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Bayesian Analysis of Kim and Warde Randomized Response Technique Using Alternative Priors

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Abstract In this paper, we developed the Bayesian estimators of the population proportion of a stigmatized attribute using Kumaraswamy and Generalised Beta prior distributions when data were obtained through the Randomized Response Technique (RRT) proposed by Kim and Warde [15]. We validated our newly developed Bayesian estimators for a wide range of the designed values of the population proportion at varying sample sizes. It was observed that our newly developed Bayesian estimators performed significantly better than the Bayesian estimator developed by Hussain and Shabbir [12] for relatively small as well as moderate sample sizes. However, the reverse was the case for very large sample sizes.

Keywords Bayesian estimation, Alternative priors, Stigmatized attribute, Mean Square Error (MSE), Absolute Bias

1. Introduction

Obtaining information about a stigmatized (induced abortion, use of drugs, tax evasion, etc.) attribute rampant in a human population is a complicated issue. Direct interrogating approach generally lead to doctoring of the true answers. The reason may be fear of social disgrace or counter attacks. But due to socioeconomic reasons, information about prevalence of such attributes in the population becomes essential. Warner [22] introduced an ingenious method of survey to obtain information about stigmatized attributes by guaranteeing confidentiality to the respondents. Numerous developments and improvements on Warner's Randomized Response Technique have been suggested by many researchers. Greenberg et al. [9], Folsom et al. [8], Christofides [7], Mangat [16], Kim and Warde [15], Adebola and Adepetun [1], Adebola and Adepetun [2], Adebola and Adepetun [3], Adepetun and Adebola [4] are some of the many to be mentioned.

At times, prior information about the unknown parameter may be available and can be used along with the sample information for the determination of that unknown parameter. This is called the Bayesian approach of estimation. The work on Bayesian analysis of randomized response techniques is not very enormous. Nonetheless, attempts have been made on the Bayesian analysis of randomized response techniques. Winkler and Franklin [23], Spurrier and Padgett [20], O'Hagan [18], Oh [19], Migon and Tachibana [17], and Unnikrishnan and Kunte [21], Barabesi and Marcheselli [5, 6], Hussain and Shabbir [10, 11], Hussain et al [13], Hussain and Shabbir [12] and Kim et al. [14] are the major references on the Bayesian analysis of the Randomized Response Techniques.

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2. Presentation of the Existing Technique

Hussain and Shabbir [12] in their paper presented a Bayesian analysis to the Randomized Response Technique (RRT) proposed by Kim and Warde [15] using a simple beta prior distribution to estimate the population proportion of respondents possessing stigmatized attribute.

Let the simple beta prior be defined as follows

$$f(\pi) = \frac{1}{B(a,b)}\pi^{a-1}(1-\pi)^{b-1} \;;\; 0 < \pi < 1 \qquad (2.1)$$

where (a,b) are the shape parameters of the distribution and π is the population proportion of respondents possessing the stigmatized attribute.

By letting $X = \sum x_i$ denotes the total number of the yes response in a sample of size n drawn from the population with simple random sampling with replacement (srswr). The conditional distribution of X given π is

$$f(X|\pi) = \binom{n}{x}\phi^x(1-\phi)^{n-x} \qquad (2.2)$$

where $\phi = T\pi + 1 - T$ is the probability of "yes response" in a sample of size n and T and $1-T$ are the pre-assigned probabilities respectively.

Then

$$f(X|\pi) = \binom{n}{x}(T\pi + 1 - T)^x(1 - T\pi - 1 + T)^{n-x}$$

$$f(X|\pi) = \binom{n}{x}T[\pi + \frac{[1-T]}{T}])^x(T - T\pi)^{n-x}$$

$$= \binom{n}{x}T^x\left[\pi + \frac{1-T}{T}\right]^x[T(1-\pi)]^{n-x}$$

$$= \binom{n}{x}T^xT^{n-x}\left[\pi + \frac{[1-T]}{T}\right](1-\pi)^{n-x}$$

Let $d = \frac{1-T}{T}$

$$f(X|\pi) = \binom{n}{x}T^n(\pi + d)^x(1-\pi)^{n-x}$$

Using binomial series expansion

$$(\pi + d)^x = \sum_{j=0}^{x}\binom{x}{j}\pi^j d^{x-j}$$

$$f(X|\pi) = \binom{n}{x}T^n\sum_{j=0}^{x}\binom{x}{j}\pi^j d^{x-j}(1-\pi)^{n-x}$$

$$= \binom{n}{x}T^n\sum_{j=0}^{x}\binom{x}{j}\pi^j d^{x-j}(1-\pi)^{n-x} \quad (2.3)$$

The joint density function of π and X was derived as follows

$$f(X,\pi) = \frac{\pi^{a-1}(1-\pi)^{b-1}}{B(a,b)}\binom{n}{x}T^n\sum_{j=0}^{x}\binom{x}{j}\pi^j d^{x-j}(1-\pi)^{n-x}$$

$$f(X,\pi) = \frac{\binom{n}{x}T^n}{B(a,b)}\sum_{j=0}^{x}\binom{x}{j}d^{x-j}\pi^{a-1+j}(1-\pi)^{n-x+b-1} \quad (2.4)$$

The marginal probability density function was found using

$$f(X) = \int_0^1 f(X,\pi)d\pi = \binom{n}{x}\frac{T^n}{B(a,b)}\sum_{j=0}^{x}\binom{x}{j}d^{x-j}\int_0^1(1-\pi)^{n-x+b-1}\pi^{a-1+j}d\pi$$

$$= \binom{n}{x}\frac{T^n}{B(a,b)}\sum_{j=0}^{x}\binom{x}{j}d^{x-j}B(a+j, n-x+b) \quad (2.5)$$

Thus, the posterior distribution of π given X was

$$f(X|\pi) = \frac{f(X,\pi)}{f(X)} = \frac{\frac{\binom{n}{x}T^n}{B(a,b)}\sum_{j=0}^{x}\binom{x}{j}d^{x-j}\pi^{a-1+j}(1-\pi)^{n-x+b-1}}{\binom{n}{x}\frac{T^n}{B(a,b)}\sum_{j=0}^{x}\binom{x}{j}d^{x-j}B(a+j,n-x+b)}$$

$$f(X|\pi) = \frac{\sum_{j=0}^{x}\binom{x}{j}d^{x-j}\pi^{a-1+j}(1-\pi)^{n-x+b-1}}{\sum_{j=0}^{x}\binom{x}{j}d^{x-j}B(a+j,n-x+b)} \quad (2.6)$$

