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Integral Collocation Approximation Methods for the Numerical Solution of High-Orders Linear Fredholm-Volterra Integro-Differential Equations

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Abstract In this paper, we employed the use of Standard Integral Collocation Approximation Method to obtain numerical solutions of special higher orders linear Fredholm-Volterra Integro-Differential Equations. Power Series, Chebyshev and Legendre's Polynomials forms of approximations are used as basis functions. From the computational view points, the method is efficient, convenient, reliable and superior to many existing methods. Two examples each of first and second orders and one of third order linear Fredholm - Volterra Integro-Differential Equations are considered to illustrate the method. We observed from the results obtained that the method performed better when compared with the results obtained in Mustafa and Yalcin (2012).

Keywords Fredholm-Volterra integro-differential equations, Chebyshev polynomials, Standard integral collocation method

1. Introduction

Integro-Differential Equations have been discussed in many applied fields such as Biology, Physical and Engineering problems. These equations are either Fredholm or Volterra or Fredholm- Volterra Integro Differential Equations are contained in many mathematical formulations of physical phenomena.Therefore, their numerical treatment is desired. .Although, it is generally difficult in finding exact solutions of Fredholm- Volterra Integro Differential Equations (FVIDE) especially when it is nonlinear. Several numerical methods have been used to solve FVIDE among them are Approximation Method (Ezzati and Najafalizadeh, 2011) and (Salih and Mehemet, 2000). Meanwhile, in recent years, the matrix method has been developed to solve Fredholm - Volterra Integral Equations For example, the method is used to solve system of Differential Equations (Akyuz and Sezer, 2003) and Differential Algebraic Equation (Karamete and Sezer, 2002). Other methods include Block Pulse function and Operational Matrices (Leyla, Bijan and Mohammed, 2011), Chebyshev polynomials approach is used in (Yuksel, Gulsu and Sezer, 2012) to mention just a few. In this research work, the work and idea of (Aliyu, 2012) is revisited and modified as our new proposed numerical method to solve the nth order linear Fredholm-Volterra Integro-Differential Equations. The general nth order linear Volterra - Fredholm Integro Differential Equations considered in this work are in two types given as:

$$Ly(x) \equiv \sum_{i=0}^{n} \left(P_i x^i \frac{d^i}{dx^i}\right) y(x) = f(x)$$
$$+\lambda_1 \int_a^x K_1(x,t)y(t)dt + \int_a^b K_2(x,t)y(t)dt \quad (1)$$

And,

$$Ly(x) \equiv \sum_{i=0}^{n} \left(P_i x^i \frac{d^i}{dx^i}\right) y(x) =$$
$$f(x) + \lambda \int_a^x \int_a^b K(x,t)y(t)dtdx \quad (2)$$

Thus, equation (1) contains disjoint integrals while equation (2) contains mixed integrals. Here, both equations (1) and (2) are subjected to the conditions

$$Ly(x) \equiv \sum_{i=0}^{n} a_i y^i(x_i) = \alpha_k \quad (3)$$

In equations (1), (2) and (3), $P_i, \lambda_1, \lambda_2$ are real numbers, $f(x), K_1(x,t)$ and $K_2(x,t)$ are known given smooth functions and $y(x)$ is the unknown function to be determined.

For the purpose of our discussion, we let

$$\iiint k \ldots . \int g(x)dx$$

denote the indefinite integration applied to $g(x)$ k-times and is denoted by

$$I_L = \iiint k \ldots\ldots \int L(.)dx \quad (4)$$

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2. Description of Numerical Method of Solution

In this section, we discussed Standard Integral Collocation Method to solve equations (1) and (2) using the following basis functions:

* Power Series
* Chebyshev Polynomials
* Legendre's Polynomials

This method used the idea reported in (Aliyu, 2012) as applied to solve non over determined differential equation. The method was proposed and used by (Taiwo and Abubakar, 2013) to find numerical solutions to first and second orders linear and non-linear integro-differential equations.

