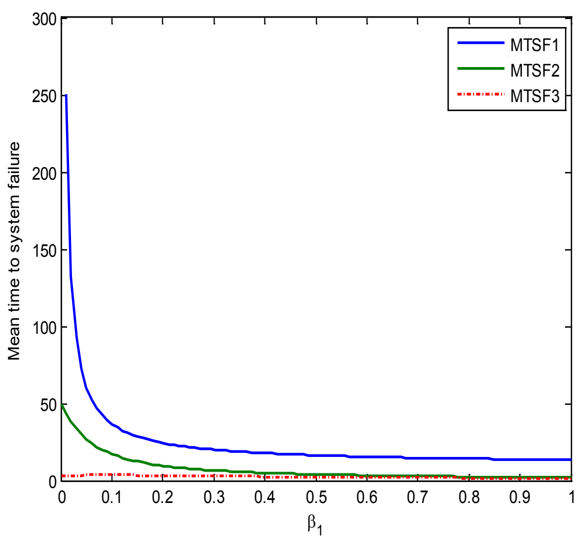

Figure 2. MTSF against β_1

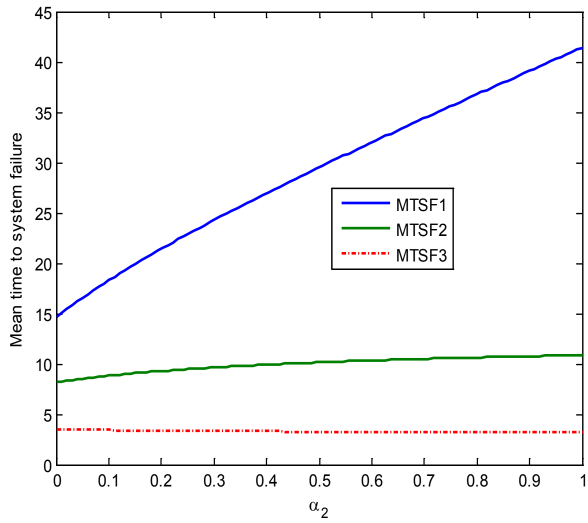

Figure 3. MTSF against α_2

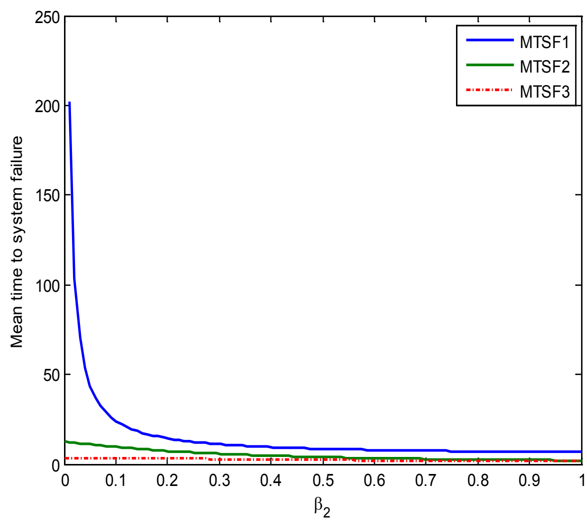

Figure 4. MTSF against β_2

5. Conclusions

In this paper, we studied three dissimilar systems each consisting of two subsystems A and B each containing two units with supporting unit attached to the systems. We first formulated models three different repairable redundant systems that used an external supporting device for their operation and obtained the explicit expressions for mean time to system failure (MTSF) for each system and performed comparative analysis analytically and numerically to determine the optimal system. It is evident from case I to V that system I is better than the other two systems. The study of mean time to system measures will help the engineers and designers to develop sophisticated models and to design more critical system in interest of human kind.

REFERENCES

[1] Bhardwj, R.K., and S.C. Malik. (2010). MTSF and Cost effectiveness of 2-out-of-3 cold standby system with probability of repair and inspection. Int. J. of Eng. Sci. and Tech. 2(1), 5882-5889.

[2] Khatab, A., Nahas, N., and Nourelfath, M., (2009). Availability of k-out-of-n: G systems with non identical components subject to repair priorities, Reliab. Eng. Syst. Safety. 94, 142-151.

[3] Khalili-Damghani K, Amiri M (2012) Solving binary-state multi-objective reliability redundancy allocation seriesparallel problem using efficient epsilonconstraint, multi-start partial bound enumeration algorithm, and DEA. Reliability Engineering and system safety103: 35-44.

[4] Khalili-Damghani K, Abtahi AR, Tavana M (2013) A new multi-objective particle swarm optimization method for solving reliability redundancy allocation problems, Reliability Engineering and system safety 111:58-75.

[5] Ke, J.C. and Chu, Y.K. (2007). Comparative analysis of availability for a redundant repairable system. Applied Mathematics and Computation, 188, pp 332-338.

[6] Mokaddis, G.S., El Sherbeny, M.S. and Al-Esayey, Entesar. (2010). Compare between two unit cold standby and warm standbyb outdoor electric power systems in changing weather, *Journal of Mathematics and Statistics,* 6(1), 17-22.

[7] Wang, K.H. and Chen, Y.J. (2009). Comparative analysis of availability between three systems with general repair times, reboot delay and switching failures, *Applied Mathematics and Computation*, 215, 384-394.

[8] Wang, K,-H., Yen, T,-C. and Fang, Y,-C. (2012). Comparison of Availability between two systems with warm standby units and different imperfect coverage, Quality technology and quantitative management, 9(3), pp 265-282.

[9] Yusuf, I. (2013). Comparison of some reliability characteristics between redundant systems requiring supporting units for their operation. Journal of Mathematical and Computational Sciences, 3(1), pp 216-232.

[10] Yusuf, I. and Bala, S.I. (2013). Modeling and Evaluation of MTSF of a repairable 2-out-of-4 warm standby system attended by repair machines and repairmen, Engineering Mathematics Letters, 2(2), 81-89.

[11] Yusuf, I. (2014). Comparative analysis of profit between three dissimilar repairable redundant systems using supporting external device for operation, Journal of Industrial Engineering International, Vol. 10, No. 77, 2-9, DOI 10.1007/s40092-014-0077-3.