Under the Square error loss, the Bayes estimator i.e the posterior mean was found using

$$\hat{\pi}_H = \int_0^1 \pi f(\pi|X)d\pi = \frac{\sum_{j=0}^{x}\binom{x}{j}d^{x-j}\int_0^1(1-\pi)^{n-x+b-1}\pi^{a+j}d\pi}{\sum_{j=0}^{x}\binom{x}{j}d^{x-j}B(a+j,n-x+b)}$$

$$\hat{\pi}_H = \frac{\sum_{j=0}^{x}\binom{x}{j}d^{x-j}B(a+j+1,n-x+b)}{\sum_{j=0}^{x}\binom{x}{j}d^{x-j}B(a+j,n-x+b)} \quad (2.7)$$

The bias as well as the Mean Square Error (MSE) of $\hat{\pi}_H$ corresponding to the sample of size *n* was given by

$$B(\hat{\pi}_H) = \hat{\pi}_H - \pi \quad (2.8)$$

$$MSE(\hat{\pi}_H) = \sum_{x=0}^{n} (\hat{\pi}_H - \pi)^2 \, \phi^x (1-\phi)^{n-x} \tag{2.9}$$

3. Presentation of the Proposed Techniques

In this section, we present a Bayesian analysis to Kim and Warde [15] Randomized Response Technique using both Kumaraswamy and Generalised Beta prior distributions as our alternative prior distributions in addition to the simple Beta prior distribution used by Hussain and Shabbir [12].

3.1. Estimation of π Using Kumaraswamy Prior

The Kumaraswamy prior distribution of π is given as

$$f(\pi) = ab\pi^{b-1}(1-\pi^b)^{a-1} \;\; ; a, b > 0 \tag{3.1}$$

The joint density function of π and X with Kumaraswamy Prior is as follows

$$f(X,\pi) = ab\binom{n}{x} T^n \sum_{j=0}^{x} \binom{x}{j} \pi^j d^{x-j} (1-\pi)^{n-x} \pi^{b-1}(1-\pi^b)^{a-1} \tag{3.2}$$

The marginal probability density function is found using

$$f(X) = \int_0^1 f(X,\pi)\, d\pi$$

Recall that

$$(1-\pi^b)^{a-1} = \sum_{k=0}^{a-1} (-1)^k \binom{a-1}{k} \pi^{bk}$$

Then

$$f(X) = \binom{n}{x} T^n ab \sum_{j=0}^{x} \sum_{k=0}^{a-1} (-1)^k \binom{x}{j} d^{x-j} \int_0^1 (1-\pi)^{n-x} \pi^{bk+b+j-1} d\pi$$

$$f(X) = \binom{n}{x} T^n ab \sum_{j=0}^{x} \sum_{k=0}^{a-1} (-1)^k \binom{x}{j} d^{x-j} B(bk+b+j, n-x+1) \tag{3.3}$$

The posterior distribution is

$$f(\pi|X) = \frac{\sum_{j=0}^{x} \sum_{k=0}^{a-1} \binom{x}{j} (-1)^k d^{x-j} \pi^{bk+b+j-1} (1-\pi)^{n-x}}{\sum_{j=0}^{x} \sum_{k=0}^{a-1} \binom{x}{j} (-1)^k d^{x-j} B(bk+b+j, n-x+1)}$$

Thus the posterior mean is

$$\hat{\pi}_{prop\,1} = \frac{\sum_{j=0}^{x} \sum_{k=0}^{a-1} \binom{x}{j} (-1)^k d^{x-j} B(bk+b+j+1, n-x+1)}{\sum_{j=0}^{x} \sum_{k=0}^{a-1} \binom{x}{j} (-1)^k d^{x-j} B(bk+b+j, n-x+1)} \tag{3.4}$$

The bias as well as Mean Square Error (MSE) of $\hat{\pi}_{prop\,1}$ is computed as

$$B(\hat{\pi}_{prop\,1}) = \hat{\pi}_{prop\,1} - \pi \tag{3.5}$$

$$MSE(\hat{\pi}_{prop\,1}) = \sum_{x=0}^{n} \left(\hat{\pi}_{prop\,1} - \pi\right)^2 \phi^x (1-\phi)^{n-x} \tag{3.6}$$

3.2. Estimation of π Using Generalised Beta Prior

The Generalised Beta prior is defined as

$$f(\pi) = \frac{c}{B(a,b)} \pi^{ac-1} (1-\pi^c)^{b-1}; \quad a, b, c > 0 \tag{3.7}$$

where a, b, c are the shape parameters of the prior distribution as given in formula (3.7)

We recall from Binomial series expansion that

$$(1-\pi^c)^{b-1} = \sum_{k=0}^{b-1} (-1)^k \binom{b-1}{k} (\pi^c)^k$$

So that

$$f(\pi) = \frac{c}{B(a,b)} \sum_{k=0}^{b-1} (-1)^k \binom{b-1}{k} \pi^{c(k+a)-1}$$

The joint density function of π and X with generalized beta prior is

$$f(X,\pi) = \frac{c}{B(a,b)} \binom{n}{x} T^n \sum_{j=0}^{x} \binom{x}{j} \pi^j d^{x-j} \pi^{ac-1} (1-\pi^c)^{b-1} (1-\pi)^{n-x}$$

which simplify to

$$f(X,\pi) = A \sum_{j=0}^{x} \sum_{k=0}^{b-1} (-1)^k \binom{x}{j} \binom{b-1}{k} d^{x-j} \pi^{ac+j-1+ck} (1-\pi)^{n-x} \tag{3.8}$$

where $A = \frac{c}{B(a,b)} \binom{n}{x} T^n$

The marginal probability density function is

$$f(X) = A \sum_{j=0}^{x} \sum_{k=0}^{b-1} (-1)^k \binom{x}{j} \binom{b-1}{k} d^{x-j} B(ac+j+ck, n-x+1) \tag{3.9}$$

Thus, the posterior distribution of π given X is

$$f(\pi|X) = \frac{f(X,\pi)}{f(X)} = \frac{\sum_{j=0}^{x} \sum_{k=0}^{b-1} (-1)^k \binom{x}{j} \binom{b-1}{k} d^{x-j} \pi^{ac+j-1+ck} (1-\pi)^{n-x}}{\sum_{j=0}^{x} \sum_{k=0}^{b-1} (-1)^k \binom{x}{j} \binom{b-1}{k} d^{x-j} B(ac+j+ck, n-x+1)}$$

The posterior mean which is the Bayes estimator is found by using

$$\hat{\pi}_{prop\ 2} = \int_0^1 \pi f(\pi|X) d\pi$$

$$\hat{\pi}_{prop\ 2} = \frac{\sum_{j=0}^{x} \sum_{k=0}^{b-1} \binom{x}{j} (-1)^k d^{x-j} B(ck+c+j+1, n-x+1)}{\sum_{j=0}^{x} \sum_{k=0}^{b-1} \binom{x}{j} (-1)^k d^{x-j} B(ck+c+j, n-x+1)} \tag{3.10}$$