2.1. Method of Solution by Power Series

In order to apply this method, we integrated both sides of equation (1) to have:

$$I_L = \iiint \dots n \dots . \int \left[f(x) + \lambda_1 \int_a^x K_1(x,t) y(t) dt + \lambda_2 \int_a^b K_2(x,t) y(t) dt \right] dx \tag{5}$$

This implies

$$\iiint n \dots \dots \int \sum_{i=0}^{n} \left(P_i x^i \frac{d^i}{dx^i} \right) y(x)\, dx = \iiint \dots n \dots . \int \left[f(x) + \lambda_1 \int_a^x K_1(x,t) y(t) dt + \ \lambda_2 \int_a^b K_2(x,t) y(t) dt \right] dx \tag{6}$$

We assumed an approximate solution of the form

$$y(x) = y_N(x) = \sum_{r=0}^{N} a_r x^r \tag{7}$$

Thus, equation (7) is substituted into equation (6) to have

$$\iiint n \dots \dots \int \sum_{i=0}^{n} \left(P_i x^i \frac{d^i}{dx^i} \right) y_N(x)\, dx = \ \iiint \dots n \dots . \int \left[f(x) + \lambda_1 \int_a^x K_1(x,t) y_N(t) dt + \lambda_2 \int_a^b K_2(\, x,t) y_N(t) dt \right] dx \tag{8}$$

This implies

$$\iiint n \dots \dots \int \sum_{i=0}^{n} \left(P_i x^i \frac{d^i}{dx^i} \right) \sum_{r=0}^{N} a_r x^r \, dx =$$

$$\iiint \dots n \dots . \int \left[f(x) + \lambda_1 \int_a^x K_1(x,t) \sum_{r=0}^{N} a_r t^r \, dt + \lambda_2 \int_a^b K_2(x,t) \sum_{r=0}^{N} a_r t^r \, dt \right] dx \tag{9}$$

Thus, we collocated equation (9) at the point x = x $_k$ to obtain

$$\iiint n \dots \dots \int \sum_{i=0}^{n} \left(P_i x_k^i \frac{d^i}{dx_k^i} \right) \sum_{r=0}^{N} a_r x_k^i \, dx_k =$$

$$\iiint \dots n \dots . \int \left[f(x_k) + \lambda_1 \int_a^x K_1(x_k,t) \sum_{r=0}^{N} a_r t^r \, dt + \lambda_2 \int_a^b K_2(x_k,t) \sum_{r=0}^{N} a_r t^r \, dt \right] dx_k \tag{10}$$

Where,

$$x_k = a + \frac{(b-a)k}{N+1}, \ k = 1,2, \dots . . N+1 \tag{11}$$

Thus, equation (11) gives (N+1) algebraic linear system of equations in (N+1) unknown constants $a_r(r \geq 0)$. These (N+1) algebraic linear system of equations are then solved by Gaussians Elimination Method to obtain the unknown constants $a_r(r \geq 0)$ which are then substituted back into equation (7) to obtain the approximate solution for case N. (the degree of the approximant)

REMARK: 1:

We demonstrated this method further by choosing $n = 1$, thus, putting n = 1 in equation (1), we have:

$$P_0 y(x) + P_1 t y'(x) = f(x) + \lambda_1 \int_a^x K_1\,(x,t) y(t) dt + \int_a^b K_2(x,t) y(t) dt \tag{12}$$

We integrated equation (12) to have

$$\int_0^x P_0 y(t) dt + \int_a^x P_1 y'(t)\, dt = \int_0^x f(z) dz + \lambda_1 \int_0^x \int_a^z \ K_1(z,t) y(t) dt dz + \ \lambda_2 \int_0^x \int_a^b \ K_2(z,t) y(t) dt dz \tag{13}$$

This implies

$$P_0 \int_0^x y(t) dt + P_2 \int_0^x t y'(t) dt = \int_0^x f(z) dz + \lambda_1 \int_0^x \int_a^z \ K_1(z,t) y(t) dt dz + \ \lambda_2 \int_0^x \int_a^b \ K_2(z,t) y(t) dt dz$$

After simplification, we have

$$P_0 \int_0^x y(t) dt + P_1 \left[x y(x) - \int_0^x y(t) dt \right] = \int_0^x f(z) dz + \lambda_1 \int_0^x \int_a^z \ K_1(z,t) y(t) dt dz + \ \lambda_2 \int_0^x \int_a^b \ K_2(z,t) y(t) dt dz \tag{14}$$