The bias of $\hat{\pi}_{prop\ 2}$ is

$$B(\hat{\pi}_{prop\ 2}) = \hat{\pi}_{prop\ 2} - \pi \tag{3.11}$$

The Mean Square Error (MSE) of $\hat{\pi}_{prop\ 2}$ is

$$MSE(\hat{\pi}_{prop\ 2}) = \sum_{x=0}^{n} (\hat{\pi}_{prop\ 2} - \pi)^2 \phi^x (1-\phi)^{n-x} \tag{3.12}$$

4. Presentation and Comparison of Results

In this section, we present as well as compare our results with the existing Hussain and Shabbir [12] under the same values of parameters in the estimators at different sample sizes. In order to overcome the computational difficulties and generate these results, we have written computer programs using R-statistical software. To save spaces, we present few results in tables and figures as follows:

Table 4.1a. Table showing the Mean Square Errors (MSEs) for Kim and Warde [15] RRT at $n = 15, x = 9, a = 1, b = 2, c = 4, T = 0.1$

π	MSE BETA	MSE KUMA	MSE GLS
0.1	1.049992E-10	5.061297E-09	1.761953E-08
0.2	7.629774E-10	2.993967E-10	3.936878E-09
0.3	1.951074E-09	1.581123E-10	3.758000E-10
0.4	2.073713E-09	6.080834E-10	1.253651E-11
0.5	1.410892E-09	6.195110E-10	1.457447E-10
0.6	6.495136E-10	3.491115E-10	1.393658E-10
0.7	1.854809E-10	1.126355E-10	5.733950E-11
0.8	2.456579E-11	1.618939E-11	9.487008E-12
0.9	5.536666E-13	3.869328E-13	2.485900E-13

Table 4.1b. Table showing the Absolute Bias for Kim and Warde [15] RRT at $n = 15, x = 9, a = 1, b = 2, c = 4, T = 0.1$

π	|BIAS|BETA	|BIAS| KUMA	|BIAS| GLS
0.1	0.02148717	0.14918224	0.27834495
0.2	0.07851283	0.04918224	0.17834495
0.3	0.17851283	0.05081776	0.07834495
0.4	0.27851283	0.15081776	0.02165505
0.5	0.37851283	0.25081776	0.12165505
0.6	0.47851283	0.35081776	0.22165505
0.7	0.57851283	0.45081776	0.32165505
0.8	0.67851283	0.55081776	0.42165505
0.9	0.77851283	0.65081776	0.52165505

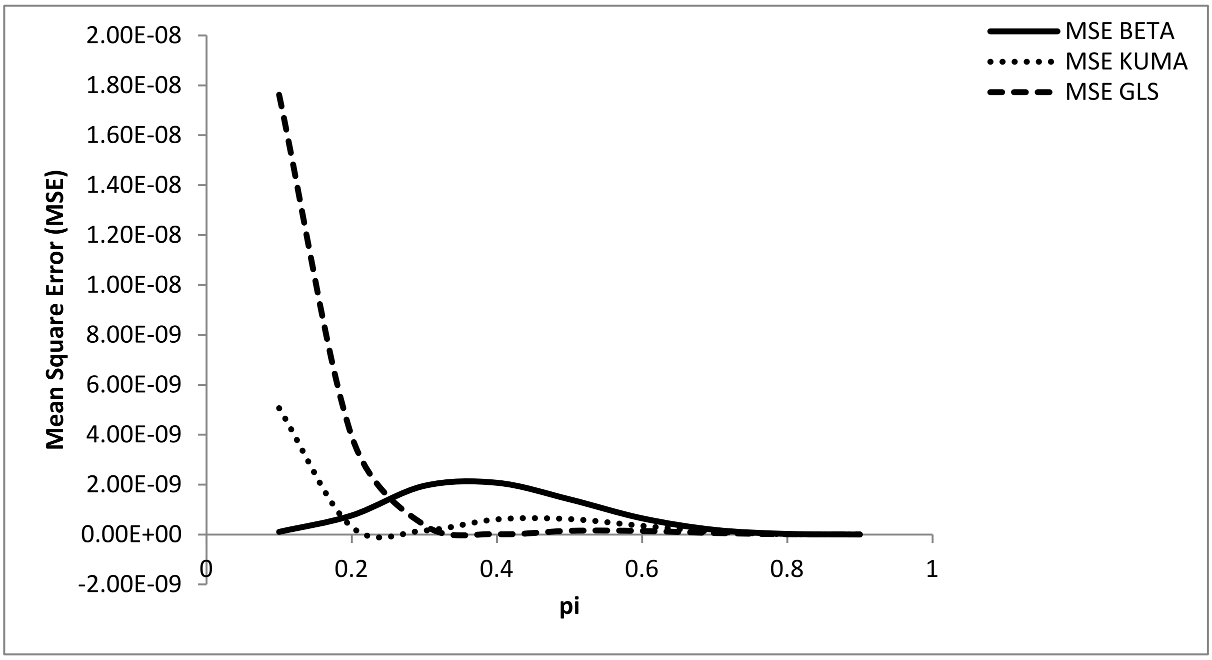

Figure 4.1a. Graph showing the Mean Square Errors (MSEs) for Kim and Warde [15] RRT at $n = 15, x = 9, a = 1, b = 2, c = 4, T = 0.1$

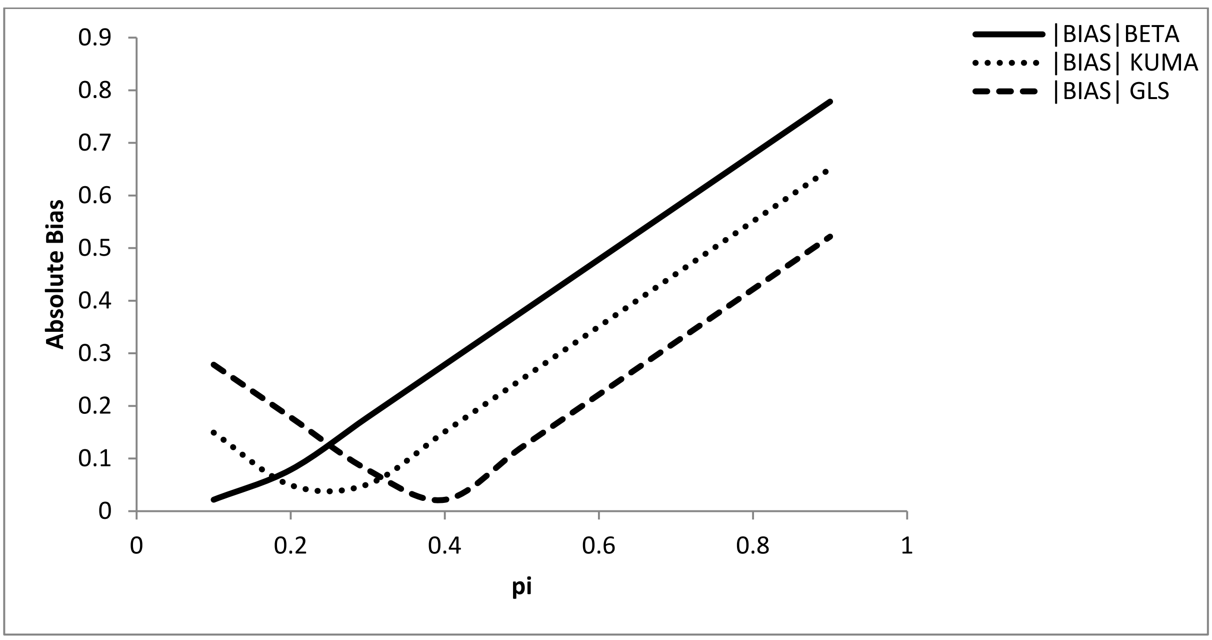

Figure 4.1b. Graph showing the Absolute Bias for Kim and Warde [15] RRT at $n = 15, x = 9, a = 1, b = 2, c = 4, T = 0.1$

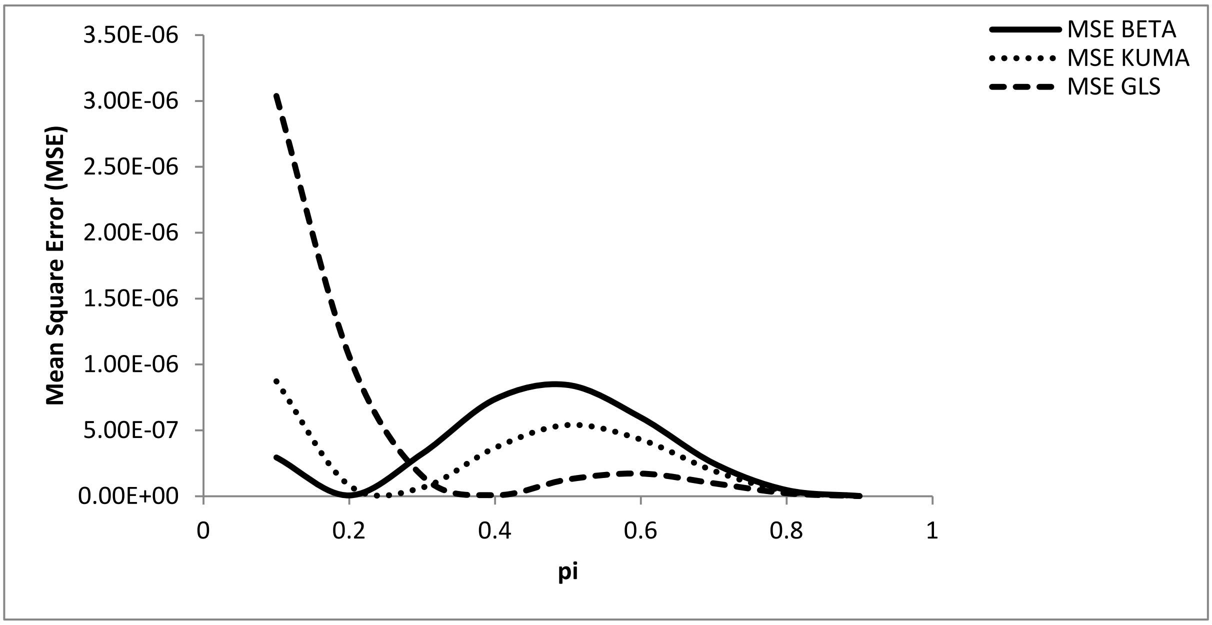

Figure 4.2a. Graph showing the Mean Square Errors (MSEs) for Kim and Warde [15] RRT at $n = 15, x = 9, a = 1, b = 2, c = 4, T = 0.4$

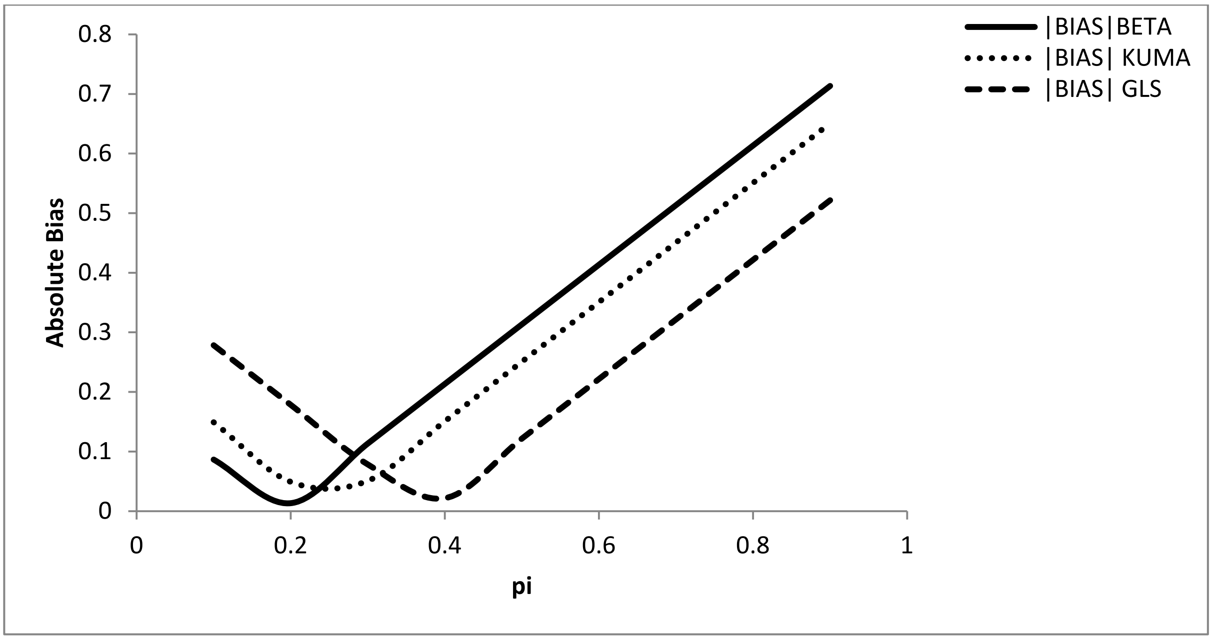

Figure 4.2b. Graph showing the Absolute Bias for Kim and Warde [15] RRT at $n = 15, x = 9, a = 1, b = 2, c = 4, T = 0.4$

Table 4.2a. Table showing the Mean Square Errors (MSEs) for Kim and Warde [15] RRT at $n = 15, x = 9, a = 1, b = 2, c = 4, T = 0.4$

π	MSE BETA	MSE KUMA	MSE GLS
0.1	2.939763E-07	8.727082E-07	3.038096E-06
0.2	6.007747E-09	8.074149E-08	1.061700E-06
0.3	3.223202E-07	6.471004E-08	1.538022E-07
0.4	7.362797E-07	3.677009E-07	7.580682E-09
0.5	8.437845E-07	5.403889E-07	1.271306E-07
0.6	5.970457E-07	4.299289E-07	1.716282E-07
0.7	2.490978E-07	1.920585E-07	9.777150E-08
0.8	4.657365E-08	3.755314E-08	2.200620E-08
0.9	1.443732E-09	1.201489E-09	7.719122E-10