Substituting the approximate solution given in equation (7) into (14), we have

$$P_0 \int_0^x \sum_{r=0}^{N} a_r t^r \, dt + P_1\left[x \sum_{r=0}^{N} a_r x^r - \int_0^x \sum_{r=0}^{N} a_r t^r \, dt\right] =$$

$$\int_0^x f(z)dz + \int_0^x \int_a^z K_1(z,t) \sum_{r=0}^{N} a_r t^r \, dtdz + \lambda_2 \int_0^x \int_a^b K_2(z,t) \sum_{r=0}^{N} a_r t^r \, dtdz$$

This implies,

$$P_0 \sum_{r=0}^{N} a^r \frac{x^{r+1}}{r+1} + P_1[\sum_{r=0}^{N} a_r x^{r+1} - \sum_{r=0}^{N} a^r \frac{x^{r+1}}{r+1}] =$$

$$\int_0^x f(z)dz + \lambda_1 \int_0^x \int_a^z K_1(z,t) \sum_{r=0}^{N} a_r t^r \, dtdz + \lambda_2 \int_0^x \int_a^b K_2(z,t) \sum_{r=0}^{N} a_r t^r \, dtdz$$

Further simplification gives

$$P_0 \sum_{r=0}^{N} a^r \frac{x^{r+1}}{r+1} + P_1[\sum_{r=0}^{N} a^r (\frac{(r+1)x^{r+1} - x^{r+1}}{r+1})] =$$

$$\int_0^x f(z)dz + \lambda_1 \int_0^x \int_a^z K_1(z,t) \sum_{r=0}^{N} a_r t^r \, dtdz + \lambda_2 \int_0^x \int_a^b K_2(z,t) \sum_{r=0}^{N} a_r t^r \, dtdz$$

$$P_0 \sum_{r=0}^{N} a^r \frac{x^{r+1}}{r+1} + P_1 \sum_{r=0}^{N} a^r (\frac{r x^{r+1}}{r+1})] =$$

$$\int_0^x f(z)dz + \lambda_1 \int_0^x \int_a^z K_1(z,t) \sum_{r=0}^{N} a_r t^r \, dtdz + \lambda_2 \int_0^x \int_a^b K_2(z,t) \sum_{r=0}^{N} a_r t^r \, dtdz$$

This implies,

$$\sum_{r=0}^{N} (P_0 + rP_1) a_r \frac{x^{r+1}}{r+1} - G_1(a,x) - G_2(a,x) = F(x) \tag{15}$$

Where,

$$G_1(a,x) = \lambda_1 \int_0^x \int_a^z K_1(z,t) \sum_{r=0}^{N} a_r t^r \, dtdz$$

$$G_2(a,x) = \lambda_2 \int_0^x \int_a^b K_2(z,t) \sum_{r=0}^{N} a_r t^r \, dtdz$$

And,

$$F(x) = \int_0^x f(z)dz$$

Thus from equation (14), we have

$$\begin{aligned} &P_0 x + (P_0 + P_1) a_1 \frac{x^2}{2} + (P_0 + 2P_1) a_1 \frac{x^3}{3} + (P_0 + 3P_1) a_1 \frac{x^4}{4} + \dots \dots \dots .\\ &+ (P_0 + NP_1) a_1 \frac{x^{N+1}}{N+1} - G_1(a,x) - G_2(a,x) = F(x) \end{aligned} \tag{16}$$

Thus, equation (16) is collocated at the point $x = x_k$ to have

$$\begin{aligned} &P_0 x_k + (P_0 + P_1) a_1 \frac{x_k^2}{2} + (P_0 + 2P_1) a_1 \frac{x_k^3}{3} + (P_0 + 3P_1) a_1 \frac{x_k^4}{4} + \dots \dots \dots .\\ &+ (P_0 + NP_1) a_1 \frac{x_k^{N+1}}{N+1} - G_1(a,x_k) - G_2(a,x_k) = F(x_k) \end{aligned} \tag{17}$$

Where

$$x_k = a + \frac{(b-a)k}{N+1}, \qquad k = 1,2, \dots .. N+1$$

Thus, equation (17) is put in matrix form as

$$AX = b,$$

Where,

$$A = \begin{bmatrix} A_{11} & A_{12} \cdots & A_{1N} \\ A_{21} & A_{22} \cdots & A_{2N} \\ \vdots & & \\ A_{N1} & A_{N2} \cdots & A_{NN} \end{bmatrix}, X = \begin{bmatrix} a_0 \\ a_1 \\ \vdots \\ a_N \end{bmatrix} \; and \; b = \begin{bmatrix} F(x_1) \\ F(x_2) \\ \vdots \\ \vdots \\ F(x_{N+1}) \end{bmatrix}$$