Table 4.2b. Table showing the Absolute Bias for Kim and Warde [15] RRT at $n = 15, x = 9, a = 1, b = 2, c = 4, T = 0.4$

π	|BIAS|BETA	|BIAS| KUMA	|BIAS| GLS
0.1	0.08658422	0.14918224	0.27834495
0.2	0.01341578	0.04918224	0.17834495
0.3	0.11341578	0.05081776	0.07834495
0.4	0.21341578	0.15081776	0.02165505
0.5	0.31341578	0.25081776	0.12165505
0.6	0.41341578	0.35081776	0.22165505
0.7	0.51341578	0.45081776	0.32165505
0.8	0.61341578	0.55081776	0.42165505
0.9	0.71341578	0.65081776	0.52165505

Table 4.3a. Table showing the Mean Square Errors (MSEs) for Kim and Warde [15] RRT at $n = 25, x = 15, a = 1, b = 2, c = 4, T = 0.1$

π	MSE BETA	MSE KUMA	MSE GLS
0.1	1.642680E-15	3.765088E-14	2.574744E-13
0.2	3.989745E-14	3.417051E-15	1.697907E-14
0.3	4.352535E-14	1.691010E-14	6.272743E-16
0.4	2.355475E-14	1.301395E-14	3.775473E-15
0.5	7.751658E-15	5.027236E-15	2.304332E-15
0.6	1.501294E-15	1.067398E-15	6.029121E-16
0.7	1.409345E-16	1.063646E-16	6.775754E-17
0.8	3.854736E-18	3.033647E-18	2.089951E-18
0.9	5.697643E-21	4.625288E-21	3.366931E-21

Table 4.3b. Table showing the Absolute Bias for Kim and Warde [15] RRT at $n = 25, x = 15, a = 1, b = 2, c = 4, T = 0.1$

π	|BIAS|BETA	|BIAS| KUMA	|BIAS| GLS
0.1	0.01392366	0.06665989	0.17431873
0.2	0.11392366	0.03334011	0.07431873
0.3	0.21392366	0.13334011	0.02568127
0.4	0.31392366	0.23334011	0.12568127
0.5	0.41392366	0.33334011	0.22568127
0.6	0.51392366	0.43334011	0.32568127
0.7	0.61392366	0.53334011	0.42568127
0.8	0.71392366	0.63334011	0.52568127
0.9	0.81392366	0.73334011	0.62568127

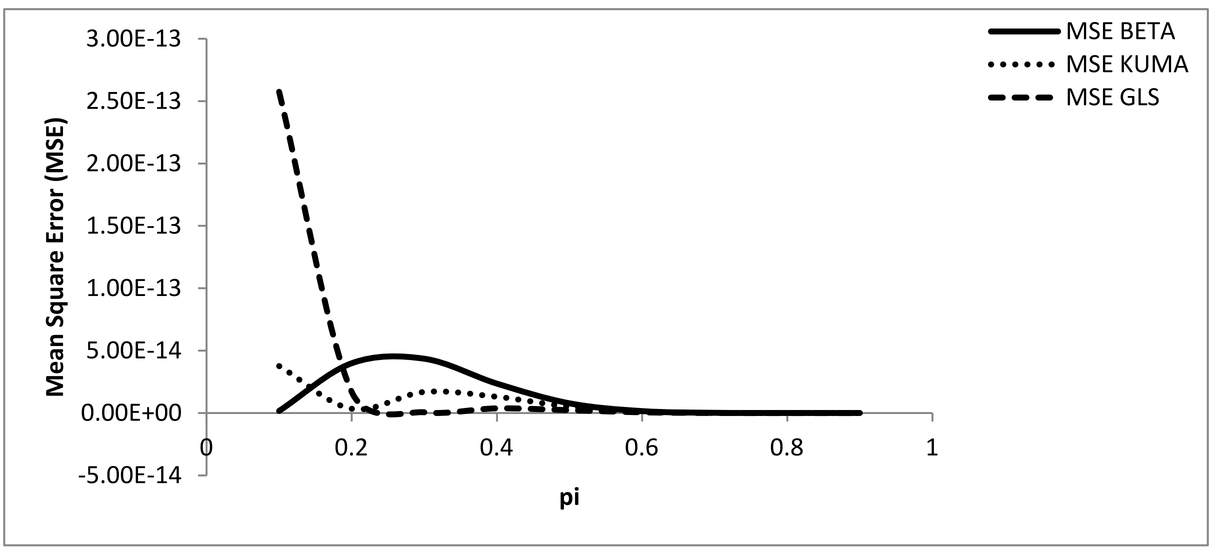

Figure 4.3a. Graph showing the Mean Square Errors (MSEs) for Kim and Warde [15] RRT at $n = 25, x = 15, a = 1, b = 2, c = 4, T_1 = 0.1$

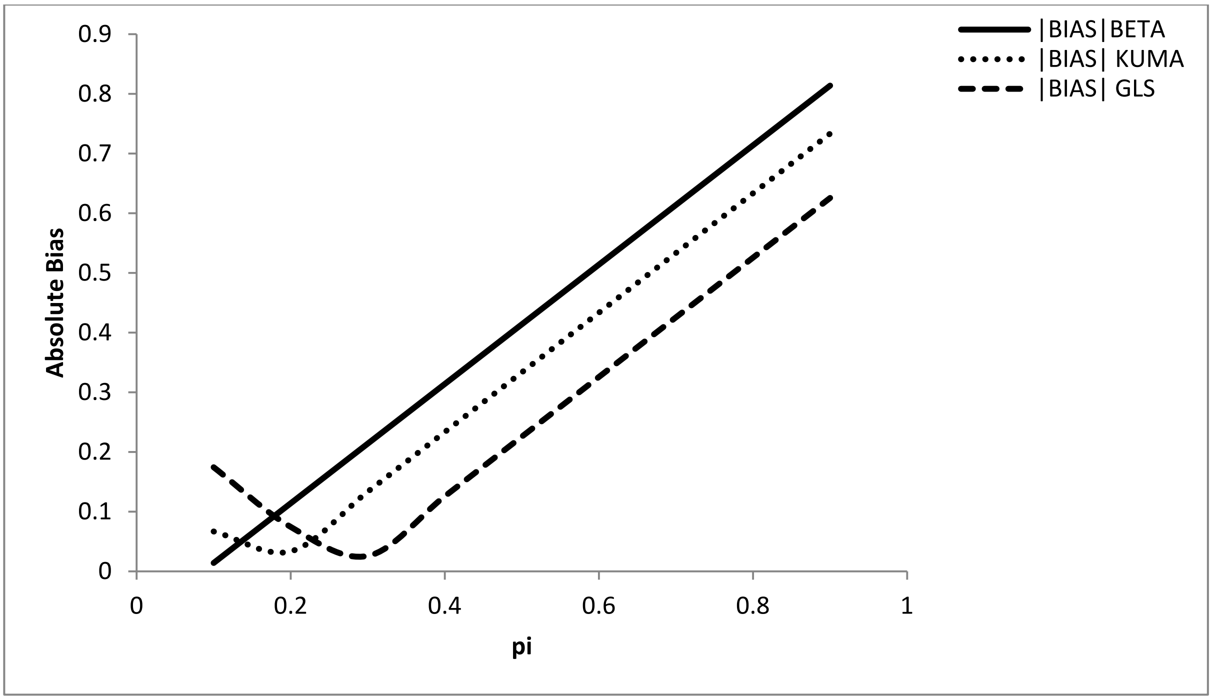

Figure 4.3b. Graph showing the Absolute Bias for Kim and Warde [15] RRT at $n = 25, x = 15, a = 1, b = 2, c = 4, T_1 = 0.1$

From the results presented in the above tables and figures 4.1a to 4.3b, we can deduce that the proposed Bayesian estimators obtained using alternative priors are more efficient than the usual Bayesian estimator when a simple beta prior is used for obtaining high response from respondents with respect to the stigmatized attribute for small as well as moderate sample sizes respectively.

Table 4.4a. Table showing the Mean Square Errors (MSEs) for Kim and Warde [15] RRT at $n = 150, x = 90, a = 1, b = 2, c = 4, T_1 = 0.6$

π	MSE BETA	MSE KUMA	MSE GLS
0.1	1.971166E-48	1.792857E-49	5.739803E-50
0.2	3.205850E-47	5.808426E-47	3.950071E-47
0.3	7.691577E-48	9.097240E-46	7.208294E-46
0.4	4.885086E-47	1.162610E-45	9.840733E-46
0.5	1.493015E-47	1.061968E-46	9.326105E-47
0.6	9.242000E-50	3.928256E-49	3.531784E-49
0.7	5.111422E-54	1.617388E-53	1.478042E-53
0.8	1.283509E-61	3.347694E-61	3.096105E-61
0.9	6.172953E-77	1.403987E-76	1.310423E-76

Table 4.4b. Table showing the Absolute Bias for Kim and Warde [15] RRT at $n = 150, x = 90, a = 1, b = 2, c = 4, T_1 = 0.6$

π	|BIAS|BETA	|BIAS| KUMA	|BIAS| GLS
0.1	0.22461894	0.06774194	0.03832951
0.2	0.12461894	0.16774194	0.13832951
0.3	0.02461894	0.26774194	0.23832951
0.4	0.07538106	0.36774194	0.33832951
0.5	0.17538106	0.46774194	0.43832951
0.6	0.27538106	0.56774194	0.53832951
0.7	0.37538106	0.66774194	0.63832951
0.8	0.47538106	0.76774194	0.73832951
0.9	0.57538106	0.86774194	0.83832951

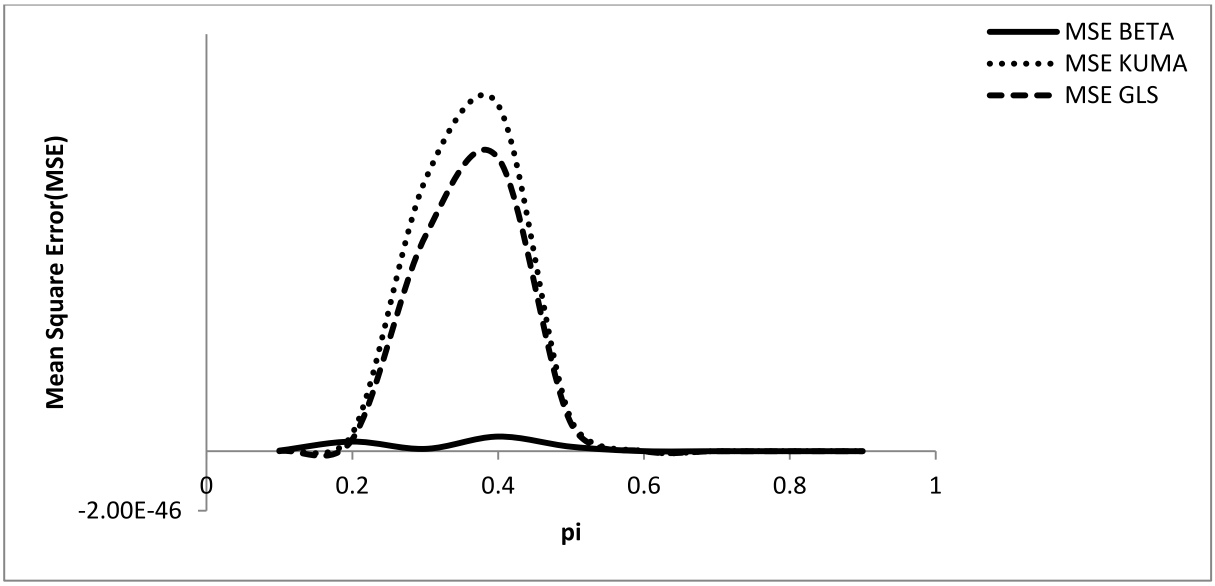

Figure 4.4a. Graph showing the Mean Square Errors (MSEs) for Kim and Warde [15] RRT at $n = 150, x = 90, a = 1, b = 2, c = 4, T_1 = 0.6$