Where,

$$A_{11} = P_0 x_1 - G_1(x_1) - G_2(x_1)$$

$$A_{12} = (P_0 + P_1) \frac{x_1^2}{2} - G_1(x_1) - G_2(x_1)$$

$$A_{13} = (P_0 + 2P_1)\frac{x_1^3}{3} - G_1(x_1) - G_2(x_1)$$

$$A_{1N} = (P_0 + NP_1)\frac{x_1^{N+1}}{N+1} - G_1(x_1) - G_2(x_1)$$

$$A_{21} = P_0 x_2 - G_1(x_2) - G_2(x_2)$$

$$A_{22} = (P_0 + P_1)\frac{x_2^2}{2} - G_1(x_2) - G_2(x_2)$$

$$A_{23} = (P_0 + 2P_1)\frac{x_2^3}{3} - G_1(x_2) - G_2(x_2)$$

$$A_{2N} = (P_0 + NP_1)\frac{x_2^{N+1}}{N+1} - G_1(x_2) - G_2(x_2)$$

$$\vdots$$

$$A_{N1} = P_0 x_N - G_1(x_N) - G_2(x_N)$$

$$A_{N2} = (P_0 + P_1)\frac{x_N^2}{2} - G_1(x_N) - G_2(x_N)$$

$$A_{N3} = (P_0 + 2P_1)\frac{x_N^3}{3} - G_1(x_N) - G_2(x_N)$$

$$A_{NN} = (P_0 + NP_1)\frac{x_N^{N+1}}{N+1} - G_1(x_N) - G_2(x_N)$$

Remark 2:

The above matrix is solved by Gaussian Elimination Method to obtain the unknown Constants $a_r (r \geq 0)$ which are then substituted back into the approximate Solution given in equation (7) to obtain the approximate solution for case N.

2.2. Method of Solution by Chebyshev Polynomials

We demonstrated this method by assuming the approximate solution of the form

$$y(x) = y_N(x) = \sum_{r=0}^{N} a_r T_r(x), \qquad a \leq x \leq b \tag{18}$$

Where, $a_r, r = 0,1,2, \dots \dots \dots N$ are unknown constants and $T_r(x)$ are the Chebushev Polynomial of degrees r of first the kind which is valid in the interval $1 - \leq x \leq 1$ and is defined by

$$T_r(x) = cos(cos^{-1}x),$$

Here,

$$T_0(x) = 1, T_1(x) = x \tag{19}$$

The recurrence relation is given by

$$T_{r+1}(x) = 2x\, T_r(x) - T_{r-1}(x), r \geq 1 \tag{20}$$

$$T_r(x) = cos\left[rcos^{-1}\left(\frac{2x-a-b}{b-a}\right)\right], a \leq x \leq b \tag{21}$$

and this satisfies the recurrence relation

$$T_r(x) = 2\left(\frac{2x-a-b}{b-a}\right)T_r(x) - T_{r-1}(x), r \geq 0, a \leq x \leq b \tag{22}$$

Substituting equation (18) into equation (6), we have

$$\iiint n \dots \dots \int \sum_{i=0}^{n}\left(P_i x^i \frac{d^i}{dx^i}\right)\sum_{r=0}^{N} a_r T_r(x)\, dx =$$

$$\iiint \dots n \dots . \int\left[f(x) + \int_a^x K_1(x,t)\sum_{r=0}^{N} a_r T_r(t)\, dt + \lambda_2 \int_a^b K_2(x,t)\sum_{r=0}^{N} a_r T_r(t)\, dt\right]dx\ ) \tag{23}$$

Hence, we collocated (23) at the point $x = x_k$ to have

$$\iiint n \dots \dots \int \sum_{i=0}^{n}\left(P_i x_k^i \frac{d^i}{dx^i}\right)\sum_{r=0}^{N} a_r T_r(x_k)\, dx_k =$$

$$\iiint \dots n \dots . \int\left[f(x_k) + \lambda_1 \int_a^x K_1(x_k,t)\sum_{r=0}^{N} a_r T_r(t)\, dt + \lambda_2 \int_a^b K_2(x_k,t)\sum_{r=0}^{N} a_r T_r(t)\, dt\right]dx_k \tag{24}$$

Where $x_k = a + \frac{(b-a)k}{N+1},\ k = 1,2, \dots \dots N+1$

Thus, (24) gives (N + 1) algebraic linear system of equations in (N + 1) unknown constants $a_r (r \geq 0)$. The (N + 1) algebraic system of linear equations are then solved by Gaussian elimination method to obtain the unknown constants $a_r(r \geq 0)$ which are then substituted back into equation (18) to obtain the approximate solution for case N.

2.3. Method of Solution by Legendre's Polynomials

The Legendre's Polynomial is denoted and defined by

$$\mathrm{P}_{r+1}(x) = \frac{1}{r+1}\{(2r+1)x\mathrm{P}_r(x) - r\mathrm{P}_{r-1}(x)\} \tag{25}$$

$$\mathrm{P}_r(x) = \frac{1}{2^r r! dx^r}(x^2-1)^r; r = 0,1, \dots.. \tag{26}$$

With,

$$\mathrm{P}_0(x) = 1, \mathrm{P}_1(x) = x, \mathrm{P}_2(x) = \frac{1}{2}(3x^2-1) \text{ etc}$$

In this method, we assumed an approximate solution of the form

$$y(x) = y_N(x) = \sum_{r=0}^{N} a_r \mathrm{P}_r(x), \qquad a \leq x \leq b \tag{27}$$

This method is similar to the two methods discussed above (the same procedure discussed as in sections (2.1.1) and (2.1.2)). We have (N + 1) algebraic linear system of equations in (N + 1) unknown constants $a_r(r \geq 0)$. The (N + 1) algebraic system of linear equations are then solved by Gaussian elimination method to obtain the unknown constants $a_r(r \geq 0)$ which are then substituted back intoequation (27) to obtain the approximate solution for case N.

3. Numerical Experiment

In this section, we have demonstrated the Standard Integral Collocation Approximation Method on first, second and third orders Fredholm - Volterra Integro-Differential Equations using Power series, Chebyshev and Legendre's Polynomials as the basis functions. The examples are solved to illustrate the accuracy, efficiency and time of execution of the method.

We have defined absolute error as

$$\boldsymbol{Error = |y(x) - y_N(X)|, \qquad a \leq x \leq b, \qquad N = 1, 2, 3, \dots \dots \dots \dots}$$

Example 1.

We considered first order FVIDE given as

$$y'(x) - y(x) = e^x - e + \int_0^1 y(t)dt + \int_0^x y(t)dt; \;\; 0 \leq x \leq 1$$

With the mixed condition

$$y(0) + \int_0^1 y(t)dt = e$$

The exact solution is given as $y(x) = e^x$

Example 2.

We considered second order FVIDE given as

$$y''(x) = -8 + 6x - 3x^2 + x^3 + \int_0^x y(t)dt + \int_{-1}^1 (1-2xt)y(t)dt, \qquad -1 \leq x \leq 1$$

With the conditions

$$y(0) = 2, \; y'(0) = 6$$

The exact solution is given as $y(x) = 2 + 6x - 3x^2$

Example 3.

We considered the third order FVIDE given as

$$y'''(x) = \frac{1}{2}x^2 + \int_0^x y(t)dt + \int_{-\pi}^{\pi} xy(t)dt$$

With the conditions

$$y(0) = y'(0) = -y''(0) = 1$$

The exact solution is given $y(x) = x + cosx$

4. Table of Results

Table 1. Numerical results for example 1

X	Result by Mustafa and Yalcin, (2012) N=6	Standard Integral Collocation Approximation Method, for case N=6		
		Power Series	Chebyshev Polynomial	Legendre's Polynomial
0	5.930E-5	4.570E-5	6.320E-6	2.630E-5
-0.1	4.830E-5	3.780E-5	5.010E-6	2.630E-5
-0.2	3.880E-5	3.060E-5	3.270E-6	7.310E-6
-0.3	3.050E-5	2.990E-5	1.320E-6	4.070E-6
-0.4	2.330E-5	6.450E-6	9.780E-7	3.400E-6
-0.5	1.700E-5	3.720E-6	7.510E-7	1.730E-6
-0.6	1.660E-5	2.480E-6	5.260E-7	8.620E-7
-0.7	6.800E-6	8.610E-7	2.070E-7	5.370E-7
-0.8	2.460E-6	5.030E-7	1.040E-7	4.110E-7
-0.9	1.530E-6	3.720E-7	8.350E-8	1.080E-7
-1.0	4.950E-6	1.790E-7	7.120E-8	8.640E-8