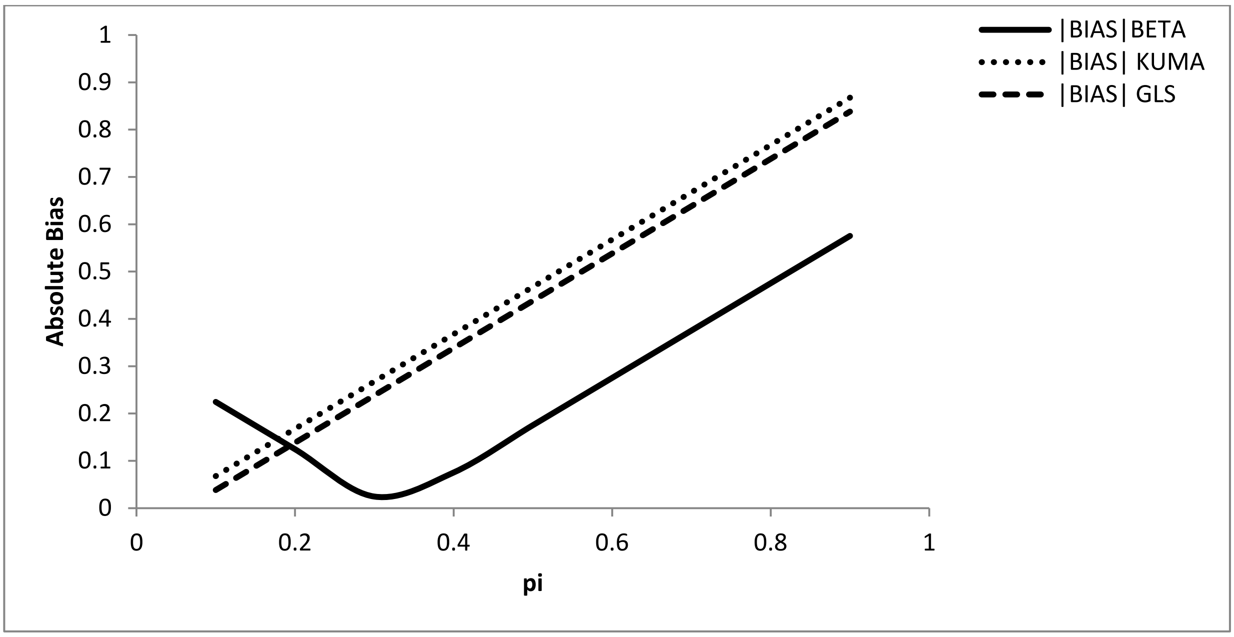

Figure 4.4b. Graph showing the Absolute Bias for Kim and Warde [15] RRT at $n = 150, x = 90, a = 1, b = 2, c = 4, T_1 = 0.6$

Table 4.5a. Table showing the Mean Square Errors (MSEs) for Kim and Warde [15] RRT at $n = 250, x = 150, a = 1, b = 2, c = 4, T_1 = 0.6$

π	MSE BETA	MSE KUMA	MSE GLS
0.1	2.339706E-79	2.907221E-80	1.722040E-80
0.2	5.488760E-77	1.089082E-76	8.769410E-77
0.3	5.437556E-77	5.428249E-75	4.734861E-75
0.4	1.867015E-76	5.220466E-75	4.724511E-75
0.5	8.862759E-78	6.918681E-77	6.395512E-77
0.6	1.028250E-81	4.683883E-81	4.389735E-81
0.7	5.498783E-89	1.840123E-88	1.741313E-88
0.8	8.675471E-102	2.372200E-101	2.260944E-101
0.9	1.990669E-127	4.716886E-127	4.520526E-127

Table 4.5b. Table showing the Absolute Bias for Kim and Warde [15] RRT at $n = 250, x = 150, a = 1, b = 2, c = 4, T_1 = 0.6$

π	|BIAS|BETA	|BIAS| KUMA	|BIAS| GLS
0.1	0.22806324	0.08039216	0.06187230
0.2	0.12806324	0.18039216	0.16187230
0.3	0.02806324	0.28039216	0.26187230
0.4	0.07193676	0.38039216	0.36187230
0.5	0.17193676	0.48039216	0.46187230
0.6	0.27193676	0.58039216	0.56187230
0.7	0.37193676	0.68039216	0.66187230
0.8	0.47193676	0.78039216	0.76187230
0.9	0.57193676	0.88039216	0.86187230

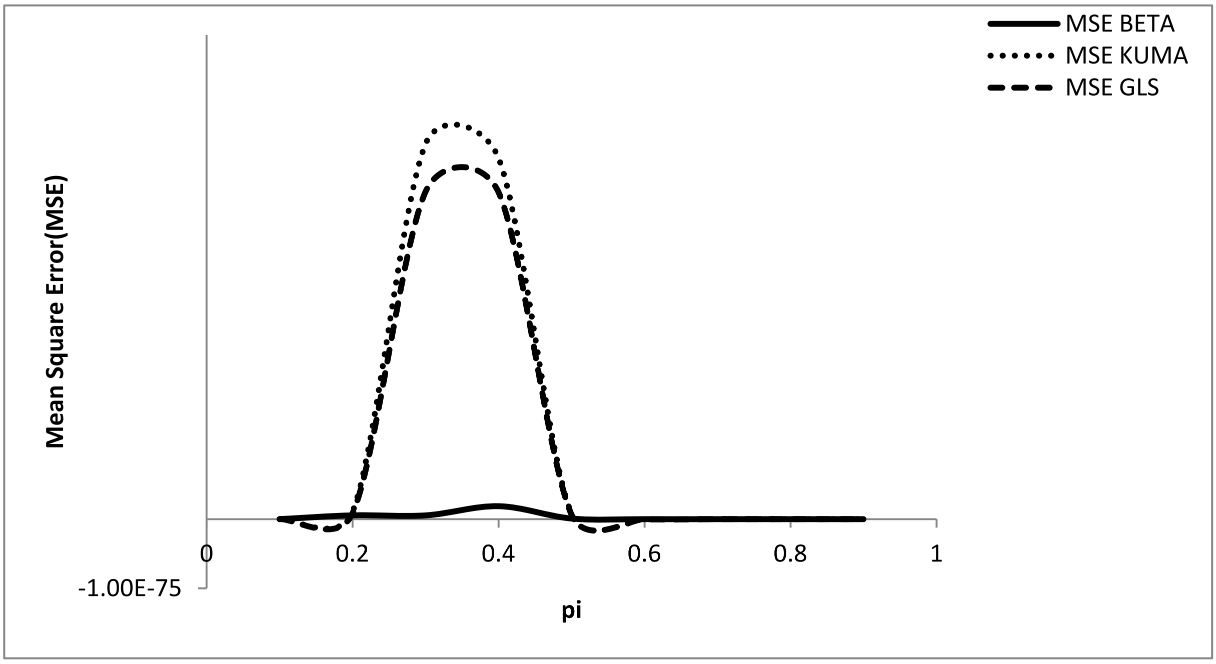

Figure 4.5a. Graph showing the Mean Square Errors (MSEs) for Kim and Warde [15] RRT at $n = 250, x = 150, a = 1, b = 2, c = 4, T_1 = 0.6$