Table 2. Numerical results for example 2

X	Exact Solution	Standard Integral Collocation Approximation Method					
		Power Series		Chebyshev Polynomial		Legendre's Polynomial	
		N=6	Error	N=6	Error	N=6	Error
0.0	2.00000	2.00000	0.00000	2.00016	1.000 E-4	2.00251	2.510 E-3
0.1	2.57000	2.55934	1.066 E-3	2.57014	1.400 E-4	2.57113	1.130 E-3
0.2	3.08000	3.07856	1.440 E-3	3.08056	5.600 E-4	3.08164	1.640 E-3
0.3	3.53000	3.55621	2.621 E-4	3.53027	2.700 E-4	3.54712	1.712 E-3
0.4	3.92000	3.94830	2.830 E-3	3.91945	5.500 E-4	3.91867	1.400 E-3
0.5	4.25000	4.28391	3.391 E-3	4.24932	6.800 E-4	4.24801	1.990 E-3
0.6	4.52000	4.54167	2.167 E-3	4.51814	1.860 E-3	4.51772	2.280 E-3
0.7	4.73000	4.74893	1.893 E-3	4.73004	4.000 E-5	4.73105	1.050 E-3
0.8	4.88000	4.89642	1.642 E-3	4.88151	1.510 E-3	4.88240	2.400 E-3
0.9	4.97000	4.98341	1.341 E-3	4.97793	7.930 E-3	4.97806	8.060 E-3
1.0	5.00000	4.99672	3.280 E-3	4.99996	4.000 E-5	4.99507	1.930 E-3

Table 3. Numerical results for example 3

X	Exact Solution	Standard Integral Collocation Approximation Method					
		Power Series		Chebyshev Polynomial		Legendre's polynomial	
		N=6	Error	N=6	Error	N=6	Error
0.0	1.000000000	1.000000000	0.0000000	1.000000000	0.0000000	1.00000000	0.000000
0.1	1.099998477	1.099816320	1.821 E- 4	1.099981301	1.718 E- 5	1.099871453	.270 E- 3
0.2	1.199939080	1.198634511	1.305 E- 3	1.199947062	7.982 E- 6	1.199756321	1.828 E- 4
0.3	1.299986292	1.299417352	5.689 E- 4	1.299941073	4.522 E- 5	1.299861415	1.249 E- 4
0.4	1.399975631	1.399094514	8.811 E- 4	1.399949310	2.632 E- 5	1.399715631	2.600 E- 4
0.5	1.499945169	1.497183216	2.779 E- 3	1.499960178	1.745 E- 6	1.499783118	1.788 E- 4
0.6	1.599945169	1.587160052	2.785 E- 3	1.599945831	4.128 E- 6	1.598345142	1.600 E- 3
0.7	1.699925370	1.693861451	6.064 E- 3	1.699951006	2,564 E- 5	1.699453417	1.472 E- 3
0.8	1.799902524	1.798885324	1.017 E- 3	1.799960145	5.762 E- 5	1.799368164	5.344 E- 4
0.9	1.899876632	1.889956321	9.920 E- 3	1.899916315	3.968 E- 5	1.898964154	9.125 E- 4
1.0	1.999847695	1.998794562	1.053 E- 3	1.999970382	1.227 E- 4	1.995300161	4.550 E- 3

5. Conclusions

We have demonstrated Standard Integral Collocation Approximation Method to solve special higher orders linear Fredholm - Volterra Integro Differential Equations by Power Series, Chebyshev and Legendre's Polynomials as the basis functions of approximations. The results obtained by Chebyshev and Legendres's Polynomials as basis functions proved superior than that of power series.

However, this method yields the desired accuracy when the results obtained are compared with the exact solutions. The simplicity is an added advantage to the method and hence, it is reliable and a powerful tool for the classes of the problems considered.