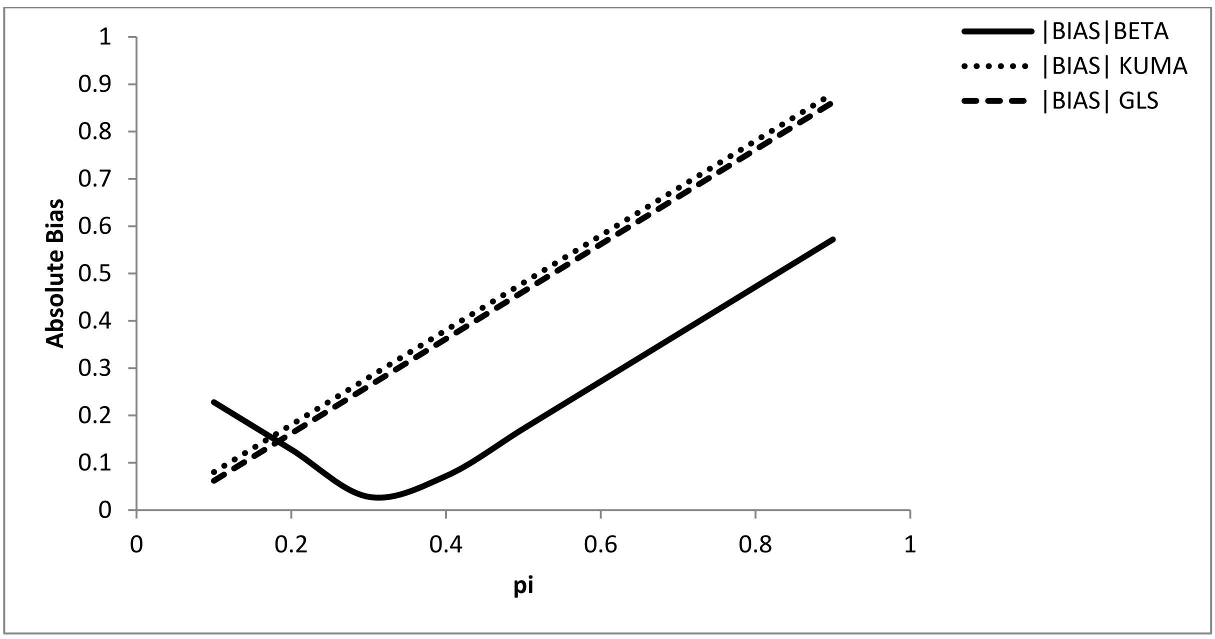

Figure 4.5b. Graph showing the Absolute Bias for Kim and Warde [15] RRT at $n = 250, x = 150, a = 1, b = 2, c = 4, T_1 = 0.6$

From the results of tables and figures 4.4a to 4.5b, we observed the reverse case for the performances of the proposed Bayesian estimators in obtaining response from respondents possessing stigmatized attribute for large sample sizes. Hence, the proposed Bayesian estimators were not suitable in this case.

5. Conclusions

We have presented the Bayesian estimation of the population proportion when the data were gathered through the Kim and Warde [15] Randomized Response Technique using both Kumaraswamy (KUMA) and Generalised (GLS) Beta priors as our alternative prior distributions in addition to simple Beta prior distribution used by Hussain and Shabbir [12]. We presented our results in tables and figures for some selected values of the design parameters and population proportion. We observed that for relatively small sample as well as moderate sample size, the proposed Bayesian estimators performed significantly better than that of Hussain and Shabbir [12]. However, the reverse was the case for very large sample sizes.

REFERENCES

[1] Adebola, F.B. and Adepetun, A.O. (2011): *A new Tripartite Randomized Response Technique*. Journal of the Nigerian Association of Mathematical Physics, Volume 19: pp 119-122.

[2] Adebola, F.B. and Adepetun, A.O. (2012): *On a Numerical Comparison of the Proposed Randomized Response Technique with Hussain and Shabbir*. Journal of the Nigerian Association of Mathematical Physics, Volume 20: pp 379-384.

[3] Adebola, F.B. and Adepetun, A.O. (2012): *On a Qualitative Comparison of the Proposed Randomized Response Technique with Hussain and Shabbir (2007)*. International Journal of Mathematical Theory and Modeling, Volume 2:pp 61-67.

[4] Adepetun, A.O. and Adebola, F.B. (2014): *On the Relative Efficiency of the Proposed Reparametized Randomized Response Model*. International Journal of Mathematical Theory and Modeling, Volume 4: pp 58-67.

[5] Barabesi, L., Marcheselli, M. (2006): *A practical implementation and Bayesian estimation in Franklin's randomized response procedure*. Communication in Statistics- Simulation and Computation, 35, 365-573.

[6] Barabesi, L., Marcheselli, M. (2010): *Bayesian estimation of proportion and sensitivity level in randomized response procedures*. Metrika, 72, 75-88.

[7] Christofides, T.C. (2003): *A generalized randomized response technique*. Metrika, 57, 195-200.

[8] Folsom, R. E., Greenberg, B. G., Horvitz, D. G., Abernathy, J. R. (1973): *The two alternate question randomized response model for human surveys*. Journal of the American Statistical Association, 68, 525-530.

[9] Greenberg, B., Abul-Ela, A., Simmons, W., Horvitz, D. (1969): *The unrelated question randomized response: theoretical framework*. Journal of the American Statistical Association, 64, 529-539.

[10] Hussain, Z., Shabbir, J. (2009a): *Bayesian estimation of population proportion of a sensitive characteristic using simple Beta prior*. Pakistan Journal of Statistics, 25(1), 27-35.

[11] Hussain, Z., Shabbir, J. (2009b): *Bayesian Estimation of population proportion in Kim and Warde (2005) Mixed Randomized Response using Mixed Prior Distribution*. Journal of probability and Statistical Sciences, 7(1), 71-80.

[12] Hussain, Z., Shabbir, J. (2012): *Bayesian Estimation of population proportion in Kim and Warde Mixed Randomized Response Technique*. Electronic Journal of Applied Statistical Analysis, Vol. 5, Issue 2, 213 – 225.

[13] Hussain, Z. Shabbir, J., Riaz, M. (2011): *Bayesian Estimation Using Warner's randomized Response Model Through Simple and Mixture Prior Distributions*. Communications in Statistics- Simulation and Computation, 40(1), 159-176.

[14] Kim, J. M., Tebbs, J. M., An, S. W. (2006): *Extension of Mangat's randomized response model*. Journal of Statistical Planning and Inference, 36(4), 1554-1567.

[15] Kim, J.M. and Warde, D.W. (2004): *A stratified Warner's Randomized Response Model*. J. Statist. Plann. Inference, 120(1-2), 155-165.

[16] Mangat, N.S. (1994): *An improved randomized response strategy*. J. Roy. Statist. Soc. Ser. B, 56(1), 93-95.

[17] Migon, H., Tachibana, V. (1997): *Bayesian approximations in randomized response models*. Computational Statistics and Data Analysis, 24, 401-409.

[18] O'Hagan, A. (1987): *Bayes linear estimators for randomized response models*. Journal of the American Statistical Association, 82, 580-585.

[19] Oh, M. (1994): *Bayesian analysis of randomized response models: a Gibbs sampling approach*. Journal of the Korean Statistical Society, 23, 463-482.

[20] Spurrier, J., Padgett, W. (1980): *The application of Bayesian techniques in randomized response*. Sociological Methodology, 11, 533-544.

[21] Unnikrishnan, N., Kunte, S. (1999): *Bayesian analysis for randomized response models*. Sankhya, B, 61, 422-432.

[22] Warner, S.L. (1965): *Randomized Response: a survey technique for eliminating evasive answer bias*. J. Amer. Statist. Assoc., 60, 63-69.

[23] Winkler, R., Franklin, L. (1979): *Warner's randomized response model: A Bayesian approach*. Journal of the American Statistical Association, 74, 207-214.