REFERENCES

[1] Aliyu, I. M. (2012): A Computational Error Estimation of the Integral Formulation of the Tau Method for Some Class of Ordinary Differential Equations Ph.D.Thesis, (Unpublished), University of Ilorin, Ilorin.

[2] Akyuz, A. and Sezer, M. (2003): Chebyshev polynomial Solutions of System of High Order Linear Differential Equations with Variable Coefficients. Appl. Math. Comput. 144, 237-247.

[3] Arikogulu, A. and Ozkol, I. (2005): Solutions of boundary Value Problems for Integro-Differential Equations by Using Di_erential Transform Method. Appl. Math.Comput 168, 1145-1158.

[4] Asady, B. and Kanjani, M.T. (2005): Direct Method for Solving Integro-Differential Equations Using Hybrid Fourier and Block Functions. Int. J. Computer Math. 82(7), 989-995.

[5] Ezzati, R. and Najafalizadeh, S. (2011): Numerical Solution of Volterra-Fredholm Integral Equation by Using Chebyshev Polynomials. Mathematical Sciences, 5(1):1-12.

[6] Hashim, I. (2006): Adomian Decomposition Method for Solving BVPs for Fourth-Order Integro-Differential Equations. J. Comput. Appl. Math.193:658-664.

[7] Karamete, A. and Sezer, M. (2002): A Taylor Collocation Method for the Solution of Linear Integro-Differential Equations. int. J. Comput. Math. 79(9): 987-1000.

[8] Leyla, R. Bijan, R.and Mohammad, M. (2011): Numerical Solution of Volterra Fredholm Integro-Differential Equation by Block Pulse Functions and Operational Matrices. Gen. Math. Notes, 4(2):37-48.

[9] Malejnejad, K. and Mahmoudi, Y. (2003): Taylor polynomial Solution of Higher-Order Nonlinear Volterra-Fredholm Integro-Differential Equations. Appl. Math.Comput., 155: 317-328.

[10] Mustafa, G. and Yalcin, O. (2012): On Numerical Solution of Linear Fredholm-Volterra Integro-Differential Difference Equations with Piecewise Intervals. Application and Applied Mathematics: An International Journal (AAM), 7: 556-570.

[11] Pachpatte, B. G. (2008): On Volterra and Fredholm Type of Integro-Differential Equations. Tamsui Oxford Journal of Mathematical Science, 24(3): 289-310.

[12] Rabbani, M. and Kiasoltani. S. H. (2011): Solving of Nonlinear System of Fredholm- Volterra Integro-Differential Equations by Using Discrete Collocation Method. The journal of Mathematics and Computer Science, 3(4): 382-389.

[13] Rabbani, M. and Zarali, B. (2012): Solution of Fredholm Integro-Differential Equation System by Modifed Decomposition Method. The Journal of Mathematics and Computer Science, 5(4): 258-264.

[14] Raji, M. T. (2013): Collocation Approximation for Solving Special Linear and Nonlinear Integro-Differential Equations. Ph. D. Thesis, (Unpublished), University of Ilorin, Ilorin, Nigeria.

[15] Salih, Y. and Mehmet, S. (2000): The Approximate Solution of Higher Order Linear Volterra-Fredholm Integro-Differential Equations in Term of Taylor Polynomials. Appl.Math. Comput. 112, 291-308.

[16] Taiwo, O. A. and Abubakar, A. (2013): Integral Collocation Approximation Methods for the Numerical Solution of Linear Integro-Differential Equations. IOSR Journal of Mathematics, 8(5): 01-14.

[17] Taiwo, O. A. and Ishola, C. Y. (2009): Collocation Approximation Methods for the Numerical Solution of Integral Equations. Int. Journal Pure and Applied Science, 2,:29-37.

[18] Yuksel, G., Gulsu, M. and Sezer, M. (2012): A Chebyshev Polynomial Approach for Higher-Order Linear Fredholm-Volterra Integro-Differential Equations. Gazi University Journal of Science, 25(2): 393-401.

[19] Wazwaz, A. (2011): Linear and Nonlinear Integral Equations, Methods an Applications. Higher Education Press, Benjing and Springer-Verlag Berlin Heidelberg.

[20] Volk, W. (1985): The Numerical Solution of Linear Integro-Differential Equations by Projection Method. Journal of Integral Equations, 9(1):171-190.